(L1)

- Signals part -

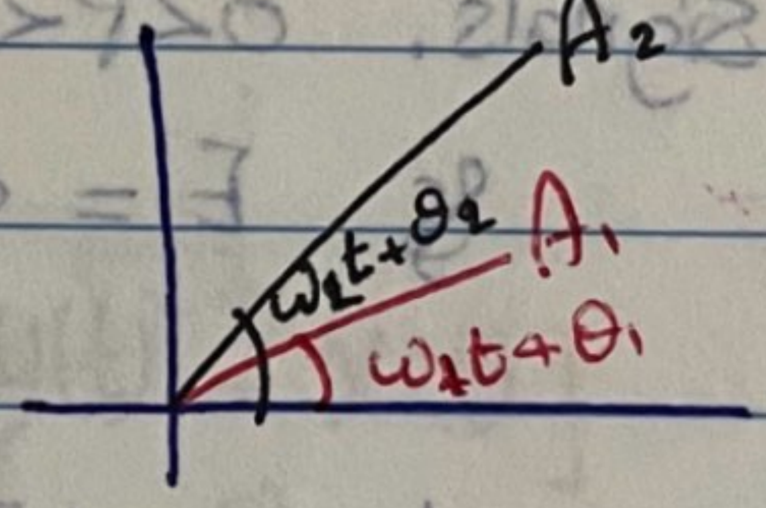
Review.

- Signals → function → 1D, 2D, 3D.

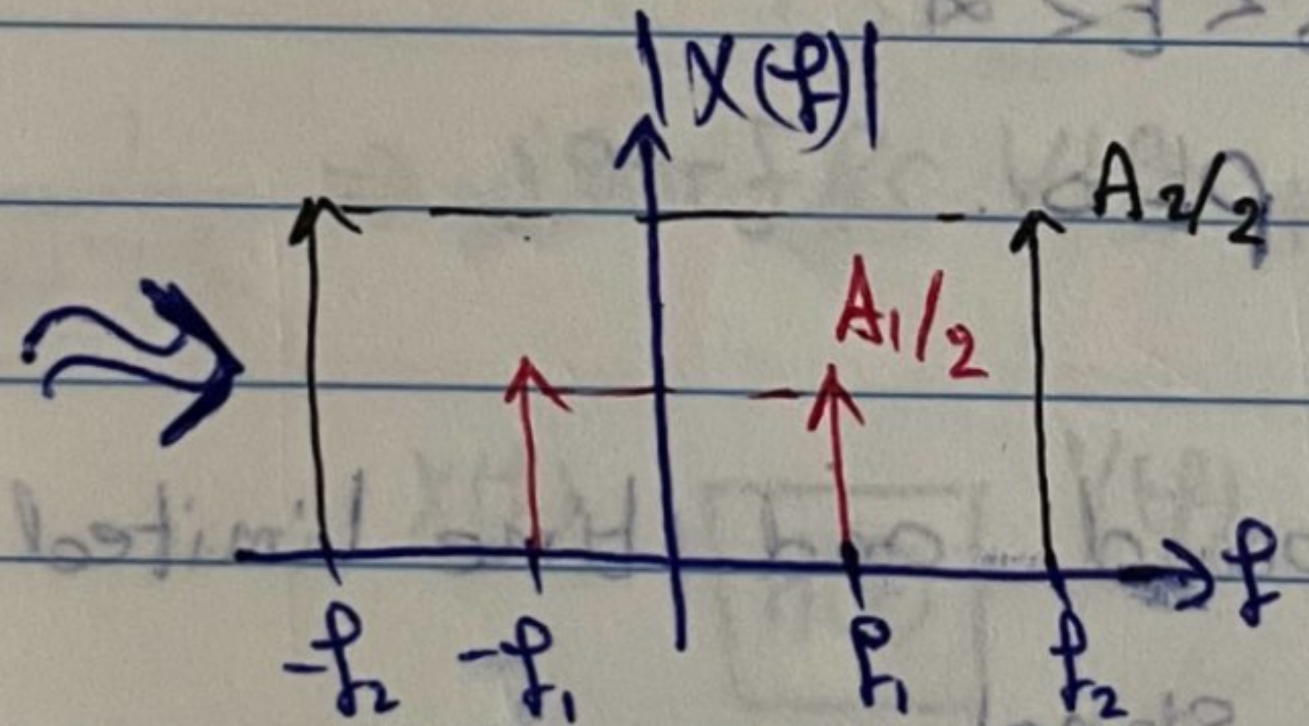
- Types of signals → step function, ramp function, pulse function, impulse function

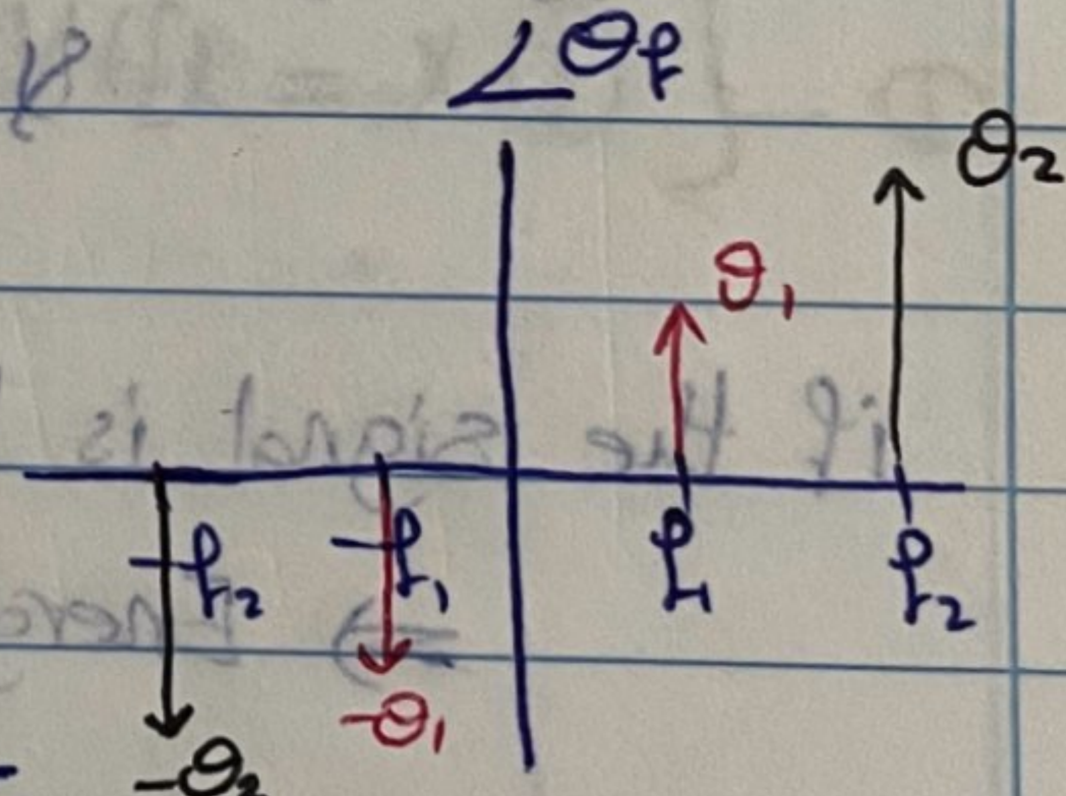
$x(t) = Re\{Ae^{j\theta}e^{j\omega t}\}$

$= A\cos(\omega t+\theta)$



(Time domain).





$|X(-f)| = |X(f)|$ - even -

$\angle\theta_{-f} = -\angle\theta_f$ - odd -

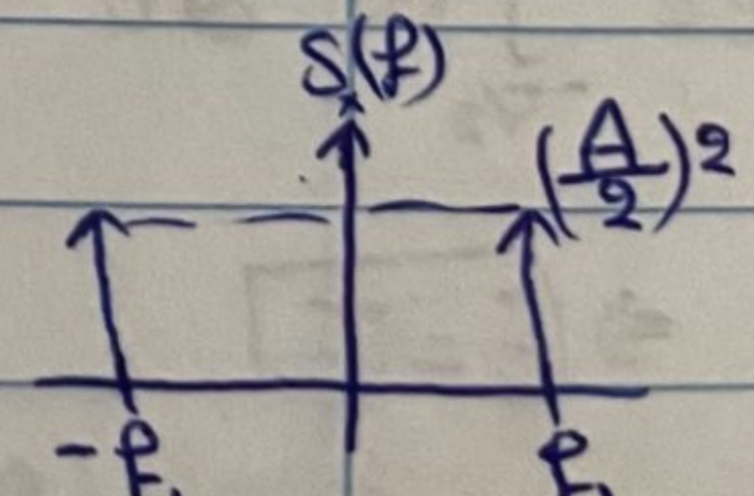
- To evaluate power.

In Time domain:- $x(t) = A\cos(\omega t+\theta)$

$P_{avg} = \frac{A^2}{2}$

In freq domain :-

- power spectral density (PSD).



- $S_x(f) = (\frac{A}{2})^2\delta(f-f_1) + (\frac{A}{2})^2\delta(f+f_1)$
- $P = 2(\frac{A}{2})^2 \rightarrow$ $\boxed{P = \frac{A^2}{2}}$

≡ Power & Energy signals.

- $E = \lim_{T_0 \to \infty} \int |x(t)|^2 \, dt$

- $P = \lim_{T_0 \to \infty} \frac{1}{2T_0} \int_{-T_0}^{T_0} |x(t)|^2 \, dt$ — $RMS = \sqrt{P_{avg}}$.

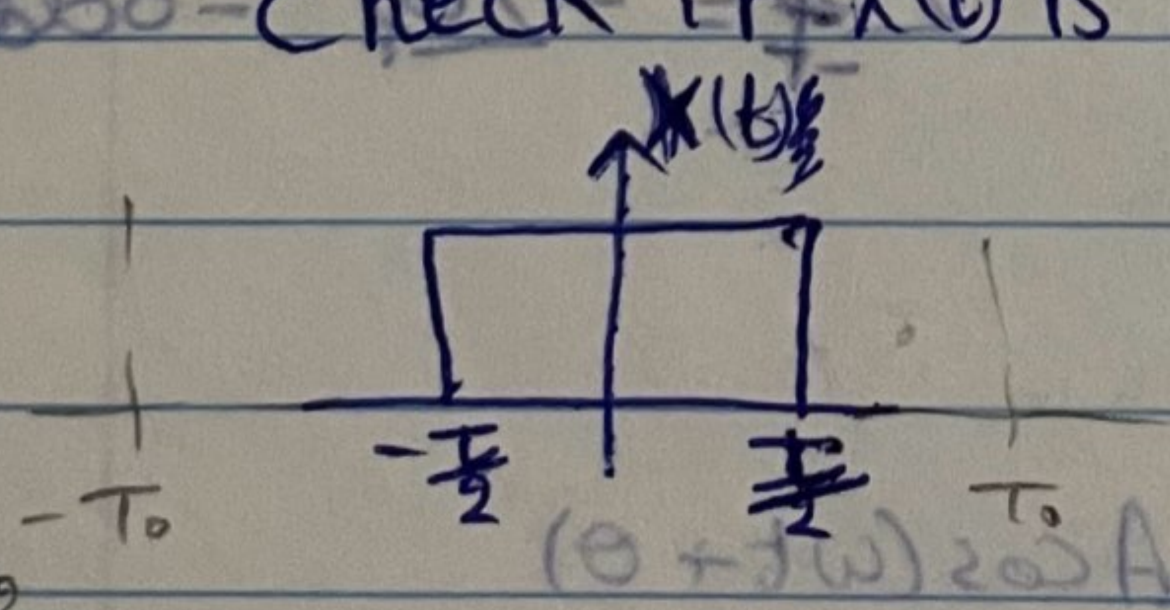
  · $P = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{0}^{T_0} |x(t)|^2 \, dt$.

• Power signals. $0 < P < \infty$ & $E = \infty$

• Energy signal. $0 < E < \infty$ & $P = 0$.

if the signal is bound and time limited ⇒ Energy signal.

- Ex - check if $x(t)$ is power or energy signal.



$$P = \lim_{T_0 \to \infty} \frac{1}{2T_0} \int_{-T_0}^{T_0} |x(t)|^2 \, dt$$

$$= \lim_{T_0 \to \infty} \frac{1}{2T_0} \left[\int_{-T_0}^{-\tau/2} (0)^2 \, dt + \int_{-\tau/2}^{\tau/2} (1)^2 \, dt + \int_{\tau/2}^{T_0} (0)^2 \, dt \right]$$

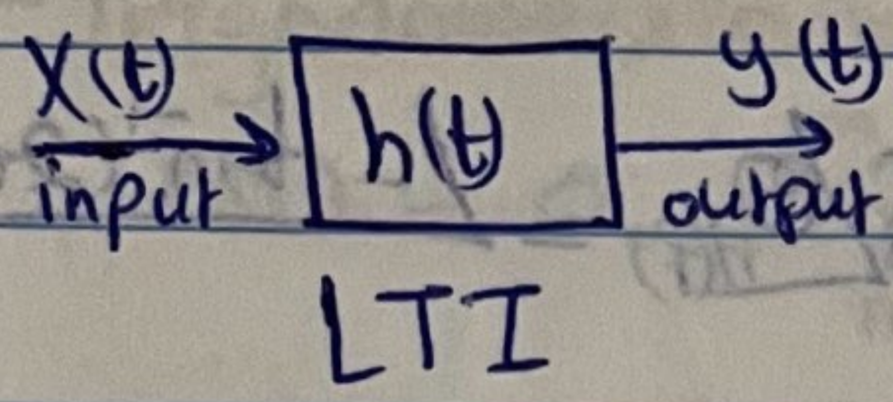
$\Rightarrow P = 0$

$$E = \int_{-\tau/2}^{\tau/2} (1)^2 \, dt$$

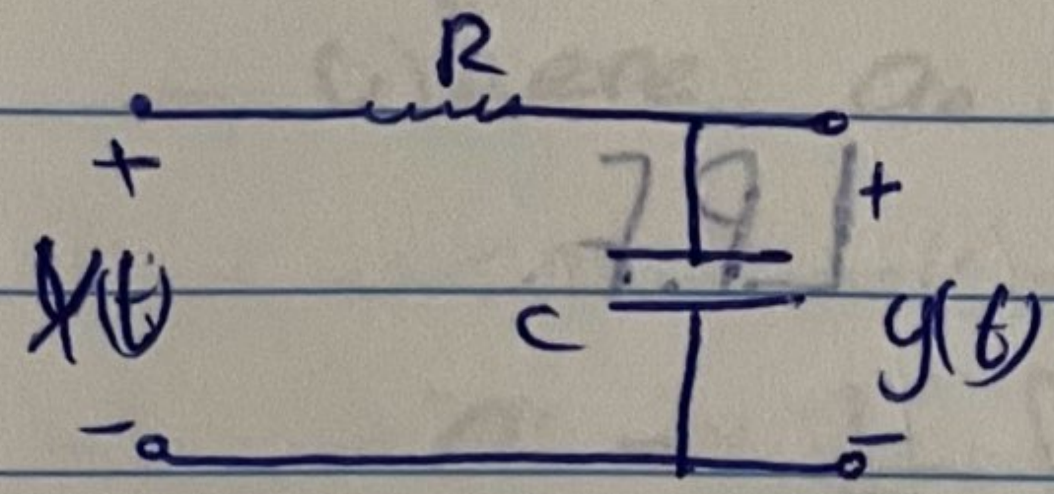
$\Rightarrow \boxed{E = \tau}$

→ Energy signal.

- **Systems.**



$$y(t) = X(t) * h(t)$$



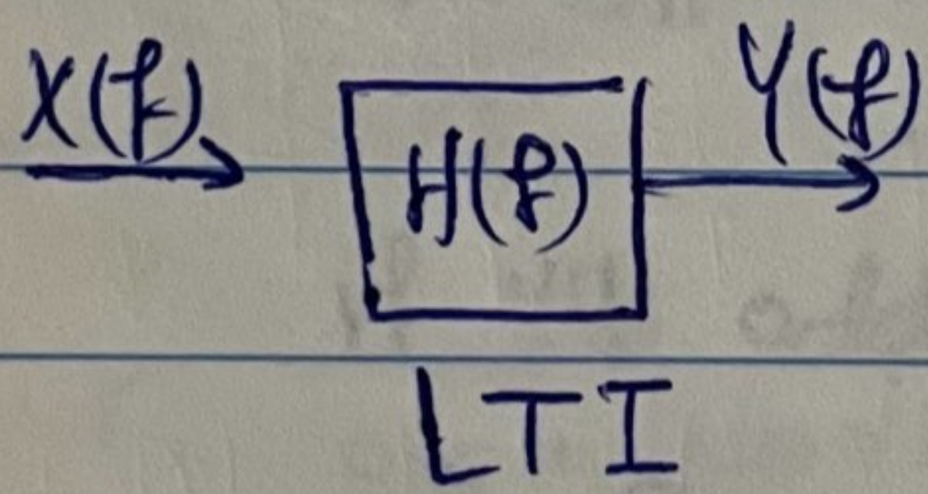
$$RC\frac{dy(t)}{dt} + y(t) = X(t)$$

$$\Rightarrow y(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

In freq. domain

$$\mathcal{F}\left[RC\frac{dy(t)}{dt} + y(t) = X(t)\right]$$

$$\Rightarrow j2\pi f RC\, Y(f) + Y(f) = X(f) \quad \text{— (1)}$$



$$Y(f) = X(f)H(f) \Rightarrow H(f) = \frac{Y(f)}{X(f)}$$

from Eq (1) :-

$$[1 + j2\pi f RC]\, Y(f) = X(f)$$

$$\Rightarrow H(f) = \frac{1}{1 + j2\pi f RC}$$

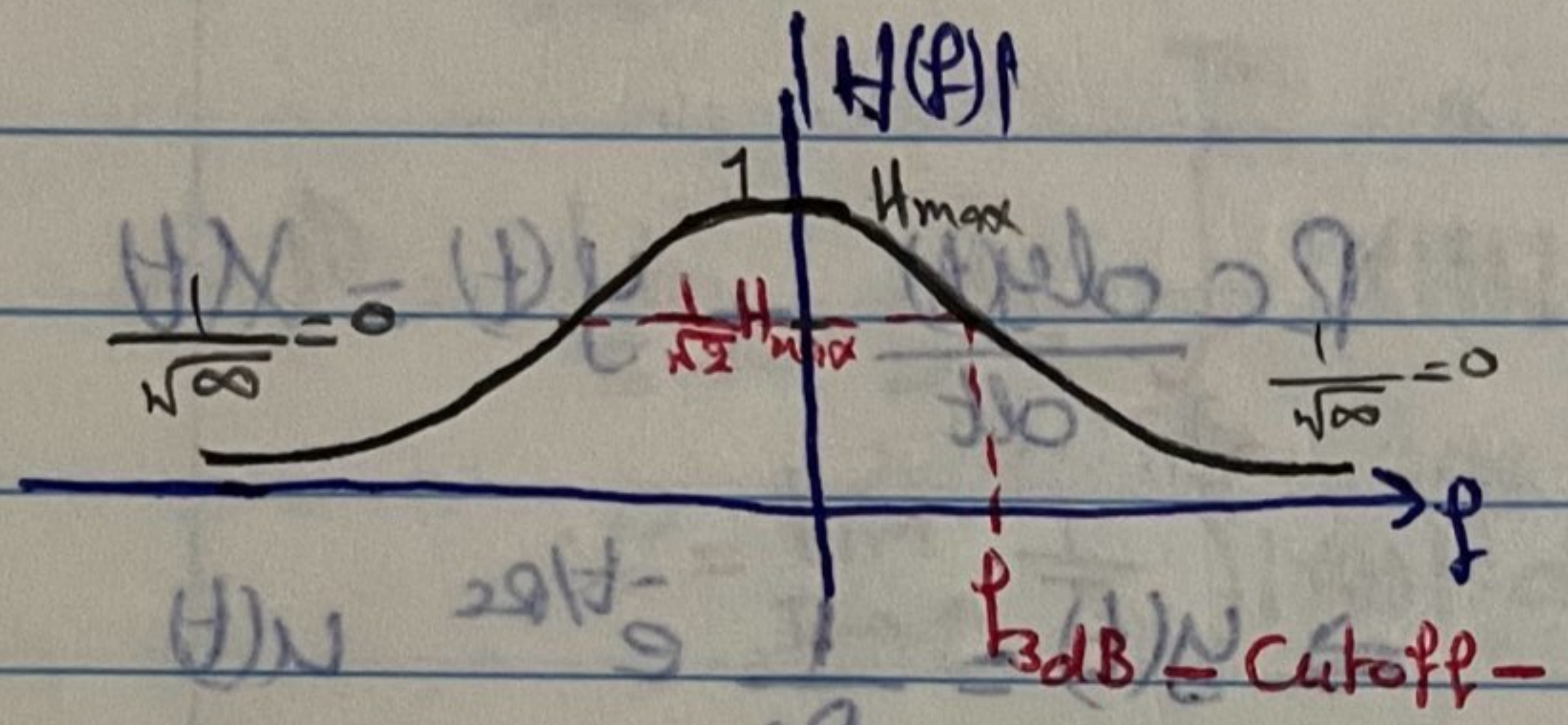
$$= \frac{1 \angle 0}{\sqrt{1^2 + (2\pi f RC)^2} \angle \tan^{-1}(2\pi f RC)}$$

$\Longrightarrow$

$\rightarrow$

$$H(f) = |H(f)| \angle \theta_{H(f)}$$

$$|H(f)| = \frac{1}{\sqrt{1+(2\pi f RC)^2}} \quad \& \quad \angle \theta_{H(f)} = \angle -\tan^{-1}(2\pi f RC)$$



L.P.F.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(2\pi f_{3dB} RC)^2}}$$

$$2 = 1 + (2\pi f_{3dB} RC)^2$$

$$1 = 2\pi f_{3dB} RC \Rightarrow \boxed{f_{3dB} = \frac{1}{2\pi RC}}$$

(L 2)

≡ Fourier Series.

↳ [1] Trigonometric fourier Series.

$$X(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

where a_0, a_n, b_n Trigonometric Coefficients fourier series.

$$a_0 = \frac{1}{T_0}\int_{T_0} X(t)\,dt = \text{average Value or DC Value}$$

$$a_n = \frac{2}{T_0}\int_{T_0} X(t)\cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T_0}\int_{T_0} X(t)\sin(n\omega_0 t)\,dt$$

<u>Note:-</u> if $X(t)$ even function.

$\Rightarrow b_n = 0$, a_n ✓

if $X(t)$ odd function.

$\Rightarrow a_n = 0$, b_n ✓

→ [2] $X(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$ — Complex Exponential fourier series —

where X_n : Complex Exponential Coefficient fourier series.

$$X_n = \frac{1}{T_0}\int_{T_0} X(t)\, e^{-jn\omega_0 t}\,dt$$

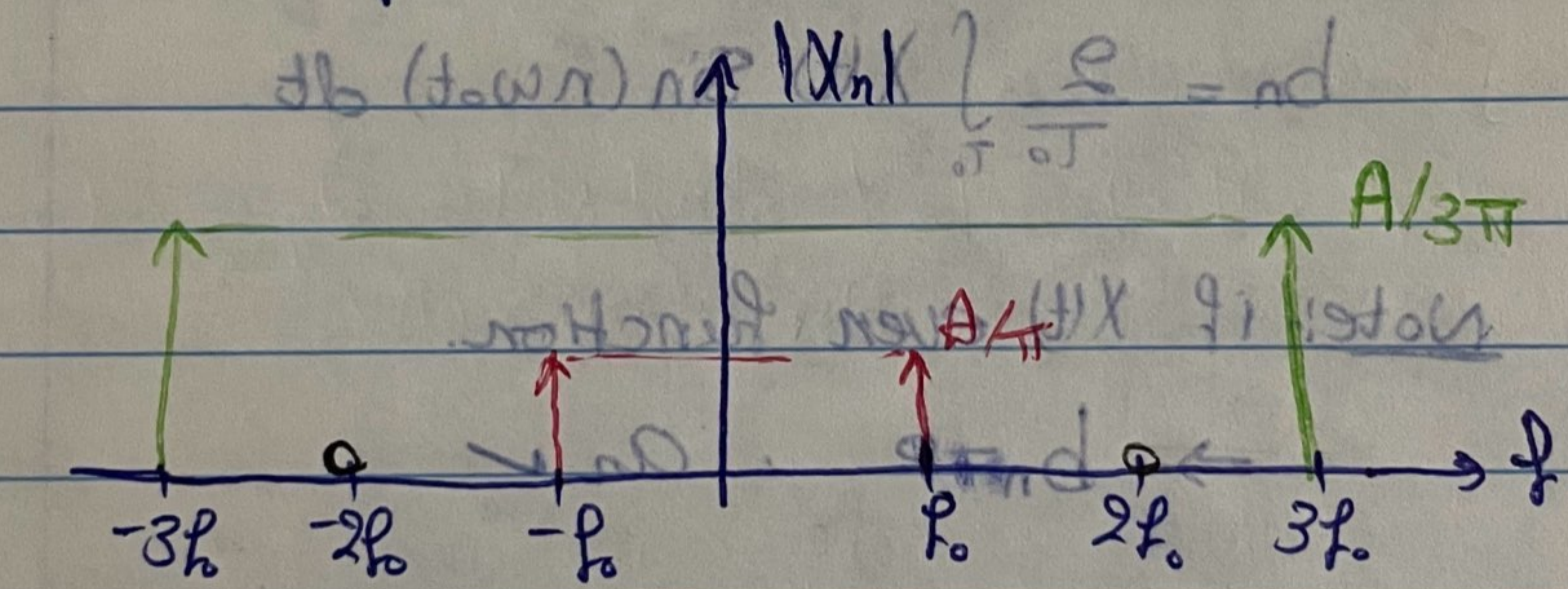
Note:- for Complex Exponential fourier Series

- if $x(t)$ even $\Rightarrow X_n =$ real
- if $x(t)$ odd $\Rightarrow X_n =$ Imagonary.

