PROBLEMS

•**22–1.** A spring is stretched 175 mm by an 8-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 1.50 m/s, determine the differential equation which describes the motion. Assume that positive displacement is downward. Also, determine the position of the block when $t = 0.22$ s.

22–2. When a 2-kg block is suspended from a spring, the spring is stretched a distance of 40 mm. Determine the frequency and the period of vibration for a 0.5-kg block attached to the same spring.

22–3. A block having a weight of 8 lb is suspended from a spring having a stiffness $k = 40$ lb/ft. If the block is pushed $y = 0.2$ ft upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What are the amplitude and the natural frequency of the vibration? Assume that positive displacement is downward.

***22–4.** A spring has a stiffness of 800 N/m. If a 2-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation that describes the block's motion. Assume that positive displacement is downward.

•**22–5.** A 2-kg block is suspended from a spring having a stiffness of 800 N/m. If the block is given an upward velocity of 2 m/s when it is displaced downward a distance of 150 mm from its equilibrium position, determine the equation which describes the motion. What is the amplitude of the motion? Assume that positive displacement is downward.

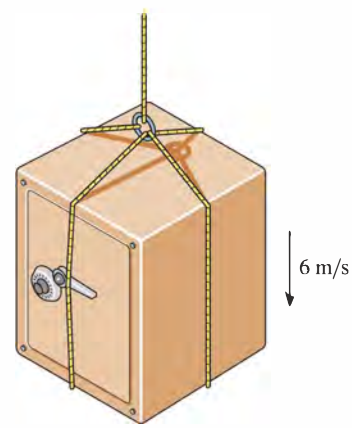
22–6. A spring is stretched 200 mm by a 15-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 0.75 m/s, determine the equation which describes the motion. What is the phase angle? Assume that positive displacement is downward.

22

22–7. A 6-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 75 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is downward.

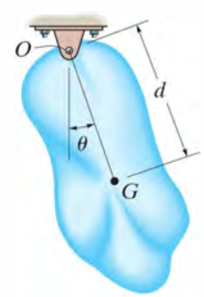
***22–8.** A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the frequency of the vibration? Assume that positive displacement is downward.

•**22–9.** A cable is used to suspend the 800-kg safe. If the safe is being lowered at 6 m/s when the motor controlling the cable suddenly jams (stops), determine the maximum tension in the cable and the frequency of vibration of the safe. Neglect the mass of the cable and assume it is elastic such that it stretches 20 mm when subjected to a tension of 4 kN.



Prob. 22–9

22–10. The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.

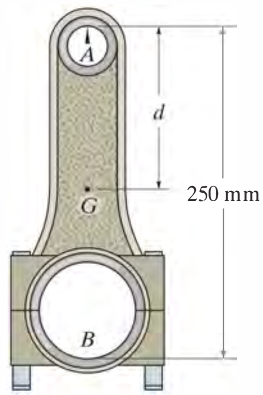


Prob. 22–10

22–11. The circular disk has a mass m and is pinned at O. Determine the natural period of vibration if it is displaced a small amount and released.

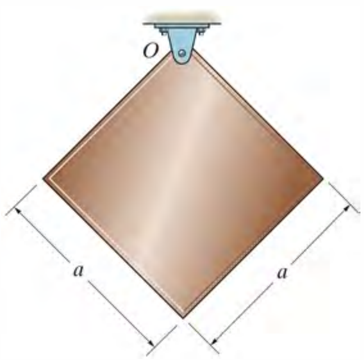
Prob. 22–11

•22–13. The connecting rod is supported by a knife edge at A and the period of vibration is measured as $\tau_A = 3.38$ s. It is then removed and rotated 180° so that it is supported by the knife edge at B. In this case the perod of vibration is measured as $\tau_B = 3.96$ s. Determine the location d of the center of gravity G, and compute the radius of gyration k_G.



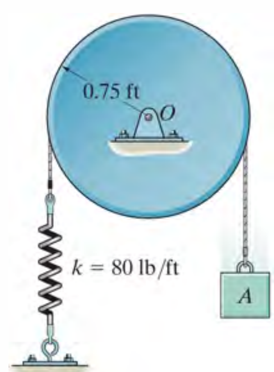
Prob. 22–13

***22–12.** The square plate has a mass m and is suspended at its corner from a pin O. Determine the natural period of vibration if it is displaced a small amount and released.



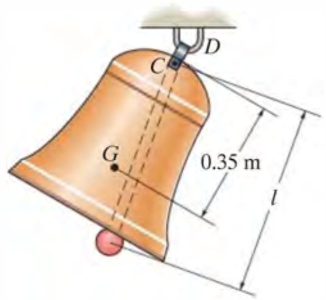
Prob. 22–12

22–14. The disk, having a weight of 15 lb, is pinned at its center O and supports the block A that has a weight of 3 lb. If the belt which passes over the disk does not slip at its contacting surface, determine the natural period of vibration of the system.



