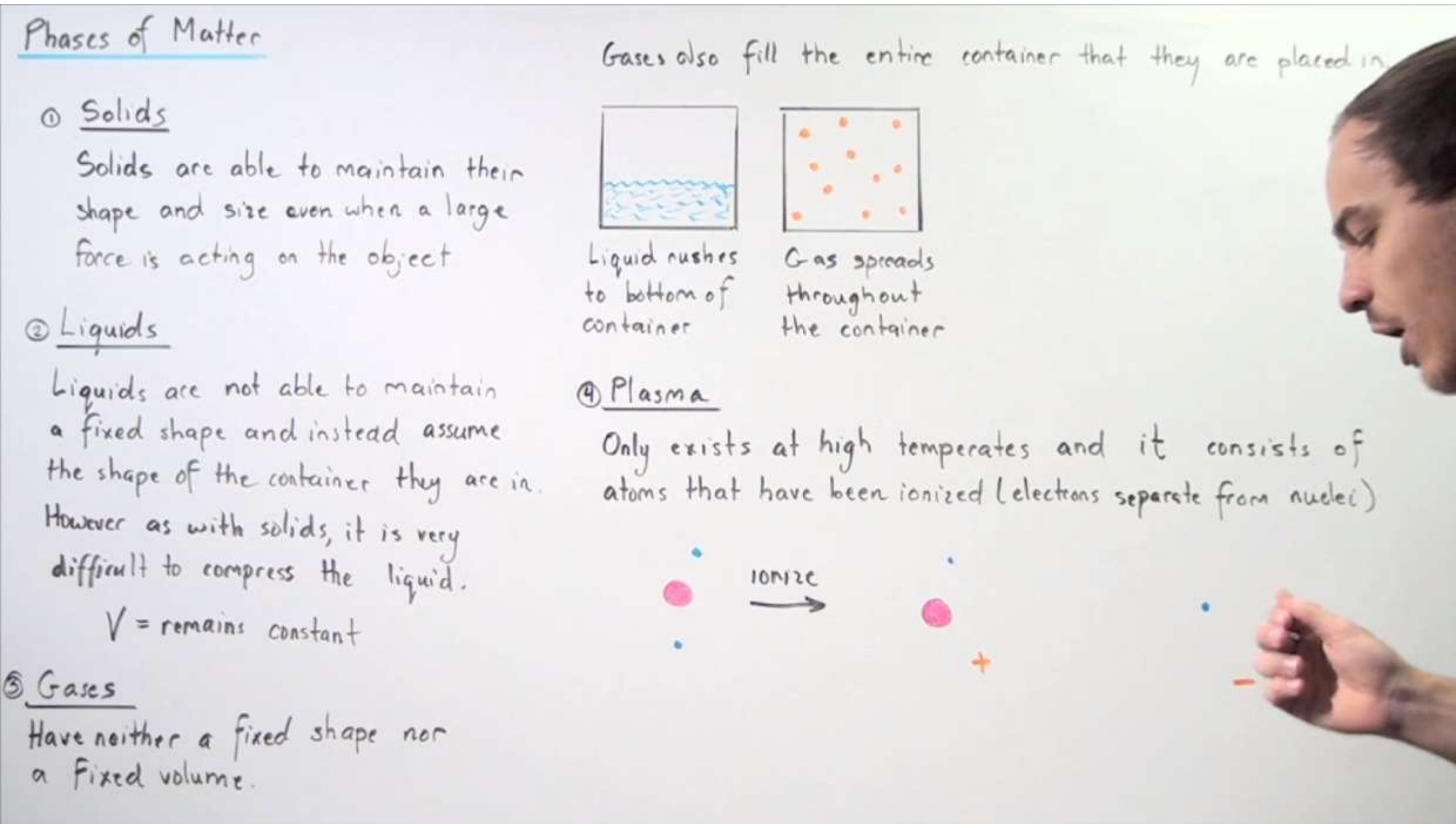


Phases of Matter

① Solids

Solids are able to maintain their shape and size even when a large force is acting on the object

② Liquids

Liquids are not able to maintain a fixed shape and instead assume the shape of the container they are in. However as with solids, it is very difficult to compress the liquid.

V = remains constant

③ Gases

Have neither a fixed shape nor a Fixed volume.

Gases also fill the entire container that they are placed in

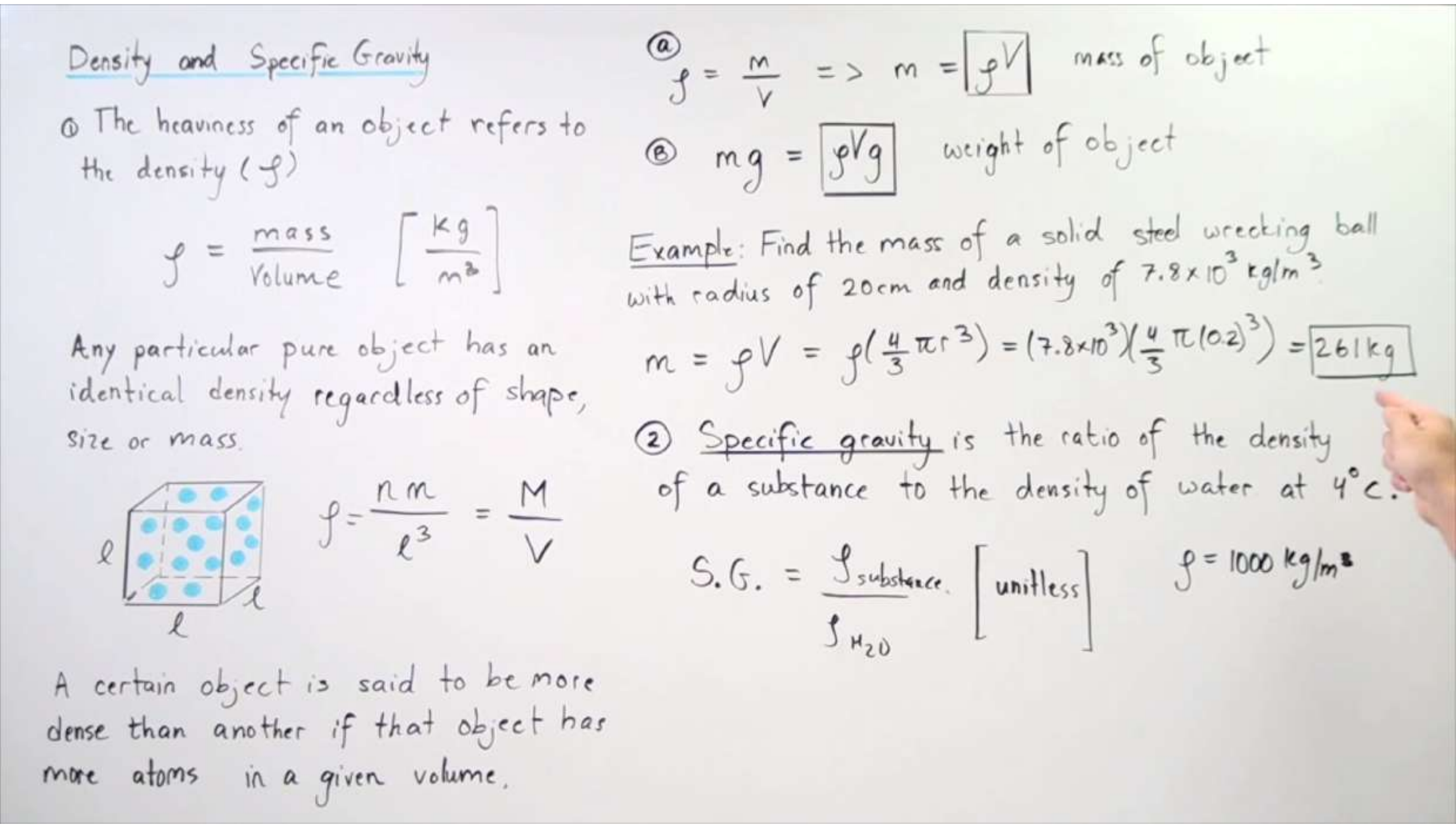
Liquid rushes to bottom of container

Gas spreads throughout the container

④ Plasma

Only exists at high temperates and it consists of atoms that have been ionized (electrons separate from nuclei)

ionize



Density and Specific Gravity

① The heaviness of an object refers to the density (ρ)

$$\rho = \frac{\text{mass}}{\text{Volume}} \quad \left[\frac{kg}{m^3}\right]$$

Any particular pure object has an identical density regardless of shape, size or mass.

$$\rho = \frac{n m}{l^3} = \frac{M}{V}$$

A certain object is said to be more dense than another if that object has more atoms in a given volume.

ⓐ $\rho = \frac{m}{V} \Rightarrow m = \boxed{\rho V}$ mass of object

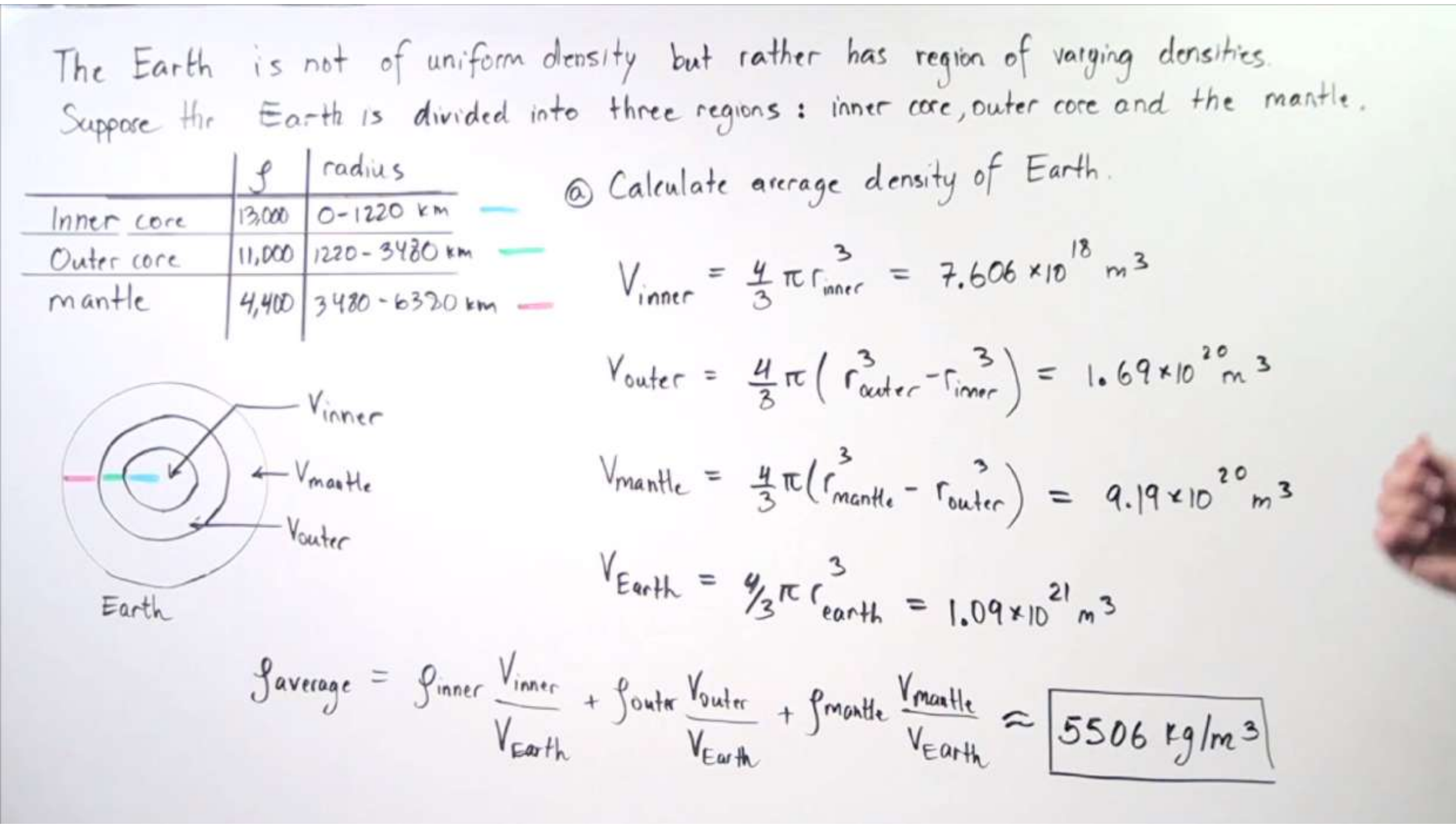
ⓑ $mg = \boxed{\rho V g}$ weight of object

Example: Find the mass of a solid steel wrecking ball with radius of 20cm and density of 7.8×10^3 kg/m^3

$$m = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = (7.8\times10^3)\left(\frac{4}{3}\pi(0.2)^3\right) = \boxed{261\,kg}$$

② Specific gravity is the ratio of the density of a substance to the density of water at 4°C.

$$S.G. = \frac{\rho_{substance}}{\rho_{H_2O}} \quad [\text{unitless}] \qquad \rho = 1000\ kg/m^3$$



The Earth is not of uniform density but rather has region of varying densities. Suppose the Earth is divided into three regions: inner core, outer core and the mantle.

	ρ	radius
Inner core	13,000	0-1220 km
Outer core	11,000	1220-3480 km
mantle	4,400	3480-6370 km

ⓐ Calculate average density of Earth.

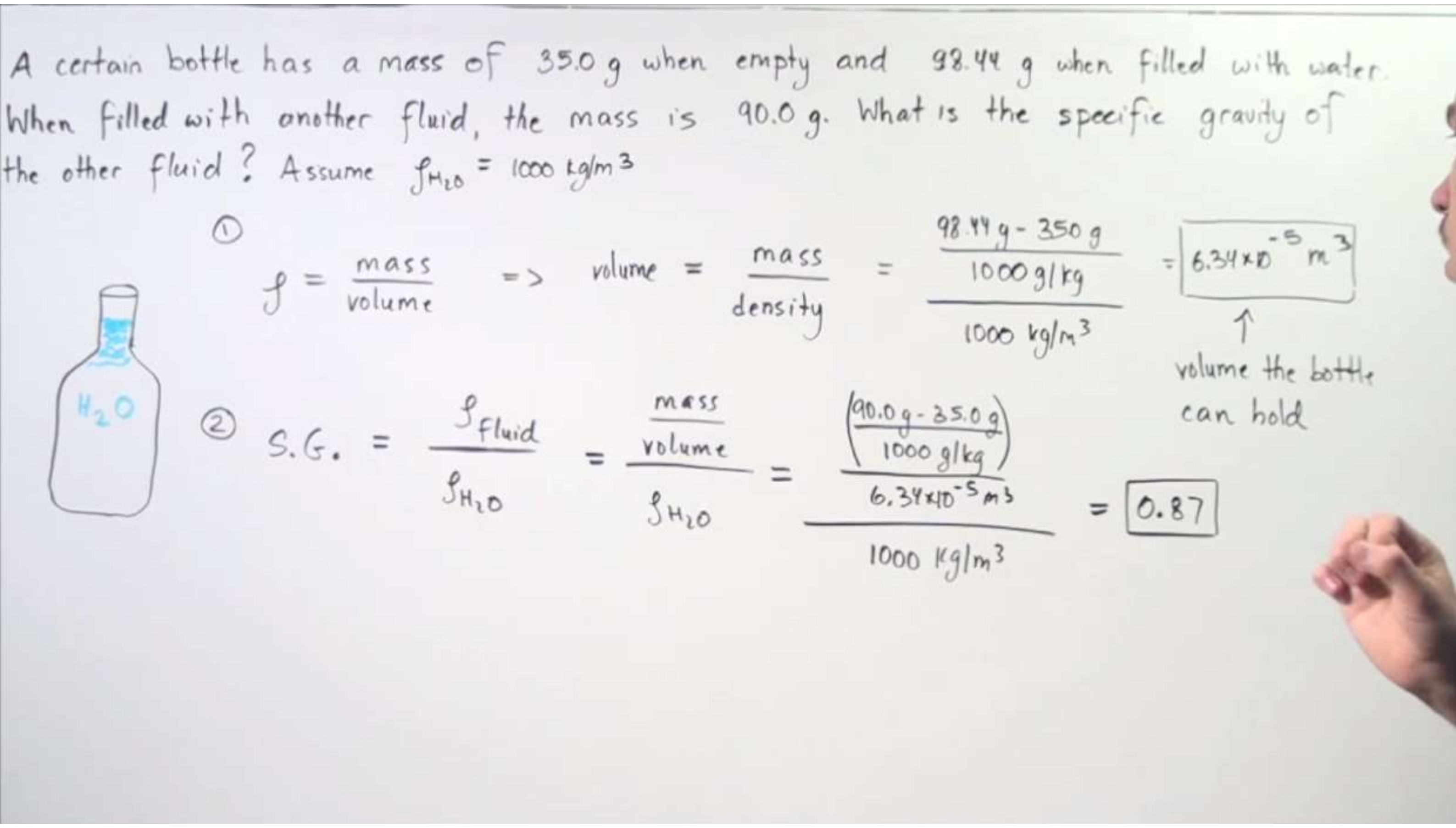
$$V_{inner} = \frac{4}{3}\pi r_{inner}^3 = 7.606\times10^{18}\ m^3$$

$$V_{outer} = \frac{4}{3}\pi\left(r_{outer}^3 - r_{inner}^3\right) = 1.69\times10^{20}\ m^3$$

$$V_{mantle} = \frac{4}{3}\pi\left(r_{mantle}^3 - r_{outer}^3\right) = 9.19\times10^{20}\ m^3$$

$$V_{Earth} = 4/3\,\pi r_{earth}^3 = 1.09\times10^{21}\ m^3$$

$$\rho_{average} = \rho_{inner}\frac{V_{inner}}{V_{Earth}} + \rho_{outer}\frac{V_{outer}}{V_{Earth}} + \rho_{mantle}\frac{V_{mantle}}{V_{Earth}} \approx \boxed{5506\ kg/m^3}$$