In general

$$X_n = \begin{cases} \frac{1}{2}(a_n - jb_n), & n>0 \\ \frac{1}{2}(a_n + jb_n), & n<0 \\ a_0, & n=0 \end{cases}$$

≡ Line Spectra.



$$X_n = \begin{cases} \frac{A}{j\pi n}, & n(\text{odd}), n \geq 0 \\ \frac{-A}{j\pi n}, & n(\text{odd}), n<0 \\ 0, & \text{o.w} \end{cases}$$

≡ Parseval's Theorem.

$$P_{avg} = \sum_{n=-\infty}^{\infty} |X_n|^2$$

Since $|X_{-n}| = |X_n|$

$$\Rightarrow P_{avg} = X_0^2 + 2\sum_{n=1}^{\infty} |X_n|^2$$

- We can write expression for PSD.

$$S_x(f) = \sum_{n=-\infty}^{\infty} |X_n|^2 \, \delta(f - nf_0)$$

≡ Fourier Transform.

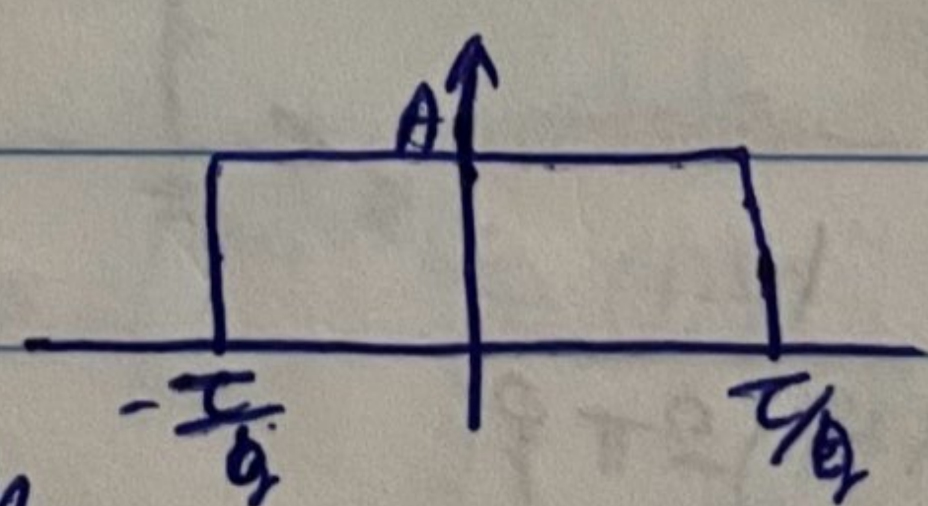
$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt.$$

&

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df.$$

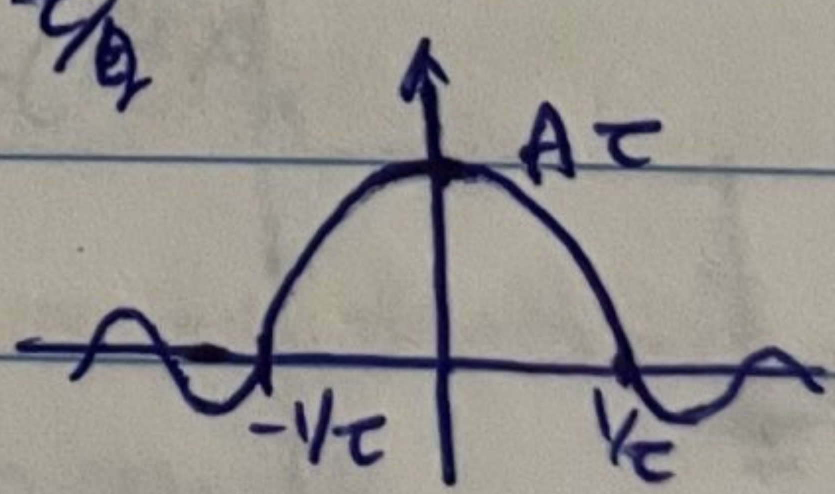
Time domain.

1. $x_1(t) = A\,\Pi\left(\frac{t}{\tau}\right)$

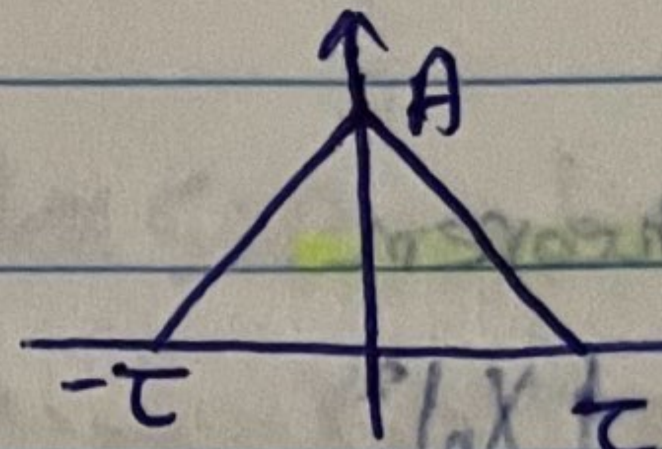


↳ Freq domain.

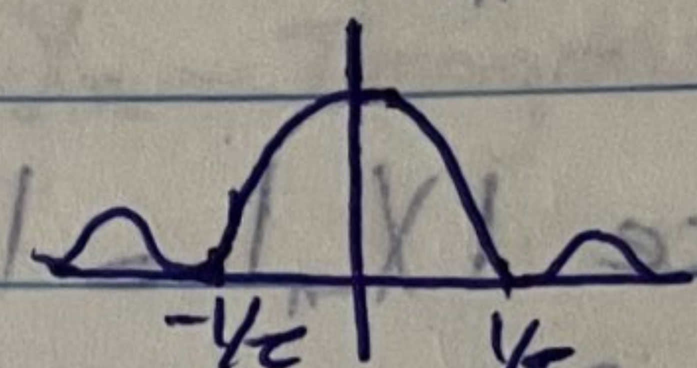
$$X_1(f) = A\tau \, \mathrm{Sinc}(\tau f)$$



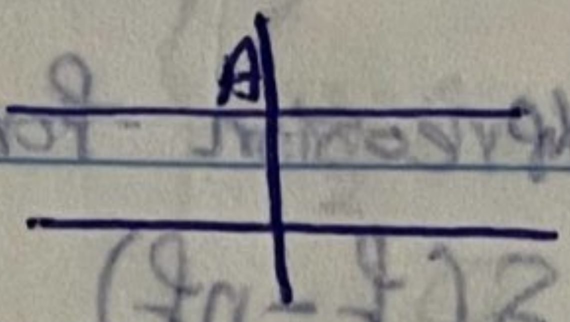
(Time domain) $X_2(t) = A\,\Lambda\left(\frac{t}{\tau}\right)$



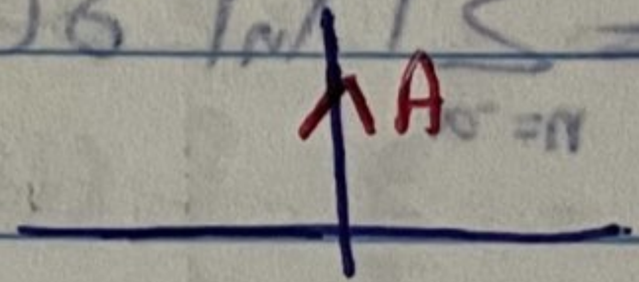
(Freq. domain) $\rightarrow X_2(f) = A\tau\, \text{sinc}^2(\tau f)$.



(Time domain) $X_3(t) = A$



(Freq. domain) $\rightarrow X_3(f) = A\,\delta(f)$



Example: Evaluate FT of the following signal.

$x(t) = e^{-\alpha t}\, u(t)$.

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-\alpha t}\, e^{-j2\pi f t}\, dt = \int_{0}^{\infty} e^{-(\alpha + j2\pi f)t}\, dt$$

$$= \frac{1}{\alpha + j2\pi f}$$

FT Theorems

1- Linearity:-

$$\mathcal{F}[x_1(t) + x_2(t)] = \mathcal{F}[x_1(t)] + \mathcal{F}[x_2(t)]$$
$$= X_1(f) + X_2(f)$$

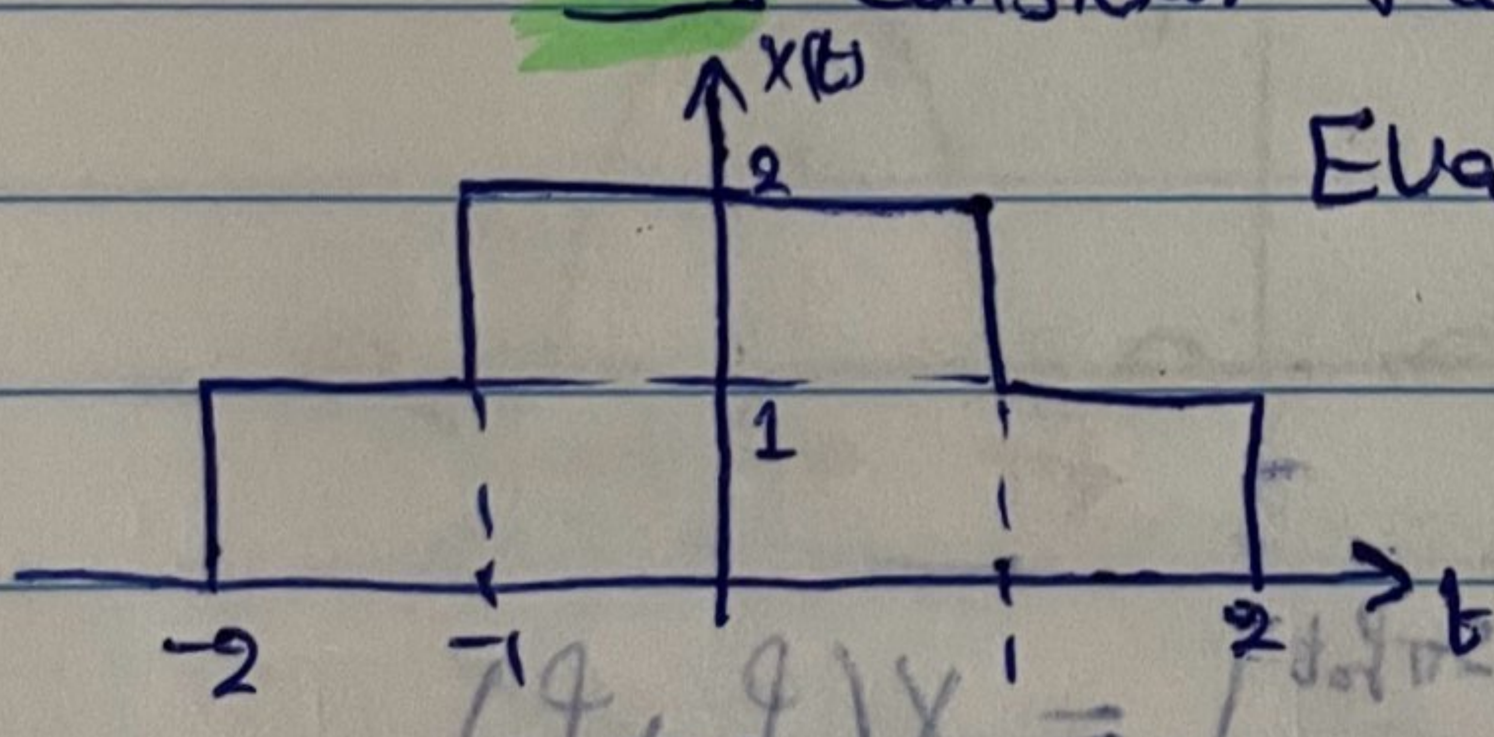
Ex| Evaluat FT. for the following signal.

$$x(t) = \mathcal{F}[\Pi(t) + \Lambda(t)]$$
$$= \mathcal{F}[\Pi(t)] + \mathcal{F}[\Lambda(t)]$$
$$= \text{Sinc}(f) + \text{Sinc}^2(f)$$

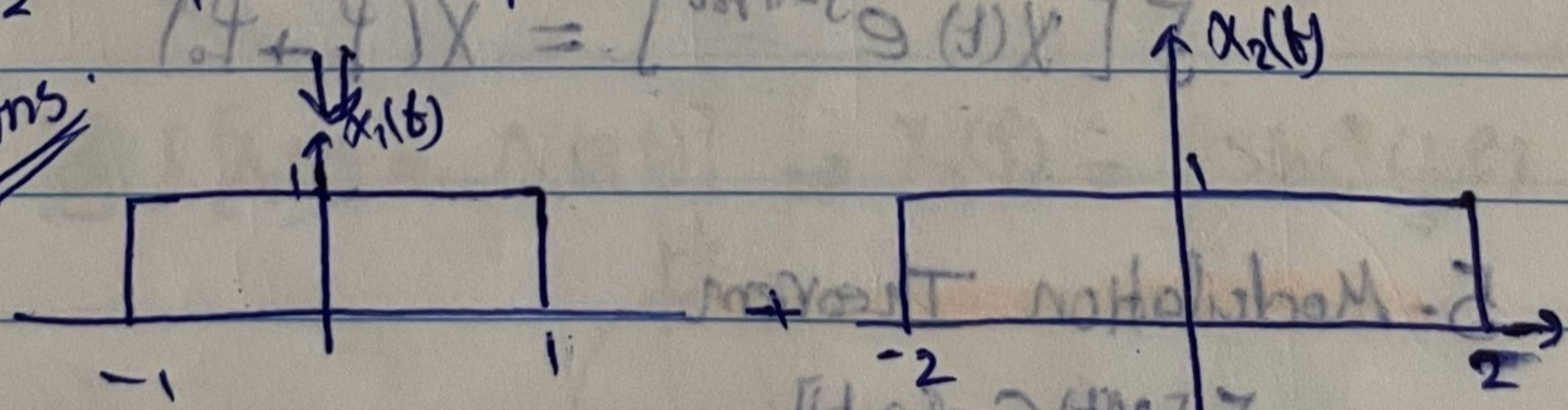
2- Scaling:-

$$\mathcal{F}[x(at)] = \frac{1}{a} X\left(\frac{f}{a}\right)$$

Ex| Consider the following signal. Evaluate FT.



Ans:

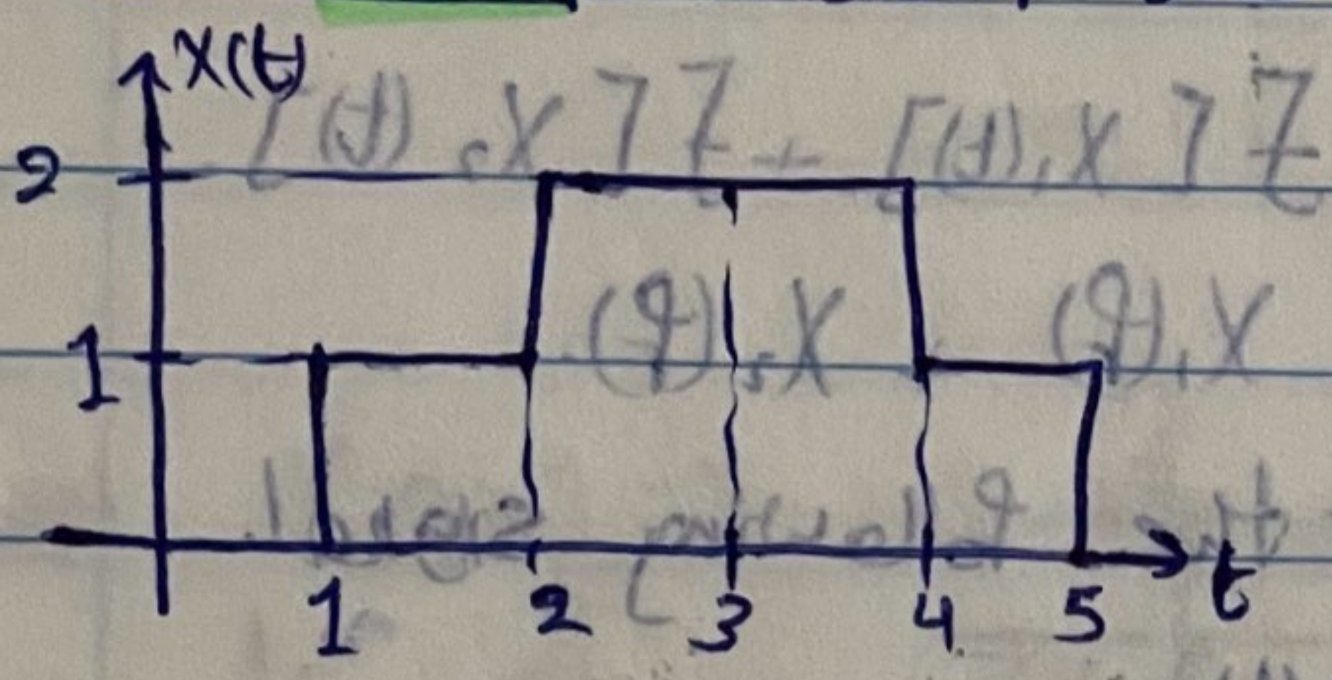


$$x(t) = x_1(t) + x_2(t)$$
$$= \mathcal{F}\left[\Pi\left(\frac{t}{2}\right) + \Pi\left(\frac{t}{4}\right)\right] \Rightarrow 2\,\text{Sinc}(2f) + 4\,\text{Sinc}(4f).$$

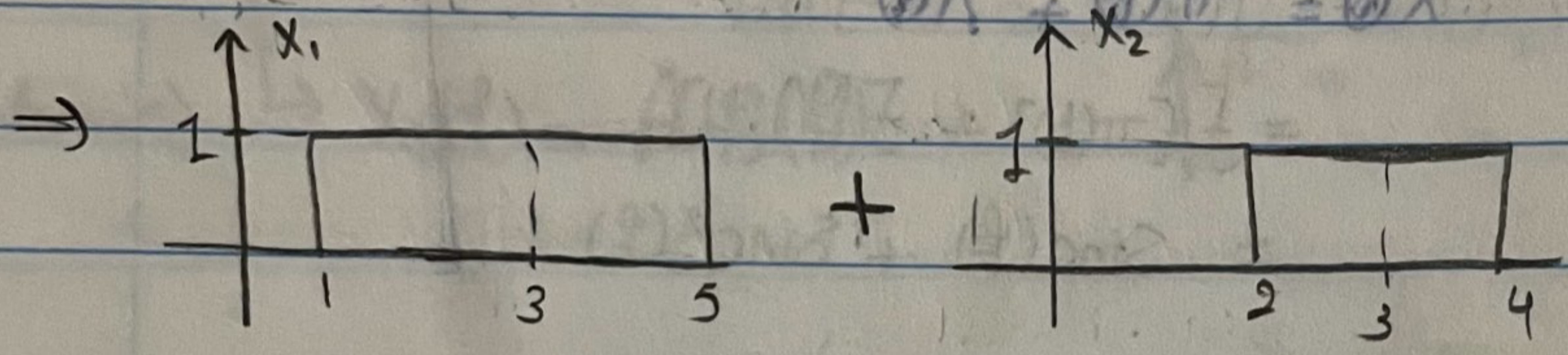
3- Time-delay :-

$$\mathcal{F}[x(t-t_0)] = X(f)\, e^{-j2\pi f t_0}$$

Ex Consider the following signal. Evaluate F.T.



$\Rightarrow$



$$x(t) = x_1(t) + x_2(t)$$

$$= \mathcal{F}\left[\Pi\left(\frac{t-3}{4}\right) + \Pi\left(\frac{t-3}{2}\right)\right]$$

$$X(f) = 4\,\text{Sinc}(4f)\, e^{-j6\pi f} + 2\,\text{sinc}(2f)\, e^{-j6\pi f}$$

4- Freq. Translation

$$\mathcal{F}[x(t)\, e^{-j2\pi f_0 t}] = X(f+f_0)$$

5- Modulation Theorem

$$\mathcal{F}[x(t)\cos(\omega_0 t)]$$

Since :- $\cos(\omega_0 t) = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

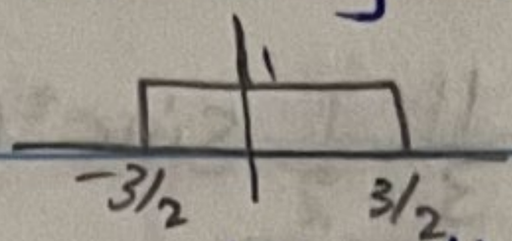
$$\mathcal{F}\left[x(t)\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right)\right] = \mathcal{F}\left[\tfrac{1}{2}x(t)e^{j\omega_0 t}\right] + \mathcal{F}\left[\tfrac{1}{2}x(t)e^{-j\omega_0 t}\right]$$

$$= \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)$$

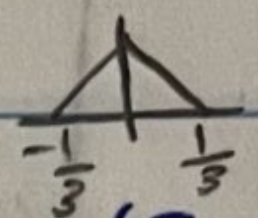
(L3)

EX] Consider the following signals.

1. $x_1(t) = \Pi\left(\frac{t}{3}\right)$



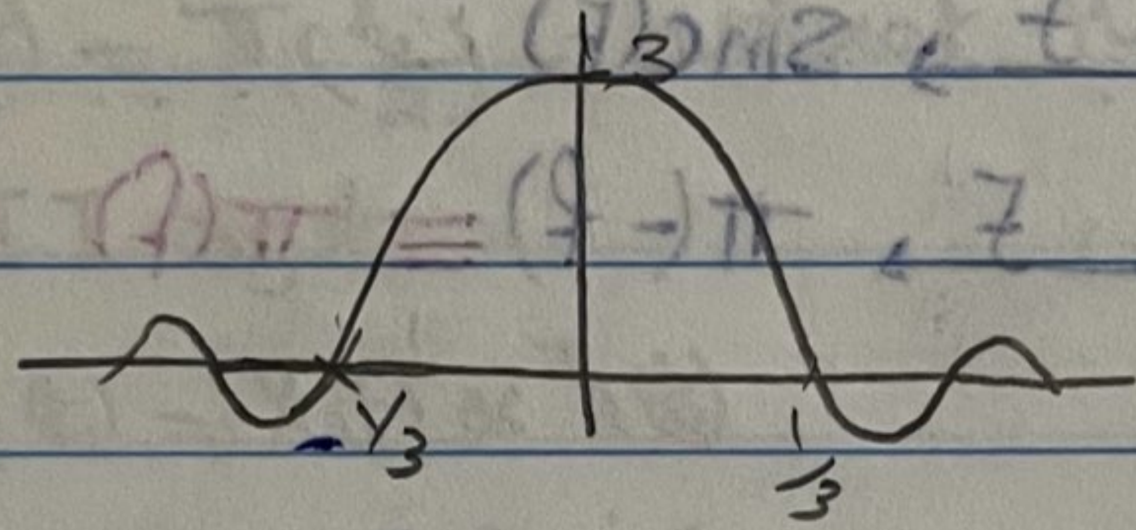
2. $x_2(t) = \Pi\left(\frac{t}{3}\right)\cos(20\pi t)$
3. $x_3(t) = \Lambda(3t)$



4. $x_4(t) = \Lambda(3t)\cos(80\pi t)$.

