6.4 Linear Polarizers & Jones Matrices

(1)

Polarizer can be represented by a matrix ~~that~~ operates on a Jones vector.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$ (polarizer with transmission along x-axis)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \checkmark$$ allows passage

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \checkmark$$ Kills light.

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$ (transmission along y-axis).

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix} \checkmark$$

Compound Polarization Systems:

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = J_{system}\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix}$$

output   input.

$$J_{system} = J_N J_{N-1} \cdots J_2 J_1$$

J_n = matrix for n^{th} polarizing optical element.

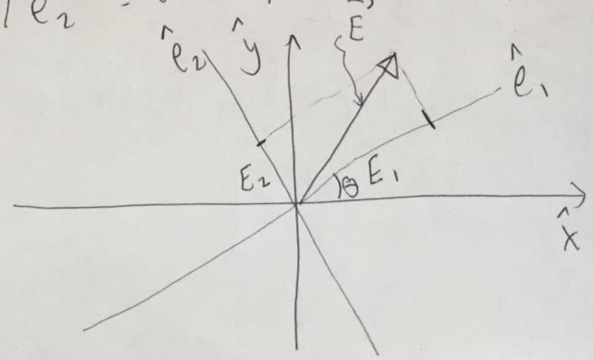
6.5 Jones Matrix for a Polarizer

- Jones Matrix for ideal polarizer with transmission axis at an arbitrary Θ from x-axis.

Input field: $\vec{E}(z,t) = (E_x \hat{x} + E_y \hat{y}) e^{i(kz - wt)}$

let $\begin{cases} \hat{e}_1 = \text{transmission axis.} \\ \hat{e}_2 = \text{absorption axis } (\perp \text{ to } \hat{e}_1). \end{cases}$

unit vectors



(3)

$$\hat{x} = \hat{e}_1 \cos\theta - \hat{e}_2 \sin\theta$$
$$\hat{y} = \hat{e}_1 \sin\theta + \hat{e}_2 \cos\theta$$

$$\therefore \; \vec{E}(z,t) = \left(E_1 \hat{e}_1 + E_2 \hat{e}_2\right) e^{i(kz-wt)}$$

$$E_1 \equiv E_x \cos\theta + E_y \sin\theta$$
$$E_2 \equiv -E_x \sin\theta + E_y \cos\theta$$

E_1 is transmitted.

E_2 is Killed (extinguished).

$$\vec{E}_{after}(z,t) = \left(E_1 \hat{e}_1 + \overset{\approx 0}{\xi} E_2 \hat{e}_2\right) e^{i(kz-wt)}$$

using
$$\hat{e}_1 = \cos\theta\, \hat{x} + \sin\theta\, \hat{y}$$
$$\hat{e}_2 = -\sin\theta\, \hat{x} + \cos\theta\, \hat{y}$$

$$\vec{E}_{after}(z,t) = \Big[\left(E_x \cos\theta + E_y \sin\theta\right)\left(\cos\theta\, \hat{x} + \sin\theta\, \hat{y}\right) + \xi\left(-E_x \sin\theta + E_y \cos\theta\right)\left(-\sin\theta\, \hat{x} + \cos\theta\, \hat{y}\right)\Big] e^{i(kz-wt)}$$

$$= \left[E_x\left(\cos^2\theta + \xi \sin^2\theta\right) + E_y\left(\sin\theta\cos\theta - \xi \sin\theta\cos\theta\right)\right] \hat{x}\, e^{i(kz-wt)}$$
$$+ \left[E_x\left(\sin\theta\cos\theta - \xi \sin\theta\cos\theta\right) + E_y\left(\sin^2\theta + \xi \cos^2\theta\right)\right] \hat{y}\, e^{i(kz-wt)}$$

4

$\xi = 1 \rightarrow$ no polarizer.

$$E_{after}(z,t) = \begin{bmatrix} \cos^2\theta + \xi \sin^2\theta & \sin\theta\cos\theta - \xi \sin\theta\cos\theta \\ \sin\theta\cos\theta - \xi\sin\theta\cos\theta & \sin^2 + \xi\cos^2\theta \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\times e^{i(kz - wt)}$$