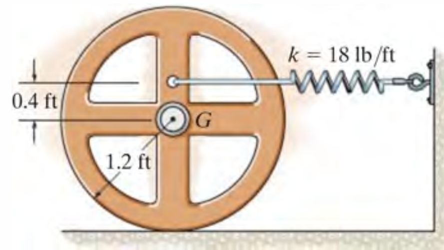
Prob. 22–14

22–15. The bell has a mass of 375 kg, a center of mass at G, and a radius of gyration about point D of $k_D = 0.4$ m. The tongue consists of a slender rod attached to the inside of the bell at C. If an 8-kg mass is attached to the end of the rod, determine the length l of the rod so that the bell will "ring silent," i.e., so that the natural period of vibration of the tongue is the same as that of the bell. For the calculation, neglect the small distance between C and D and neglect the mass of the rod.



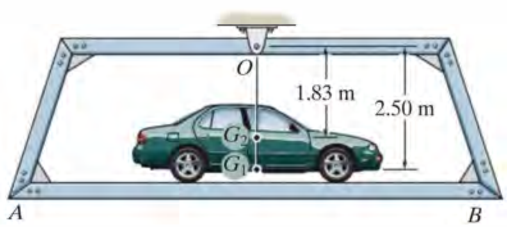
Prob. 22–15

•**22–17.** The 50-lb wheel has a radius of gyration about its mass center G of $k_G = 0.7$ ft. Determine the frequency of vibration if it is displaced slightly from the equilibrium position and released. Assume no slipping.



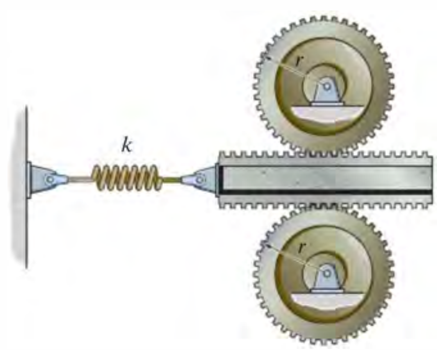
Prob. 22–17

*__22–16.__ The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



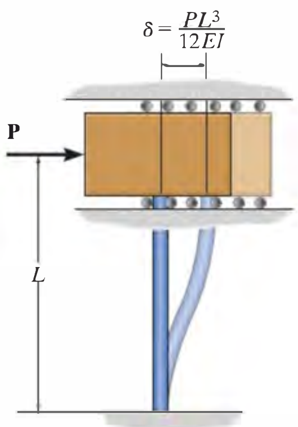
Prob. 22–16

22–18. The two identical gears each have a mass of m and a radius of gyration about their center of mass of k_0. They are in mesh with the gear rack, which has a mass of M and is attached to a spring having a stiffness k. If the gear rack is displaced slightly horizontally, determine the natural period of oscillation.



Prob. 22–18

22–19. In the "lump mass theory", a single-story building can be modeled in such a way that the whole mass of the building is lumped at the top of the building, which is supported by a cantilever column of negligible mass as shown. When a horizontal force **P** is applied to the model, the column deflects an amount of $\delta = PL^3/12EI$, where L is the effective length of the column, E is Young's modulus of elasticity for the material, and I is the moment of inertia of the cross section of the column. If the lump mass is m, determine the frequency of vibration in terms of these parameters.



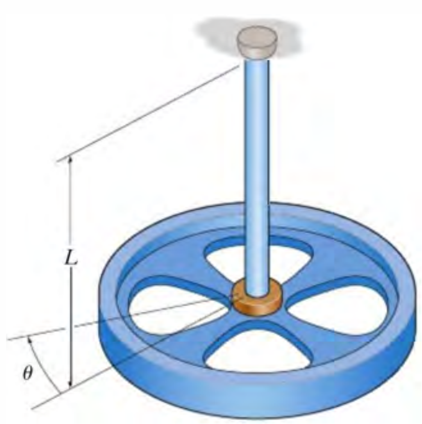
Prob. 22–19

•**22–21.** The cart has a mass of m and is attached to two springs, each having a stiffness of $k_1 = k_2 = k$, unstretched length of l_0, and a stretched length of l when the cart is in the equilibrium position. If the cart is displaced a distance of $x = x_0$ such that both springs remain in tension $(x_0 < l - l_0)$, determine the natural frequency of oscillation.

22–22. The cart has a mass of m and is attached to two springs, each having a stiffness of k_1 and k_2, respectively. If both springs are unstretched when the cart is in the equilibrium position shown, determine the natural frequency of oscillation.