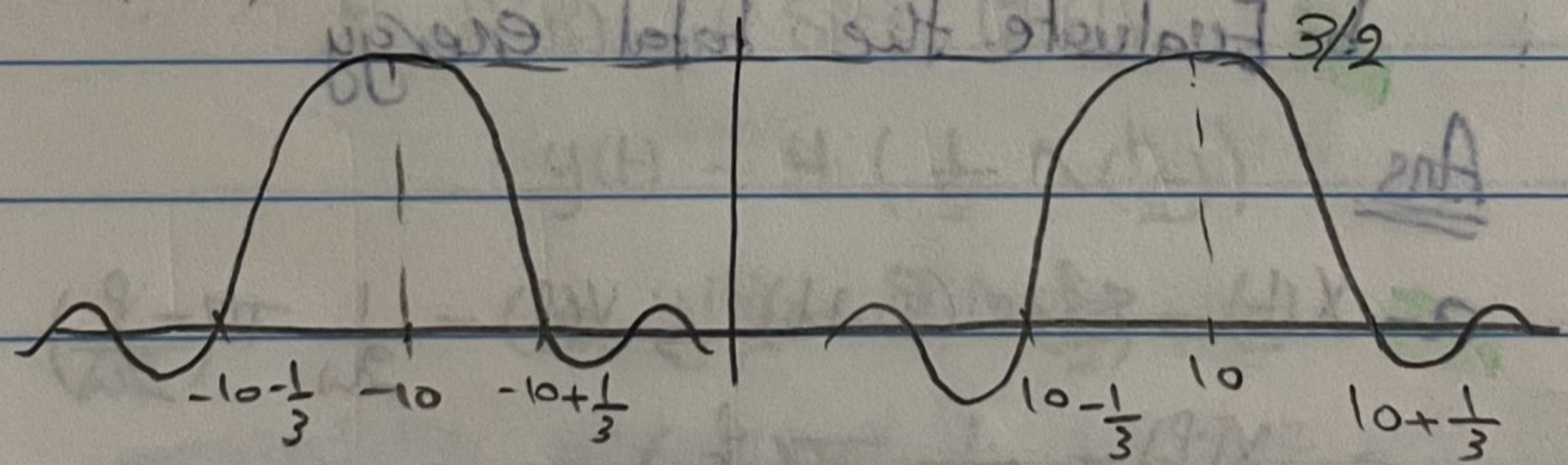
Evaluate and plot FT for each signals.

[1] $\mathcal{F}\left[x_1(t) = \Pi\left(\frac{t}{3}\right)\right] \Rightarrow X_1(f) = 3\,\text{sinc}(3f)$

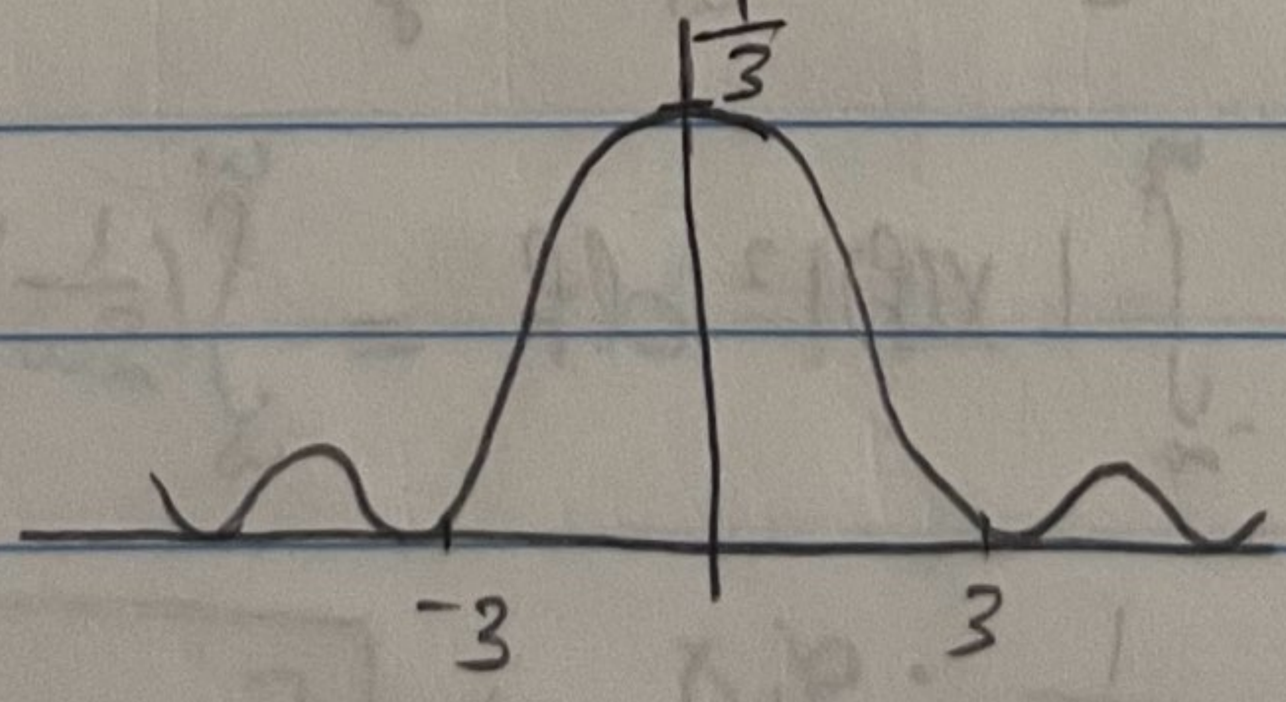


[2] $\mathcal{F}\left[x_2(t) = \Pi\left(\frac{t}{3}\right)\cos(20\pi t)\right]$

$\rightarrow X_2(f) = \frac{3}{2}\,\text{sinc}(3(f-10)) + \frac{3}{2}\,\text{sinc}(3(f+10))$

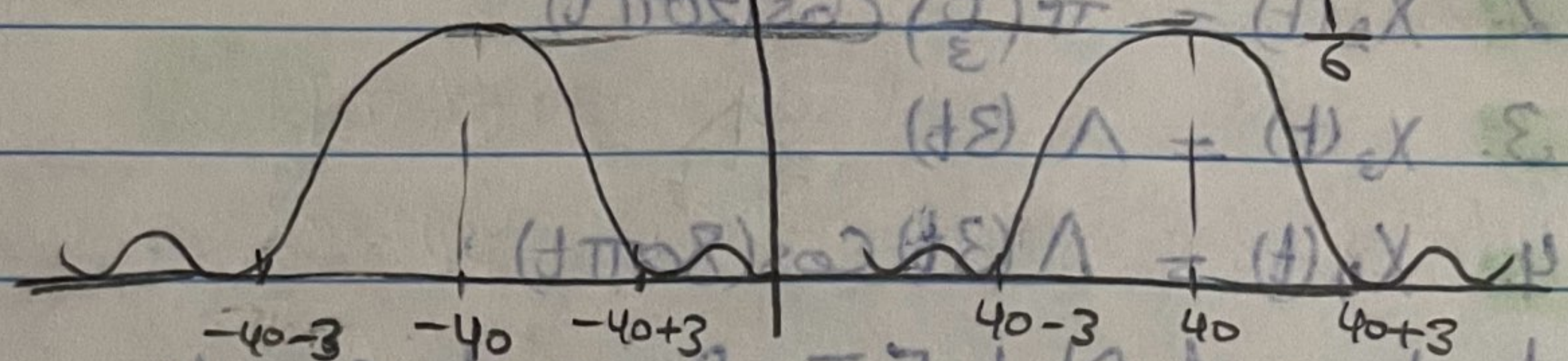


[3] $\mathcal{F}\left[x_3(t) = \Lambda(3t)\right] \rightarrow X_3(f) = \frac{1}{3}\,\text{sinc}^2\left(\frac{1}{3}f\right)$.



[4] $\mathcal{F}\{X_4(t) = \Lambda(3t)\cos(80\pi t)\}$

$$X_4(f) = \frac{1}{3}\cdot\frac{1}{2}\,\text{sinc}^2\left(\frac{1}{3}(f-40)\right) + \frac{1}{3}\cdot\frac{1}{2}\,\text{sinc}^2\left(\frac{1}{3}(f+40)\right)$$



6- Duality Theorem.

$$X(t) \xrightarrow{\mathcal{F}} \text{sinc}(f)$$

$$\text{sinc}(t) \xrightarrow{\mathcal{F}} \Pi(-f) \equiv \Pi(f) \quad \text{-even-}$$

EX| Consider the following signal.

1. $X(t) = \text{sinc}(2\omega t)$.

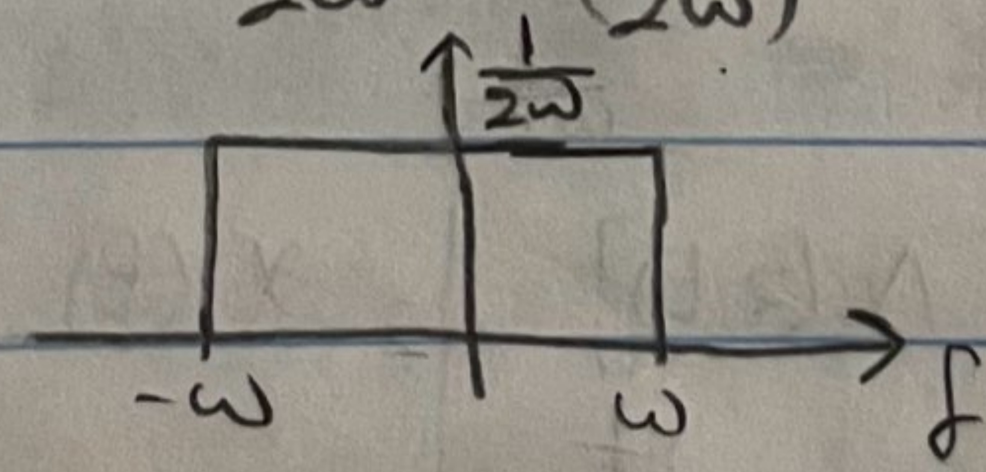
a- Evaluate and plot FT $X(t)$

b- Evaluate the total energy.

Ans.

a- $X(t) = \text{sinc}(2\omega t) \rightarrow X(f) = \frac{1}{2\omega}\,\Pi\left(\frac{-f}{2\omega}\right)$

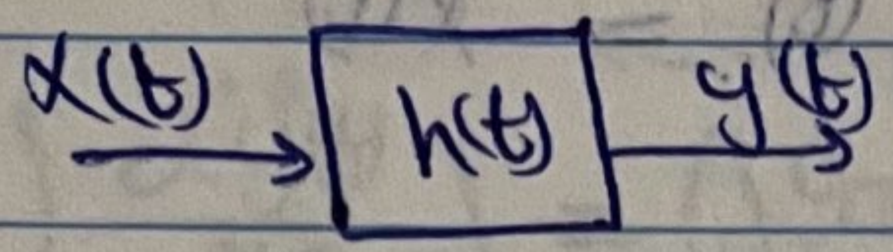
$$\Rightarrow X(f) = \frac{1}{2\omega}\,\Pi\left(\frac{f}{2\omega}\right)$$



b- $E = \int_{-\infty}^{\infty} |X(f)|^2\,df = \int_{-\omega}^{\omega}\left(\frac{1}{2\omega}\right)^2 df$

$$= \frac{1}{4\omega^2}\cdot 2\omega \Rightarrow \boxed{E = \frac{1}{2\omega}}$$

7- Convolution Theorem.



-LTI-

$y(t) = x(t) * h(t)$

$\hookrightarrow Y(f) = X(f)\,H(f) \Rightarrow H(f) = \frac{Y(f)}{X(f)}$

Ex For LTI system if the input signal $x(t) = \Pi(\frac{t}{2})$ & the response of system $h(t) = \Pi(\frac{t}{2})$. Evaluat the output signal $y(t)$

For LTI system

$y(t) = x(t) * h(t)$

$\hookrightarrow Y(f) = X(f)H(f)$

$= \mathcal{F}[\Pi(\frac{t}{2})]\,\mathcal{F}[\Pi(\frac{t}{2})]$.

$= 2\operatorname{sinc}(2f)\, 2\operatorname{sinc}(2f)$

$\mathcal{F}^{-1}[Y(f) = 4\operatorname{sinc}^2(2f)]$

$y(t) = 4\left(\frac{1}{2}\Lambda(\frac{t}{2})\right)$

$\Rightarrow y(t) = 2\Lambda(\frac{t}{2})$

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EX Consider the following signal

$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

Evaluate.

① the response of the system $h(t)$.

$$\mathcal{F}\left[RC\frac{dy(t)}{dt} + y(t) = x(t)\right]$$

$$\rightarrow RC\, j2\pi f\, Y(f) + Y(f) = X(f)$$

$$Y(f)\left[RC \cdot j2\pi f + 1\right] = X(f)$$

$$\frac{Y(f)}{X(f)} = \frac{1}{jRC2\pi f + 1} = H(f)$$

$$\rightarrow H(f) = \frac{1}{RC\left(j2\pi f + \frac{1}{RC}\right)}$$

$\mathcal{F}^{-1}$

Since :- $\mathcal{F}\left[e^{-\alpha t}u(t)\right] = \dfrac{1}{\alpha + j2\pi f}$

$$\hookrightarrow h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

② the out put of the impulse input response:

$y(t) = h(t)$ In general $y(t) = x(t) * h(t)$

For impulse Response $x(t) = \delta(t)$.

$$y(t) = x(t) * h(t) = \delta(t) * h(t)$$

$$\Rightarrow y(t) = h(t)$$

$\Rightarrow$

(4-1)

8 Differentiation & Integration Theorem.

$$\mathcal{F}\left[\frac{d^n y(t)}{dt^n}\right] = (j2\pi f)^n Y(f)$$

$$\mathcal{F}\left[\int_{-\infty}^{t} h(\tau)\, d\tau\right] = \frac{1}{j2\pi f} H(f)$$

2:→ ③ the output signal if $x(t) = e^{-2t} u(t)$.

$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f)\, H(f)$$

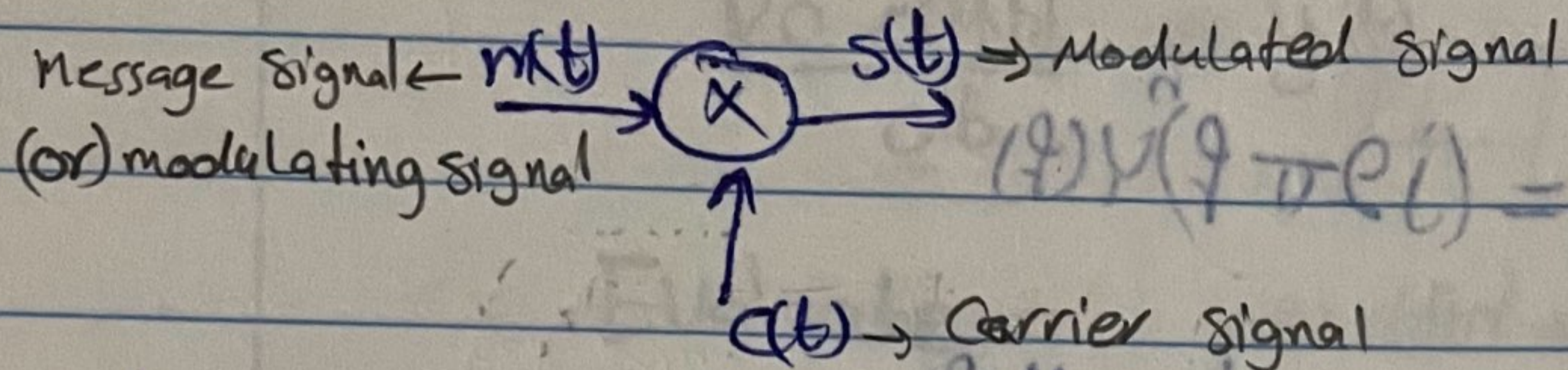
$$= \left(\frac{1}{2+j2\pi f}\right)\left(\frac{1/RC}{\frac{1}{RC}+j2\pi f}\right)$$ By using partial fraction

$$Y(f) = \frac{A}{2+j2\pi f} + \frac{(1/RC)(B)}{\frac{1}{RC}+j2\pi f}$$

$$= \frac{A\left(\frac{1}{RC}+j2\pi f\right)+\left(\frac{1}{RC}\right)(B)(2+j2\pi f)}{(2+j2\pi f)\left(\frac{1}{RC}+j2\pi f\right)}$$

(L4)

Example Consider the following system [8]



where $m(t) = 3\cos(20\pi t)$: message signal.

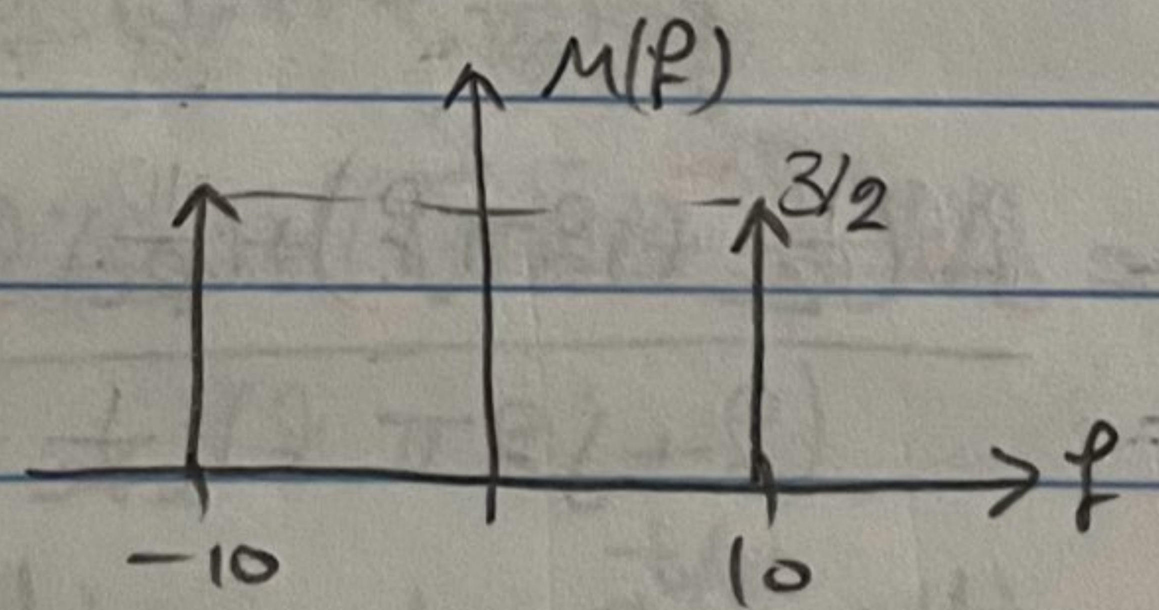
$c(t) = 4\cos(200\pi t)$: carrier signal.

a- Evaluate and plot the spectrum of each message and carrier signal

Ans:

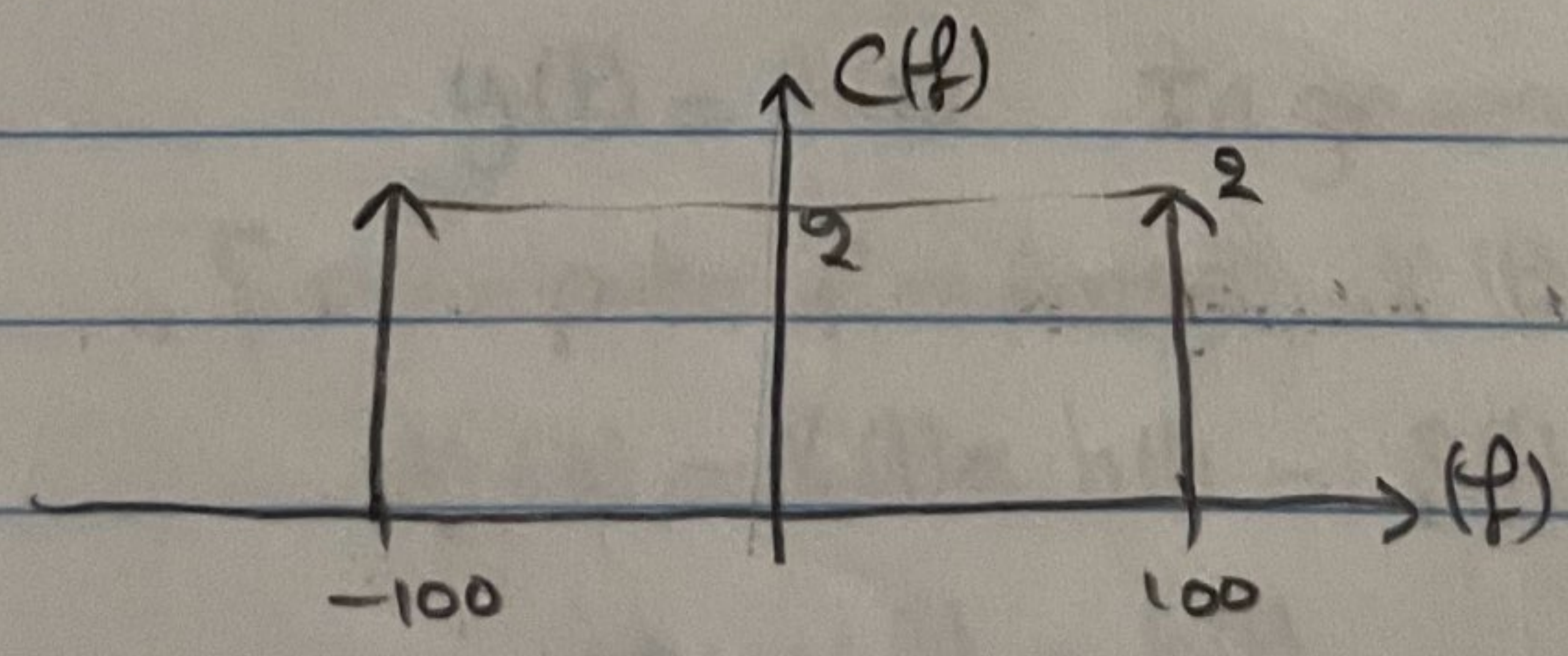
$\mathcal{F}[m(t) = 3\cos(20\pi t)]$

↳ $M(f) = \frac{3}{2}\delta(f-10) + \frac{3}{2}\delta(f+10)$



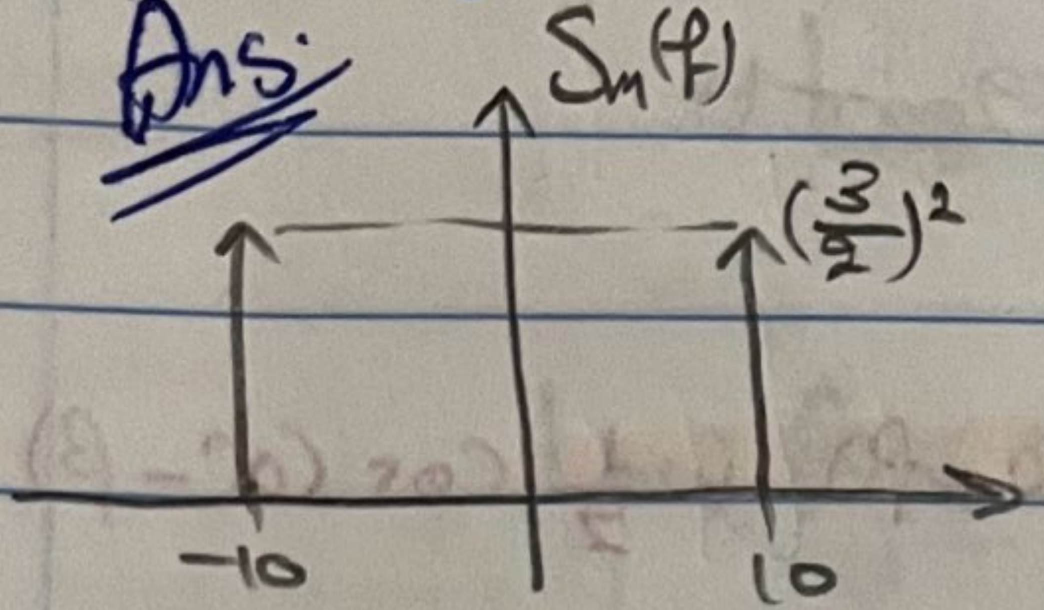
$\mathcal{F}[c(t) = 4\cos(200\pi t)]$

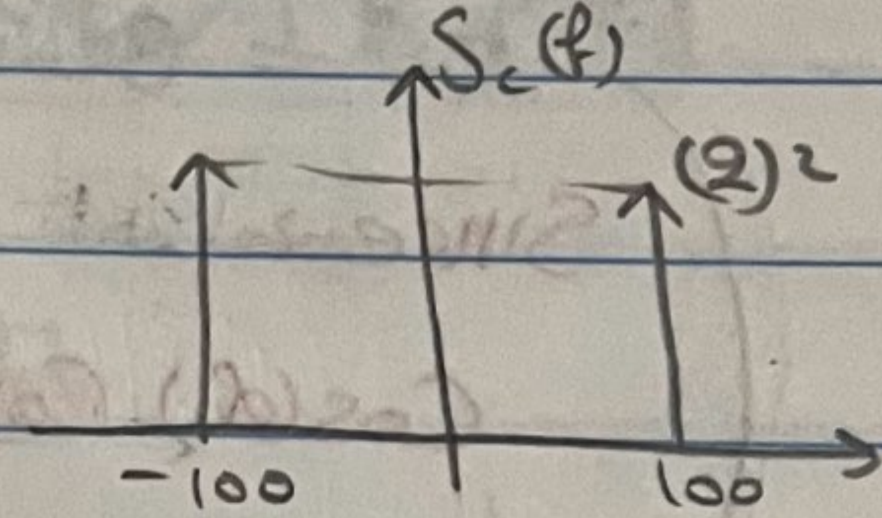
↳ $C(f) = 2\delta(f-100) + 2\delta(f+100)$



b- Evaluate and plot PSD for each message signal and carrier signal.