Case 1: $\xi = 0$

$$\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

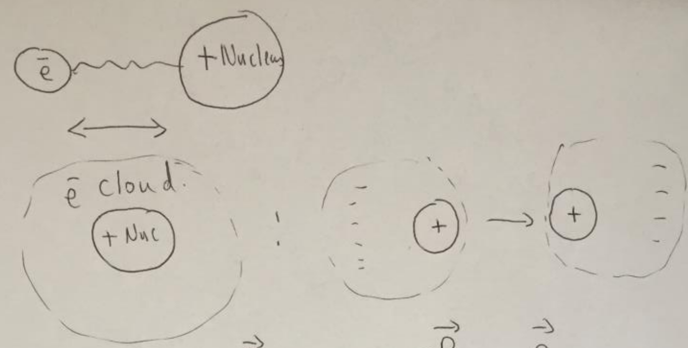
if $\theta = 0 \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ($\hat{x}$ transmission)

$\theta = \frac{\pi}{2}$ $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ ($\hat{y}$ transmission)

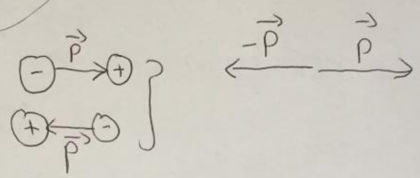
Polarization by reflection:

5

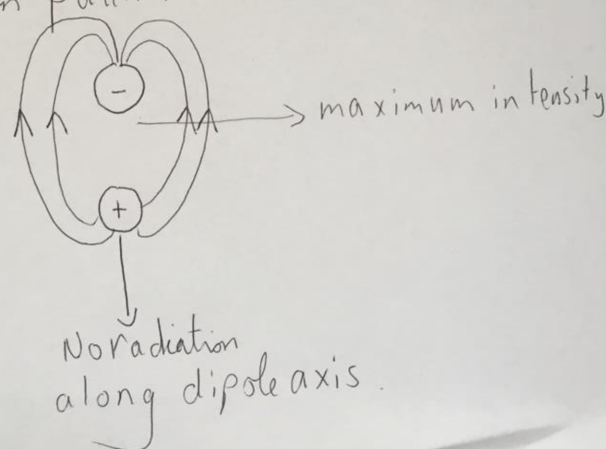
Electron-Oscillator Model.



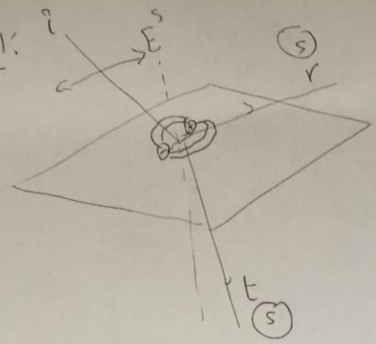
dipole



Radiation pattern:

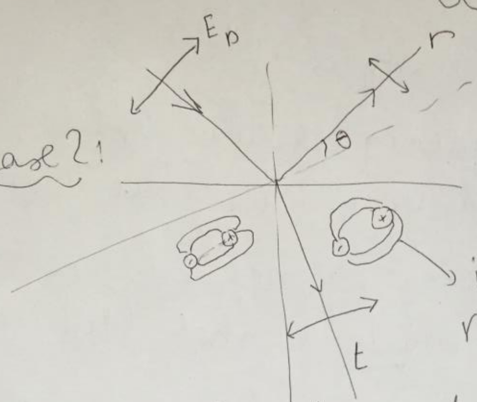


Case 1:



reflected S
transmitted S.

Case 2:

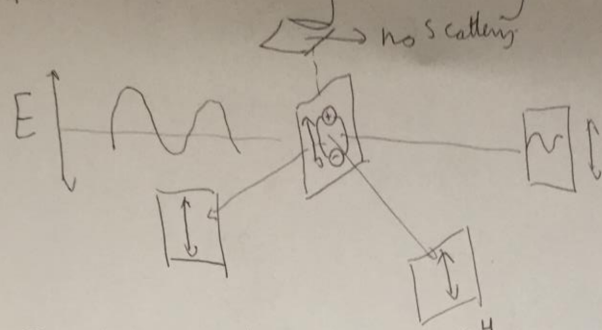


influenced by refracted wave.

mostly refracted; much less reflected.
For this light, mostly transmitted.

Case 3: Mixture of S, p;
You get S reflected. Thus
polarization by reflection.

Polarization by Scattering



Scattering intensity $\propto f^4$

high frequency e.g. blue light is scattered very well. (blue sky)

low frequency e.g. red light is scattered much weaker. (red evenings)