A certain bottle has a mass of 35.0 g when empty and 98.44 g when filled with water. When filled with another fluid, the mass is 90.0 g. What is the specific gravity of the other fluid? Assume $\rho_{H_2O} = 1000\ kg/m^3$

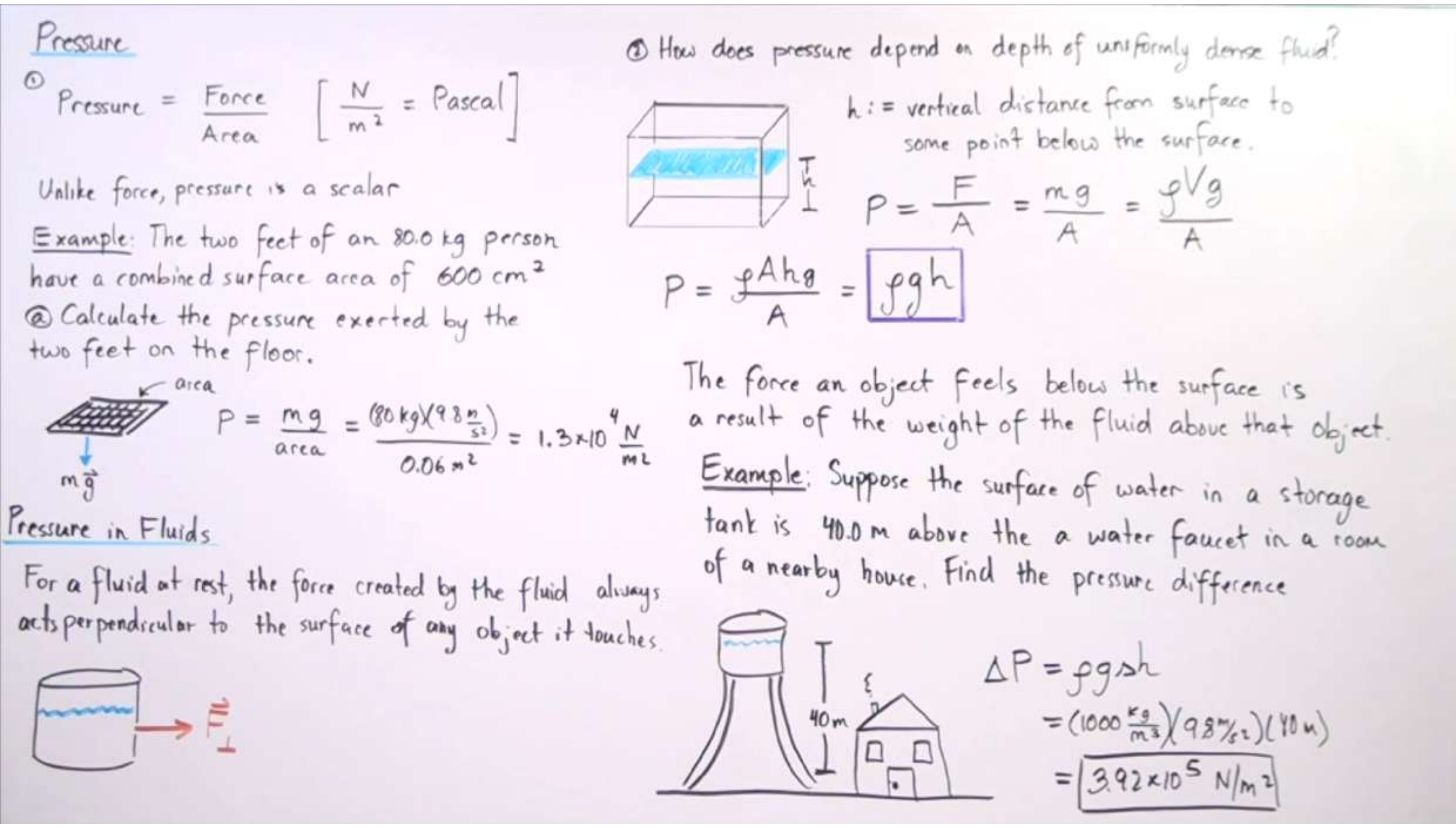
①

$$\rho = \frac{mass}{volume} \Rightarrow volume = \frac{mass}{density} = \frac{\frac{98.44\,g - 35.0\,g}{1000\,g/kg}}{1000\ kg/m^3} = \boxed{6.34\times10^{-5}\ m^3}$$

↑ volume the bottle can hold

②

$$S.G. = \frac{\rho_{Fluid}}{\rho_{H_2O}} = \frac{\frac{mass}{volume}}{\rho_{H_2O}} = \frac{\frac{\left(\frac{90.0\,g - 35.0\,g}{1000\,g/kg}\right)}{6.34\times10^{-5}\,m^3}}{1000\ kg/m^3} = \boxed{0.87}$$



Pressure

① Pressure = $\frac{\text{Force}}{\text{Area}}$ $\left[\frac{N}{m^2} = \text{Pascal}\right]$

Unlike force, pressure is a scalar

Example: The two feet of an 80.0 kg person have a combined surface area of 600 cm^2

ⓐ Calculate the pressure exerted by the two feet on the floor.

$$P = \frac{mg}{\text{area}} = \frac{(80\,kg)(9.8\,\frac{m}{s^2})}{0.06\,m^2} = 1.3\times10^4\,\frac{N}{m^2}$$

Pressure in Fluids

For a fluid at rest, the force created by the fluid always acts perpendicular to the surface of any object it touches.

② How does pressure depend on depth of uniformly dense fluid?

h := vertical distance from surface to some point below the surface.

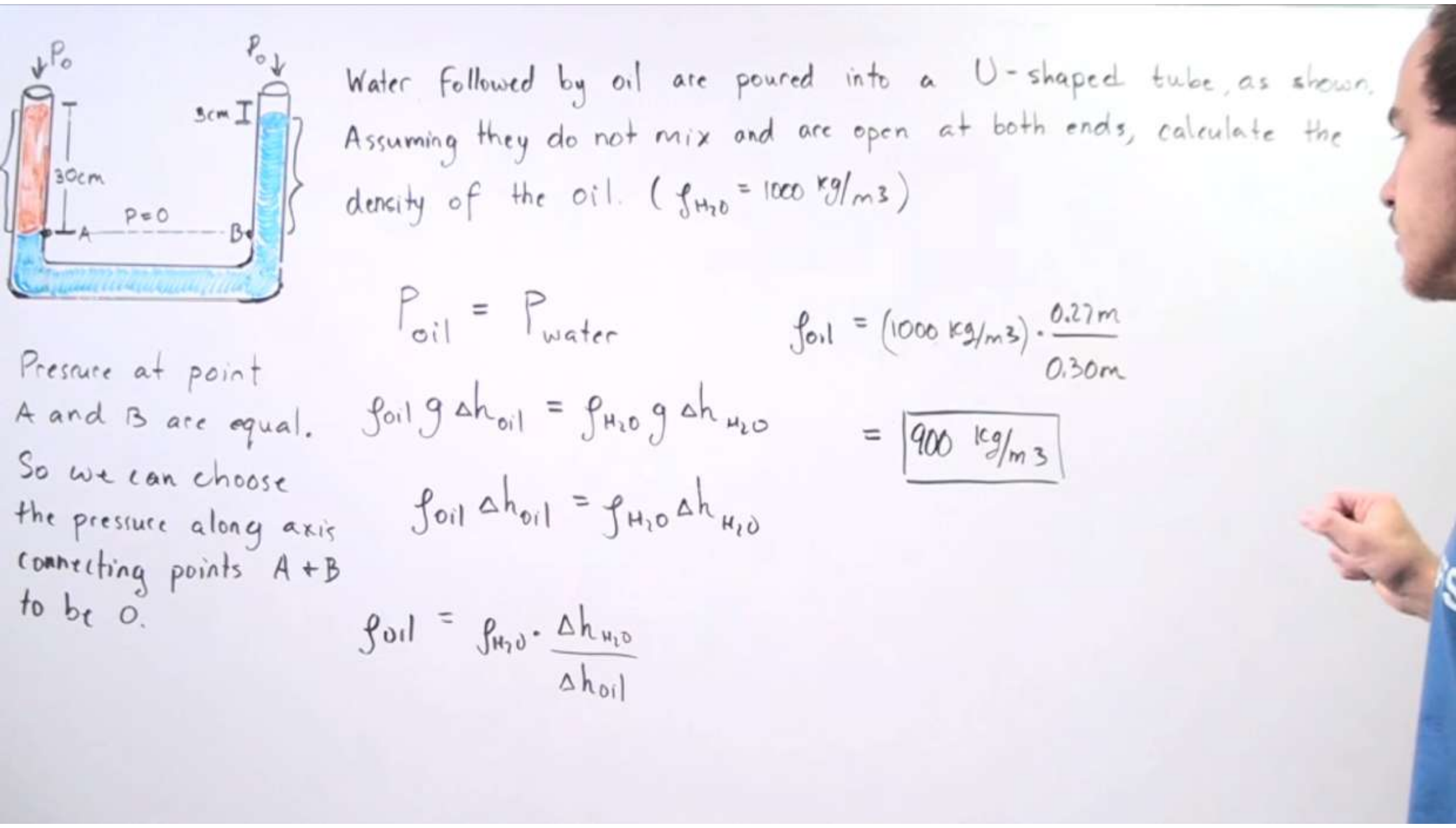
$$P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho V g}{A}$$

$$P = \frac{\rho A h g}{A} = \boxed{\rho g h}$$

The force an object feels below the surface is a result of the weight of the fluid above that object.

Example: Suppose the surface of water in a storage tank is 40.0 m above the a water faucet in a room of a nearby house. Find the pressure difference

$$\Delta P = \rho g \Delta h$$
$$= (1000\,\tfrac{kg}{m^3})(9.8\,m/s^2)(40\,m)$$
$$= \boxed{3.92\times10^5\ N/m^2}$$



Water followed by oil are poured into a U-shaped tube, as shown. Assuming they do not mix and are open at both ends, calculate the dencity of the oil. ($\rho_{H_2O} = 1000$ kg/m3)

Pressure at point A and B are equal. So we can choose the pressure along axis connecting points A + B to be 0.

$$P_{oil} = P_{water}$$

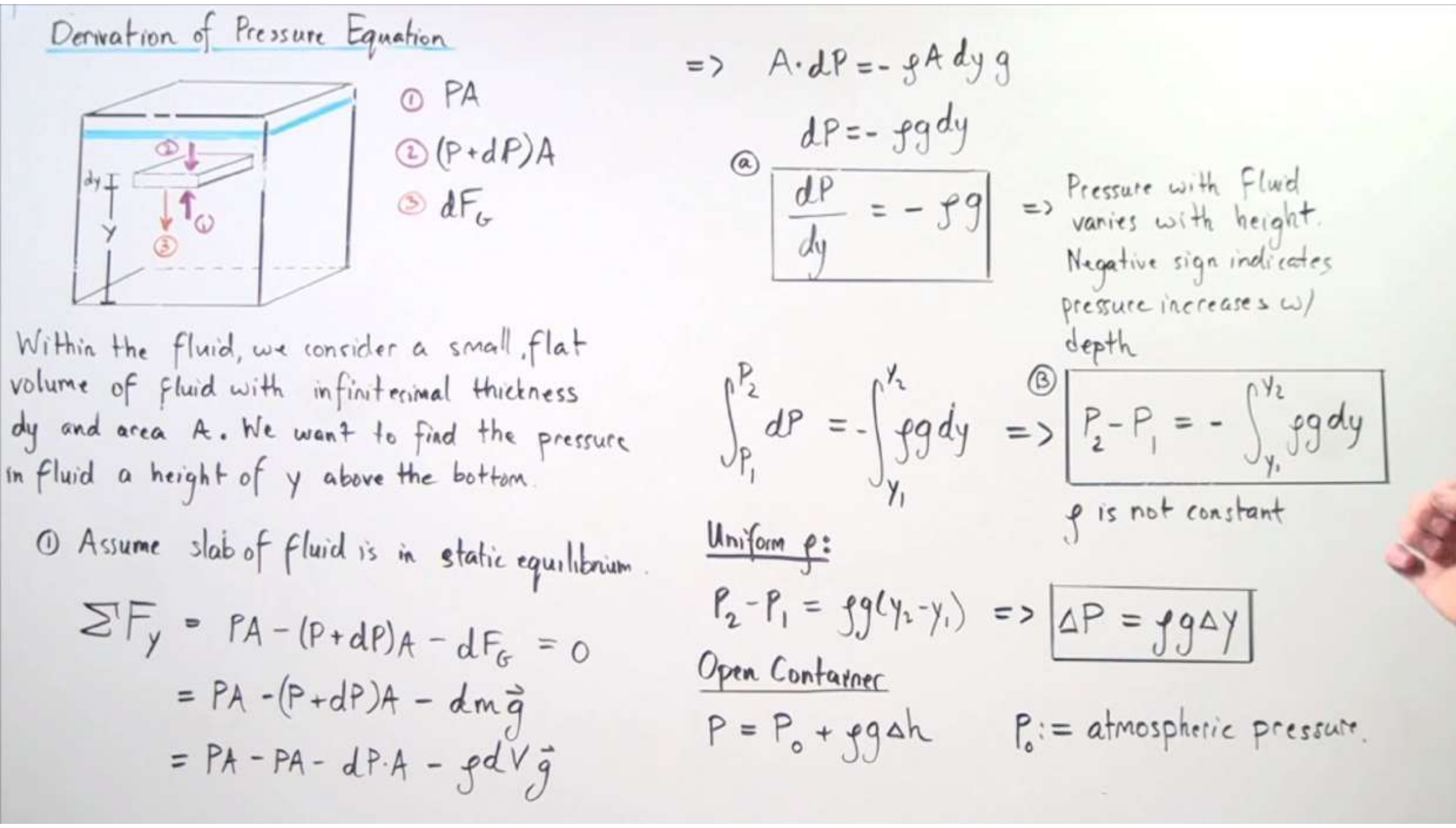
$$\rho_{oil}\, g\, \Delta h_{oil} = \rho_{H_2O}\, g\, \Delta h_{H_2O}$$

$$\rho_{oil}\, \Delta h_{oil} = \rho_{H_2O}\, \Delta h_{H_2O}$$

$$\rho_{oil} = \rho_{H_2O} \cdot \frac{\Delta h_{H_2O}}{\Delta h_{oil}}$$

$$\rho_{oil} = (1000\ \text{kg/m3}) \cdot \frac{0.27\text{m}}{0.30\text{m}}$$

$$= \boxed{900\ \text{kg/m3}}$$



Derivation of Pressure Equation

① PA

② $(P+dP)A$

③ dF_G

Within the fluid, we consider a small, flat volume of fluid with infinitesimal thickness dy and area A. We want to find the pressure in fluid a height of y above the bottom.