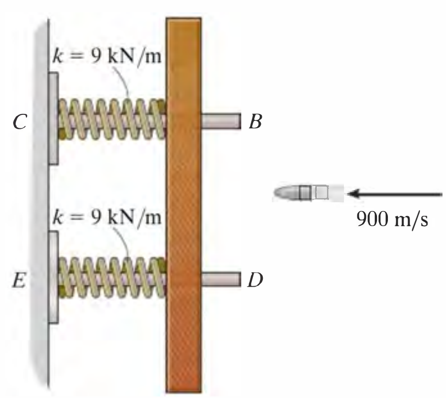
Probs. 22–21/22

*__22–20.__ A flywheel of mass m, which has a radius of gyration about its center of mass of k_O, is suspended from a circular shaft that has a torsional resistance of $M = C\theta$. If the flywheel is given a small angular displacement of θ and released, determine the natural period of oscillation.



Prob. 22–20

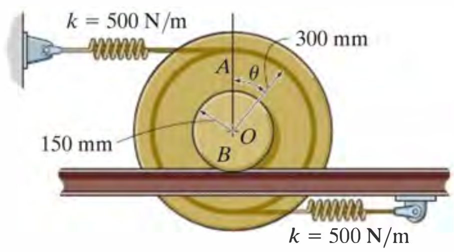
22–23. The 3-kg target slides freely along the smooth horizontal guides BC and DE, which are 'nested' in springs that each have a stiffness of $k = 9$ kN/m. If a 60-g bullet is fired with a velocity of 900 m/s and embeds into the target, determine the amplitude and frequency of oscillation of the target.



Prob. 22–23

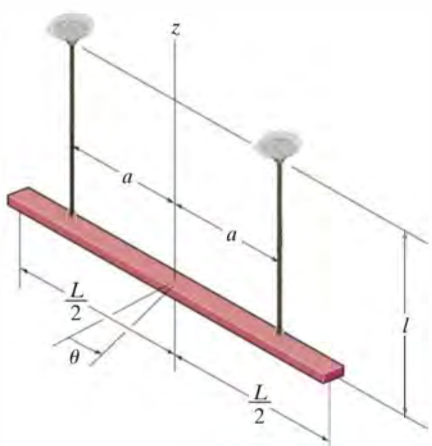
22

***22–24.** If the spool undergoes a small angular displacement of θ and is then released, determine the frequency of oscillation. The spool has a mass of 50 kg and a radius of gyration about its center of mass O of $k_O = 250$ mm. The spool rolls without slipping.



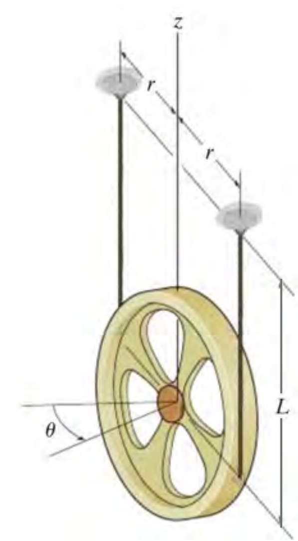
Prob. 22–24

•22–25. The slender bar of mass m is supported by two equal-length cords. If it is given a small angular displacement of θ about the vertical axis and released, determine the natural period of oscillation.



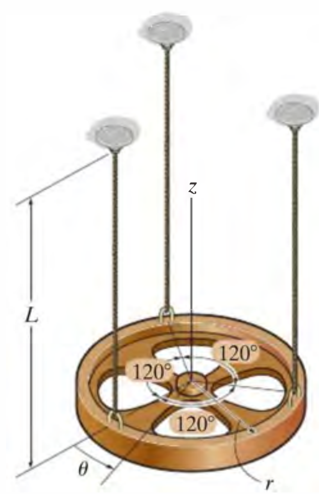
Prob. 22–25

22–26. A wheel of mass m is suspended from two equal-length cords as shown. When it is given a small angular displacement of θ about the z axis and released, it is observed that the period of oscillation is τ. Determine the radius of gyration of the wheel about the z axis.



Prob. 22–26

22–27. A wheel of mass m is suspended from three equal-length cords. When it is given a small angular displacement of θ about the z axis and released, it is observed that the period of oscillation is τ. Determine the radius of gyration of the wheel about the z axis.



Prob. 22–27

22

PROBLEMS

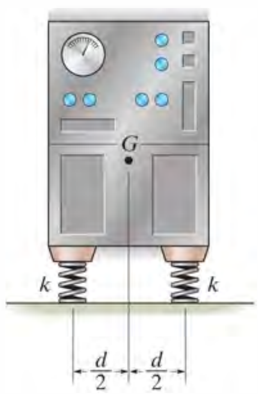
*22–28. Solve Prob. 22–10 using energy methods.

•22–29. Solve Prob. 22–11 using energy methods.