Ans:





$S_m(f) = (\frac{3}{2})^2 \delta(f-10) + (\frac{3}{2})^2 \delta(f+10)$ | $S_c(f) = 4\delta(f-100) + 4\delta(f+100)$

c- Evaluate the total power for each message signal and carrier signal

Ans:

to evaluate Total power ① → Time domain :-

By using time domain $[P = \frac{(Amp)^2}{2}]$

$P_m = \frac{(3)^2}{2} = \frac{9}{2}$

$P_c = \frac{(4)^2}{2} = 8$

② ↓ freq domain $[P = 2.(Amp)^2]$

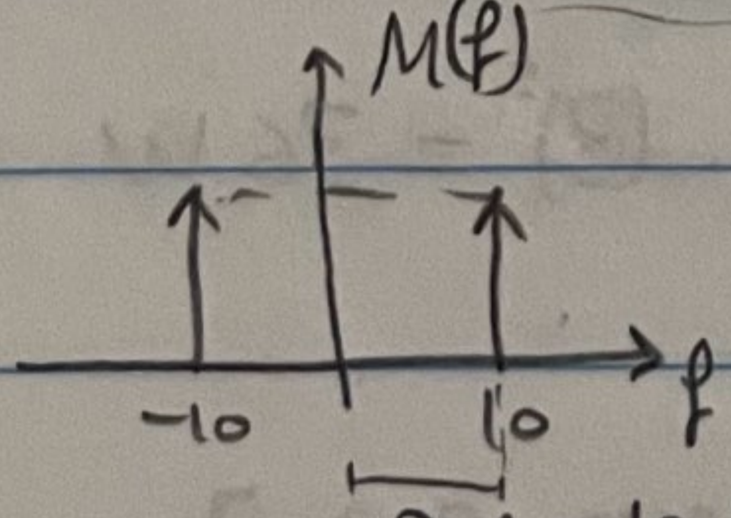
(الافتراض لازم يكون بالاصل cosin)

$P_m = 2.(\frac{3}{2})^2 = \frac{9}{2}$

$P_c = 2.(2)^2 = 8$

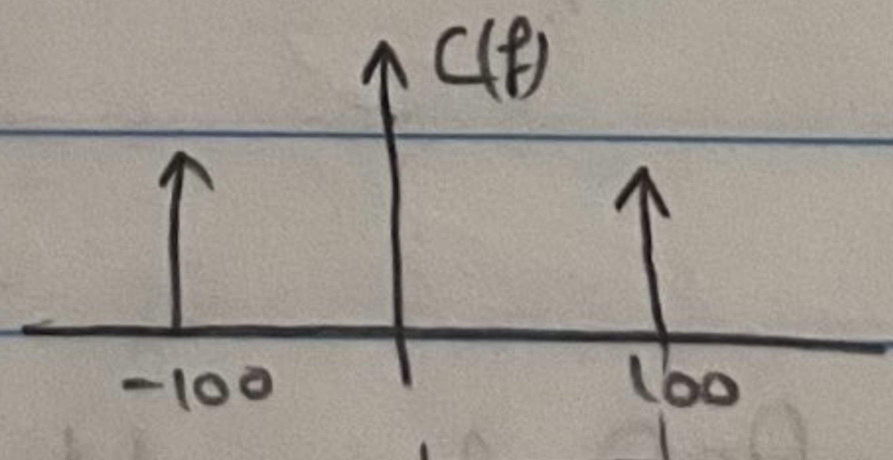
d- Evaluate the bandwidth (BW) for each message & carrier signal

Ans since message & carrier signals are base band signal] → متمحورة حول الoriginal



$BW = 10\ Hz$

↗ بعد أعلى فريكونسي عن الorigin



$BW = 100\ Hz$

e- Evaluate the modulated signal, $s(t)$.

Ans. $s(t) = m(t)\, c(t)$

$= 3\cos(20\pi t)\; 4\cos(200\pi t)$

Since :-

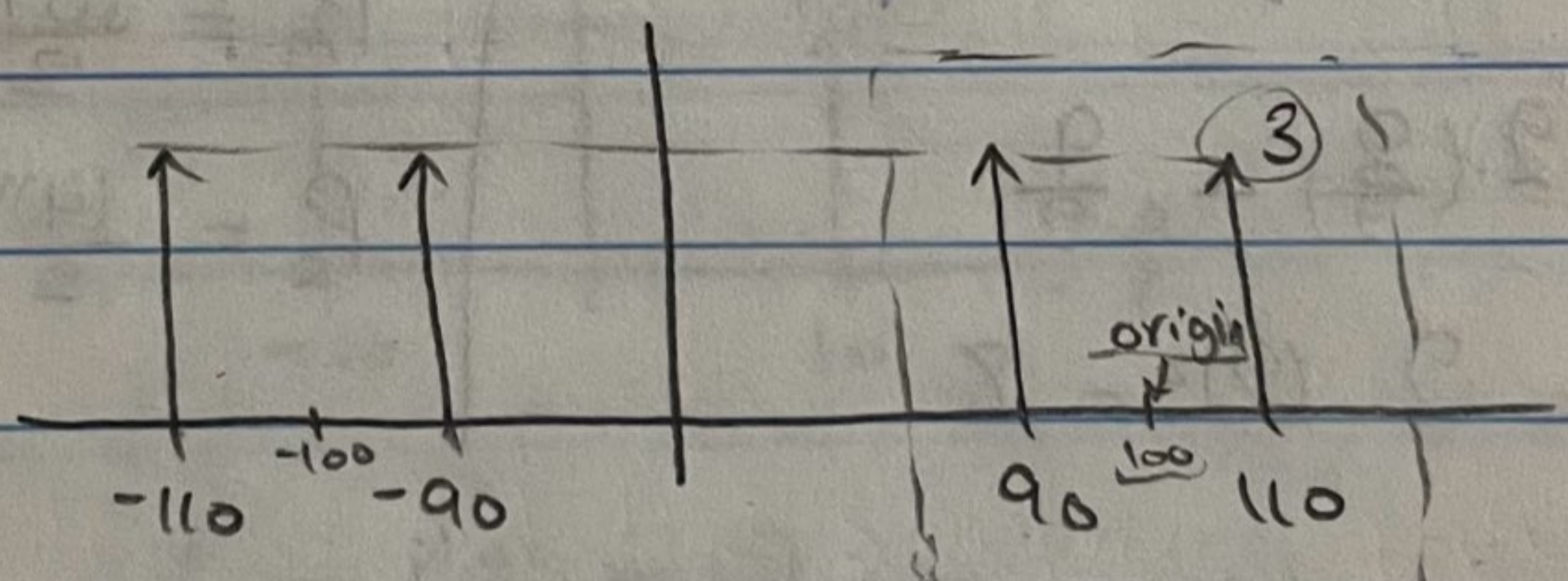
$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$.

$= 6\cos(220\pi t) + 6\cos(180\pi t)$.

f- Evaluate & Plot the spectrum of modulated signal.

Ans.

$S(f) = 3\delta(f-110) + 3\delta(f+110) + 3\delta(f-90)$
$+ 3\delta(f+90)$.



Pass band signal [محور الفوريي على نقطة غير الصفر]

$\Rightarrow BW = 110 - 90$
$= 20\ Hz$

$P = 4 \cdot (3)^2 = 36\ W$

(4 اشارات)

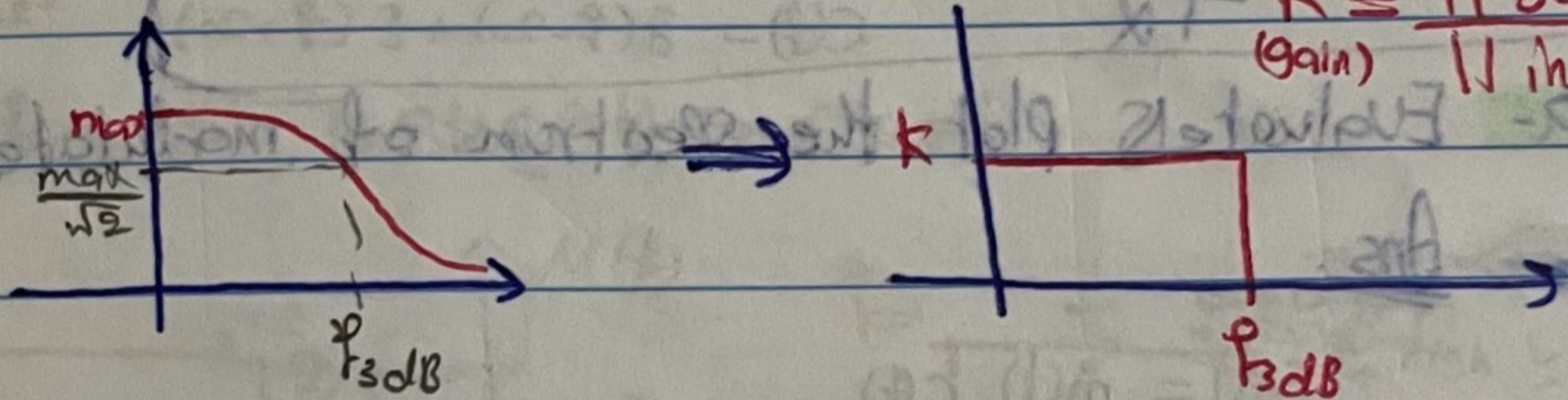
But PSD for modulated signal [Avg power].

$S_s(t) = (3)^2\delta(f-90) + (3)^2\delta(f+90) + 9\delta(f-110) + 9\delta(f+110)$.

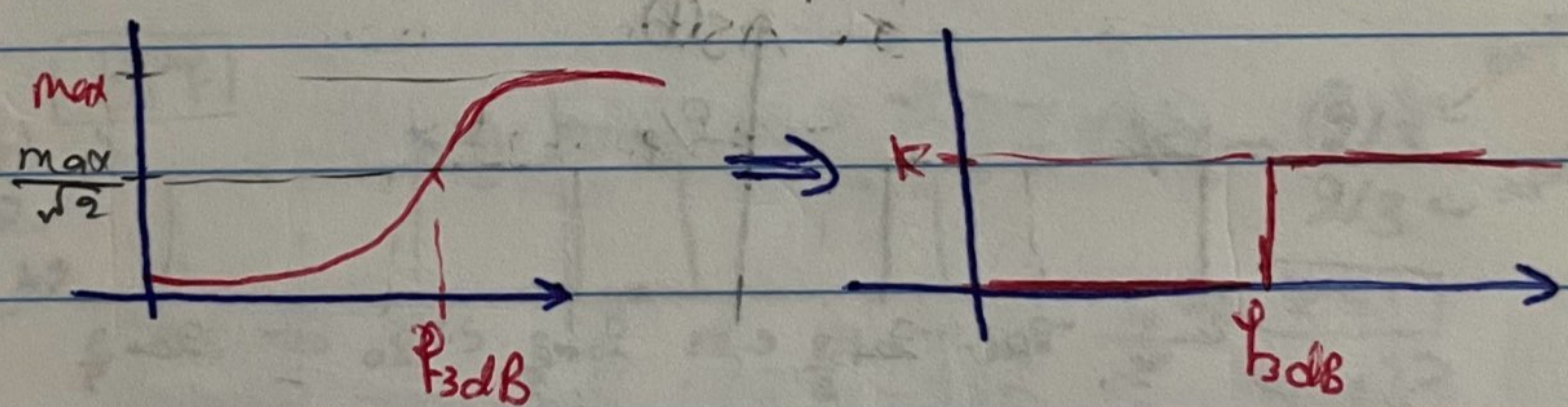
≡ Filters → Low Pass filter [L.P.f].
→ High Pass filter [H.P.f].
→ Band Pass filter [B.P.f].

_ Low Pass filter [L.P.f]

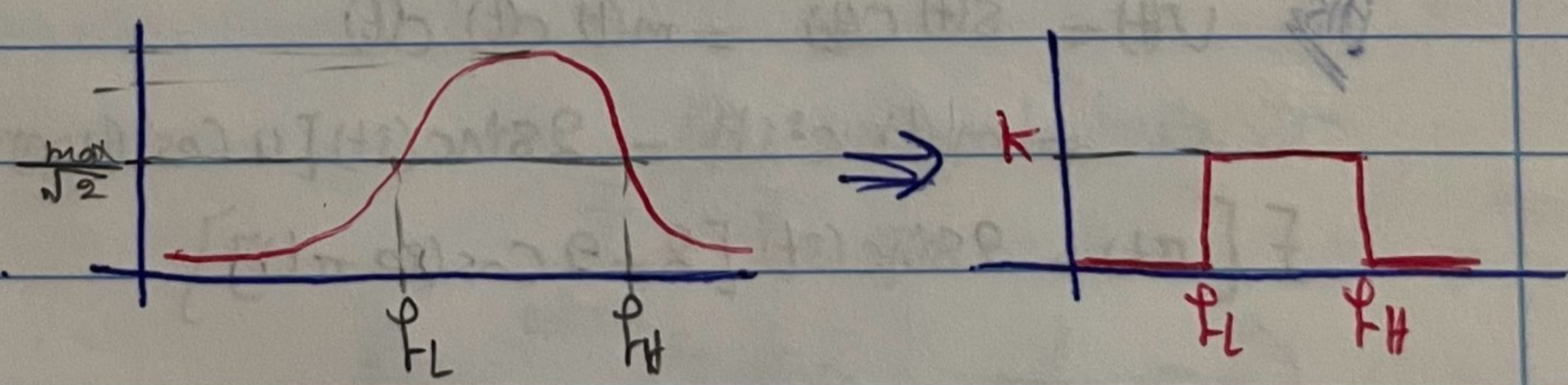
$K = \frac{|| \text{output}}{|| \text{input}}$ (gain)



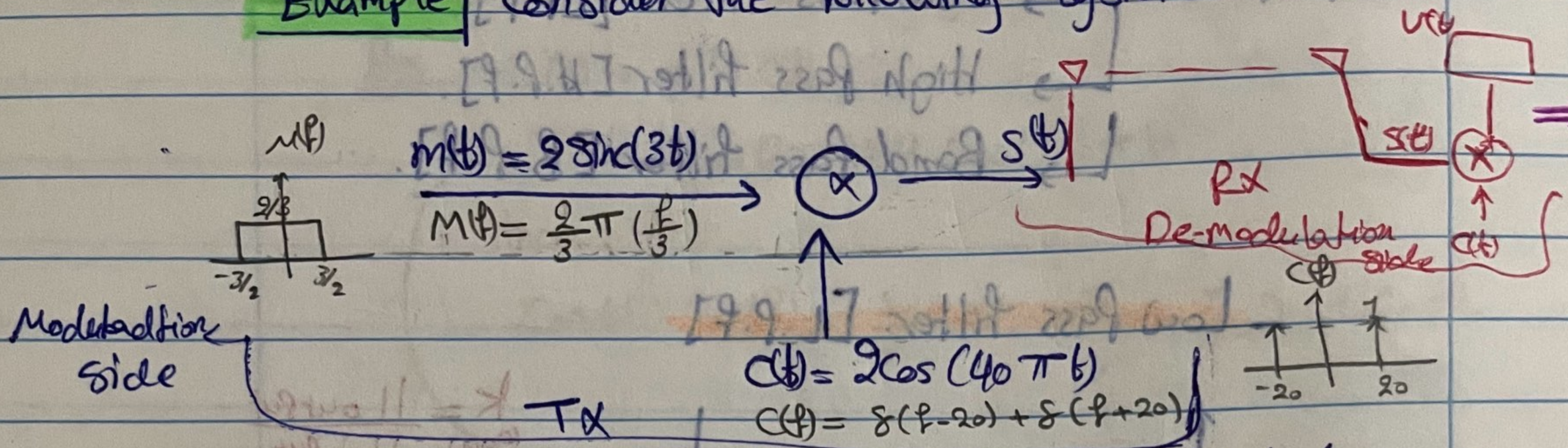
_ High Pass filter [H.P.f].



_ Band Pass filter [BPf].



Example Consider the following system.



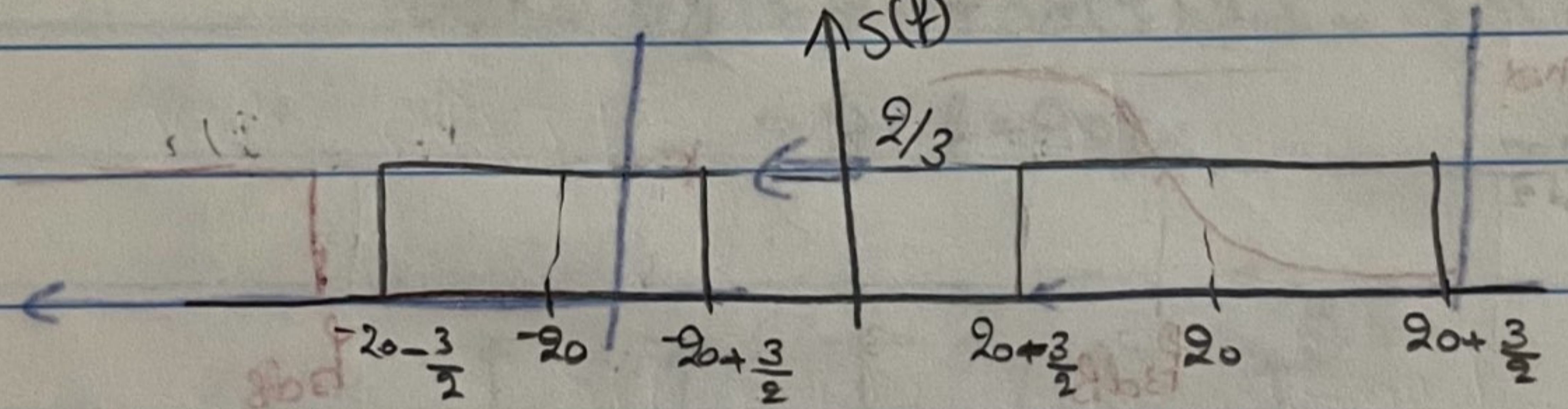
a- Evaluate & plot the spectrum of modulated signal

Ans.

$$s(t) = m(t)\,c(t)$$

$$= 2\,\text{sinc}(3t) \cdot 2\cos(40\pi t)$$

$$S(f) = 2 \cdot \frac{1}{3}\left[\Pi\left(\frac{1}{3}(f-20)\right) + \Pi\left(\frac{1}{3}(f+20)\right)\right]$$



$$E = \int_{-\infty}^{\infty} |S(f)|^2\,df$$

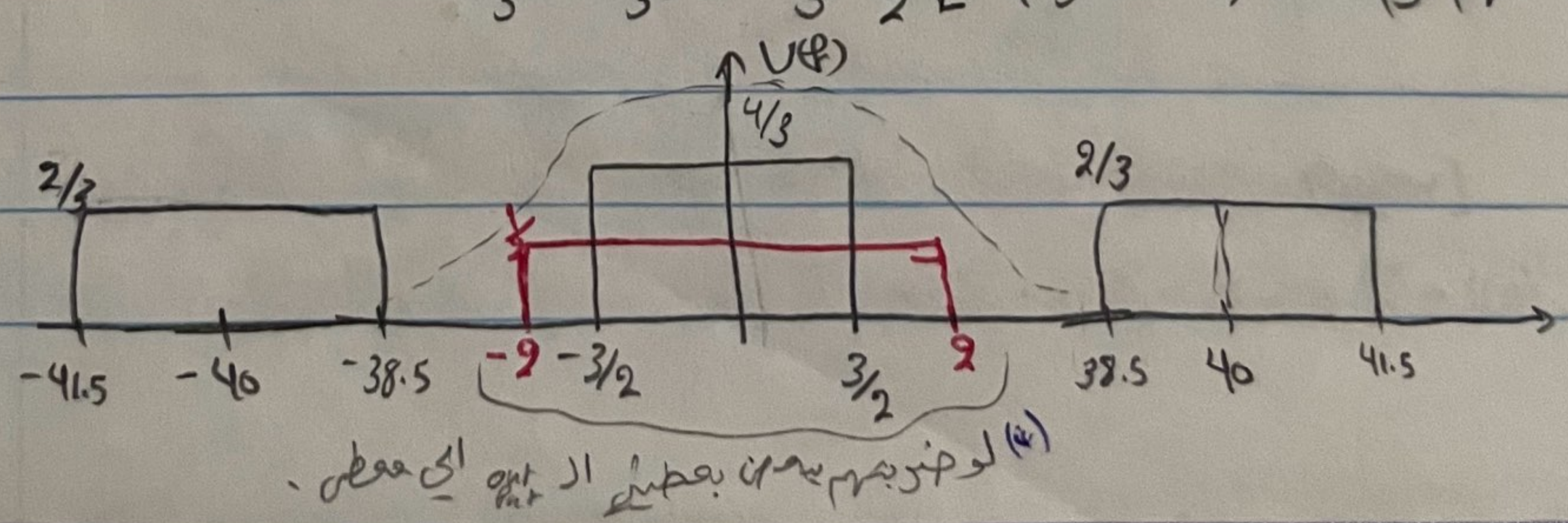
b- Evaluate and plot the spectrum v(t).