① Assume slab of fluid is in static equilibrium.

$$\Sigma F_y = PA - (P+dP)A - dF_G = 0$$

$$= PA - (P+dP)A - dm\vec{g}$$

$$= PA - PA - dP\cdot A - \rho dV\vec{g}$$

$$\Rightarrow A\cdot dP = -\rho A\,dy\,g$$

$$dP = -\rho g\,dy$$

ⓐ $\boxed{\frac{dP}{dy} = -\rho g}$ => Pressure with fluid varies with height. Negative sign indicates pressure increases w/ depth

$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g\,dy$ => ⓑ $\boxed{P_2 - P_1 = -\int_{y_1}^{y_2} \rho g\,dy}$

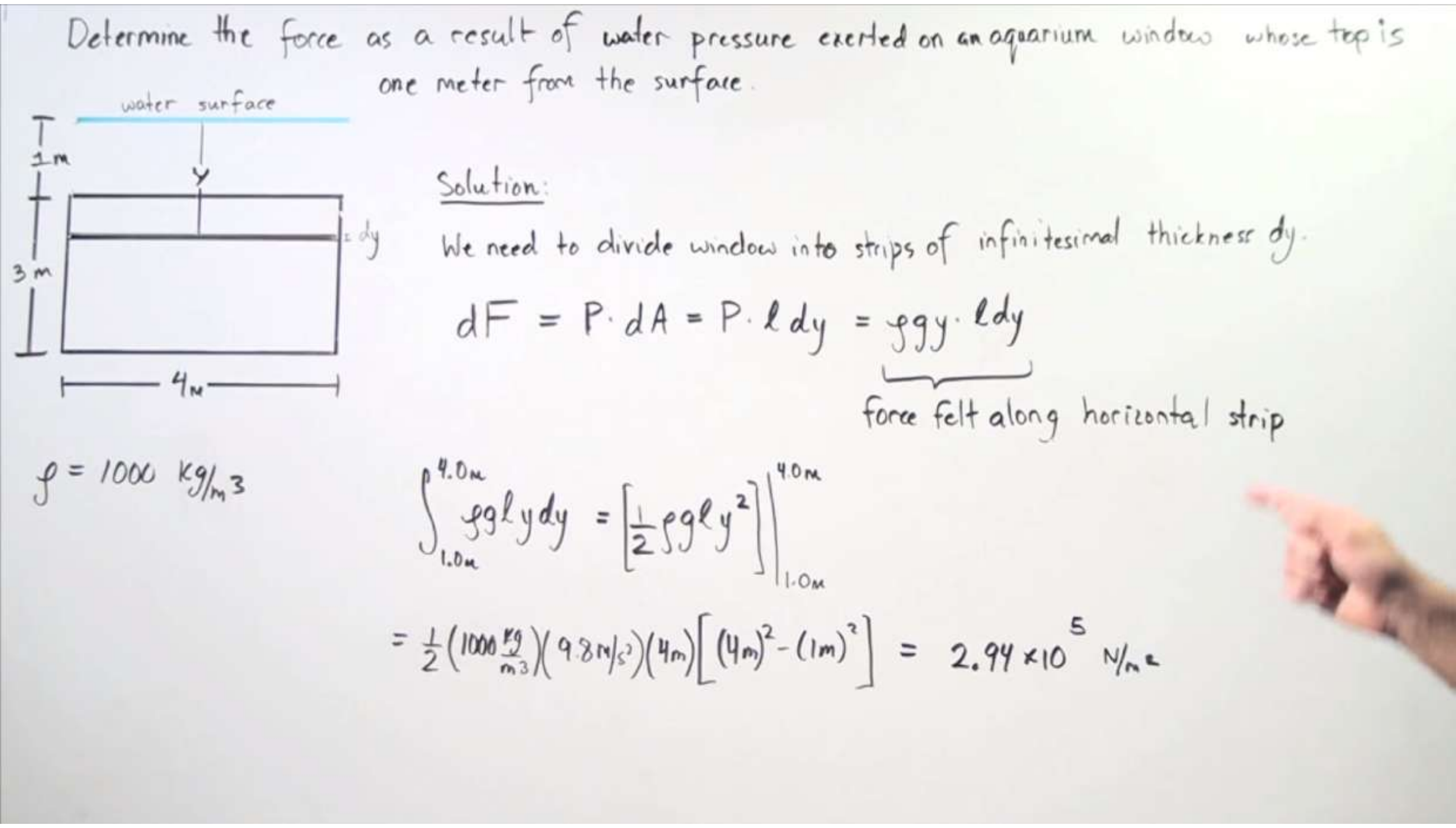
ρ is not constant

Uniform ρ:

$P_2 - P_1 = \rho g(y_2 - y_1)$ => $\boxed{\Delta P = \rho g \Delta y}$

Open Container

$P = P_0 + \rho g \Delta h$ $\quad P_0 :=$ atmospheric pressure.



Determine the force as a result of water pressure exerted on an aquarium window whose top is one meter from the surface.

$\rho = 1000\ kg/m^3$

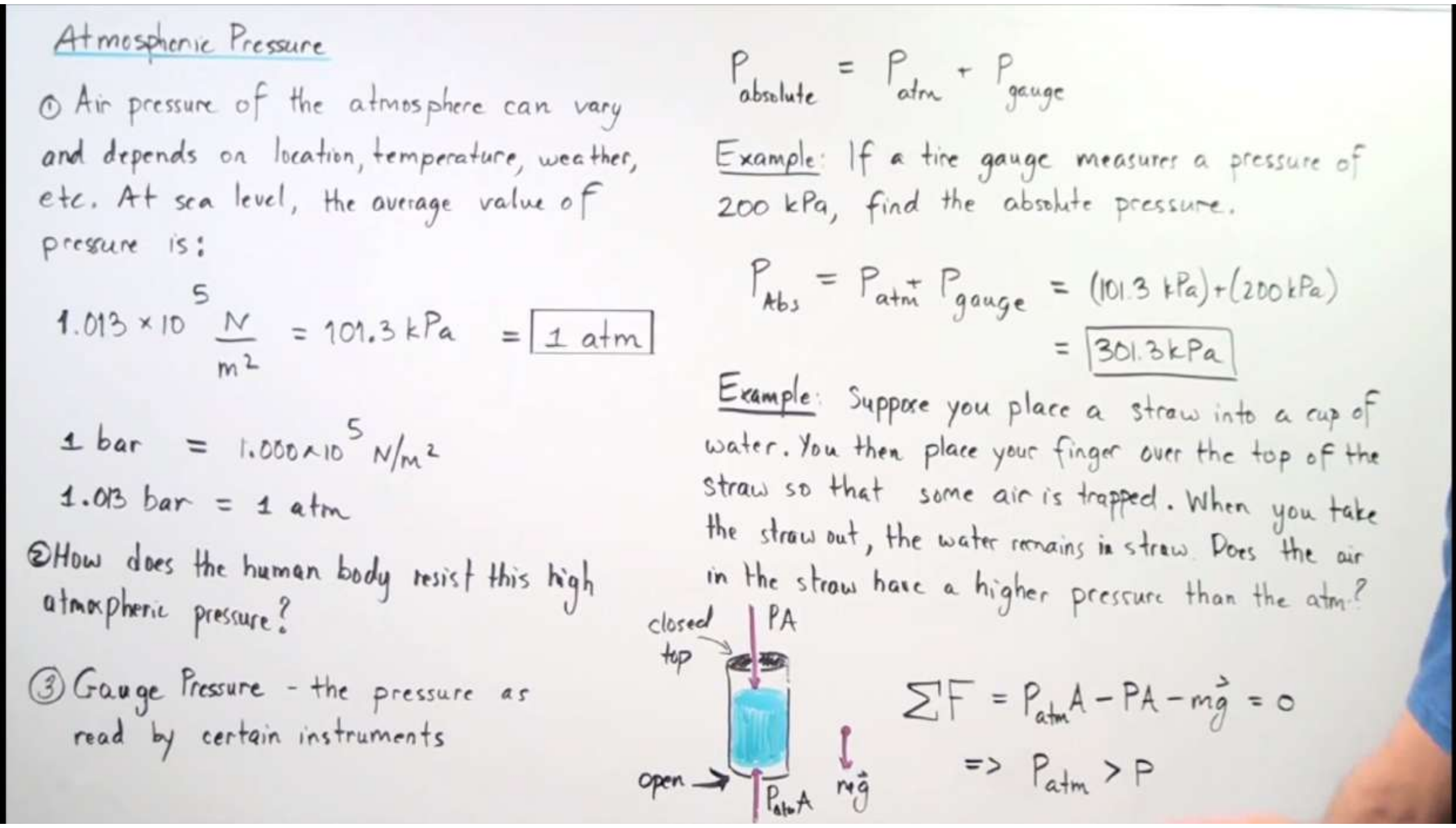
Solution:

We need to divide window into strips of infinitesimal thickness dy.

$$dF = P\cdot dA = P\cdot \ell dy = \underbrace{\rho g y\cdot \ell dy}_{\text{force felt along horizontal strip}}$$

$$\int_{1.0m}^{4.0m} \rho g \ell y\, dy = \left[\frac{1}{2}\rho g \ell y^2\right]\Bigg|_{1.0m}^{4.0m}$$

$$= \frac{1}{2}\left(1000\frac{kg}{m^3}\right)\left(9.8\, m/s^2\right)(4m)\left[(4m)^2-(1m)^2\right] = 2.94\times 10^5\ N/m^2$$



Atmospheric Pressure

① Air pressure of the atmosphere can vary and depends on location, temperature, weather, etc. At sea level, the average value of pressure is:

$$1.013 \times 10^5 \frac{N}{m^2} = 101.3\ kPa = \boxed{1\ atm}$$

$$1\ bar = 1.000 \times 10^5\ N/m^2$$

$$1.013\ bar = 1\ atm$$

② How does the human body resist this high atmospheric pressure?

③ Gauge Pressure - the pressure as read by certain instruments

$$P_{absolute} = P_{atm} + P_{gauge}$$

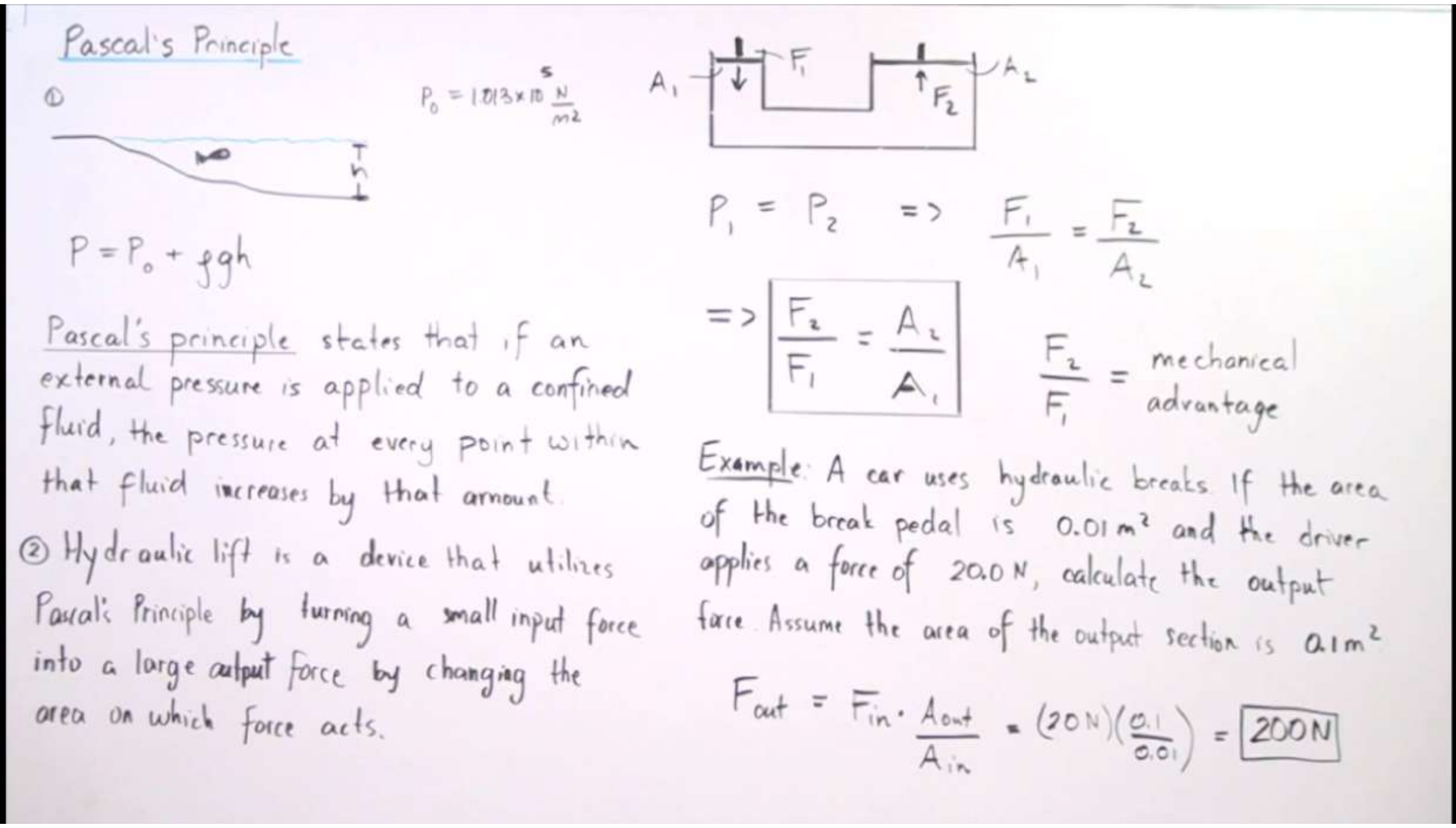
Example: If a tire gauge measures a pressure of 200 kPa, find the absolute pressure.

$$P_{Abs} = P_{atm} + P_{gauge} = (101.3\ kPa) + (200\ kPa)$$
$$= \boxed{301.3\ kPa}$$

Example: Suppose you place a straw into a cup of water. You then place your finger over the top of the straw so that some air is trapped. When you take the straw out, the water remains in straw. Does the air in the straw have a higher pressure than the atm?

$$\sum F = P_{atm}A - PA - m\vec{g} = 0$$

$$\Rightarrow P_{atm} > P$$



Pascal's Principle

①

$$P_0 = 1.013\times10^5 \frac{N}{m^2}$$

$$P = P_0 + \rho g h$$

Pascal's principle states that if an external pressure is applied to a confined fluid, the pressure at every point within that fluid increases by that amount.