22–30. Solve Prob. 22–12 using energy methods.

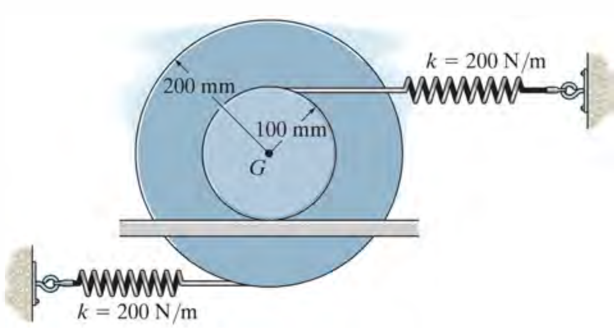
22–31. Solve Prob. 22–14 using energy methods.

*22–32. The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.



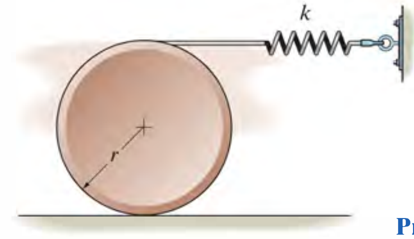
Prob. 22–32

•22–33. Determine the differential equation of motion of the 15-kg spool. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is $k_G = 125$ mm. The springs are originally unstretched.



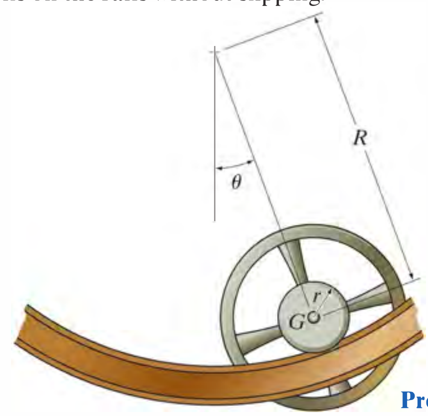
Prob. 22–33

22–34. Determine the natural period of vibration of the disk having a mass m and radius r. Assume the disk does not slip on the surface of contact as it oscillates.



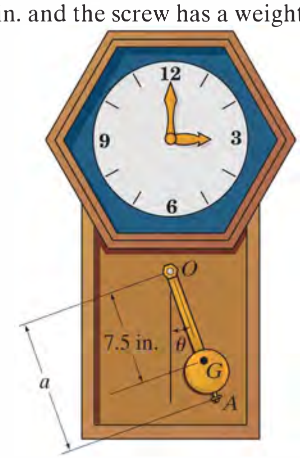
Prob. 22–34

22–35. If the wheel is given a small angular displacement of θ and released from rest, it is observed that it oscillates with a natural period of τ. Determine the wheel's radius of gyration about its center of mass G. The wheel has a mass of m and rolls on the rails without slipping.



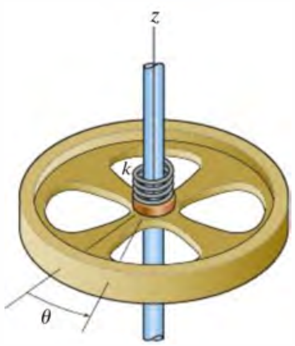
Prob. 22–35

*22–36. Without an adjustable screw, A, the 1.5-lb pendulum has a center of gravity at G. If it is required that it oscillates with a period of 1 s, determine the distance a from pin O to the screw. The pendulum's radius of gyration about O is $k_O = 8.5$ in. and the screw has a weight of 0.05 lb.



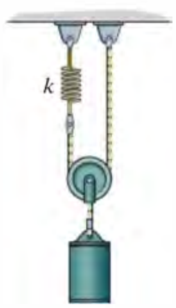
Prob. 22–36

•22–37. A torsional spring of stiffness k is attached to a wheel that has a mass of M. If the wheel is given a small angular displacement of θ about the z axis determine the natural period of oscillation. The wheel has a radius of gyration about the z axis of k_z.



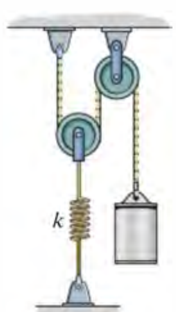
Prob. 22–37

22–38. Determine the frequency of oscillation of the cylinder of mass m when it is pulled down slightly and released. Neglect the mass of the small pulley.



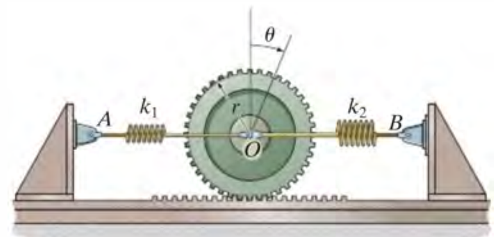
Prob. 22–38

22–39. Determine the frequency of oscillation of the cylinder of mass m when it is pulled down slightly and released. Neglect the mass of the small pulleys.