Ans: $v(t) = s(t)\,c(t) = m(t)\,c(t)\,c(t)$

$$= m(t)\,c^2(t) = 2\,\text{sinc}(3t)\left[4\cos^2(40\pi t)\right]$$

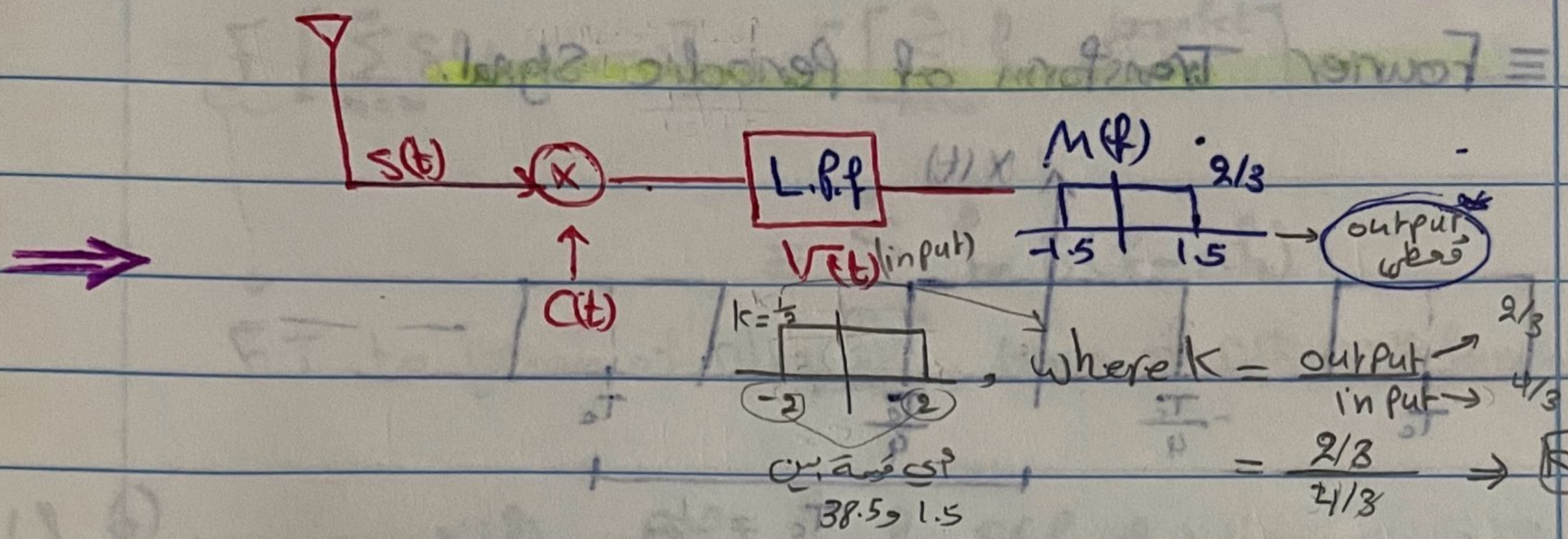
$$\mathcal{F}\left[v(t)\right] = 2\,\text{sinc}(3t)\left[2 + 2\cos(80\pi t)\right]$$

$$V(f) = \frac{4}{3}\Pi\left(\frac{f}{3}\right) + \frac{4}{3}\cdot\frac{1}{2}\left[\Pi\left(\frac{1}{3}(f-40)\right) + \Pi\left(\frac{1}{3}(f+40)\right)\right]$$



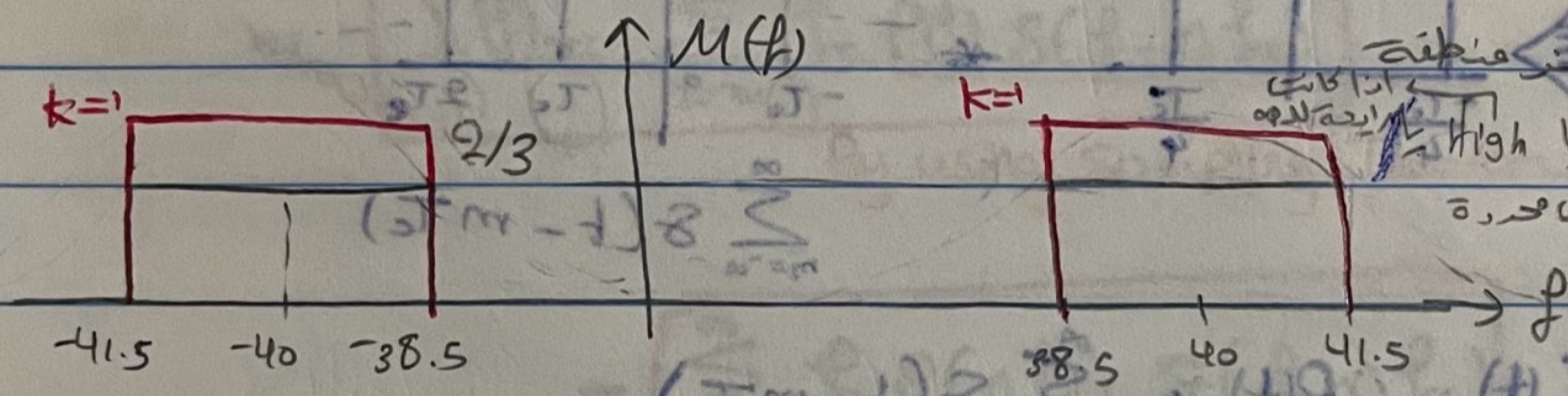
(*) لو فرضو بيعطي بعطيني ال range اي معطى.

(L5)

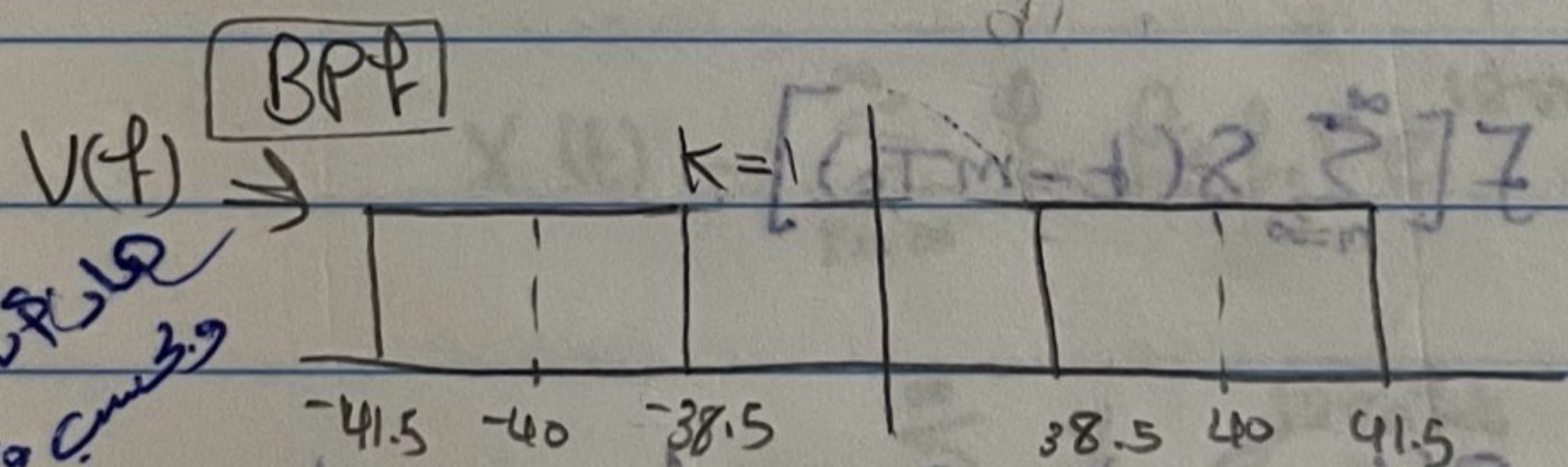


where $k = \frac{\text{output}}{\text{input}} = \frac{2/3}{4/3} \rightarrow \boxed{k = 0.5}$

c- if $m(t) = 4\,\text{sinc}(3t)\cos(80\pi t)$.



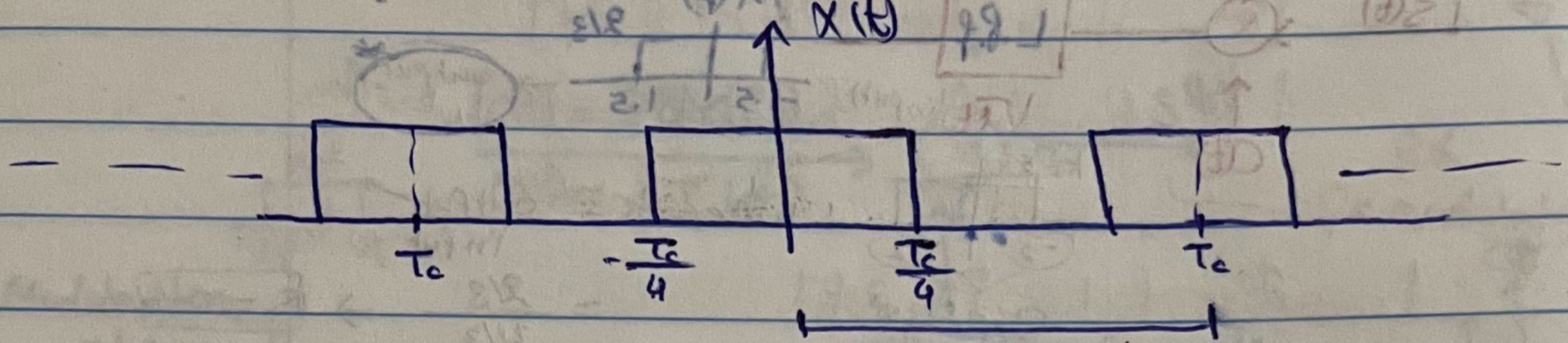
المستر مش عند منطقة إذا كانت رايحة للصفر
الـ original فبتكون يا High
band ← إذا كانت محددة



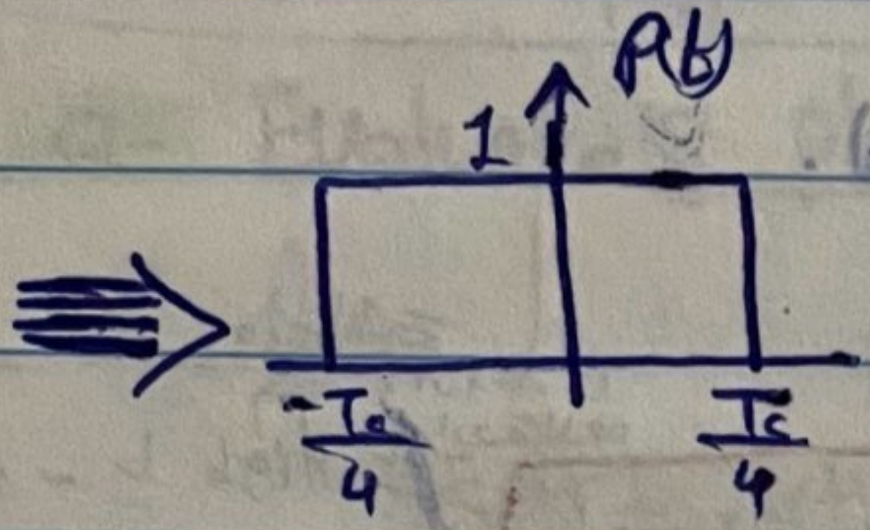
عشان أعرف وأحسب قيمة K

$k = \frac{(2/3) \leftarrow \text{output}(m(f))}{2/3 \leftarrow \text{input}(v(f))} \Rightarrow \boxed{k = 1}$

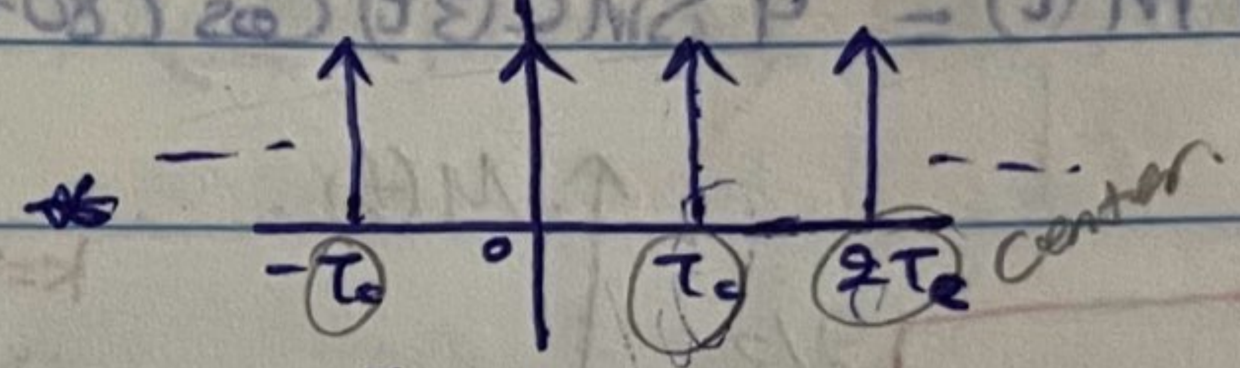
≡ Fourier Transform of Periodic Signal.



$T_c = \frac{1}{f_c}$



$\Rightarrow$ p(t) $*$



$$\sum_{m=-\infty}^{\infty} \delta(t - mT_c)$$

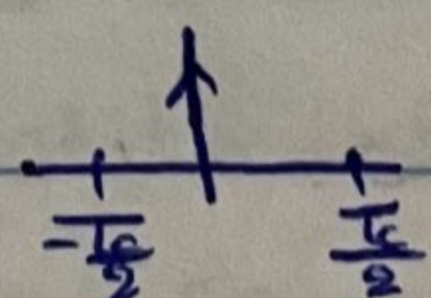
$$x(t) = p(t) * \sum_{m=-\infty}^{\infty} \delta(t - mT_c)$$

$$X(f) = P(f)\, \mathcal{F}\left[\sum_{m=-\infty}^{\infty} \delta(t - mT_c)\right]$$

Since $\sum_{m=-\infty}^{\infty} \delta(t - mT_c)$: periodic signal.

$$\Rightarrow \sum_{m=-\infty}^{\infty} \delta(t - mT_c) = \sum_{m=-\infty}^{\infty} C_n e^{j2\pi n f_c t}$$

To evaluate C_n



$$C_n = \frac{1}{T_c} \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} \delta(t)\, e^{-j2\pi n f_c t}\, dt$$

$t_0 = 0$ موجودة بين $-\frac{T_c}{2}$ و $\frac{T_c}{2}$ → $\int e^0 dt = 1$

By using sifting theorem.

$$C_n = \frac{1}{T_c} = f_c$$

$$\mathcal{F}\left[\sum_{m=\infty}^{\infty}\delta(t-mT_c)\right] = \mathcal{F}\left[\sum_{n=-\infty}^{\infty} f_c\, e^{j2\pi n f_c t}\right]$$

$$= \sum_{n=-\infty}^{\infty} f_c\, \delta(f-nf_c)$$

FT to [tream of Dlta(δ)] → tream of Dlta(δ) * fundamental freq.

(L6)

$$\Rightarrow X(f) = P(f)\sum_{n=-\infty}^{\infty} f_c\,\delta(f-nf_c)$$

$$X(f) = \sum_{n=-\infty}^{\infty} f_c\, P(f)\,\delta(f-nf_c)$$

By using Sampling Theorem.

$$= \sum_{n=-\infty}^{\infty} f_c\, P(nf_c)\,\delta(f-nf_c)$$

Constant

$$x(t) = \sum_{n=\infty}^{\infty} f_c\, P(nf_c)\, e^{j2\pi n f_c t}$$

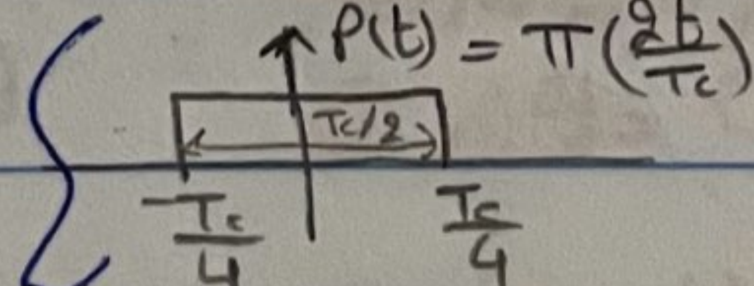
$$= \sum_{n=-\infty}^{\infty} X_n\, e^{j2\pi n f_c t} \quad ; \quad X_n = f_c\, P(nf_c)$$

For previous example,

1. Evaluate FT of x(t).

$$X(f) = \sum_{n=-\infty}^{\infty} f_c\, P(f)\,\delta(f-nf_c)$$

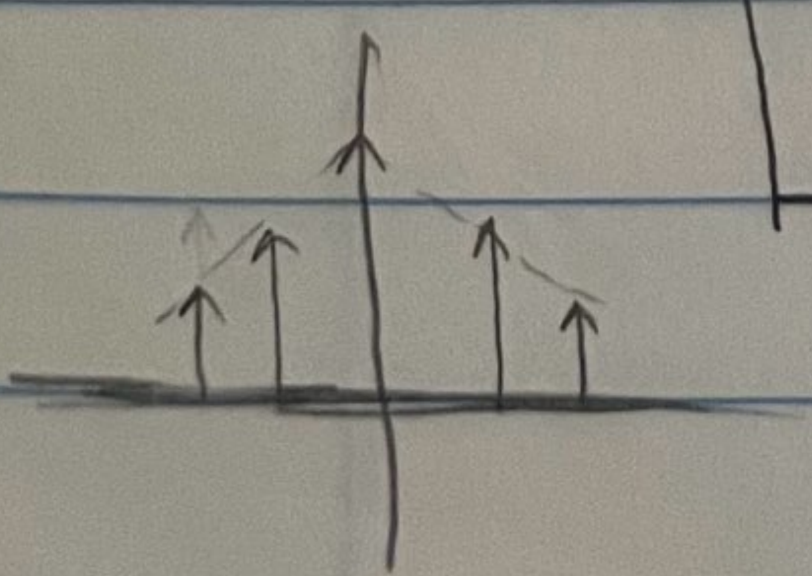
$$f_c = \frac{1}{T_c}$$



→ P(f)

$$P(nf_c) = \frac{T_c}{2}\,\text{sinc}\left(\frac{T_c}{2} n f_c\right)$$

$$X(f) = \sum_{n=\infty}^{\infty} \frac{1}{T_c}\cdot\frac{T_c}{2}\,\text{sinc}\left(\frac{T_c}{2} n \cdot \frac{1}{T_c}\right)\delta$$



$$\Rightarrow X(f) = \sum_{n=-\infty}^{\infty} \frac{1}{2}\,\text{sinc}\left(\frac{n}{2}\right)\delta(f-nf_c)$$

① بيكتب المعادلة الأساسية

② $f_c = \frac{1}{T_c}$

③ باخد بيريود وحدة وبطلع منها P(t) وبعملها باخدلها FT ← P(f)

④ ~~[illegible]~~

⑤ بعوض P(f) ← P(nf)

⑥ بعوض في المعادلة الأصلية

2- Evaluate $X(t)$.

$$X(t) = \sum_{n=-\infty}^{\infty} f_c\, P(nf_c)\, e^{j2\pi n f_c t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right) e^{j2\pi n f_c t}$$

3. Evaluate the Trigonometric Coefficient fourier series.

$$X(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t).$$

ω_0, $b_n = 0$ - even function

$$a_0 = \frac{1}{T_c}\int_{-\frac{T_c}{4}}^{\frac{T_c}{4}} (1)\, dt \rightarrow \frac{1}{T_c} \cdot \frac{T_c}{2} \Rightarrow \boxed{a_0 = \frac{1}{2}}$$

بقدر بردو هاد القانون *

$a_0 = X_0 = f\, P(0) = \frac{1}{2} \cdot \operatorname{sinc}(0)$ $\Rightarrow$ $\boxed{a_0 = \frac{1}{2}}$

Since

$$X_n = \begin{cases} \frac{1}{2}(a_n - jb_n), & n > 0 \\ \frac{1}{2}(a_n + jb_n), & n < 0 \\ a_0, & n = 0 \end{cases}$$

$a_n = 2\operatorname{Re}\{X_n\}$, $b_n = 2\operatorname{Im}\{X_n\}$, $a_0 = X_0$.

For previous example.

$a_n = 2 \cdot \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right) \rightarrow a_n = \operatorname{sinc}\left(\frac{n}{2}\right)$

4- Evaluate PSD.

$$S_x(f) = \sum_{n=-\infty}^{\infty} |X_n|^2 \delta(f-nf_c)$$

$$= |X_0|^2 \delta(f) + 2\sum_{n=1}^{\infty} |X_n|^2 \delta(f-nf_c)$$

$$S_x(f) = \left(\frac{1}{2}\right)^2 \delta(f) + 2\sum_{n=1}^{\infty}\left(\frac{1}{2}\operatorname{sinc}\left(\frac{n}{2}\right)\right)^2 \delta(f-nf_c)$$

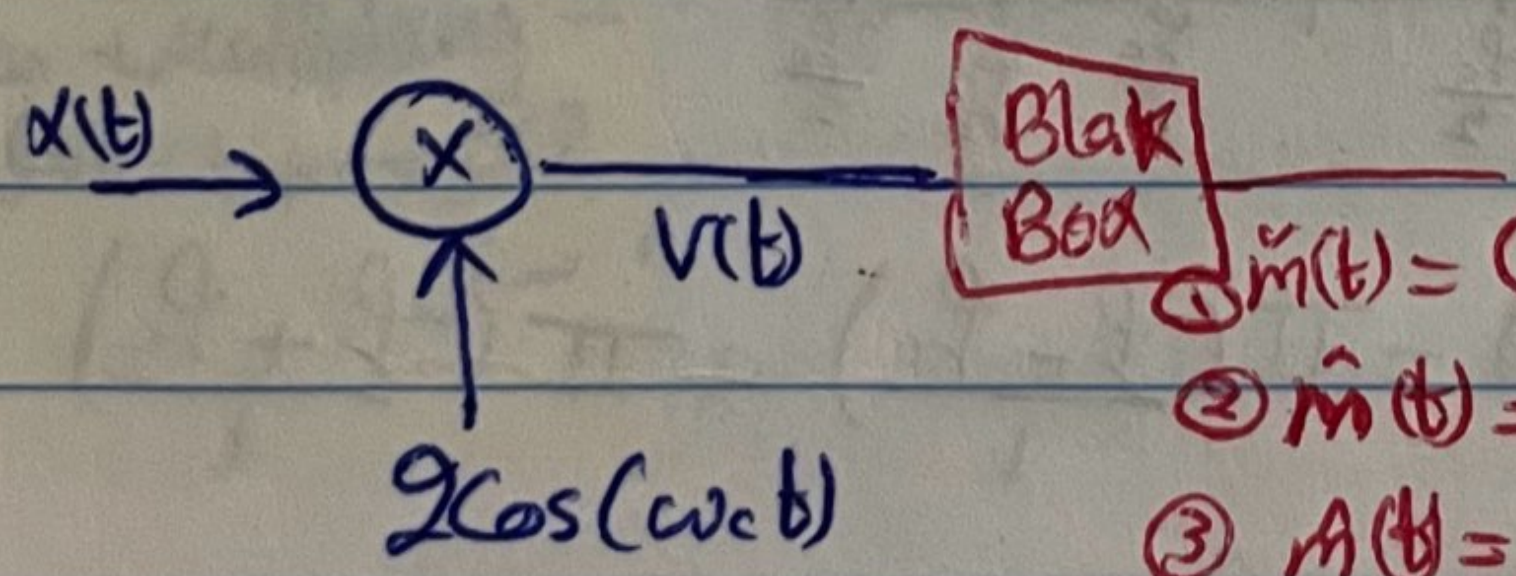
5- Evaluate the total power $(-1<n<1)$.

$$|X_0|^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$|X_1|^2 = |X_{-1}|^2 = \left(\frac{1}{2}\operatorname{sinc}\left(\frac{1}{2}\right)\right)^2$$

$$P_{-1<n<1} = |X_0|^2 + 2|X_1|^2$$

6- Based on $x(t)$ (in previous part). Consider the following system.