② Hydraulic lift is a device that utilizes Pascal's Principle by turning a small input force into a large output force by changing the area on which force acts.

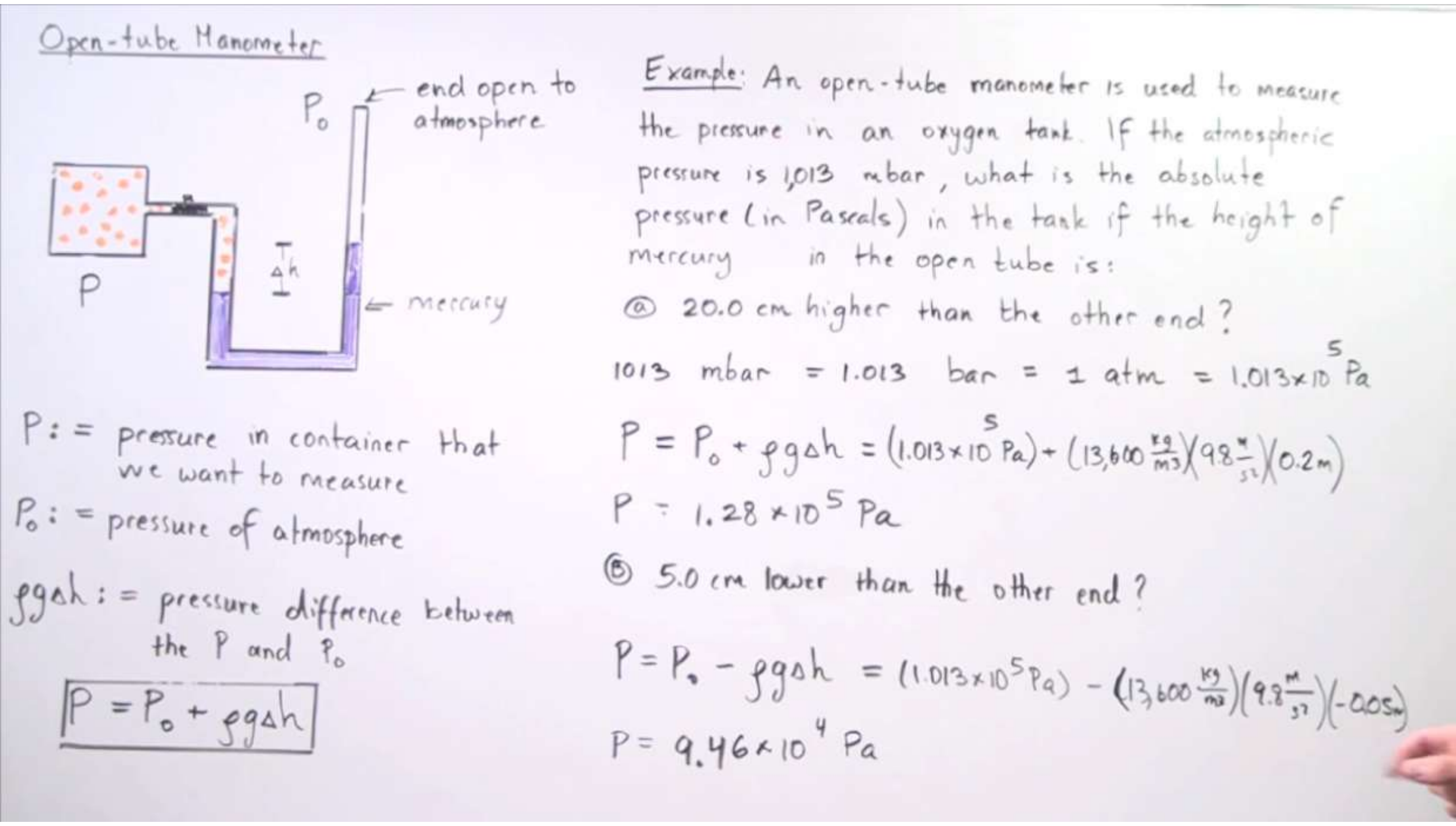
$$P_1 = P_2 \Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\Rightarrow \boxed{\frac{F_2}{F_1} = \frac{A_2}{A_1}}$$

$\frac{F_2}{F_1}$ = mechanical advantage

Example: A car uses hydraulic breaks. If the area of the break pedal is 0.01 m² and the driver applies a force of 20.0 N, calculate the output force. Assume the area of the output section is 0.1 m²

$$F_{out} = F_{in}\cdot\frac{A_{out}}{A_{in}} = (20N)\left(\frac{0.1}{0.01}\right) = \boxed{200N}$$



Open-tube Manometer

P: = pressure in container that we want to measure

P_o: = pressure of atmosphere

$\rho g \Delta h$: = pressure difference between the P and P_o

$$\boxed{P = P_o + \rho g \Delta h}$$

Example: An open-tube manometer is used to measure the pressure in an oxygen tank. If the atmospheric pressure is 1,013 mbar, what is the absolute pressure (in Pascals) in the tank if the height of mercury in the open tube is:

ⓐ 20.0 cm higher than the other end?

$$1013 \text{ mbar} = 1.013 \text{ bar} = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

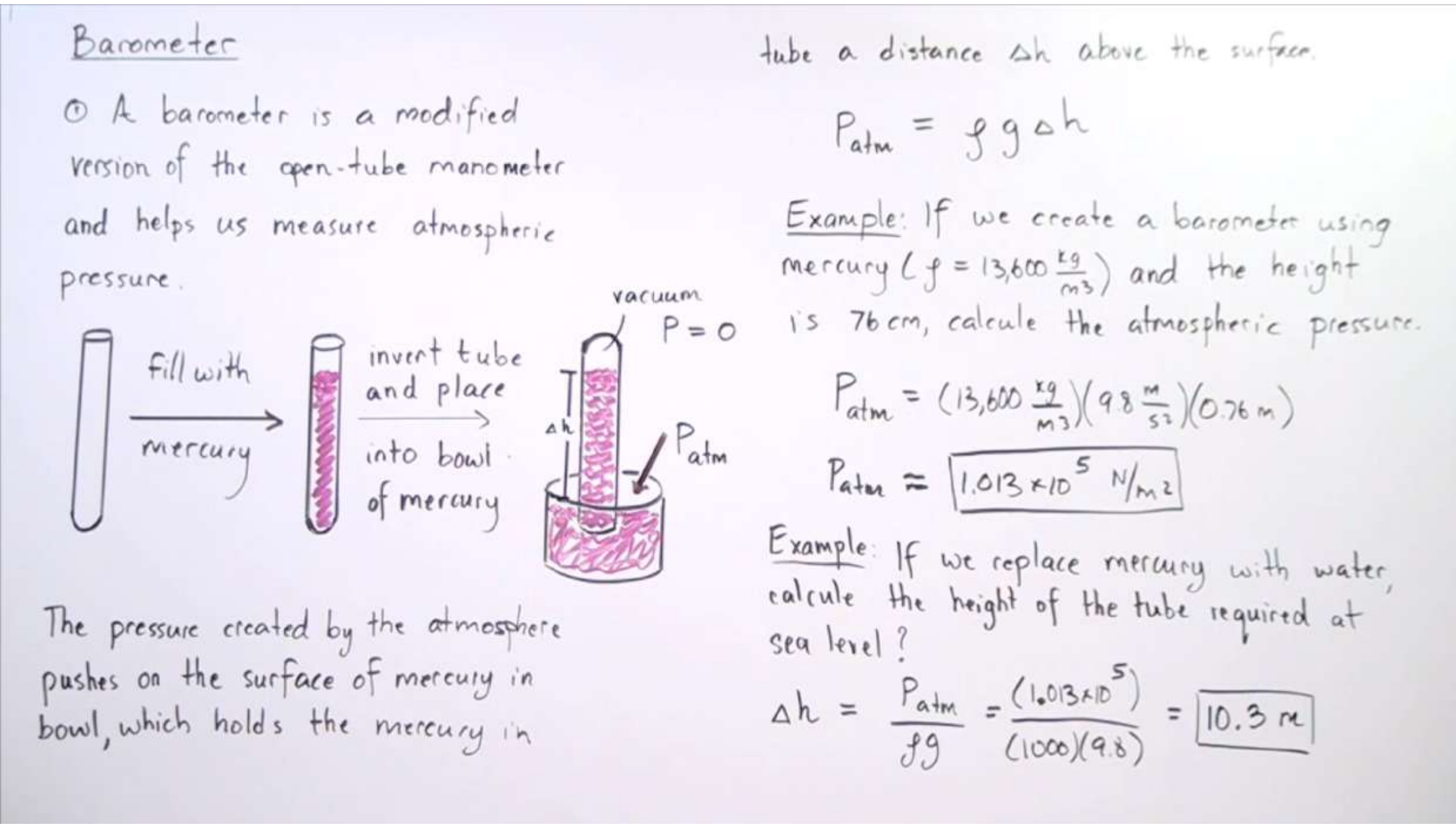
$$P = P_o + \rho g \Delta h = (1.013 \times 10^5 \text{ Pa}) + (13{,}600 \tfrac{\text{kg}}{\text{m}^3})(9.8 \tfrac{\text{m}}{\text{s}^2})(0.2 \text{m})$$

$$P = 1.28 \times 10^5 \text{ Pa}$$

ⓑ 5.0 cm lower than the other end?

$$P = P_o - \rho g \Delta h = (1.013 \times 10^5 \text{ Pa}) - (13{,}600 \tfrac{\text{kg}}{\text{m}^3})(9.8 \tfrac{\text{m}}{\text{s}^2})(-0.05 \text{m})$$

$$P = 9.46 \times 10^4 \text{ Pa}$$



Barometer

① A barometer is a modified version of the open-tube manometer and helps us measure atmospheric pressure.

The pressure created by the atmosphere pushes on the surface of mercury in bowl, which holds the mercury in tube a distance Δh above the surface.

$$P_{atm} = \rho g \Delta h$$

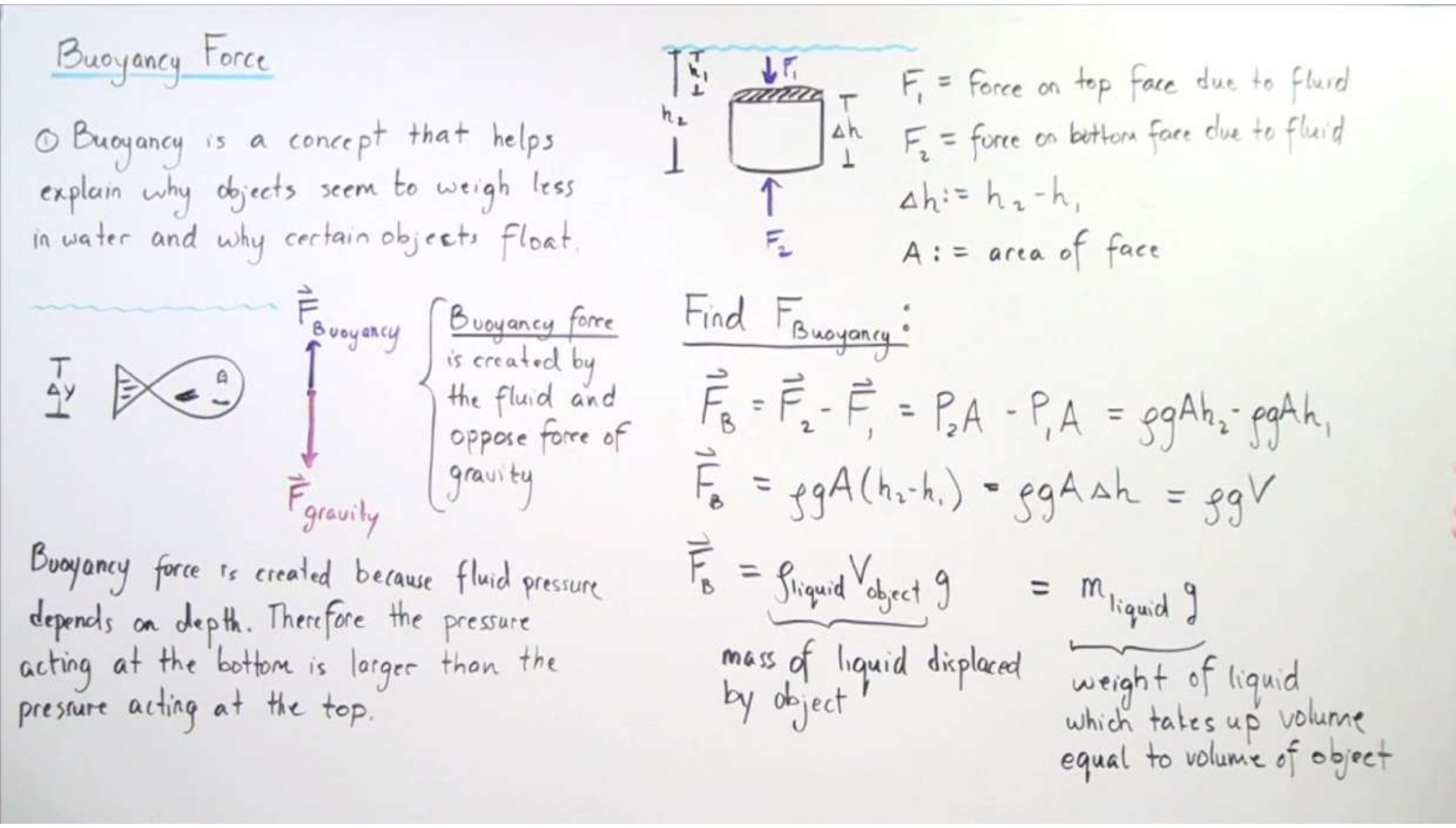
Example: If we create a barometer using mercury ($\rho = 13{,}600 \frac{kg}{m^3}$) and the height is 76 cm, calcule the atmospheric pressure.

$$P_{atm} = (13{,}600 \tfrac{kg}{m^3})(9.8 \tfrac{m}{s^2})(0.76\ m)$$

$$P_{atm} \approx \boxed{1.013 \times 10^5\ N/m^2}$$

Example: If we replace mercury with water, calcule the height of the tube required at sea level?

$$\Delta h = \frac{P_{atm}}{\rho g} = \frac{(1.013 \times 10^5)}{(1000)(9.8)} = \boxed{10.3\ m}$$



Buoyancy Force

① Buoyancy is a concept that helps explain why objects seem to weigh less in water and why certain objects float.

Buoyancy force is created by the fluid and oppose force of gravity

Buoyancy force is created because fluid pressure depends on depth. Therefore the pressure acting at the bottom is larger than the pressure acting at the top.