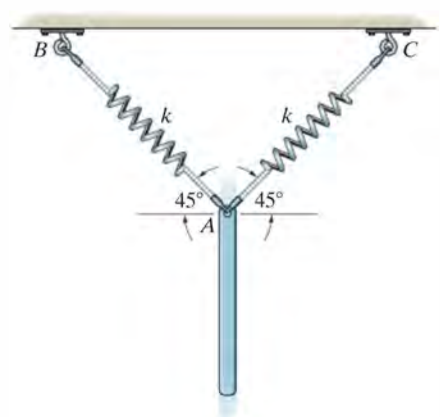
Prob. 22–39

***22–40.** The gear of mass m has a radius of gyration about its center of mass O of k_O. The springs have stiffnesses of k_1 and k_2, respectively, and both springs are unstretched when the gear is in an equilibrium position. If the gear is given a small angular displacement of θ and released, determine its natural period of oscillation.



Prob. 22–40

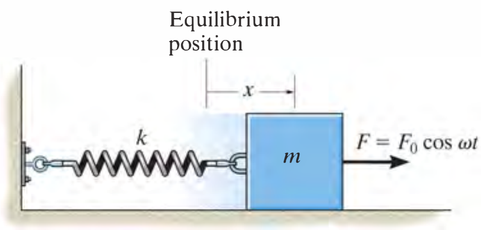
22–41. The bar has a mass of 8 kg and is suspended from two springs such that when it is in equilibrium, the springs make an angle of 45° with the horizontal as shown. Determine the natural period of vibration if the bar is pulled down a short distance and released. Each spring has a stiffness of $k = 40$ N/m.



Prob. 22–41

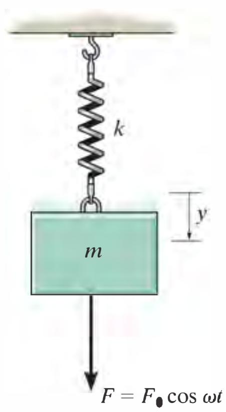
PROBLEMS

22–42. If the block-and-spring model is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{x} + (k/m)x = (F_0/m) \cos \omega t$, where x is measured from the equilibrium position of the block. What is the general solution of this equation?



Prob. 22–42

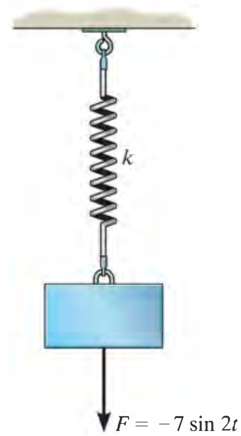
22–43. If the block is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{y} + (k/m)y = (F_0/m) \cos \omega t$, where y is measured from the equilibrium position of the block. What is the general solution of this equation?



Prob. 22–43

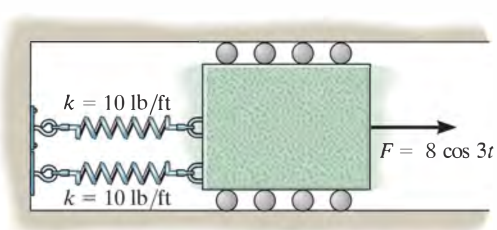
***22–44.** A block having a mass of 0.8 kg is suspended from a spring having a stiffness of 120 N/m. If a dashpot provides a damping force of 2.5 N when the speed of the block is 0.2 m/s, determine the period of free vibration.

•22–45. The spring shown stretches 6 in. when it is loaded with a 50-lb weight. Determine the equation which describes the position of the weight as a function of time if the weight is pulled 4 in. below its equilibrium position and released from rest at $t = 0$. The weight is subjected to the periodic force of $F = (-7 \sin 2t)$ lb, where t is in seconds.



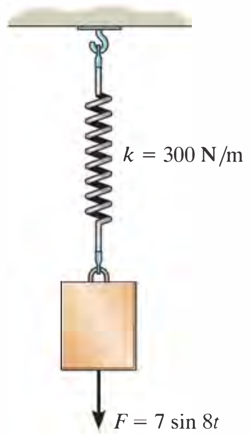
Prob. 22–45

22–46. The 30-lb block is attached to two springs having a stiffness of 10 lb/ft. A periodic force $F = (8 \cos 3t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



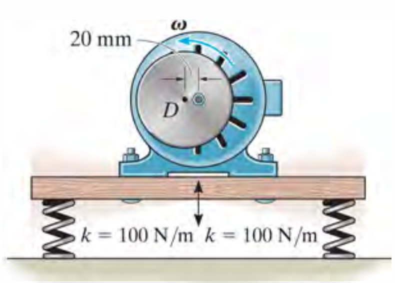
Prob. 22–46

22–47. A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical periodic force $F = (7 \sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Consider positive displacement to be downward.