① $\breve{m}(t) = C + 3\cos(\omega_c t)$
② $\hat{m}(t) = 2\cos(\omega_c t)$
③ $A(t) = C$

$$x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \operatorname{sinc}\left(\frac{n}{2}\right)\cos(n\omega_c t)$$

6-a- Evaluat $v(t) = x(t) \cdot 2\cos(\omega_c t)$

$$= \left[\frac{1}{2} + \sum_{n=1}^{\infty} \operatorname{sinc}\left(\frac{n}{2}\right)\cos(n\omega_c t)\right] \cdot 2\cos(\omega_c t).$$

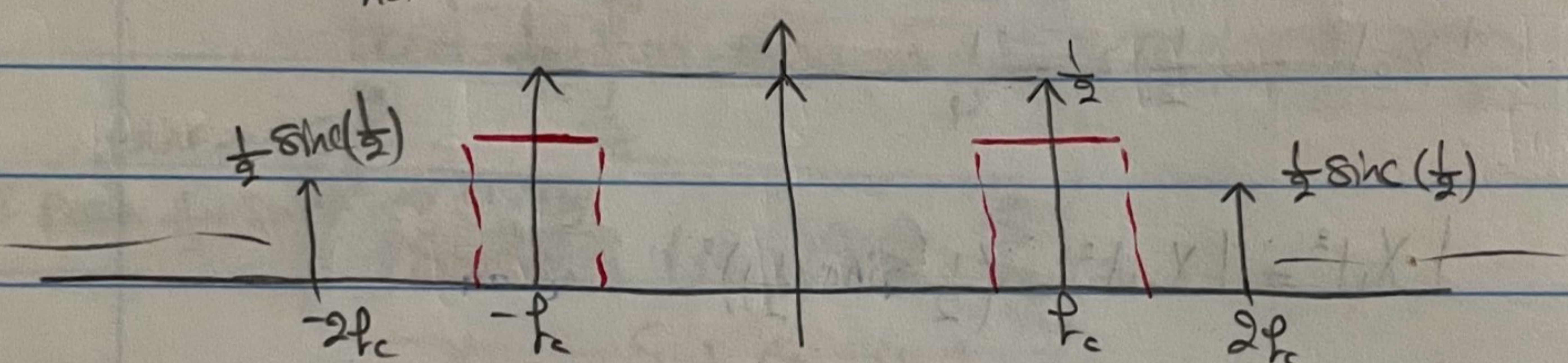
$\Rightarrow$

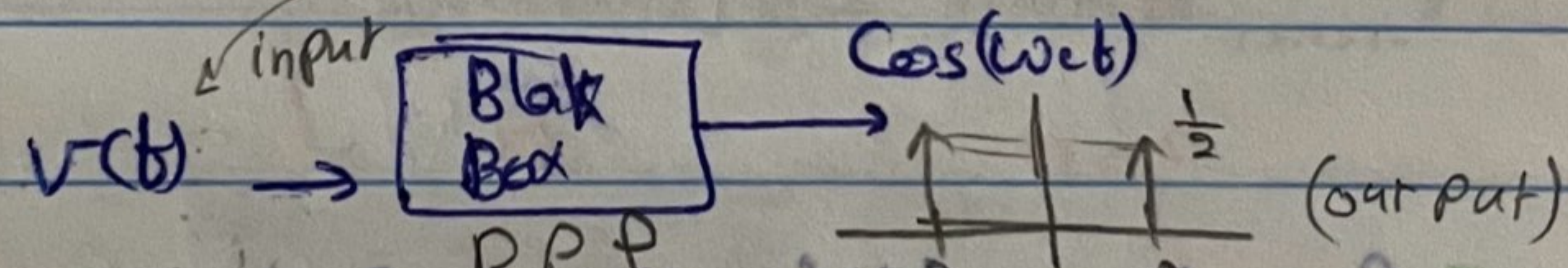
$$v(t) = \cos(\omega_c t) + \sum_{n=1}^{\infty} 2\,\text{sinc}\left(\frac{n}{2}\right) \cos(n\omega_c t)\cos(\omega_c t)$$

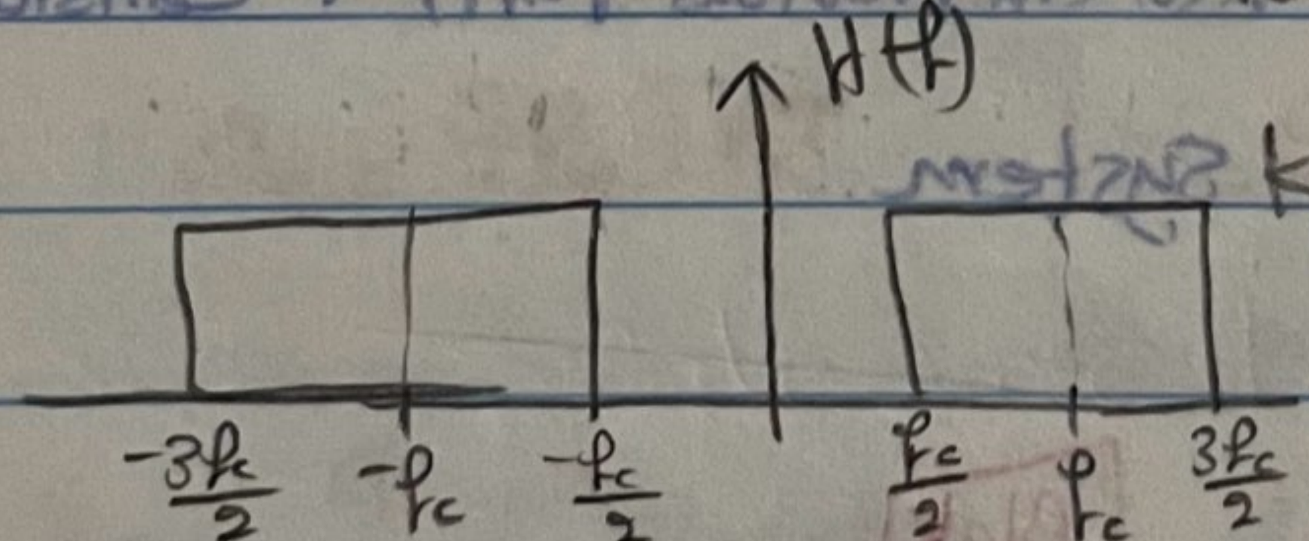
$$v(t) = \cos(\omega_c t) + \sum_{n=1}^{\infty} 2\,\text{sinc}\left(\frac{n}{2}\right) \left[\frac{1}{2}\cos((n+1)\omega_c t) + \frac{1}{2}\cos((n-1)\omega_c t)\right]$$

$$v(t) = \cos(\omega_c t) + \sum_{n=1}^{\infty} \text{sinc}\left(\frac{n}{2}\right)\cos((n+1)\omega_c t) + \sum_{n=1}^{\infty} \text{sinc}\left(\frac{n}{2}\right)\cos((n-1)\omega_c t)$$

$$V(f) = \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c) + \sum_{n=1}^{\infty} \text{sinc}\left(\frac{n}{2}\right)\left[\frac{1}{2}\delta(f-(n+1)f_c) + \frac{1}{2}\delta(f+(n+1)f_c)\right]$$
$$+ \sum_{n=1}^{\infty} \text{sinc}\left(\frac{n}{2}\right)\left[\frac{1}{2}\delta(f-(n-1)f_c) + \frac{1}{2}\delta(f+(n-1)f_c)\right].$$







$$k = \frac{1/2}{1/2} = 1$$

$$H(f) = \Pi\left(\frac{f-f_c}{1}\right) + \Pi\left(\frac{f+f_c}{1}\right)$$

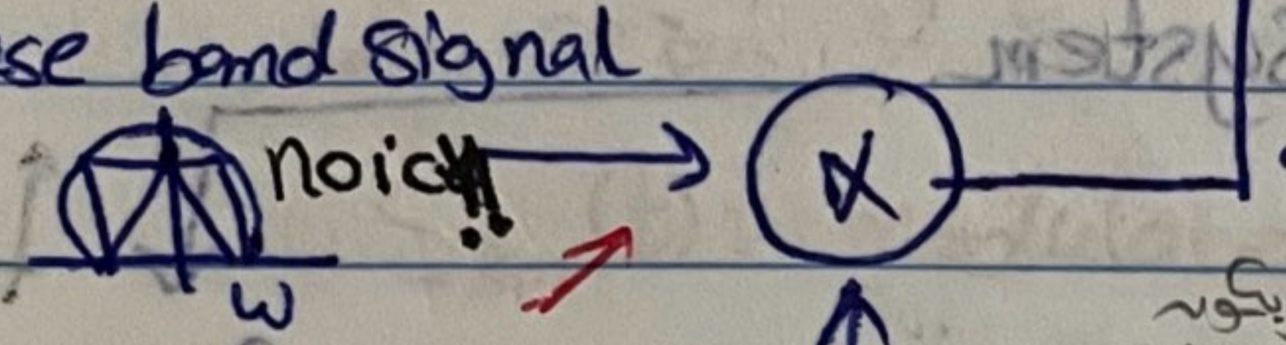
(L7)

- Communication Part -

↳ Analog Com }
↳ Digital Com } → Modulation scheme.

antinna

$\lambda_{antenna} = \frac{1}{2}\lambda$ or $\frac{\lambda}{4}$ (طول الموجة)



S(t): modulated signal.
"Pass band signal". (السنتر بكون فريكونسي عالية)

"message signal" or "Modulating signal"

Carrier
∴ $f_c \gg w$
↳ ($f_c \geq 10w$)

Why Modulation!

- التحكم في ال antenna.
لو في الوضع الطبيعي ال signal الي دائماً بنتقلها الفريكونسي الها قليلة وهذا بيعني انه طول الموجة عالي وطول الموجة عالي يعني طول ال antenna الي حنكون نشتغل عليه كبير بالتالي من الصعب انقل الاشارة عطول على ال antenna فبعمل ال Modul

- Channel: لازم تكون Pass band signal.

- Multiplexing: بقدر أعمل أكثر من Channel ولكل وحدة الها سنتر

→ eg: $m(t) = \cos(3000\pi t)$

$$\lambda = \frac{c}{f} = \frac{3*10^8}{1.5*10^3}$$
(c: سرعة الضوء)

$\lambda = 2*10^5$ } كبير جداً و صعب نقله على ال antenna ← طول

Carrier فبدخل معه قيمة الفريكونسي فيها عالية عشان نقلل ال λ

⇒

(F)

- In general:-

$$C(t) = A_c \cos(\theta) = A_c \cos(\omega_c t + \phi)$$

- A_c ← Amplitude
- θ ← Angle
- ω_c ← freq
- ϕ ← phase.

Com system.

- Analog
  - Amplitude [AM]
  - Angle
    - Freq [FM]
    - Phase [PM]
- Digital "Shift Keying"
  - Amplitude [ASK]
  - Angle
    - Freq. [FSK]
    - Phase [PSK]

- Four types of AM:-
  1. Normal AM
  2. Double sideband suppressed Carrier (DSB)
  3. SSB single side band

(L8)

≡ Normal Amplitude modulation [AM]. "DSB-FC"

عملية حمل للاشارة على الـ Carrier Amp

$$S_{Am}(t) = [1 + K_a m(t)] \cdot A_c \cos(2\pi f_c t).$$

where :-

$S_{Am}(t)$:- modulated signal.

$m(t)$:- modulating signal or message signal.

A_c :- Amplitude of carrier.

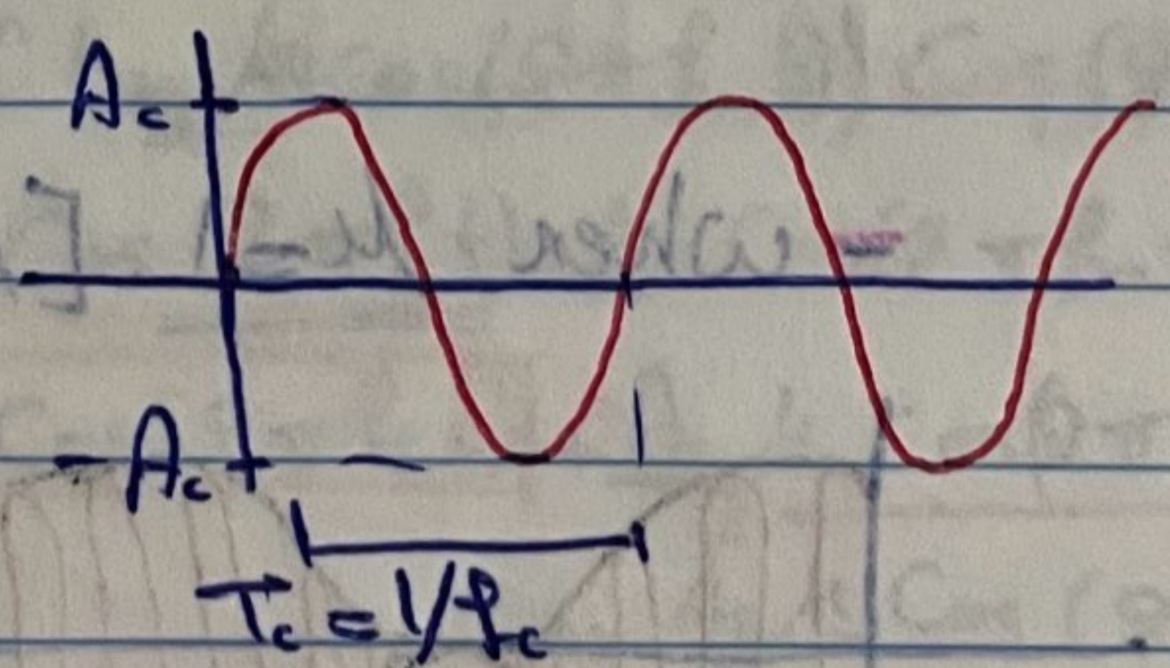
K_a :- modulation sensitivity ($\frac{1}{Volt}$)

Now, let:

$$m(t) = A_m \cos(2\pi f_m t).$$

$$\rightarrow S_{Am}(t) = A_c(1 + K_a A_m \cos(2\pi f_m t)) \cos(2\pi f_c t)$$

• For carrier signal. $c(t) = A_c \cos(2\pi f_c t)$



الـ Amp تاع الـ Carrier بطّل ثابت
صار يتغير عن طريقه :-
$1 + K_a A_m \cos(2\pi f_m t)$

• Now, let us take

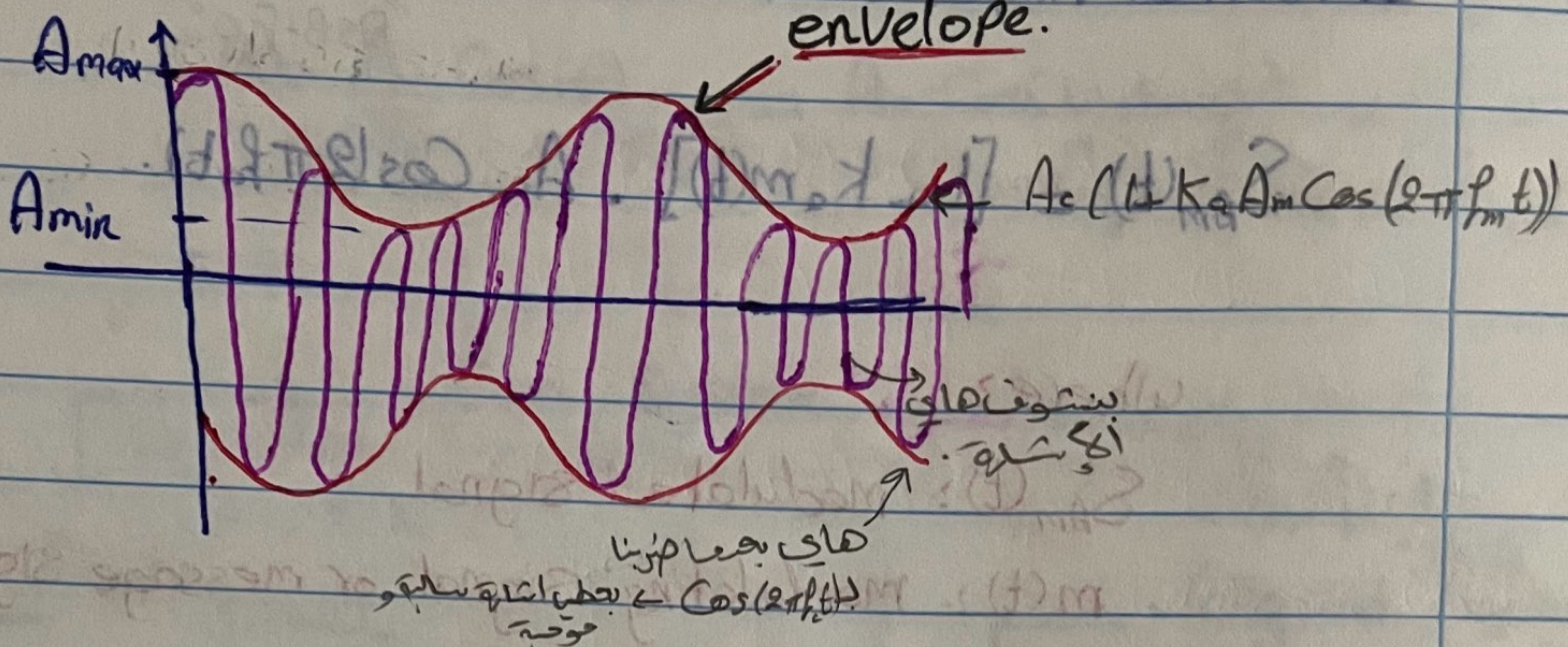
$$|A_c(1 + K_a A_m \cos(2\pi f_m t))|$$
envelop

$(1 + K_a A_m) A_c \rightarrow A_{max}$

$(1 - K_a A_m) A_c \rightarrow A_{min}$

رح يكونو بالطرف الموجب

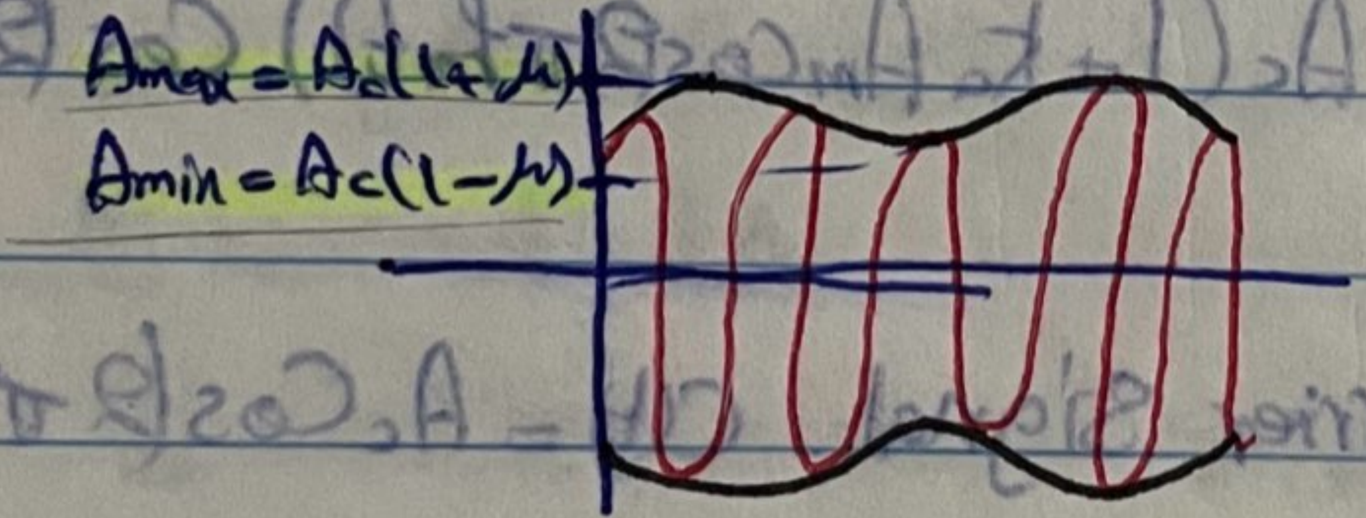
$S_{Am}(t)$:-



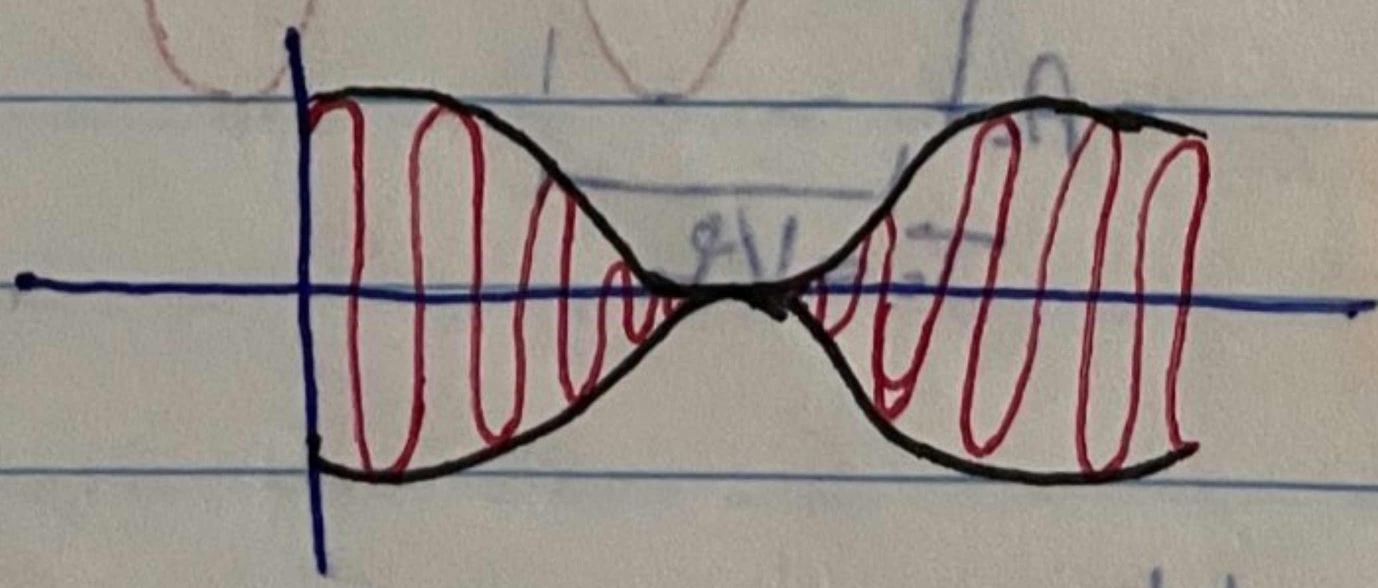
• Now, Let $\mu = K_a A_m$; μ = modulation index.

$$\rightarrow S_{Am}(t) = A_c(1 + \mu \cos(2\pi f_m t)) \cos(2\pi f_c t).$$

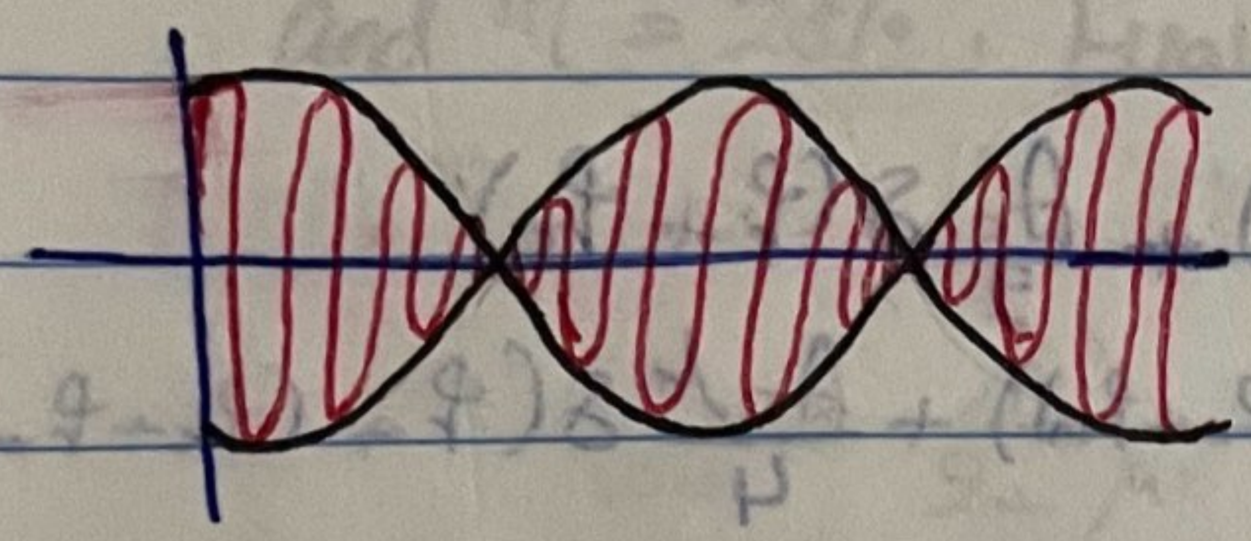
- when $\mu < 1$. [under modulation].



- when $\mu = 1$. [Normal modulation].



– When $\mu > 1$. [over modulation].



في هاي الحالة بقدر أرجع الـ message بتحكم بـ Am بحيث تكون $\mu \leq 1$

$\{0 < \mu \leq 1\}$ ← هاي الحالة بضمن اني أحصل على السيجنال الأصلية

- To evaluate modulation index. (μ)

[1] $|A_{max} = A_c(1+\mu)|$ [From figure].