F_1 = Force on top face due to fluid

F_2 = force on bottom face due to fluid

$\Delta h := h_2 - h_1$

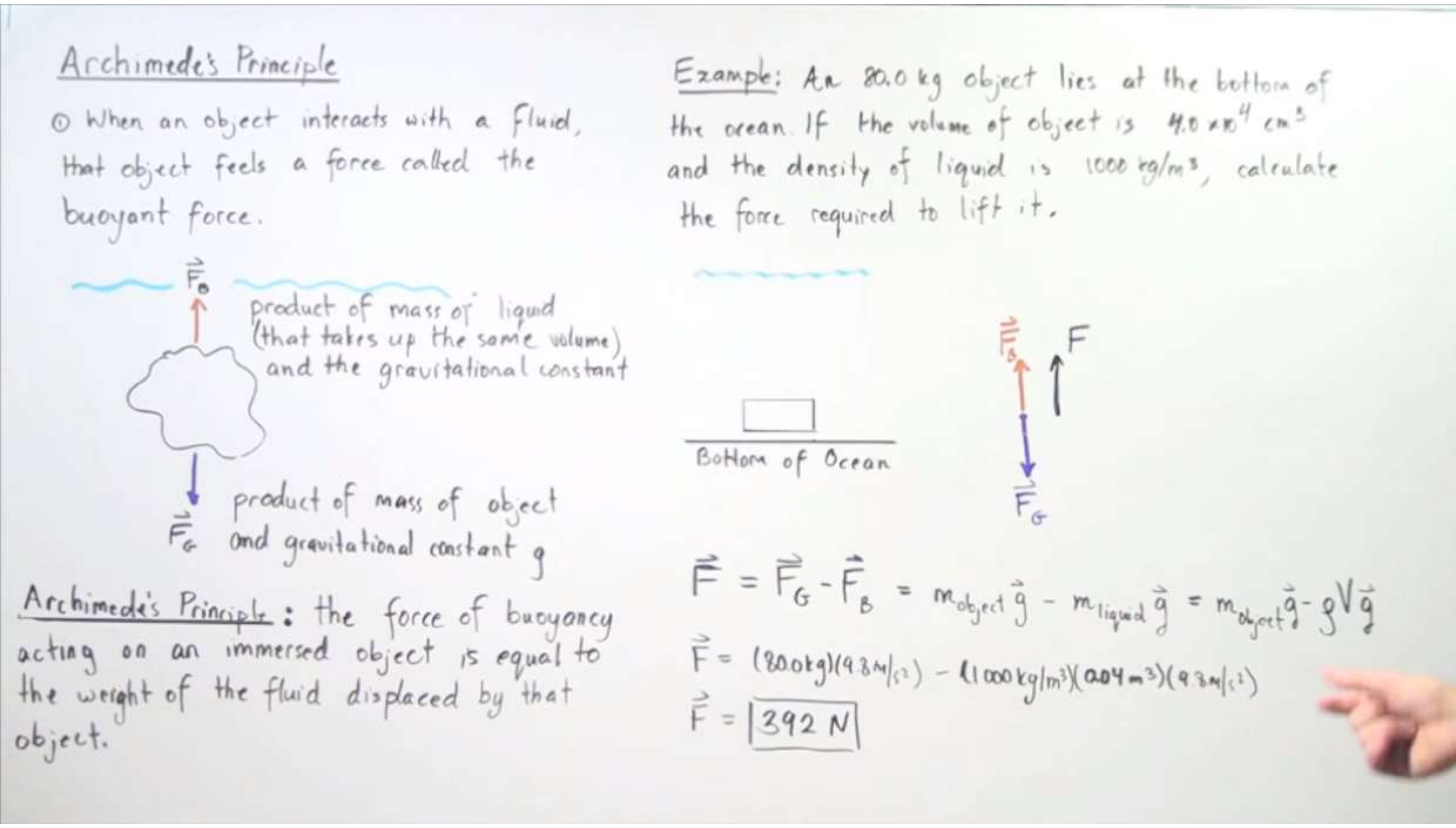
$A :=$ area of face

Find $F_{Buoyancy}$:

$$\vec{F}_B = \vec{F}_2 - \vec{F}_1 = P_2 A - P_1 A = \rho g A h_2 - \rho g A h_1$$

$$\vec{F}_B = \rho g A (h_2 - h_1) = \rho g A \Delta h = \rho g V$$

$$\vec{F}_B = \underbrace{\rho_{liquid} V_{object}}_{\text{mass of liquid displaced by object}} g = \underbrace{m_{liquid}\, g}_{\text{weight of liquid which takes up volume equal to volume of object}}$$



Archimede's Principle

① When an object interacts with a fluid, that object feels a force called the buoyant force.

$\vec{F}_B$ product of mass of liquid (that takes up the same volume) and the gravitational constant

$\vec{F}_G$ product of mass of object and gravitational constant g

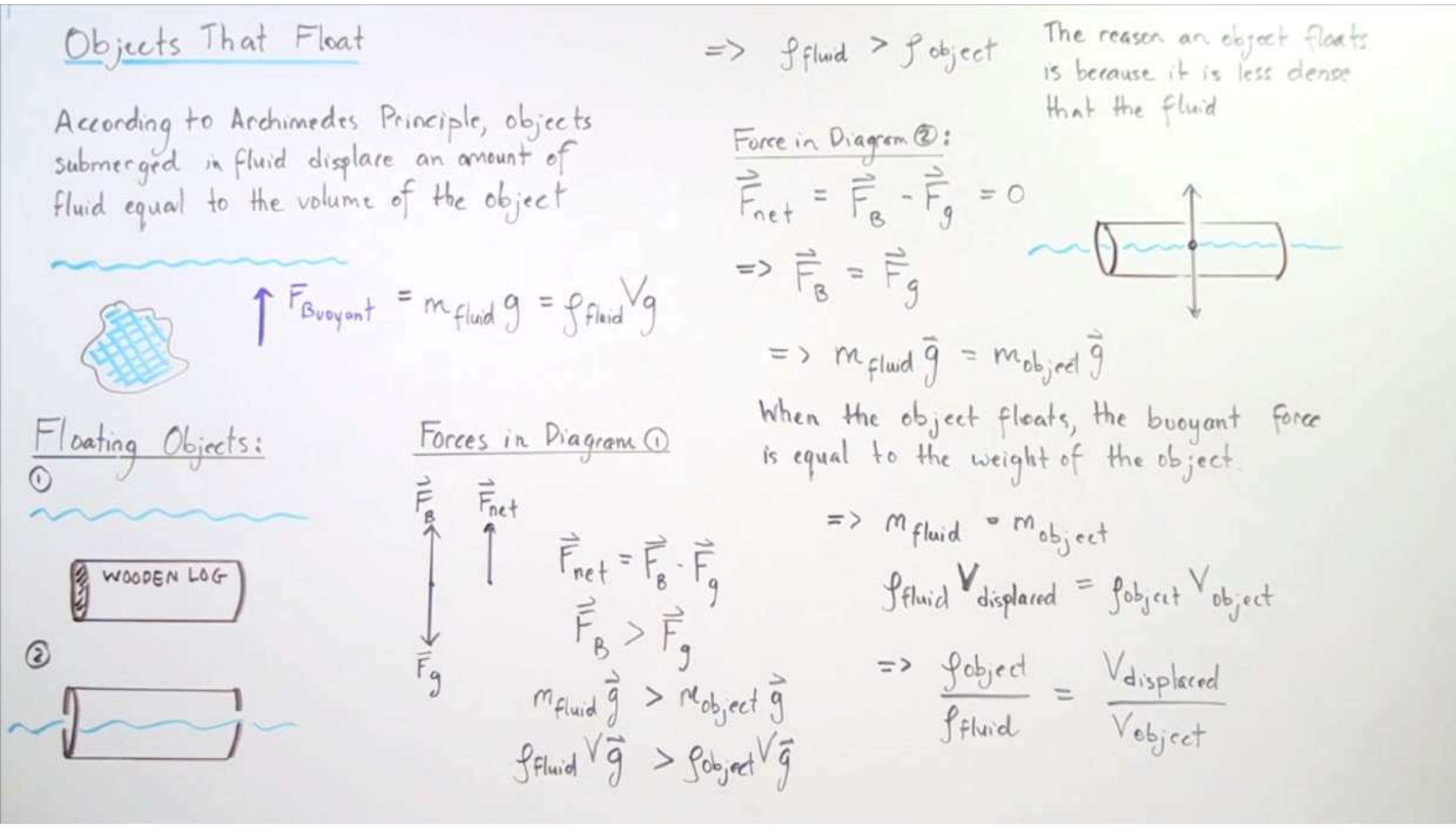
Archimede's Principle: the force of buoyancy acting on an immersed object is equal to the weight of the fluid displaced by that object.

Example: An 80.0 kg object lies at the bottom of the ocean. If the volume of object is 4.0×10^4 cm^3 and the density of liquid is 1000 kg/m^3, calculate the force required to lift it.

$$\vec{F} = \vec{F}_G - \vec{F}_B = m_{object}\vec{g} - m_{liquid}\vec{g} = m_{object}\vec{g} - \rho V \vec{g}$$

$$\vec{F} = (80.0 kg)(9.8 m/s^2) - (1000 kg/m^3)(0.04 m^3)(9.8 m/s^2)$$

$$\vec{F} = \boxed{392 N}$$



Objects That Float

According to Archimedes Principle, objects submerged in fluid displace an amount of fluid equal to the volume of the object

$$\uparrow F_{Buoyant} = m_{fluid}\, g = \rho_{fluid} V g$$

Floating Objects:

Forces in Diagram ①

$$\vec{F}_{net} = \vec{F}_B - \vec{F}_g$$

$$\vec{F}_B > \vec{F}_g$$

$$m_{fluid}\vec{g} > m_{object}\vec{g}$$

$$\rho_{fluid} V\vec{g} > \rho_{object} V\vec{g}$$

$$\Rightarrow \rho_{fluid} > \rho_{object}$$

The reason an object floats is because it is less dense that the fluid

Force in Diagram ②:

$$\vec{F}_{net} = \vec{F}_B - \vec{F}_g = 0$$

$$\Rightarrow \vec{F}_B = \vec{F}_g$$

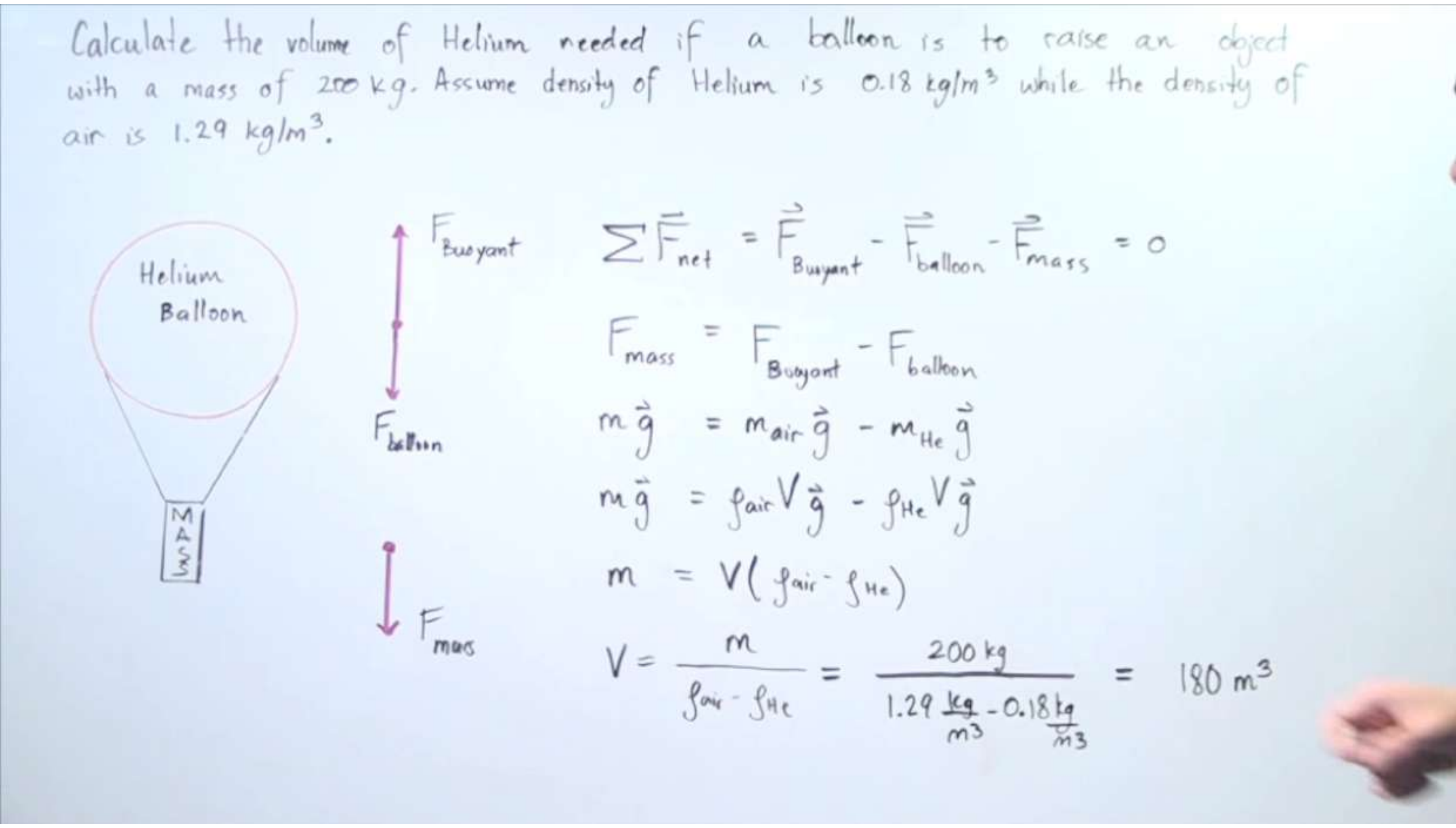
$$\Rightarrow m_{fluid}\vec{g} = m_{object}\vec{g}$$

When the object floats, the buoyant force is equal to the weight of the object

$$\Rightarrow m_{fluid} = m_{object}$$

$$\rho_{fluid} V_{displaced} = \rho_{object} V_{object}$$

$$\Rightarrow \frac{\rho_{object}}{\rho_{fluid}} = \frac{V_{displaced}}{V_{object}}$$



Calculate the volume of Helium needed if a balloon is to raise an object with a mass of 200 kg. Assume density of Helium is 0.18 kg/m³ while the density of air is 1.29 kg/m³.

$$\sum \vec{F}_{net} = \vec{F}_{Buoyant} - \vec{F}_{balloon} - \vec{F}_{mass} = 0$$