Prob. 22–47

***22–48.** The electric motor has a mass of 50 kg and is supported by *four springs,* each spring having a stiffness of 100 N/m. If the motor turns a disk D which is mounted eccentrically, 20 mm from the disk's center, determine the angular velocity ω at which resonance occurs. Assume that the motor only vibrates in the vertical direction.



Prob. 22–48

•22–49. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the angular velocity of the fan blade at which resonance will occur. *Hint:* See the first part of Example 22.8.

22–50. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the amplitude of steady-state vibration of the fan if the angular velocity of the fan blade is 10 rad/s. *Hint:* See the first part of Example 22.8.

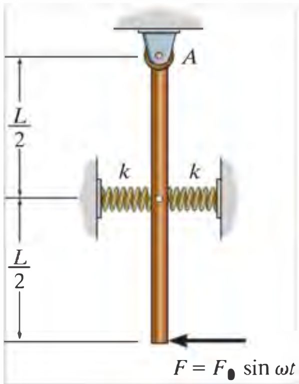
22–51. What will be the amplitude of steady-state vibration of the fan in Prob. 22–50 if the angular velocity of the fan blade is 18 rad/s? *Hint:* See the first part of Example 22.8.

Probs. 22–49/50/51

***22–52.** A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which can be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

•22–53. Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22–52.

22–54. The uniform rod has a mass of m. If it is acted upon by a periodic force of $F = F_0 \sin \omega t$, determine the amplitude of the steady-state vibration.

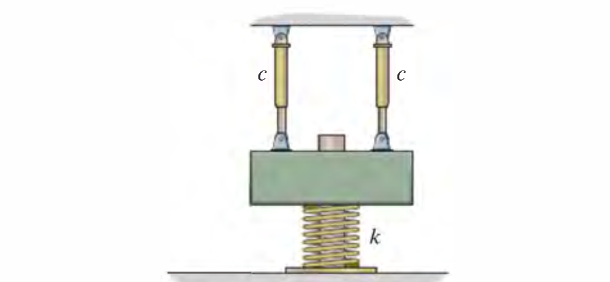


Prob. 22–54

22–55. The motion of an underdamped system can be described by the graph in Fig. 20–16. Show that the relation between two successive peaks of vibration is given by $\ln(x_n/x_{n+1}) = 2\pi(c/c_c)/\sqrt{1-(c/c_c)^2}$, where c/c_c is the *damping factor* and $\ln(x_n/x_{n+1})$ is called the *logarithmic decrement*.

***22–56.** Two successive amplitudes of a spring-block underdamped vibrating system are observed to be 100 mm and 75 mm. Determine the damping coefficient of the system. The block has a mass of 10 kg and the spring has a stiffness of $k = 1000$ N/m. Use the result of Prob. 22–55.

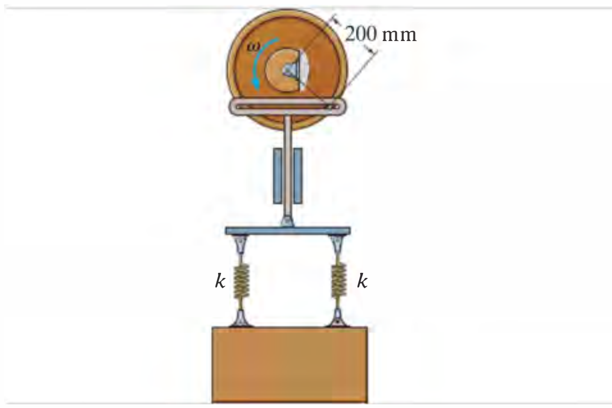
•22–57. Two identical dashpots are arranged parallel to each other, as shown. Show that if the damping coefficient $c < \sqrt{mk}$, then the block of mass m will vibrate as an underdamped system.



Prob. 22–57

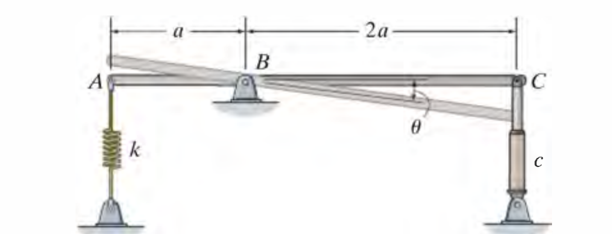
22–58. The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\boldsymbol{\omega}$. If the amplitude of the steady-state vibration is observed to be 400 mm, and the springs each have a stiffness of $k = 2500$ N/m, determine the two possible values of $\boldsymbol{\omega}$ at which the wheel must rotate. The block has a mass of 50 kg.