$|A_{min} = A_c(1-\mu)|$

$$\Rightarrow \frac{A_{max}}{A_{min}} \times \frac{1+\mu}{1-\mu} \Rightarrow A_{max} - \mu A_{max} = A_{min} + \mu A_{min}$$

$$A_{max} - A_{min} = \mu A_{max} + \mu A_{min}$$

$$\Rightarrow \boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

[2] $\mu = K_a A_m$ [From $S_{Am}(t)$].

- To evaluate Power.

$$S_{Am}(t) = A_c(1 + \mu \cos(2\pi f_m t)) \cos(2\pi f_c t).$$

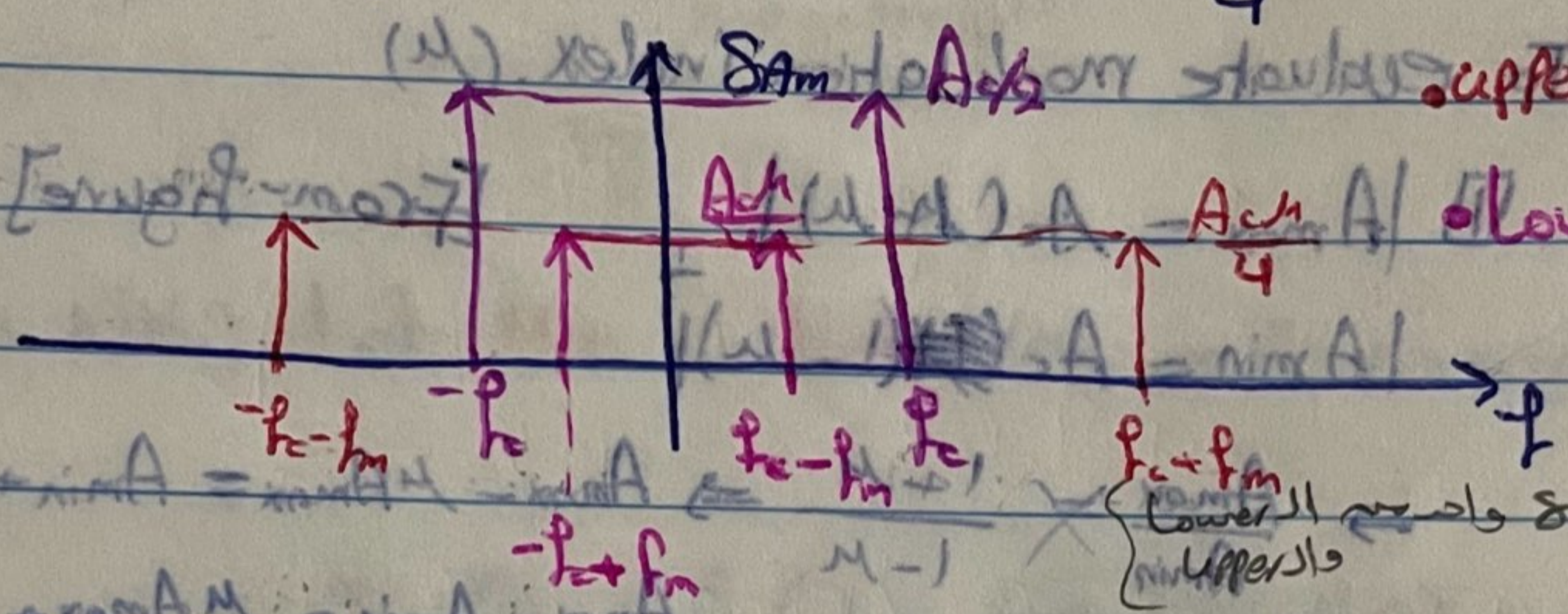
$$= A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_m t) \cos(2\pi f_c t)$$

$$= \underbrace{A_c \cos(2\pi f_c t)}_{\rightarrow P_c} + \underbrace{\frac{A_c \mu}{2} \cos(2\pi(f_c + f_m)t)}_{P_{side_1} \equiv P_{USB}}$$

$$+ \underbrace{\frac{A_c \mu}{2} \cos(2\pi(f_c - f_m)t)}_{P_{side_2} \equiv P_{LSB}}.$$

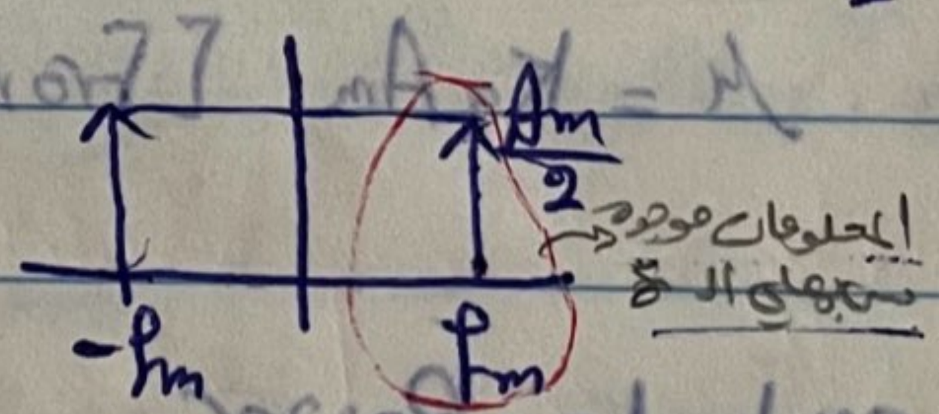
• To evaluate and sketch the spectrum of modulated signal.

$$S_{AM}(f) = \frac{A_c}{2}\delta(f-f_c) + \frac{A_c}{2}\delta(f+f_c) + \frac{A_c\mu}{4}\delta(f-(f_c+f_m)) + \frac{A_c\mu}{4}\delta(f+(f_c+f_m)) + \frac{A_c\mu}{4}\delta(f-(f_c-f_m)) + \frac{A_c\mu}{4}\delta(f+(f_c-f_m))$$



Since $m(t) = A_m \cos(2\pi f_m)$

$$\rightarrow m(f) = \frac{A_m}{2}\delta(f-f_m) + \frac{A_m}{2}\delta(f+f_m)$$



• $P_{total} = P_c + P_{USB} + P_{LSB}$

• $P_c = \frac{A_c^2}{2}$

• $P_{USB} = \frac{(A_c\mu/2)^2}{2}$

• $P_{LSB} = \frac{(A_c\mu/2)^2}{2}$

power.

بتحدد لي انا مقدار البور الي فيها كفيله اني افهم منها الـ Data → efficiency (η) :-

$$\eta = \frac{P_{sides}}{P_{total}} = \frac{P_{USB} + P_{LSB}}{P_c + P_{USB} + P_{LSB}} \times 100\%$$

$$= \frac{2\frac{(A_c\mu/2)^2}{2}}{\frac{A_c^2}{2} + (A_c\mu/2)^2} \rightarrow \boxed{\eta = \frac{\mu^2}{2+\mu^2}} \times 100\%$$

(P_1)

eg. if $m(t) = 2\cos(20\pi t)$, $c(t) = 4\cos(100\pi t)$.

and $\eta = 20\%$. Evaluate μ.

$$\frac{20}{100} = \frac{\mu^2}{2+\mu^2} \rightarrow \frac{1}{5} = \frac{\mu^2}{2+\mu^2}$$

$$2 + \mu^2 = 5\mu^2 \rightarrow 2 = 4\mu^2$$

$$\sqrt{\mu^2} = \sqrt{\frac{1}{2}}$$

$$\Rightarrow \mu = \sqrt{0.5}$$

$$\boxed{\mu = 0.71}$$

- Evaluate k_a.

$$\mu = k_a A_m \rightarrow 0.71 = k_a 2$$

$$\Rightarrow k_a = 0.35$$

• if $\mu = 1$ $\Rightarrow$ $\eta_{100\%} = \frac{1}{2+1} \times 100\% \Rightarrow \boxed{\eta = 33\%}$

(L9)

$$S_{AM}(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$$

Now, Let assume $m(t) = A_m \cos(2\pi f_m t)$.

$$\Rightarrow S_{AM}(t) = A_c(1 + k_a A_m \cos(2\pi f_m t)) \cos(2\pi f_c t)$$
$$= A_c(1 + \mu \cos(2\pi f_m t)) \cos(2\pi f_c t).$$

- Power efficiency.

كمية البور الي موجود بالجزء الي بحاكي المسج بالنسبة للبور الكلي.

$$\eta_{eff} = \frac{P_{sides}}{P_{total}} = \frac{P_{sides}}{P_c + P_{sides}}.$$

$$\Rightarrow S_{Am}(t) = A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_m t) \cos(2\pi f_c t).$$
$$= \underbrace{A_c \cos(2\pi f_c t)}_{P_c} + \underbrace{\frac{A_c \mu}{2} \cos(2\pi(f_c + f_m)t) + \frac{A_c \mu}{2} \cos(2\pi(f_c - f_m)t)}_{P_{sides}}$$

To evaluate power → Time-domain
↳ freq- domain

- In Time domain :-

$$P_c = \frac{A_c^2}{2}.$$

P_{sides} → $P_{USB} = \frac{(A_c \mu/2)^2}{2}$
↳ P_{LSB} $\frac{(A_c \mu/2)^2}{2}$

⇒

$$\eta_{eff\%} = \frac{\frac{2(A_c \mu/2)^2}{2}}{\frac{A_c^2}{2} + \frac{2(A_c \mu/2)^2}{2}} \times 100\%$$

$$= \frac{(A_c \mu/2)^2}{\frac{A_c^2}{2} + (\mu A_c/2)^2} \rightarrow \frac{A_c^2 \mu^2/4}{\frac{2 \cdot A_c^2}{2 \cdot 2} + \frac{A_c^2 \mu^2}{4}}$$

$$\boxed{\eta_{eff\%} = \frac{\mu^2}{2+\mu^2} \times 100\%}$$

For $\mu = 1$ أفضل حالة في قيمة البور بقدر نوصلها

$\eta_{eff\%} = 33.\overline{33}\%$ كل ما كانت η_{eff} عالية كل ما كانت البور الي بتجينا من المسج سجنال أعلى.

Very Small!

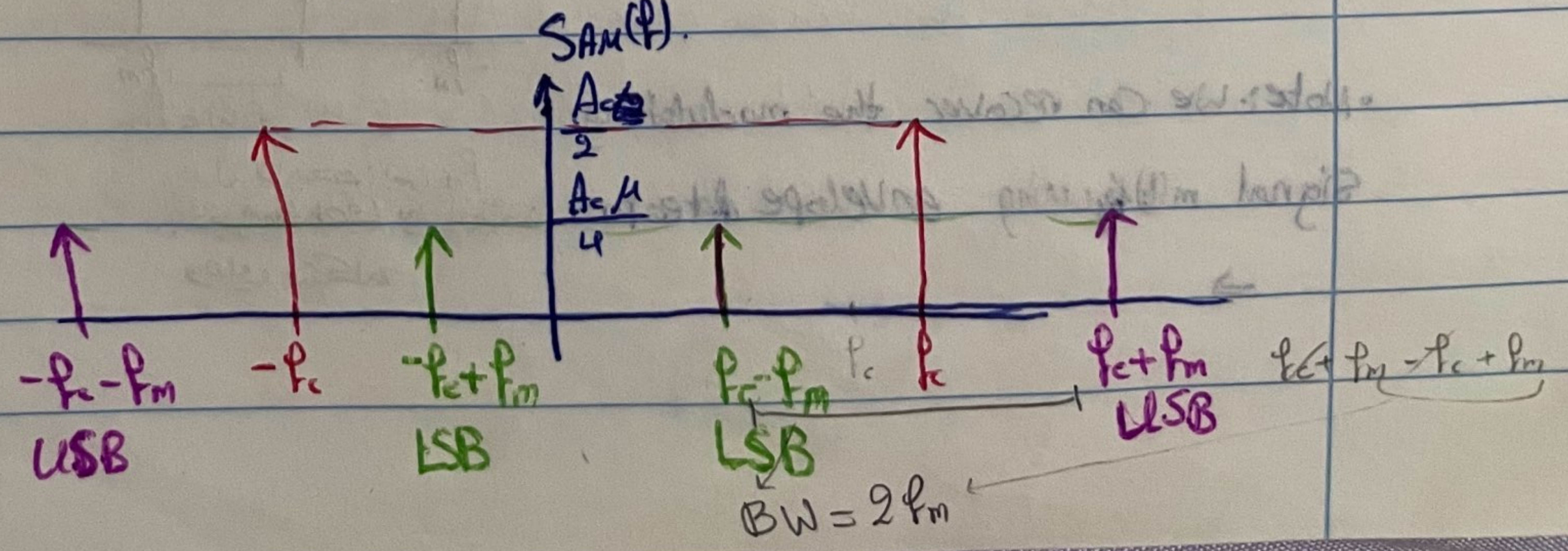
- In freq domain.

$$S_{AM}(t) = A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2} \cos(2\pi(f_c + f_m)t)$$

$$+ \frac{A_c \mu}{2} \cos(2\pi(f_c - f_m)t)$$

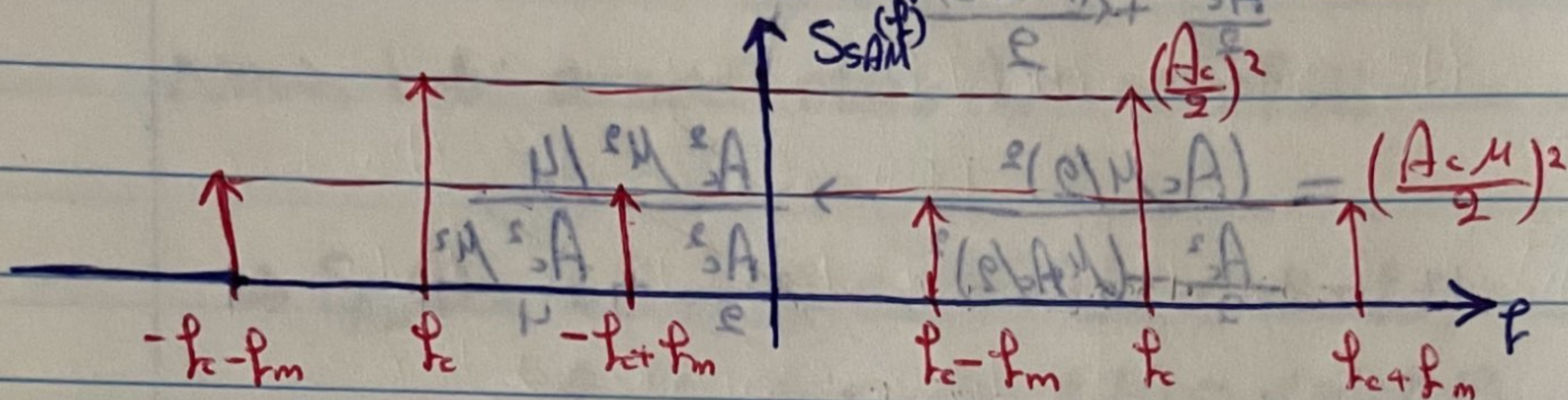
$$\rightarrow S_{AM}(f) = \frac{A_c}{2}\delta(f - f_c) + \frac{A_c}{2}\delta(f + f_c) + \frac{A_c \mu}{4}\delta(f - (f_c + f_m))$$

$$+ \frac{A_c \mu}{4}\delta(f + (f_c + f_m)) + \frac{A_c \mu}{4}\delta(f - (f_c - f_m)) + \frac{A_c \mu}{4}\delta(f + (f_c - f_m))$$



→ To evaluate Power efficiency from freq. domain

- Evaluate and plot PSD



$$S_{sAm}(f) = \left(\frac{A_c}{2}\right)^2\delta(f-f_c) + \left(\frac{A_c}{2}\right)^2\delta(f+f_c)$$
$$+\left(\frac{A_c\mu}{2}\right)^2\delta(f-(f_c-f_m)) + \left(\frac{A_c\mu}{2}\right)^2\delta(f+(f_c-f_m))$$
$$+\left(\frac{A_c\mu}{2}\right)^2\delta(f-(f_c+f_m)) + \left(\frac{A_c\mu}{2}\right)^2\delta(f+(f_c+f_m))$$

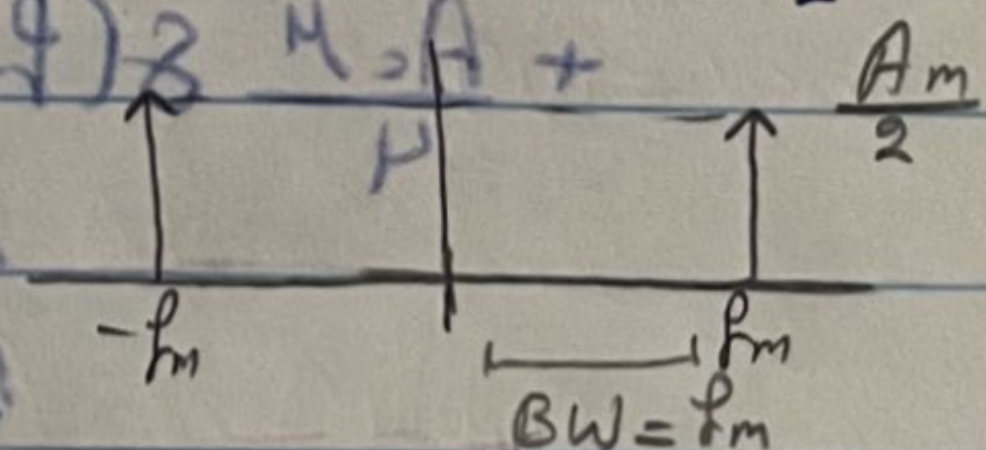
4 إشارات →

$$\eta_{eff} = \frac{P_{sides}}{P_c + P_{sides}} \times 100\% = \frac{4\left(\frac{A_c\mu}{4}\right)^2}{2\left(\frac{A_c}{2}\right)^2 + 4\left(\frac{A_c\mu}{4}\right)^2} \times 100\%$$

$$\eta_{eff} = \frac{\mu^2}{2+\mu^2} \times 100\%$$

- Remember:-

base band signal → $m(t) = A_m\cos(2\pi f_m t) \rightarrow M(f) = \frac{A_m}{2}\delta(f-f_m) + \frac{A_m}{2}\delta(f-f_m)$.



- Note:- We can recover the modulating signal m(t) using envelope detector.

بدل ما نحجز بس f_m بنحجز الضعف (2Channel) وهاي مشكلة.

→

Example| if $m(t) = 4\cos(30\pi t)$ and $c(t) = 2\cos(300\pi t)$.

Assume we have normal AM. where the power efficiency at this modulation scheme is 20%. Evaluate the amplitude sensitivity "K_a".

Ans:- For normal AM

$$\eta_{\%} = \frac{\mu^2}{2+\mu^2} = 0.2$$

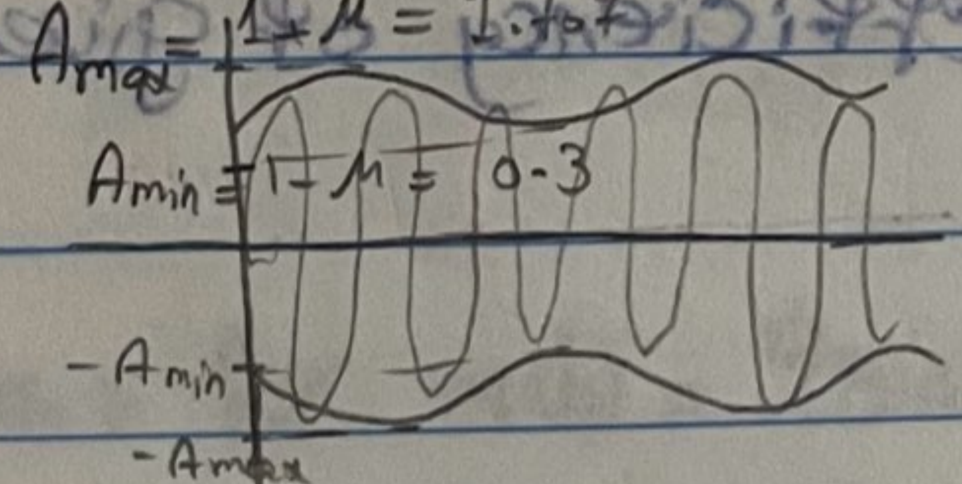
$$0.4 + 0.2\mu^2 = \mu^2 \rightarrow \frac{0.4}{0.8} = \frac{0.8}{0.8}\mu^2$$

$$\Rightarrow \boxed{\mu = 0.707} \text{ "under"}$$

Since:-

$$\mu = K_a A_m \Rightarrow K_a = \frac{\mu}{A_m} = \frac{0.707}{4}$$

$$\Rightarrow \boxed{K_a = 0.177}$$



Example 1 Consider Normal AM modulation scheme if the modulating signal [1] $m(t) = 2\cos(20\pi t)$

[2] $m(t) = 2\,\text{sinc}^2(2t)$

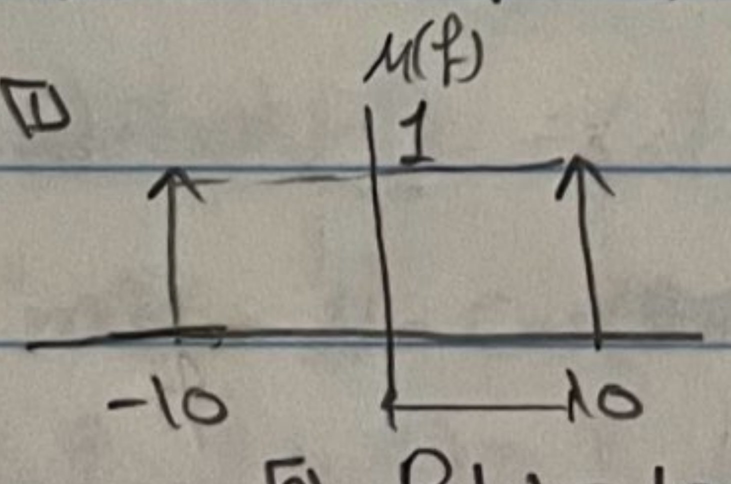
and $c(t) = 4\cos(200\pi t)$.

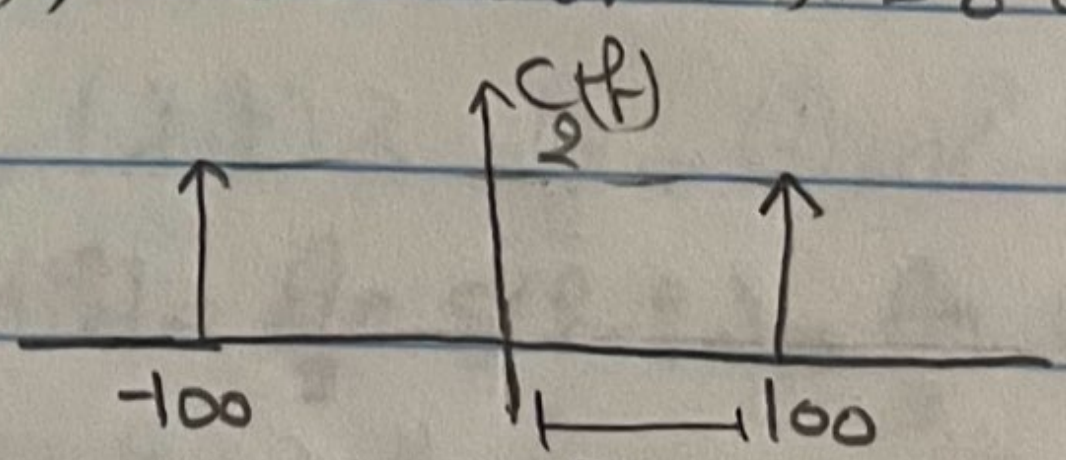
- Evaluate and plot the spectrum of modulating signal & carrier signal
- Evaluate the BW of modulating & carrier signals.
- Write the expression of the modulated signal. Assume the amplitude sensitivity ($K_a = 1$)
- Evaluate & plot the spectrum of modulated signal
- Evaluate the BW of modulated signal
- Evaluate the power efficiency of given modulation scheme.

