

$$8) \sum_{n=2}^{\infty} \frac{\ln(n^2)}{n}$$

$$f(x) = \frac{\ln x^2}{x}$$

{10.3}

$$= \frac{\ln 2^2}{2} + \sum_{n=3}^{\infty} \frac{\ln(n^2)}{n}$$

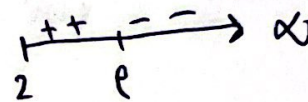
$$f'(x) = \frac{x \cdot \frac{2}{x} - \ln x^2}{x^2} = \frac{2 - \ln x^2}{x^2}$$

$$= \ln 2 + \sum_{n=3}^{\infty} \frac{\ln(n^2)}{n} = \ln 2 + \infty = \infty$$

$$2 = 2 \ln x$$

$$\ln x = 1$$

$$e^{\ln x} = e^1 \Rightarrow \boxed{x=e}$$



$$u = \ln n$$

$$du = \frac{1}{n} dn$$

$$\int 2u du = 2 \ln n^2$$

$$\sum_{n=3}^{\infty} \frac{\ln(n^2)}{n} = \lim_{b \rightarrow \infty} [2 \ln b - 2 \ln 3] = \infty$$

Haytham  
Schrader

Hence,  $\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n}$  div by Integral test

$$26) \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$$

$$f(x) = \frac{1}{\sqrt{x}(\sqrt{x}+1)}$$

$$\int_1^{\infty} \frac{1}{\sqrt{x}(\sqrt{x}+1)}$$

$$u = \sqrt{x} + 1$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$\int \frac{1}{u} \cdot 2 du = 2 \ln(\sqrt{x} + 1)$$

Hence,  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$  div by IT

$$2 \lim_{b \rightarrow \infty} [\ln(\sqrt{b} + 1) - \ln 2] = \infty$$

$$\sum_{n=1}^{\infty} n \tan \frac{1}{n}$$

34]  $\lim_{n \rightarrow \infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}}$

$$u = \frac{1}{n}$$

$$\lim_{u \rightarrow 0^+} \frac{\tan u}{u} = \lim_{u \rightarrow 0^+} \sec^2 u = 1 \neq 0 \quad \text{div by } n^{\text{th}} \text{ term test}$$

6]  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

$$f(x) = \frac{1}{x(\ln x)^2}$$

$$f'(x) = \frac{-2(\ln x)^3}{(x(\ln x)^2)^2}$$

$$\int_2^{\infty} \frac{1}{x(\ln x)^2}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \frac{du}{u^2} = -\frac{1}{u} = -\frac{1}{\ln x}$$

$$\lim_{b \rightarrow \infty} -\frac{1}{\ln b} + \frac{1}{\ln 2} = \frac{1}{\ln 2} \quad \text{conv by IT}$$

13]  $\sum_{n=1}^{\infty} \frac{n}{n+1}$

$$\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1 \neq 0 \quad \text{div by } n^{\text{th}} \text{ term test}$$

20]  $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$

$$\int_2^{\infty} \frac{\ln x}{\sqrt{x}}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\ln x = e^u$$

$$x = e^u$$

$$x^{\frac{1}{2}} = e^{\frac{u}{2}}$$

$$\int u e^{\frac{u}{2}} du$$

$$= 2u e^{\frac{u}{2}} - 4e^{\frac{u}{2}}$$

$$= 2 \ln x e^{\frac{\ln x}{2}} - 4e^{\frac{\ln x}{2}}$$

$$\begin{array}{rcl} 4 & \xrightarrow{+} & e^{\frac{u}{2}} \\ 1 & \xrightarrow{+} & 2e^{\frac{u}{2}} \\ 0 & \xrightarrow{-} & 4e^{\frac{u}{2}} \end{array}$$

$$\lim_{b \rightarrow \infty} \left[ 2be^{\frac{b}{2}} - 4e^{\frac{b}{2}} \right] - \left[ 2 \ln 2 e^{\frac{\ln 2}{2}} - 4e^{\frac{\ln 2}{2}} \right] = \infty \quad \text{div by Integral test}$$

$$22) \sum_{n=1}^{\infty} \frac{5^n}{4^n + 3}$$

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$$\lim_{n \rightarrow \infty} \frac{5^n \ln 5}{4^n \ln 4} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n \frac{\ln 5}{\ln 4} = \infty \neq 0 \quad \text{div by } n^{\text{th}} \text{ term test}$$

$$28) \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \neq 0 \quad \text{div by } n^{\text{th}} \text{ term test}$$

$$32) \sum_{n=1}^{\infty} \frac{1}{n(1 + \ln^2 n)}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$f(x) = \frac{1}{x(1 + \ln^2 x)}$$

$$\int \frac{1}{1+u^2} = \tan^{-1} u$$

$$\lim_{b \rightarrow \infty} [\tan^{-1} b - \tan^{-1} 0] = \frac{\pi}{2} \quad \text{conv by IT}$$

$$38) \sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$f(x) = \frac{x}{x^2 + 1}$$

$$\lim_{b \rightarrow \infty} \left[ \frac{1}{2} \ln(b^2 + 1) - \frac{1}{2} \ln 2 \right] = \lim_{b \rightarrow \infty} \int \frac{xdx}{2u} = \frac{1}{2} \ln(x^2 + 1)$$

$$= \infty \quad \text{div by IT}$$

$$40) \sum_{n=1}^{\infty} \text{sech}^2 n$$

$$\lim_{b \rightarrow \infty} \int_1^b \text{sech}^2 x = \lim_{b \rightarrow \infty} \left. \tanh x \right|_1^b = \lim_{b \rightarrow \infty} [\tanh b - \tanh 1] = 1 - \tanh 1$$

$$\text{conv by IT}$$

42]  $\sum_{n=3}^{\infty} \left( \frac{1}{n-1} - \frac{2a}{n+1} \right)$

4

$$\lim_{b \rightarrow \infty} \left[ \ln \left| \frac{(n-1)}{(n+1)^{2a}} \right| \right]_3^b = \lim_{b \rightarrow \infty} \left[ \ln \frac{b-1}{(b+1)^{2a}} - \ln \left( \frac{2}{4^{2a}} \right) \right]$$

$$\lim_{b \rightarrow \infty} \frac{b-1}{(b+1)^{2a}} = \lim_{b \rightarrow \infty} \frac{1}{2a(b+1)^{2a-1}} = \begin{cases} 1, & a = \frac{1}{2} \\ \infty, & a < \frac{1}{2} \end{cases}$$

conv  $\rightarrow \ln 2$