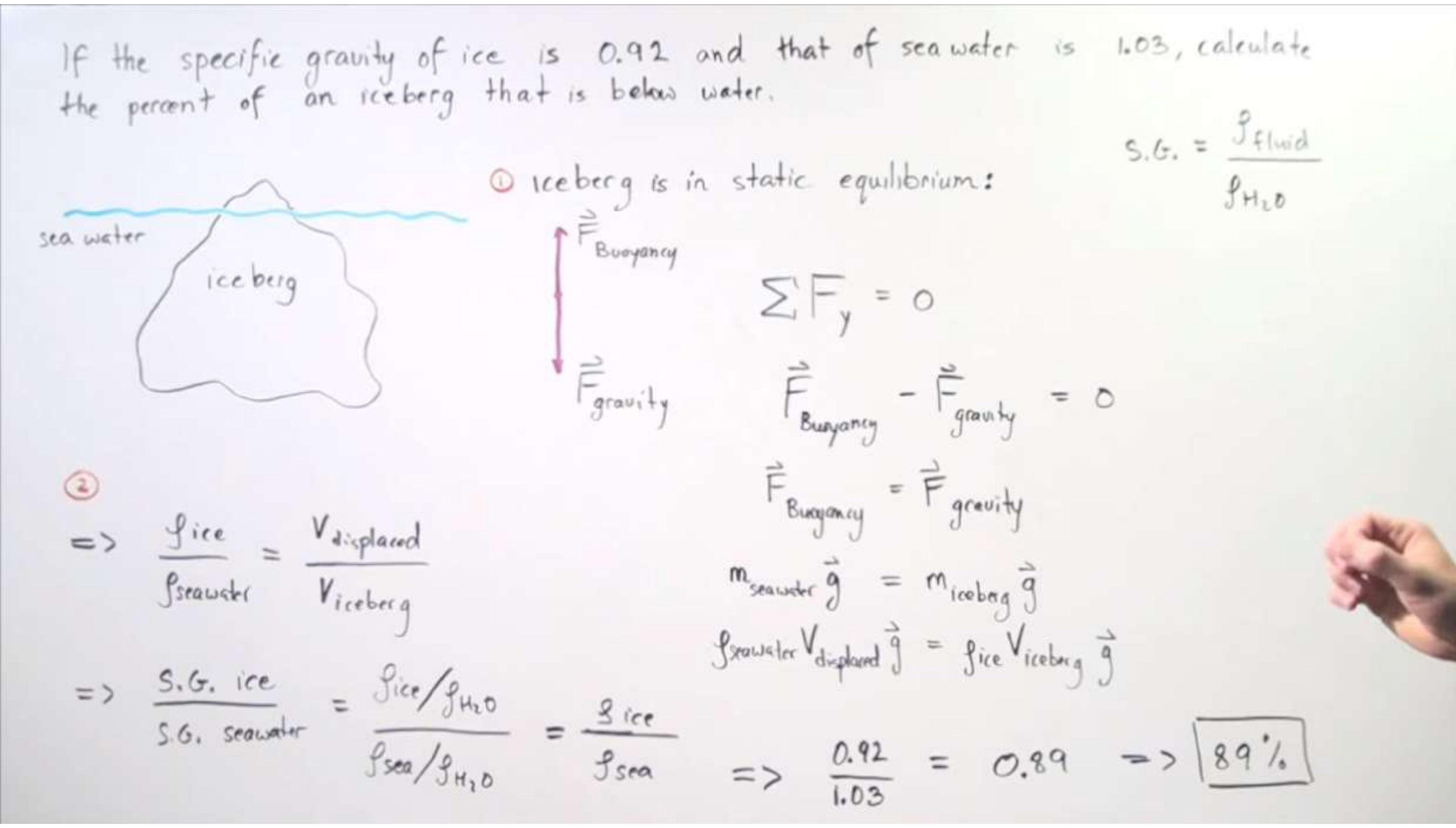
$$F_{mass} = F_{Buoyant} - F_{balloon}$$

$$m\vec{g} = m_{air}\vec{g} - m_{He}\vec{g}$$

$$m\vec{g} = \rho_{air}V\vec{g} - \rho_{He}V\vec{g}$$

$$m = V(\rho_{air} - \rho_{He})$$

$$V = \frac{m}{\rho_{air} - \rho_{He}} = \frac{200\text{ kg}}{1.29\frac{\text{kg}}{\text{m}^3} - 0.18\frac{\text{kg}}{\text{m}^3}} = 180\text{ m}^3$$



If the specific gravity of ice is 0.92 and that of sea water is 1.03, calculate the percent of an iceberg that is below water.

$$S.G. = \frac{\rho_{fluid}}{\rho_{H_2O}}$$

① iceberg is in static equilibrium:

$$\Sigma F_y = 0$$

$$\vec{F}_{Buoyancy} - \vec{F}_{gravity} = 0$$

$$\vec{F}_{Buoyancy} = \vec{F}_{gravity}$$

$$m_{seawater}\,\vec{g} = m_{iceberg}\,\vec{g}$$

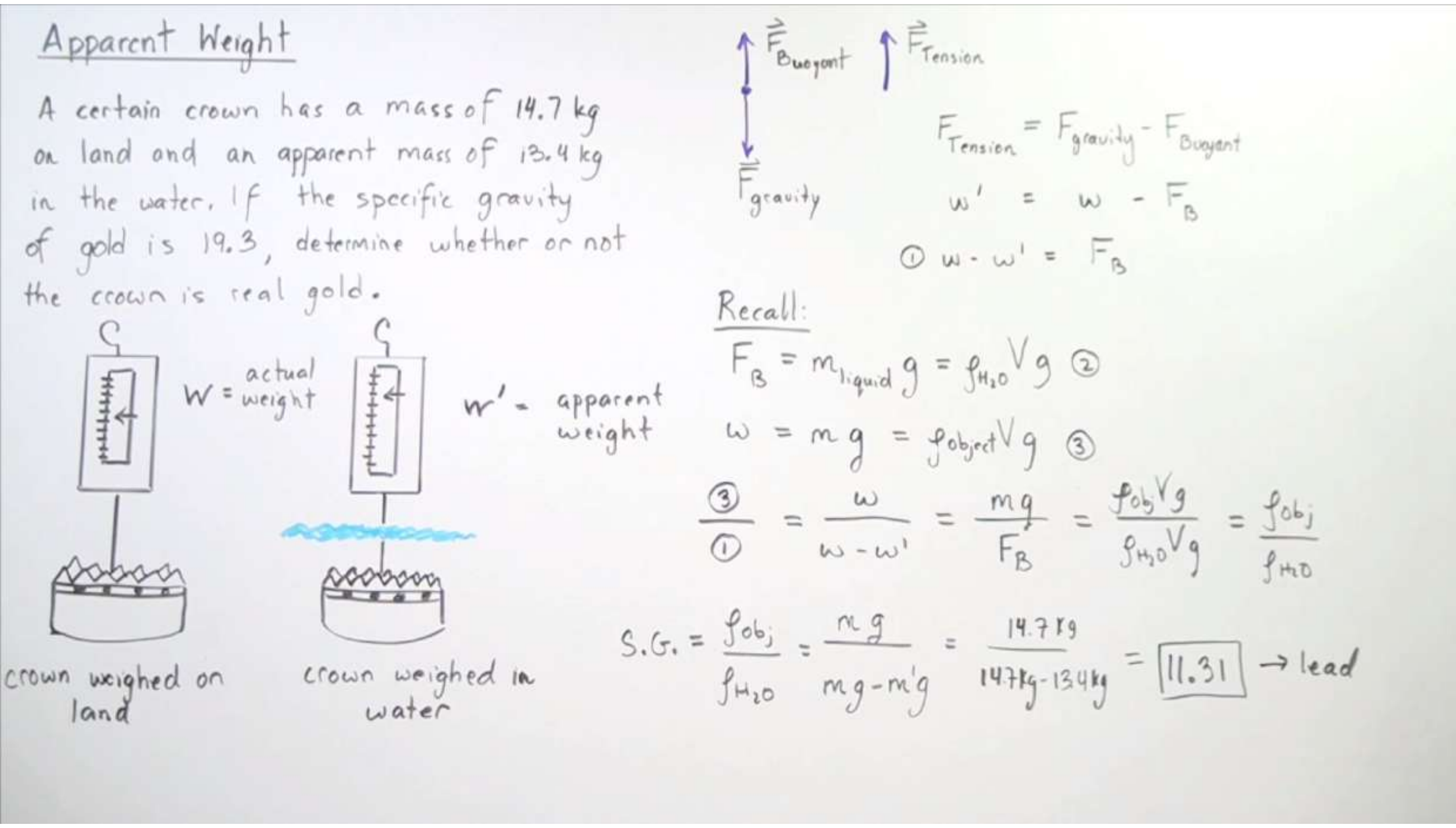
$$\rho_{seawater} V_{displaced}\,\vec{g} = \rho_{ice} V_{iceberg}\,\vec{g}$$

②

$$\Rightarrow \frac{\rho_{ice}}{\rho_{seawater}} = \frac{V_{displaced}}{V_{iceberg}}$$

$$\Rightarrow \frac{S.G.\ ice}{S.G.\ seawater} = \frac{\rho_{ice}/\rho_{H_2O}}{\rho_{sea}/\rho_{H_2O}} = \frac{\rho_{ice}}{\rho_{sea}}$$

$$\Rightarrow \frac{0.92}{1.03} = 0.89 \Rightarrow \boxed{89\%}$$



Apparent Weight

A certain crown has a mass of 14.7 kg on land and an apparent mass of 13.4 kg in the water. If the specific gravity of gold is 19.3, determine whether or not the crown is real gold.

W = actual weight

w' = apparent weight

crown weighed on land

crown weighed in water

$\vec{F}_{Buoyant}$

$\vec{F}_{gravity}$

$\vec{F}_{Tension}$

$$F_{Tension} = F_{gravity} - F_{Buoyant}$$

$$w' = w - F_B$$

$$① \; w - w' = F_B$$

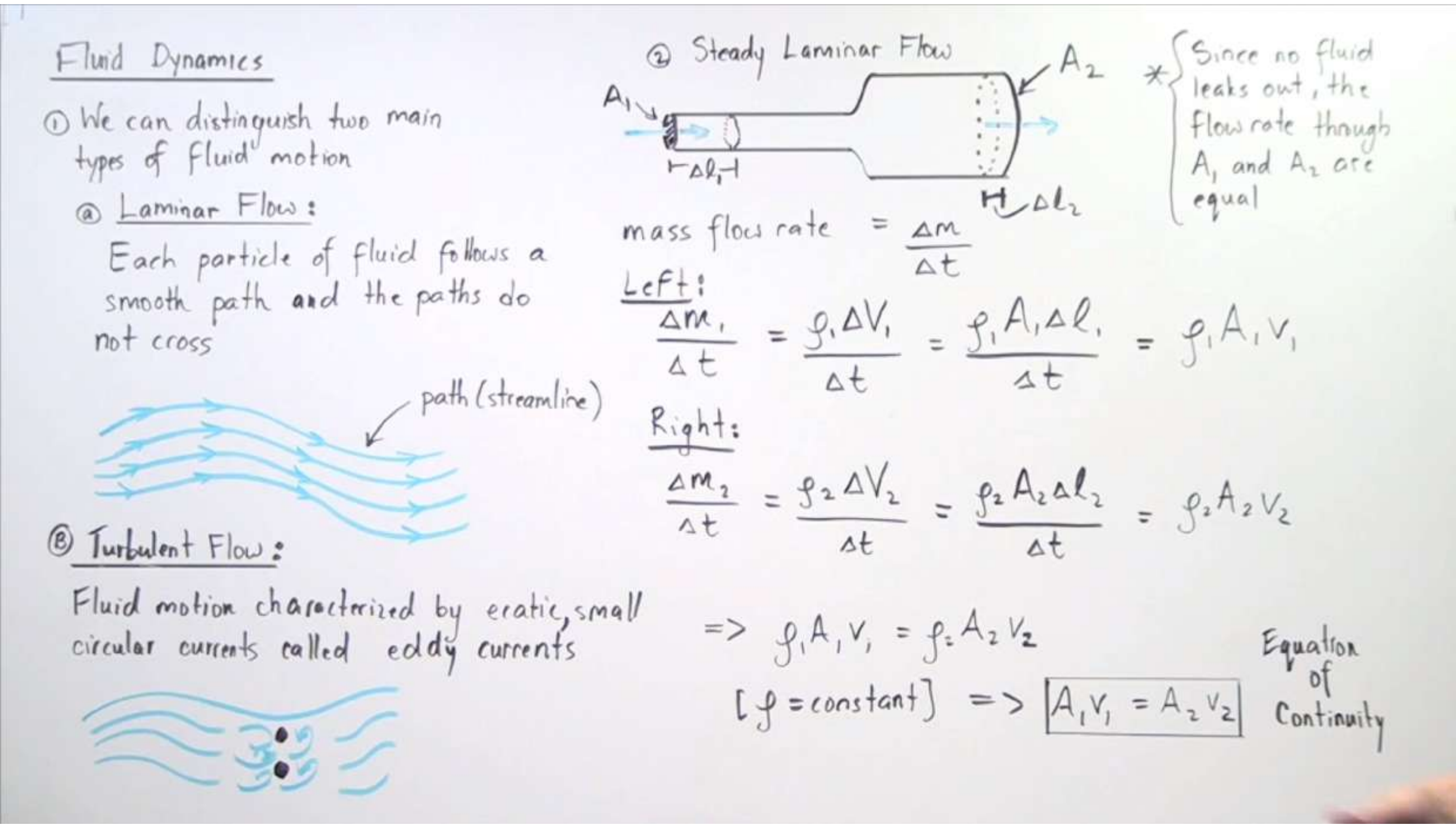
Recall:

$$F_B = m_{liquid}\, g = \rho_{H_2O} V g \; ②$$

$$w = mg = \rho_{object} V g \; ③$$

$$\frac{③}{①} = \frac{w}{w - w'} = \frac{mg}{F_B} = \frac{\rho_{obj} V g}{\rho_{H_2O} V g} = \frac{\rho_{obj}}{\rho_{H_2O}}$$

$$S.G. = \frac{\rho_{obj}}{\rho_{H_2O}} = \frac{mg}{mg - m'g} = \frac{14.7\,kg}{14.7kg - 13.4kg} = \boxed{11.31} \rightarrow \text{lead}$$



Fluid Dynamics

① We can distinguish two main types of fluid motion

ⓐ **Laminar Flow:**

Each particle of fluid follows a smooth path and the paths do not cross

ⓑ **Turbulent Flow:**

Fluid motion characterized by eratic, small circular currents called eddy currents

② Steady Laminar Flow

* Since no fluid leaks out, the flow rate through A_1 and A_2 are equal

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t}$$

Left:

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

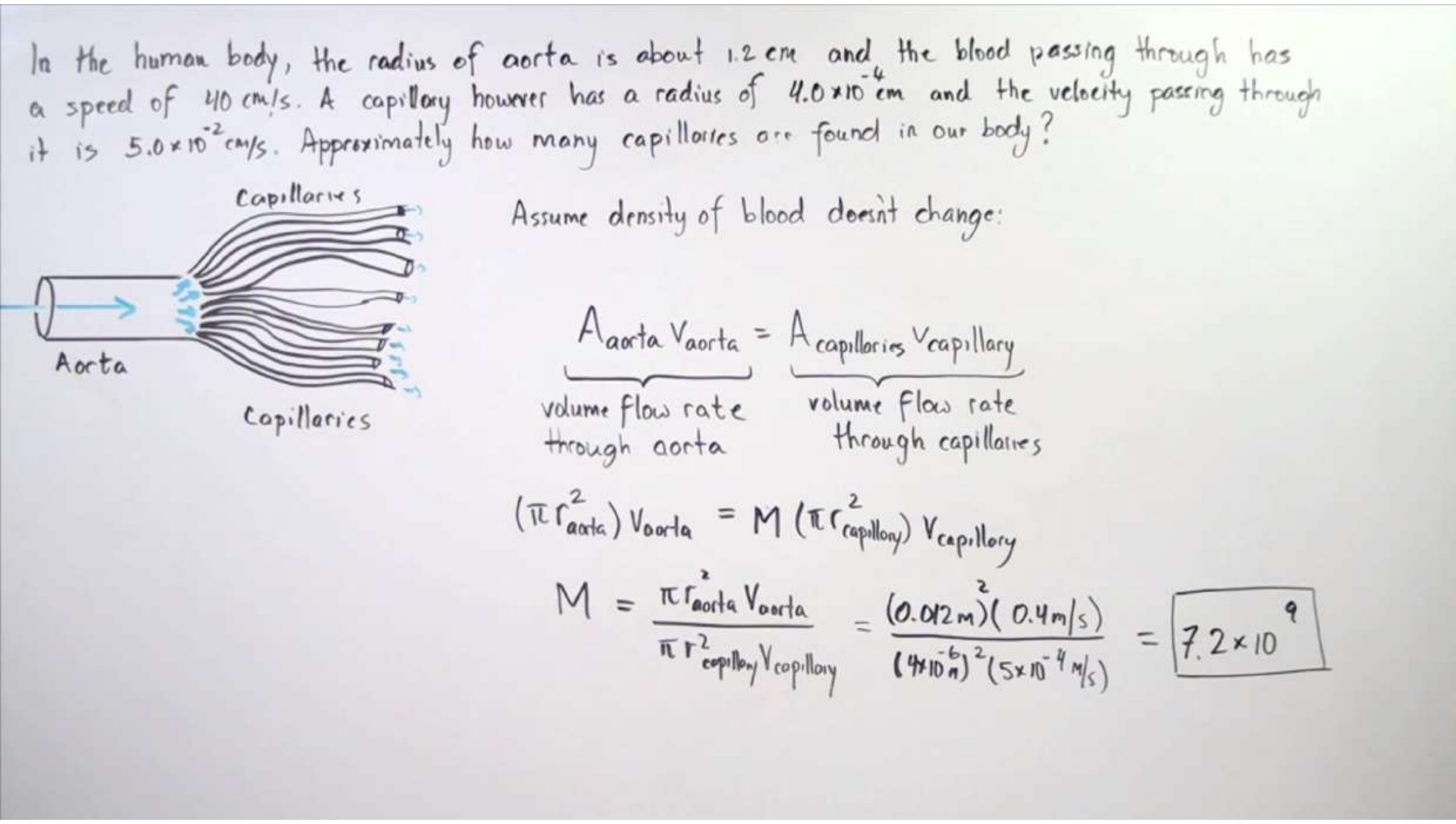
Right:

$$\frac{\Delta m_2}{\Delta t} = \frac{\rho_2 \Delta V_2}{\Delta t} = \frac{\rho_2 A_2 \Delta l_2}{\Delta t} = \rho_2 A_2 V_2$$

$$\Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 V_2$$

$$[\rho = \text{constant}] \Rightarrow \boxed{A_1 v_1 = A_2 v_2}$$

Equation of Continuity



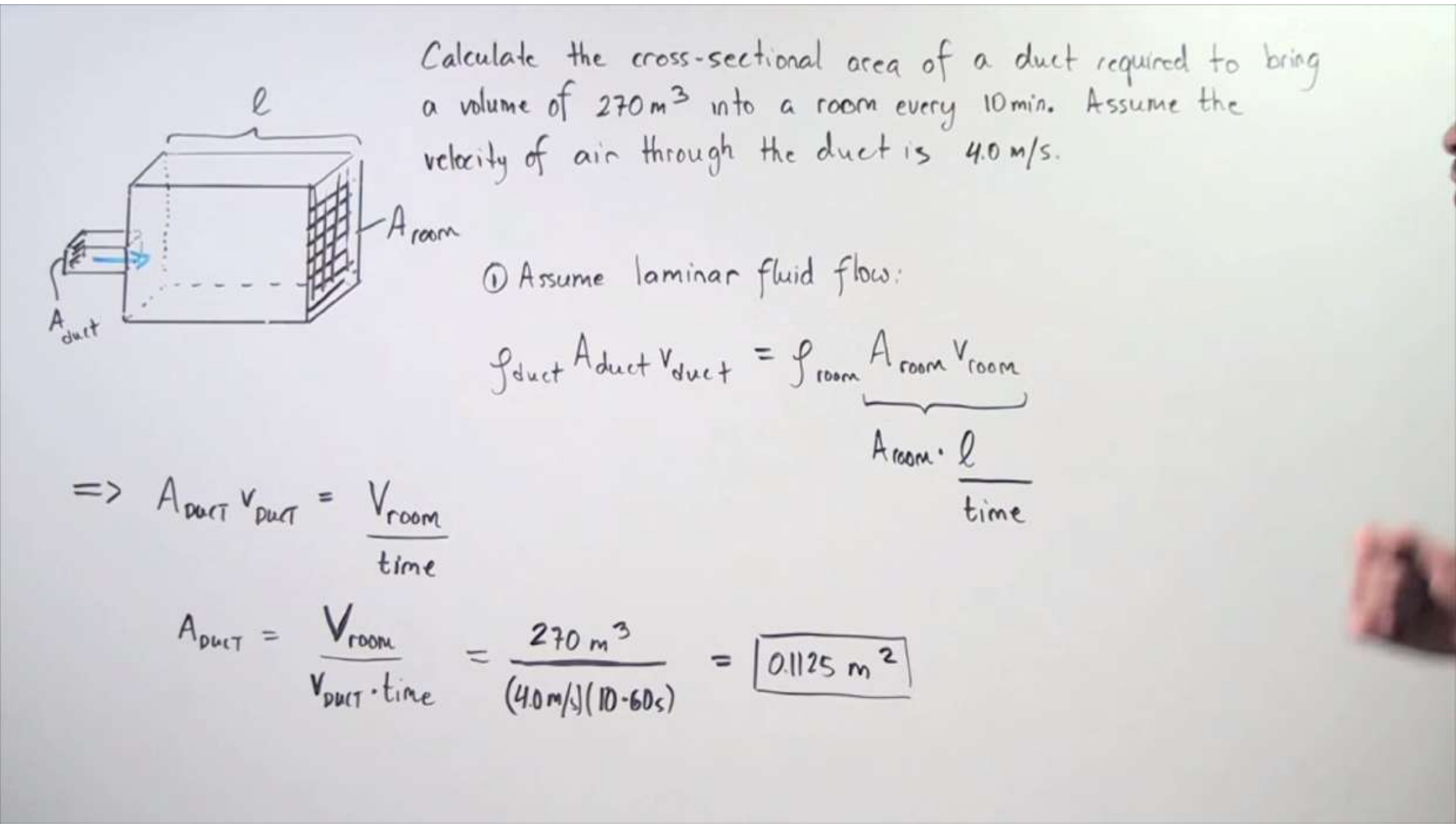
In the human body, the radius of aorta is about 1.2 cm and the blood passing through has a speed of 40 cm/s. A capillory however has a radius of 4.0×10^{-4} cm and the velocity passing through it is 5.0×10^{-2} cm/s. Approximately how many capillaries are found in our body?

Assume density of blood doesn't change:

$$\underbrace{A_{aorta} V_{aorta}}_{\text{volume flow rate through aorta}} = \underbrace{A_{capillaries} V_{capillary}}_{\text{volume flow rate through capillaries}}$$

$$(\pi r_{aorta}^2) V_{aorta} = M (\pi r_{capillary}^2) V_{capillary}$$

$$M = \frac{\pi r_{aorta}^2 V_{aorta}}{\pi r_{capillary}^2 V_{capillary}} = \frac{(0.012\,m)^2 (0.4\,m/s)}{(4 \times 10^{-6}\,m)^2 (5 \times 10^{-4}\,m/s)} = \boxed{7.2 \times 10^{9}}$$



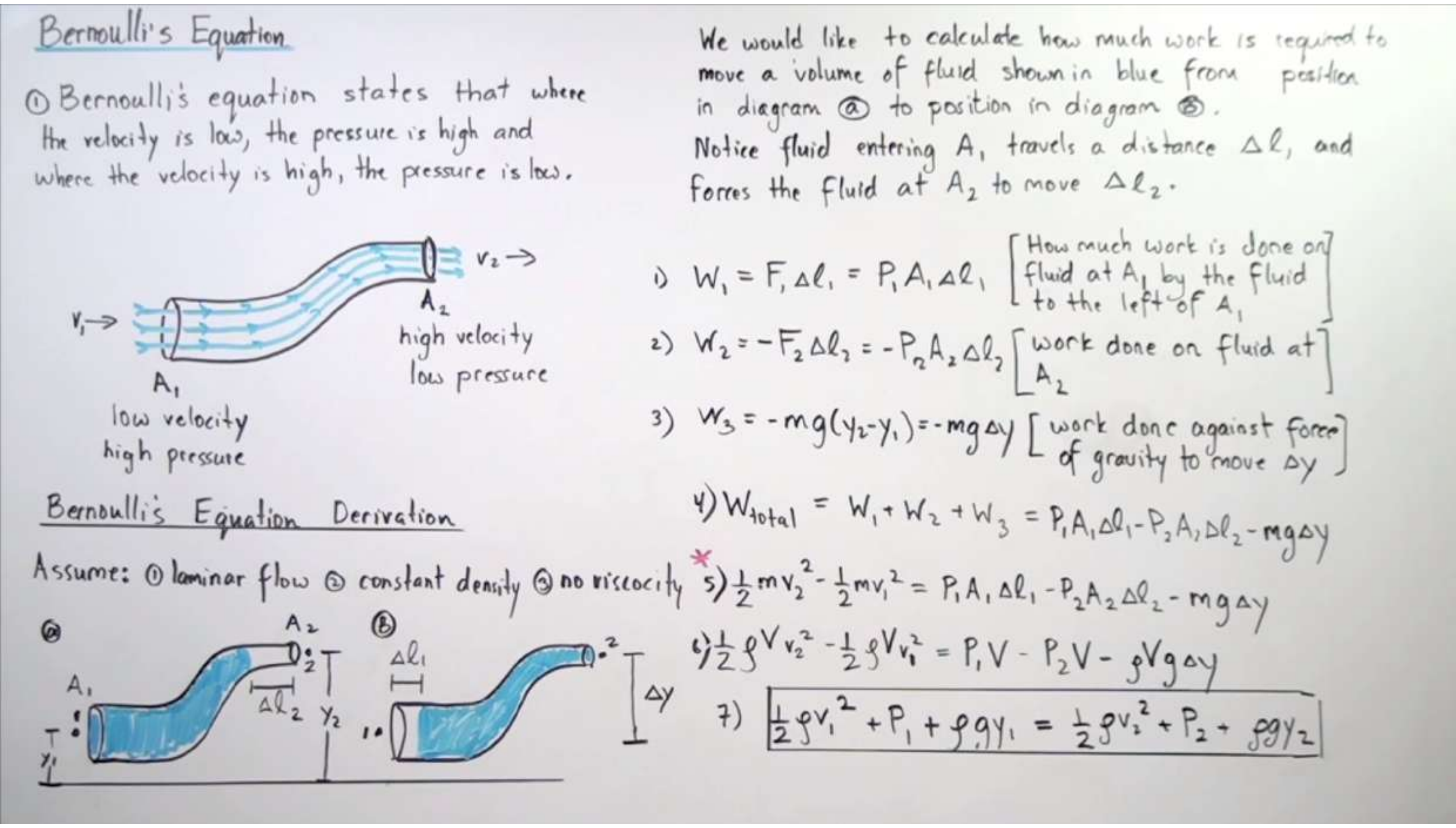
Calculate the cross-sectional area of a duct required to bring a volume of 270 m^3 into a room every 10 min. Assume the velocity of air through the duct is 4.0 m/s.

① Assume laminar fluid flow:

$$\rho_{duct} A_{duct} v_{duct} = \rho_{room} \underbrace{A_{room} v_{room}}_{\frac{A_{room}\cdot \ell}{time}}$$

$$\Rightarrow A_{DUCT} v_{DUCT} = \frac{V_{room}}{time}$$

$$A_{DUCT} = \frac{V_{room}}{v_{DUCT}\cdot time} = \frac{270\ m^3}{(4.0\ m/s)(10\cdot 60s)} = \boxed{0.1125\ m^2}$$



Bernoulli's Equation

① Bernoulli's equation states that where the velocity is low, the pressure is high and where the velocity is high, the pressure is low.

Bernoulli's Equation Derivation

Assume: ① laminar flow ② constant density ③ no viscocity

We would like to calculate how much work is required to move a volume of fluid shown in blue from position in diagram ⓐ to position in diagram ⓑ.

Notice fluid entering A_1 travels a distance Δl_1 and forces the fluid at A_2 to move Δl_2.

1) $W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1$ [How much work is done on fluid at A_1 by the fluid to the left of A_1]

2) $W_2 = -F_2 \Delta l_2 = -P_2 A_2 \Delta l_2$ [work done on fluid at A_2]

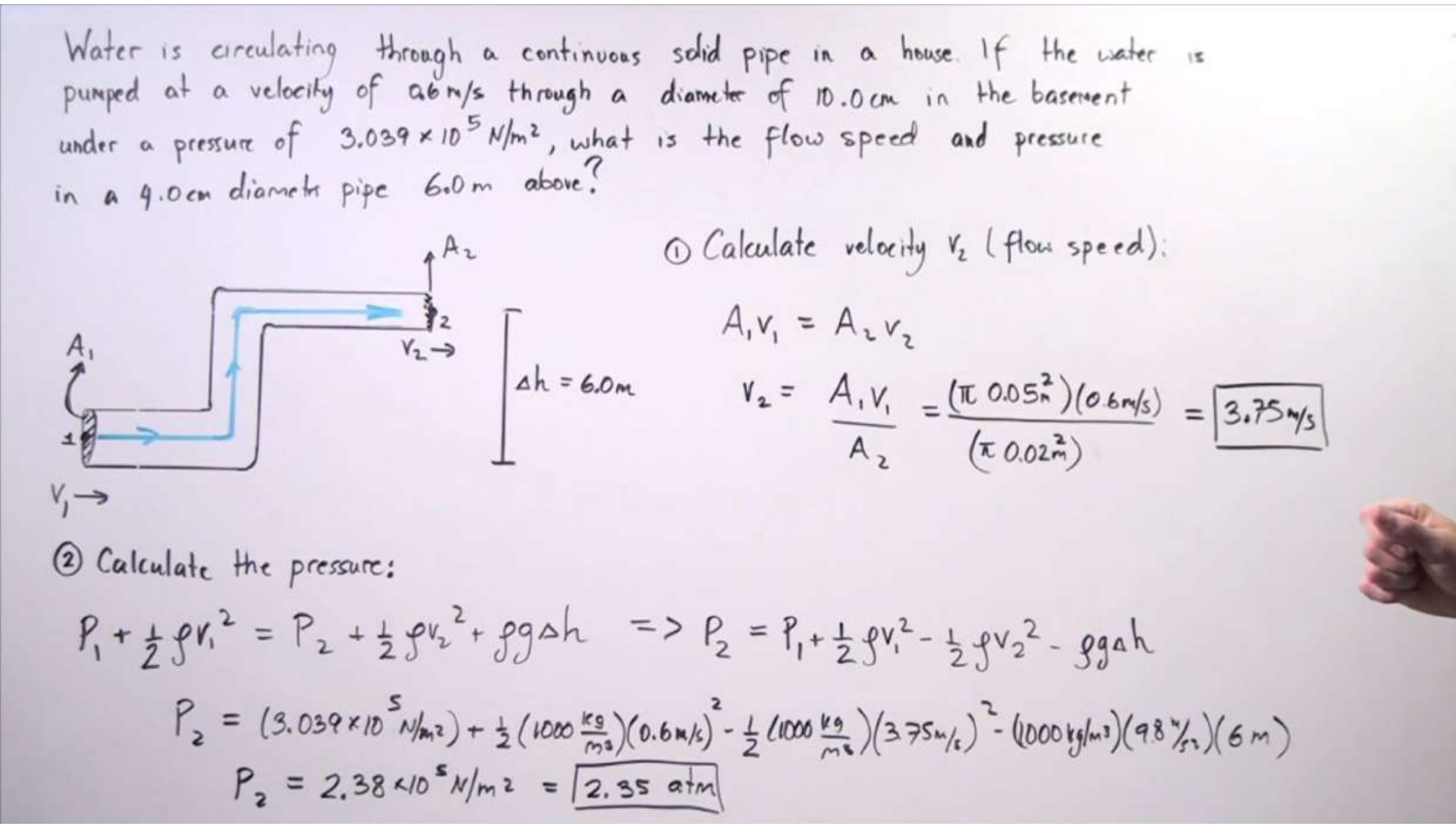
3) $W_3 = -mg(y_2 - y_1) = -mg\Delta y$ [work done against force of gravity to move Δy]

4) $W_{total} = W_1 + W_2 + W_3 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg\Delta y$

* 5) $\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg\Delta y$

6) $\frac{1}{2} \rho V v_2^2 - \frac{1}{2} \rho V v_1^2 = P_1 V - P_2 V - \rho V g \Delta y$

7) $$\boxed{\frac{1}{2} \rho v_1^2 + P_1 + \rho g y_1 = \frac{1}{2} \rho v_2^2 + P_2 + \rho g y_2}$$



Water is circulating through a continuous solid pipe in a house. If the water is pumped at a velocity of 0.6 m/s through a diameter of 10.0 cm in the basement under a pressure of $3.039 \times 10^5\ N/m^2$, what is the flow speed and pressure in a 4.0 cm diameter pipe 6.0 m above?