22–59. The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\omega = 5$ rad/s. If the amplitude of the steady-state vibration is observed to be 400 mm, determine the two possible values of the stiffness k of the springs. The block has a mass of 50 kg.



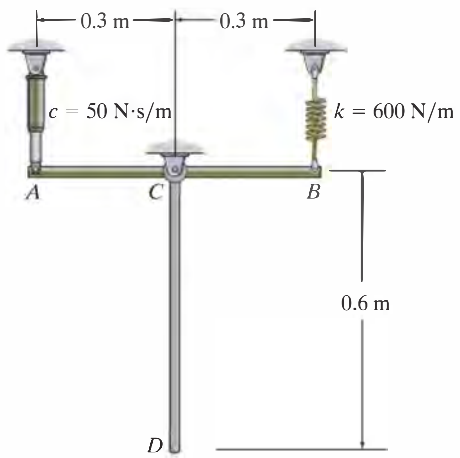
Probs. 22–58/59

***22–60.** Find the differential equation for small oscillations in terms of θ for the uniform rod of mass m. Also show that if $c < \sqrt{mk}/2$, then the system remains underdamped. The rod is in a horizontal position when it is in equilibrium.



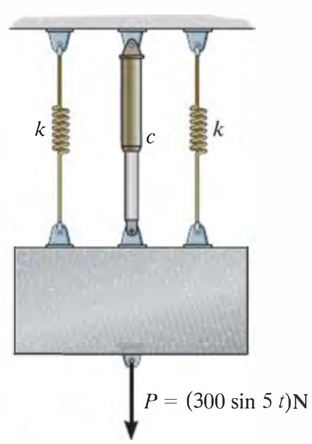
Prob. 22–60

•**22–61.** If the dashpot has a damping coefficient of $c = 50\ \text{N}\cdot\text{s/m}$, and the spring has a stiffness of $k = 600\ \text{N/m}$, show that the system is underdamped, and then find the pendulum's period of oscillation. The uniform rods have a mass per unit length of 10 kg/m.



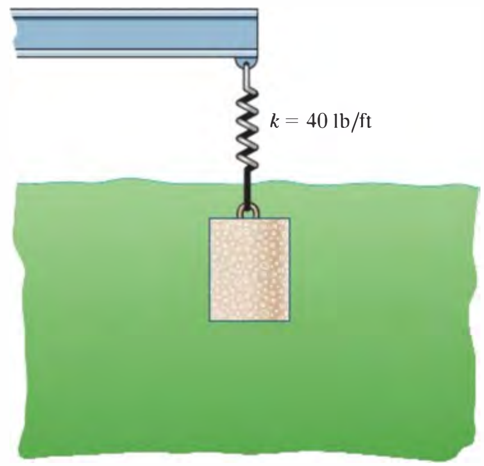
Prob. 22–61

22–62. If the 30-kg block is subjected to a periodic force of $P = (300 \sin 5t)$ N, $k = 1500\ \text{N/m}$, and $c = 300\ \text{N}\cdot\text{s/m}$, determine the equation that describes the steady-state vibration as a function of time.



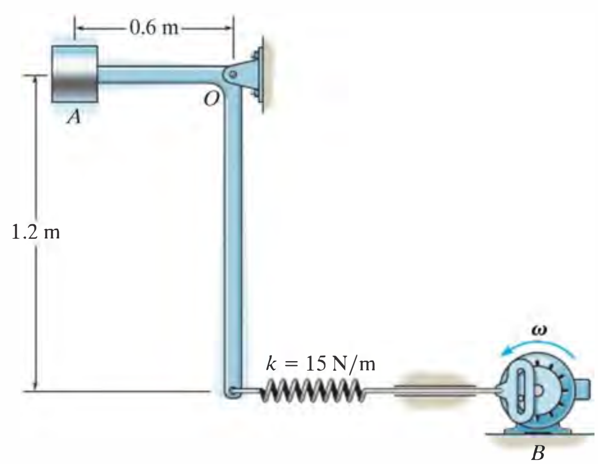
Prob. 22–62

22–63. The block, having a weight of 15 lb, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.8|v|)$ lb, where v is the velocity of the block in ft/s. If the block is pulled down 0.8 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 40$ lb/ft. Consider positive displacement to be downward.



Prob. 22–63

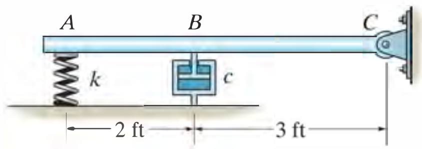
***22–64.** The small block at A has a mass of 4 kg and is mounted on the bent rod having negligible mass. If the rotor at B causes a harmonic movement $\delta_B = (0.1 \cos 15t)$ m, where t is in seconds, determine the steady-state amplitude of vibration of the block.