① Calculate velocity v_2 (flow speed):

$$A_1 v_1 = A_2 v_2$$

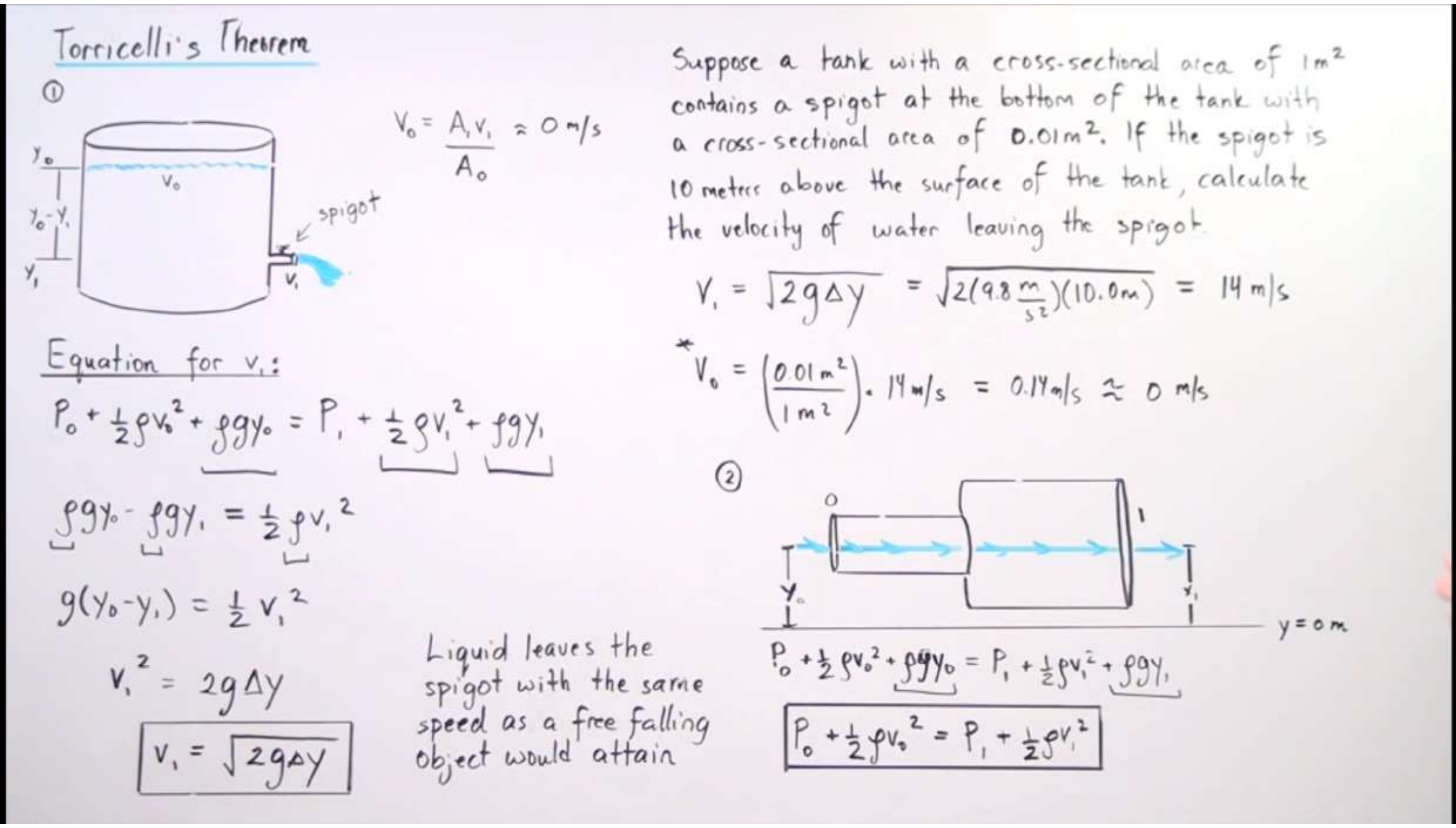
$$v_2 = \frac{A_1 v_1}{A_2} = \frac{(\pi\, 0.05^2_m)(0.6 m/s)}{(\pi\, 0.02^2_m)} = \boxed{3.75 m/s}$$

② Calculate the pressure:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g \Delta h \Rightarrow P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 - \rho g \Delta h$$

$$P_2 = (3.039 \times 10^5 N/m^2) + \frac{1}{2}(1000 \frac{kg}{m^3})(0.6 m/s)^2 - \frac{1}{2}(1000 \frac{kg}{m^3})(3.75 m/s)^2 - (1000 kg/m^3)(9.8 m/s^2)(6 m)$$

$$P_2 = 2.38 \times 10^5 N/m^2 = \boxed{2.35\ atm}$$



Torricelli's Theorem

①

$$v_0 = \frac{A_1 v_1}{A_0} \approx 0 \text{ m/s}$$

Equation for v_1:

$$P_0 + \frac{1}{2}\rho v_0^2 + \underbrace{\rho g y_0}_{} = P_1 + \underbrace{\frac{1}{2}\rho v_1^2}_{} + \underbrace{\rho g y_1}_{}$$

$$\underbrace{\rho}_{} g y_0 - \underbrace{\rho}_{} g y_1 = \frac{1}{2}\underbrace{\rho}_{} v_1^2$$

$$g(y_0 - y_1) = \frac{1}{2} v_1^2$$

$$v_1^2 = 2g\Delta y$$

$$\boxed{v_1 = \sqrt{2g\Delta y}}$$

Liquid leaves the spigot with the same speed as a free falling object would attain

Suppose a tank with a cross-sectional area of 1 m² contains a spigot at the bottom of the tank with a cross-sectional area of 0.01 m². If the spigot is 10 meters above the surface of the tank, calculate the velocity of water leaving the spigot.

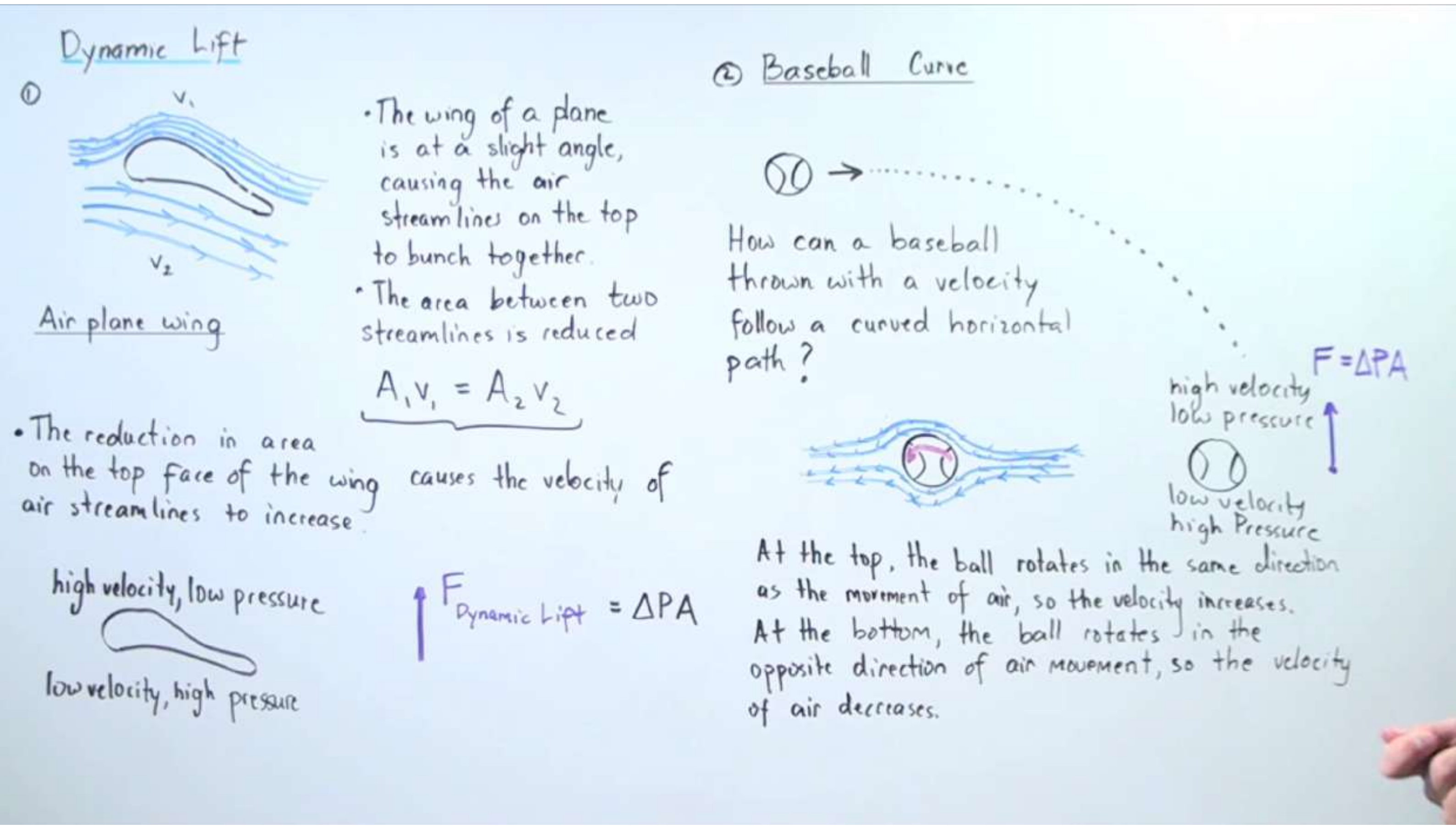
$$v_1 = \sqrt{2g\Delta y} = \sqrt{2(9.8\tfrac{m}{s^2})(10.0m)} = 14 \text{ m/s}$$

$$^{*}v_0 = \left(\frac{0.01 m^2}{1 m^2}\right) \cdot 14 \text{ m/s} = 0.14 \text{ m/s} \approx 0 \text{ m/s}$$

②

$$P_0 + \frac{1}{2}\rho v_0^2 + \underbrace{\rho g y_0}_{} = P_1 + \frac{1}{2}\rho v_1^2 + \underbrace{\rho g y_1}_{}$$

$$\boxed{P_0 + \frac{1}{2}\rho v_0^2 = P_1 + \frac{1}{2}\rho v_1^2}$$



Dynamic Lift

①

Air plane wing

- The wing of a plane is at a slight angle, causing the air streamlines on the top to bunch together.
- The area between two streamlines is reduced

$$\underbrace{A_1 v_1 = A_2 v_2}$$

- The reduction in area on the top face of the wing causes the velocity of air streamlines to increase.

high velocity, low pressure

low velocity, high pressure

$$F_{\text{Dynamic Lift}} = \Delta PA$$

② Baseball Curve

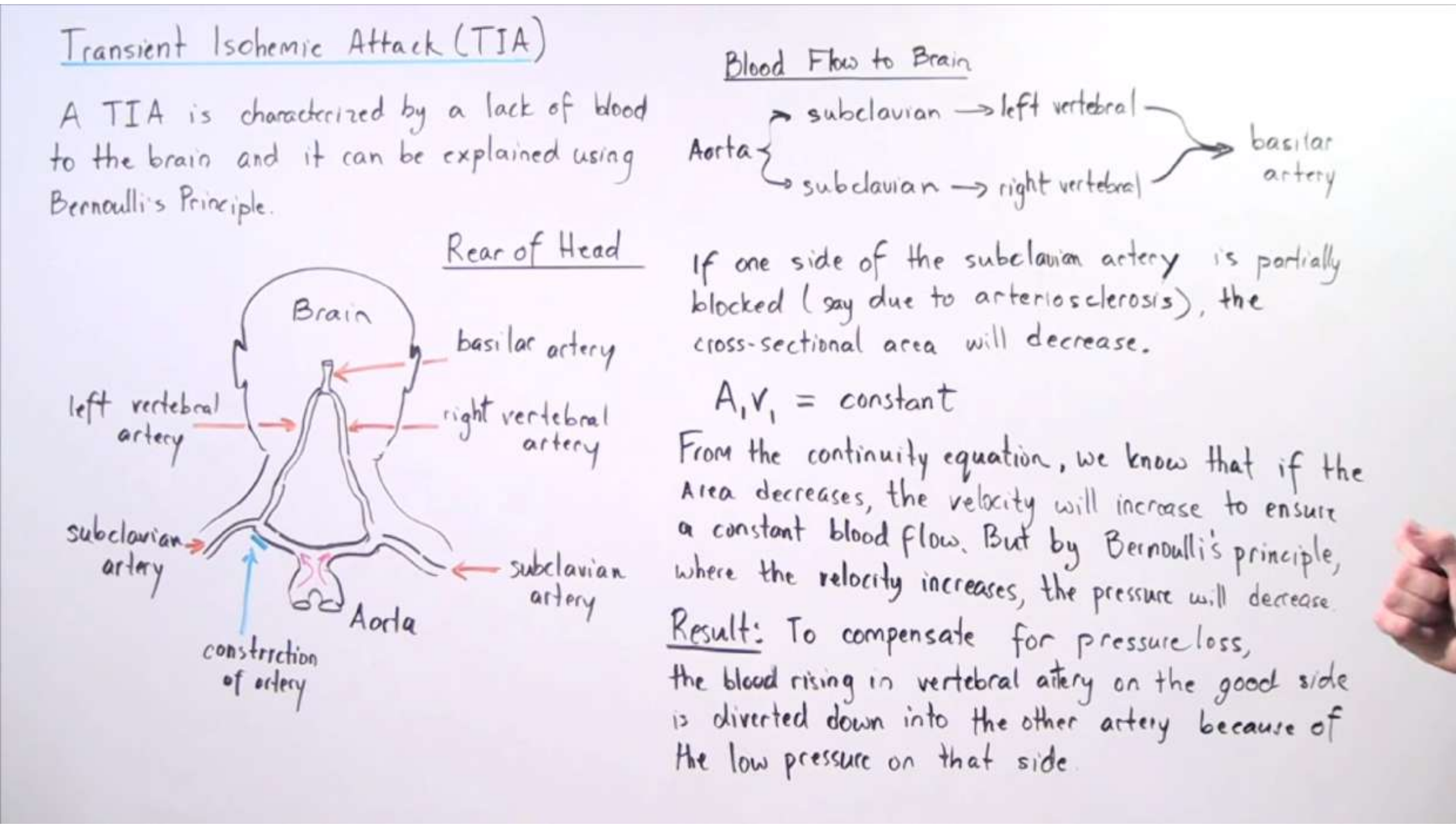
How can a baseball thrown with a velocity follow a curved horizontal path?

$F = \Delta PA$

high velocity
low pressure

low velocity
high Pressure

At the top, the ball rotates in the same direction as the movement of air, so the velocity increases. At the bottom, the ball rotates in the opposite direction of air movement, so the velocity of air decreases.



Transient Ischemic Attack (TIA)

A TIA is characterized by a lack of blood to the brain and it can be explained using Bernoulli's Principle.

Rear of Head

Blood Flow to Brain

Aorta → subclavian → left vertebral → basilar artery

Aorta → subclavian → right vertebral → basilar artery

If one side of the subclavian artery is partially blocked (say due to arteriosclerosis), the cross-sectional area will decrease.

$$A_1 v_1 = \text{constant}$$

From the continuity equation, we know that if the Area decreases, the velocity will increase to ensure a constant blood flow. But by Bernoulli's principle, where the velocity increases, the pressure will decrease.

Result: To compensate for pressure loss, the blood rising in vertebral artery on the good side is diverted down into the other artery because of the low pressure on that side.