Prob. 22–64

22

•**22–65.** The bar has a weight of 6 lb. If the stiffness of the spring is $k = 8\,\text{lb/ft}$ and the dashpot has a damping coefficient $c = 60\,\text{lb}\cdot\text{s/ft}$, determine the differential equation which describes the motion in terms of the angle θ of the bar's rotation. Also, what should be the damping coefficient of the dashpot if the bar is to be critically damped?

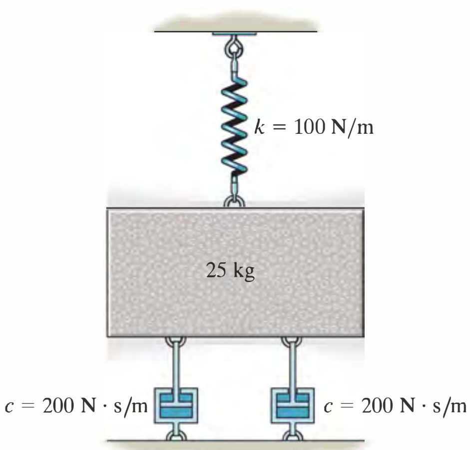


Prob. 22–65

22–66. A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$ N, where v is the velocity of the block in m/s.

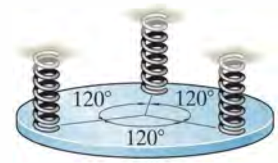
22–67. A 4-lb weight is attached to a spring having a stiffness $k = 10$ lb/ft. The weight is drawn downward a distance of 4 in. and released from rest. If the support moves with a vertical displacement $\delta = (0.5 \sin 4t)$ in., where t is in seconds, determine the equation which describes the position of the weight as a function of time.

***22–68.** Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs?



Prob. 22–68

•**22–69.** The 4-kg circular disk is attached to three springs, each spring having a stiffness $k = 180$ N/m. If the disk is immersed in a fluid and given a downward velocity of 0.3 m/s at the equilibrium position, determine the equation which describes the motion. Consider positive displacement to be measured downward, and that fluid resistance acting on the disk furnishes a damping force having a magnitude $F = (60|v|)$ N, where v is the velocity of the block in m/s.



Prob. 22–69

22–70. Using a block-and-spring model, like that shown in Fig. 22–13*a*, but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

22–71. The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. due to the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weights 150 lb. Neglect the mass of the beam.

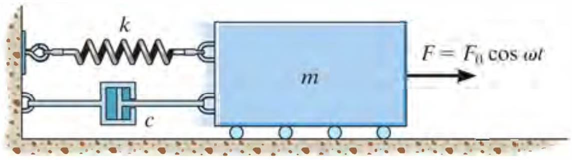
***22–72.** What will be the amplitude of steady-state vibration of the motor in Prob. 22–71 if the angular velocity of the flywheel is 20 rad/s?

•**22–73.** Determine the angular velocity of the flywheel in Prob. 22–71 which will produce an amplitude of vibration of 0.25 in.

Probs. 22–71/72/73

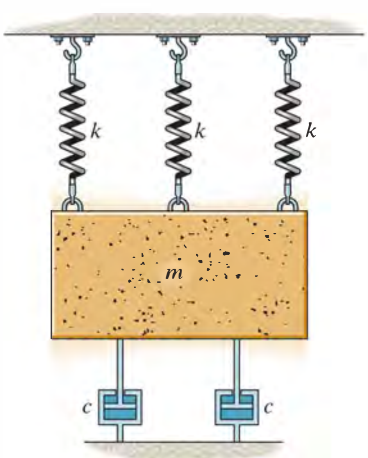
22

22–74. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



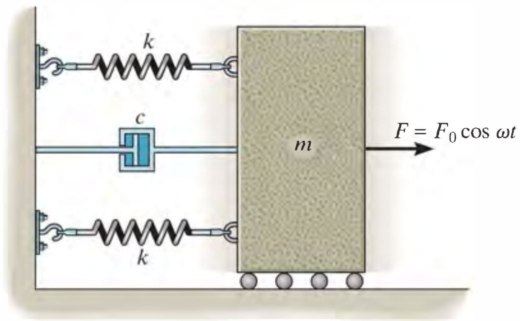
Prob. 22–74

22–75. Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N·s/m, $m = 25$ kg.



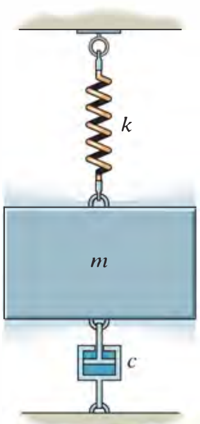
Prob. 22–75

***22–76.** Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?



Prob. 22–76

•22–77. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



Prob. 22–77

22