Ans: For $m(t) = 2\cos(20\pi t)$ & $c(t) = 4\cos(200\pi t)$

$M(f) = \delta(f-10) + \delta(f+10)$, $C(f) = 2\delta(f-100) + 2\delta(f+100)$

[1]





[2] BW = 10 BW = 100

[3] Since we have Normal AM

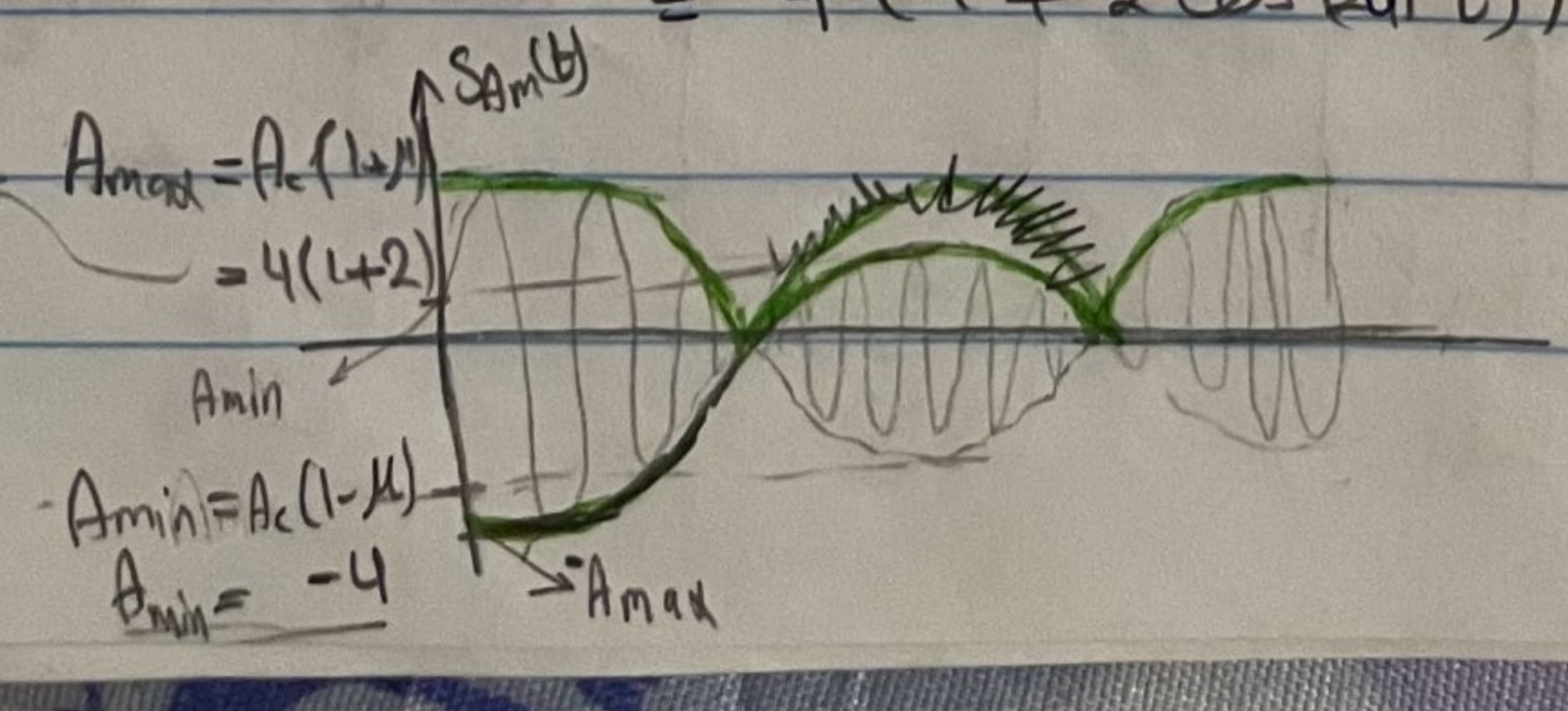
$S_{AM}(t) = A_c(1 + K_a m(t))\cos(2\pi f_c t)$

$= 4(1 + 2\cos(20\pi t))\cos(200\pi t)$.

$\mu = K_a A_m = (1)(2)$

$\mu = 2$

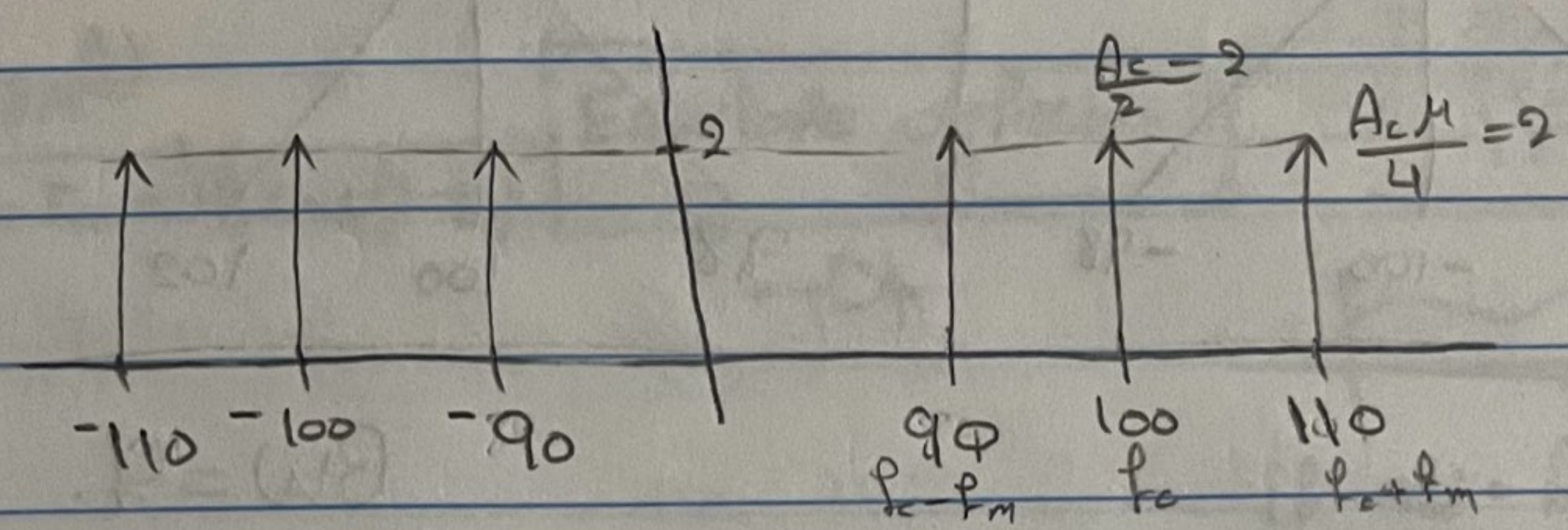
$A_{max} = 12$



4

$$S_{AM}(t) = 4\cos(200\pi t) + 8\cos(20\pi t)\cos(200\pi t)$$
$$= 4\cos(200\pi t) + 4\cos(220\pi t) + 4\cos(180\pi t)$$

$$S_{AM}(f) = 2\delta(f-100) + 2\delta(f+100) + 2\delta(f-110) + 2\delta(f+110)$$
$$+ 2\delta(f-90) + 2\delta(f+90)$$



5 $BW = 110 - 90 = 20\ Hz$

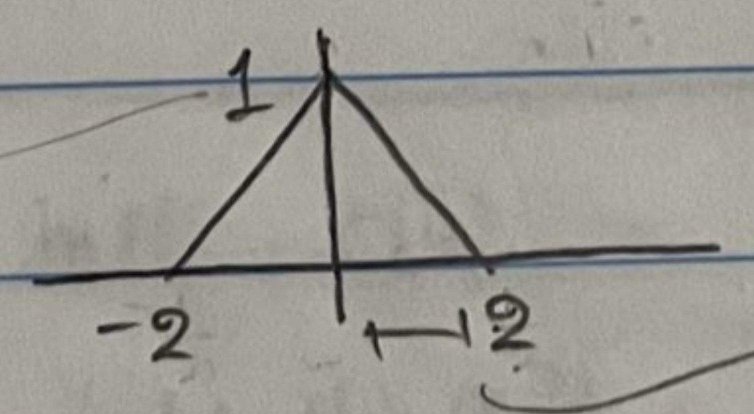
or $BW = 2 f_m$.

5 $$\eta_{eff} = \frac{\mu^2}{2+\mu^2} \times 100\% = \frac{(2)^2}{2+(2)^2} \times 100\% = \frac{4}{6} \times 100\%$$

$$\eta_{eff} = 66.\bar{6}\ \%$$

Ans: for $m(t) = 2\,\text{sinc}^2(2t)$, $c(t) = 2\cos(200\pi t)$

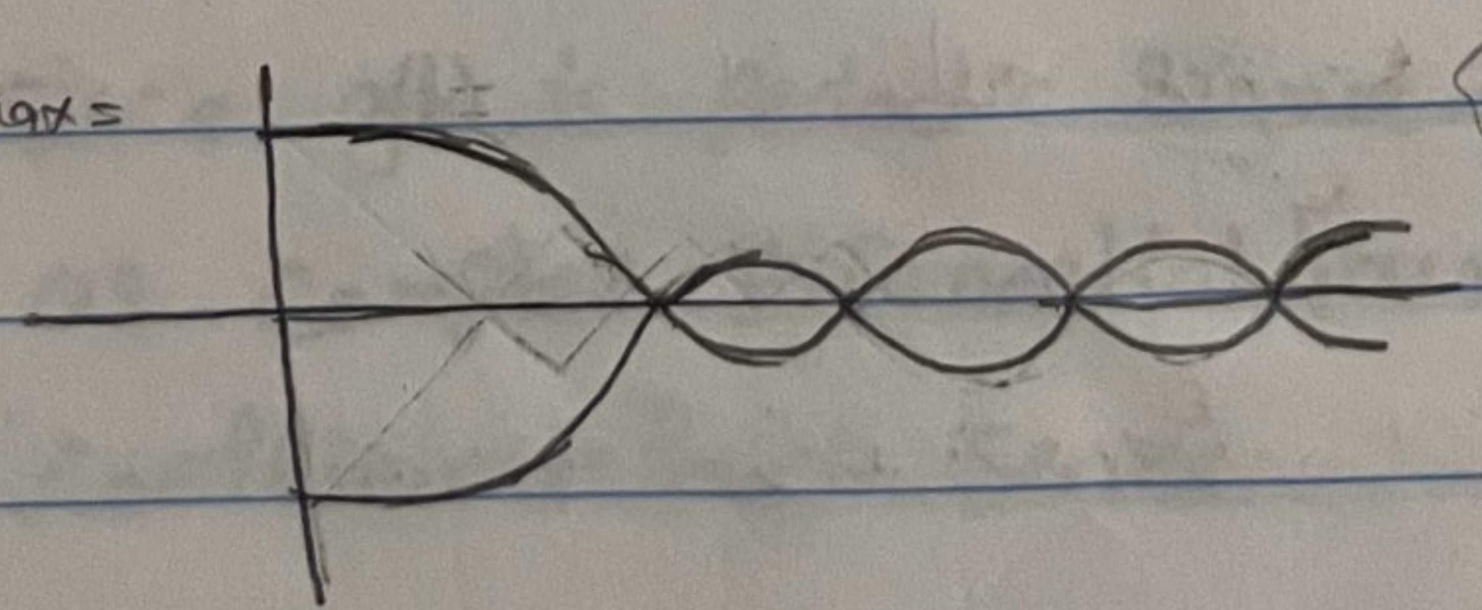
1 + 2 $M(f) = 2 \cdot \frac{1}{2} \Lambda\left(\frac{f}{2}\right) \rightarrow M(f) = \Lambda\left(\frac{f}{2}\right)$



$BW = 2$

3 $S_{AM}(t) = 4(1 + 2\,\text{sinc}^2(2t))\cos(200\pi t)$ $\mu = (1)(2) = 2$

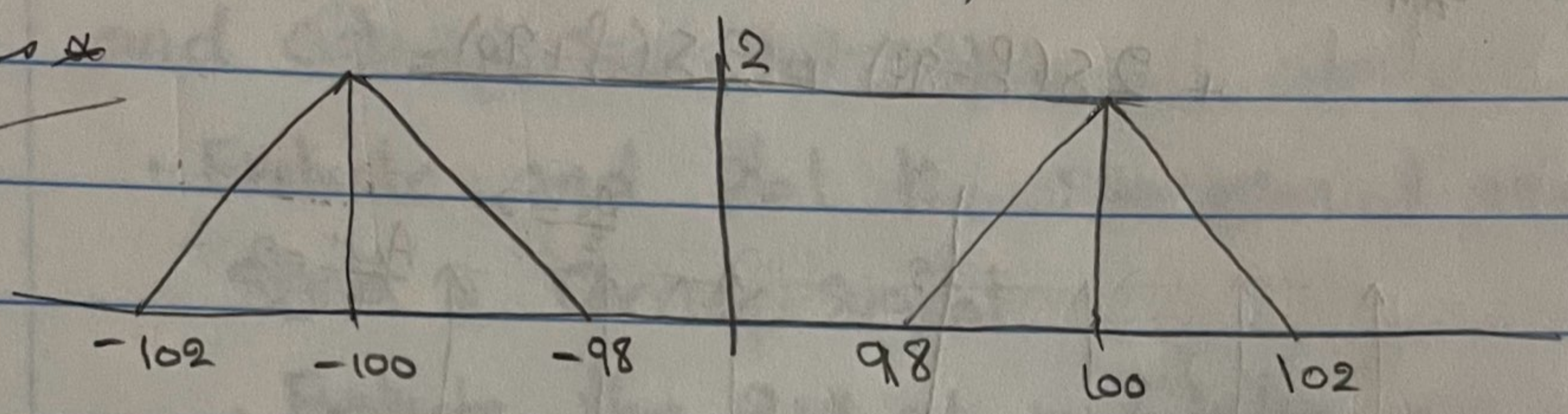
$A_{max} =$



* اذا كان الwave cos بكون الشكل كوساين
* لذا كان Λ بكون مثلث
* اذا كان Π بكون Π .

$$S_{AM}(t) = 4\cos(200\pi t) + 8\,\text{sinc}^2(2t)\cos(200\pi t)$$

$$S_{AM}(f) = 2\delta(f-100) + 2\delta(f+100) + 2\Lambda\left(\tfrac{1}{2}(f-100)\right) + 2\Lambda\left(\tfrac{1}{2}(f+100)\right)$$



$BW = 4.$

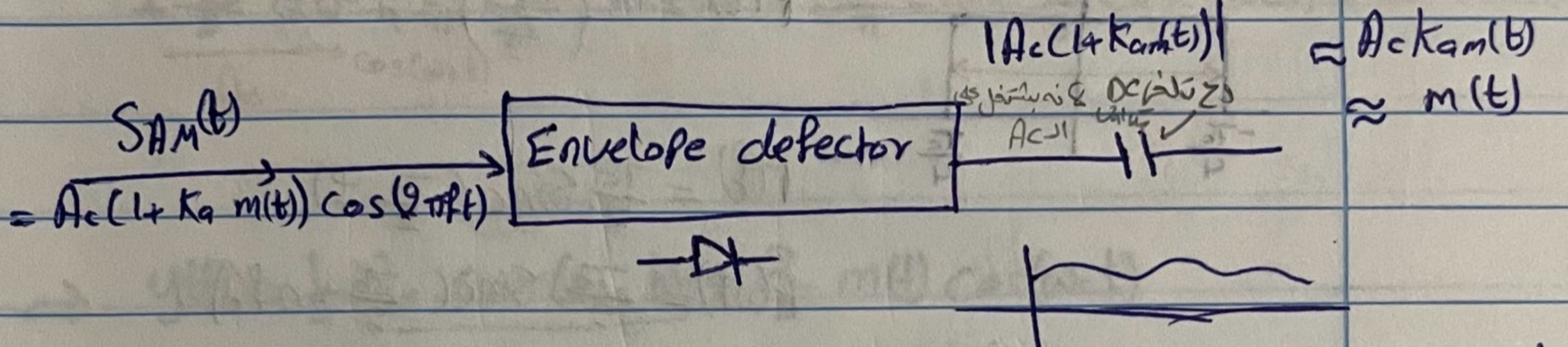
$$\mu_{eff} = \frac{\mu^2}{2+\mu^2} = \frac{2^2}{2+2^2}$$

$$\Rightarrow \mu_{eff} = 66.\bar{6}\ \%.$$

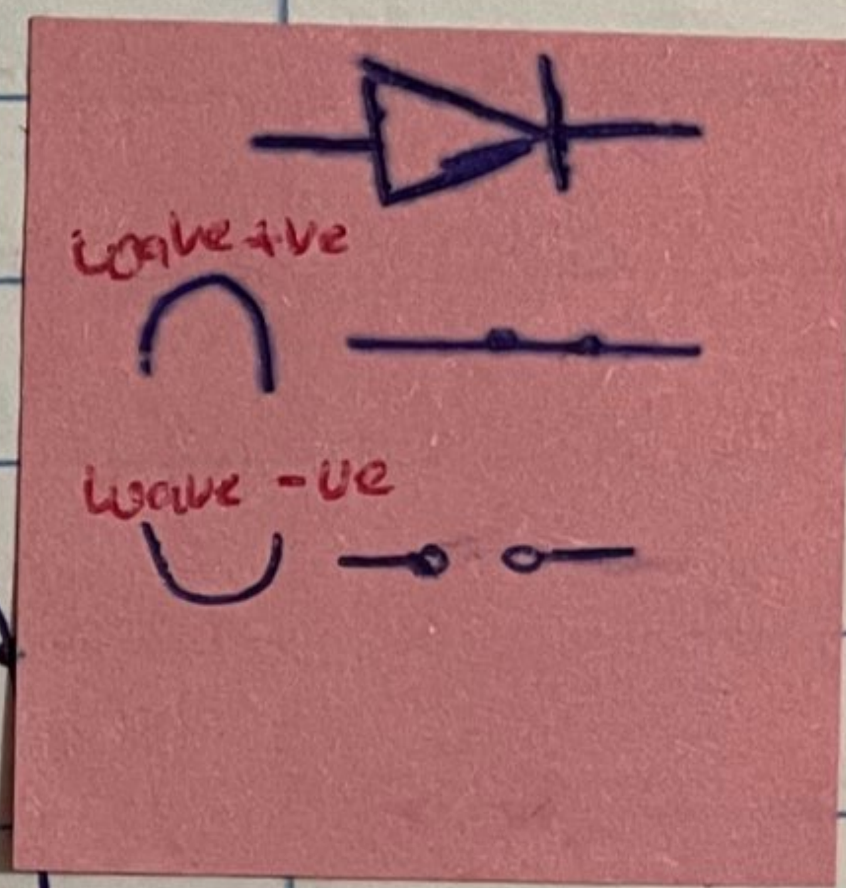
(L10)

L5 New.

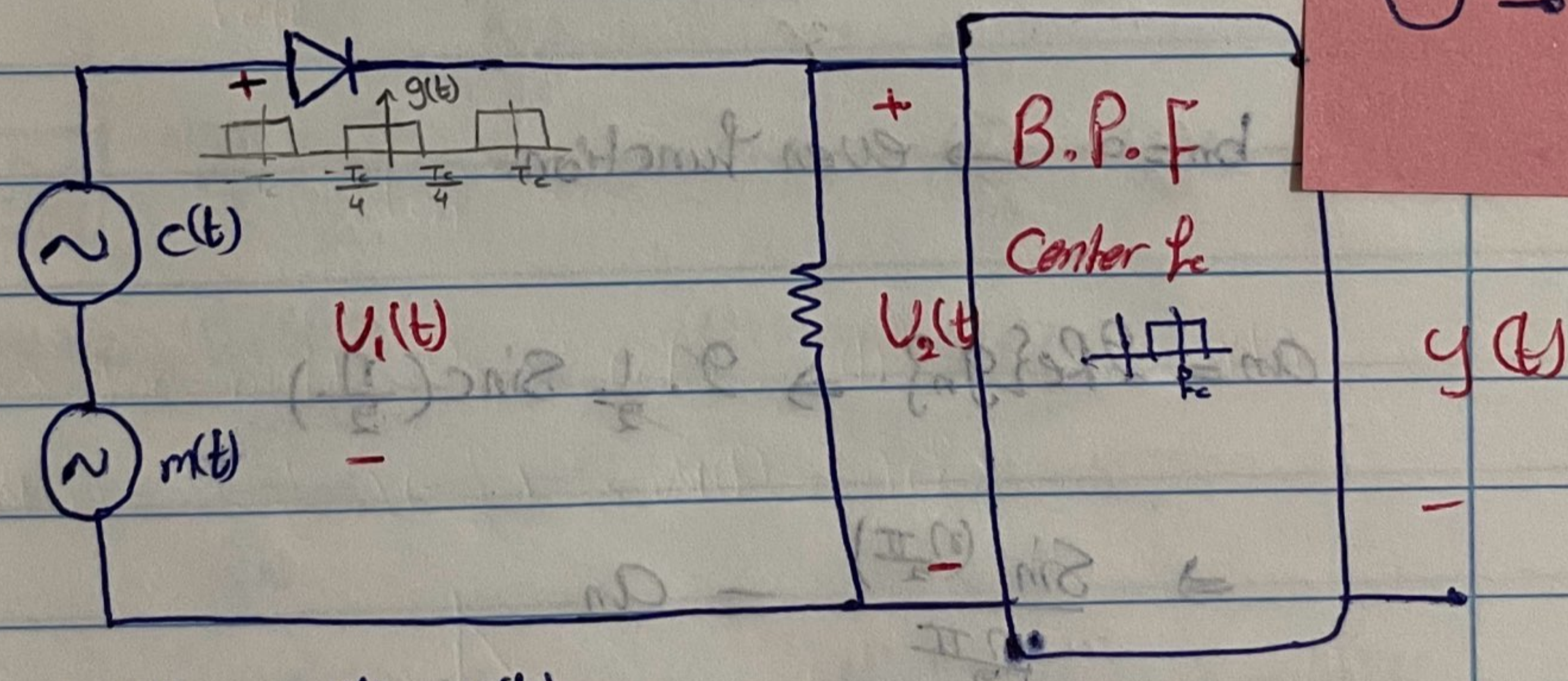
- Note: we can recover the modulation signal $m(t)$, using envelope detector.



if $m(t) = A_m\cos(2\pi f_m t)$, $\mu = k_a A_m < 1$



= Generation Normal AM.



$V_1(t) = m(t) + c(t)$

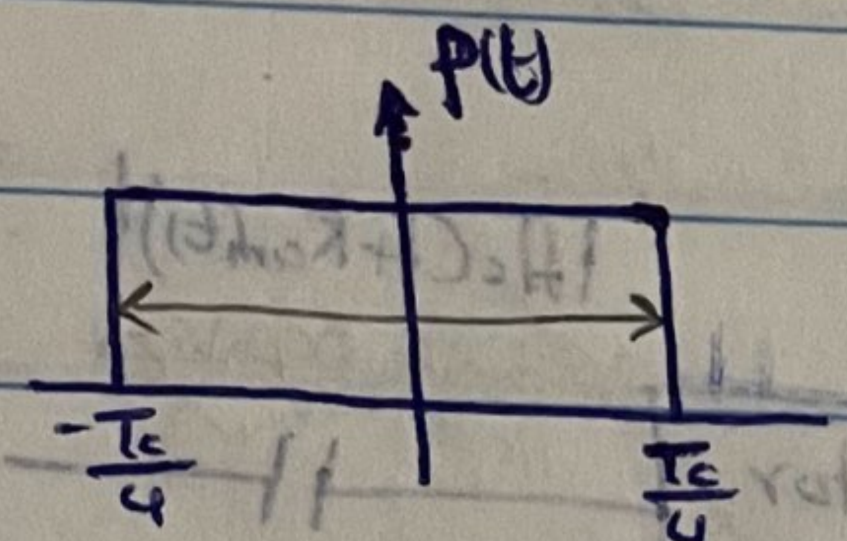
$V_2(t) = V_1(t)\, g(t)$

Since $g(t)$ is periodic signal ⇒ We can express $g(t)$. as complex Exponential Fourier Series. or Trignometric coefficient fourir Series.

⇒

by Exponential Fourier Series.

$$g(t) = \sum_{n=-\infty}^{\infty} g_n e^{j2\pi n f_c t} \quad ; \quad g_n = f_c P(nf_c)$$



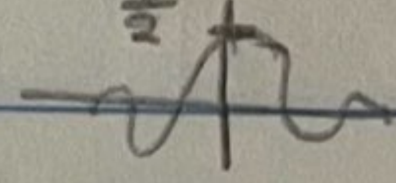
$$p(t) = \Pi\left(\frac{t}{\frac{T_c}{2}}\right) \quad ; \quad \tau = \frac{T_c}{2}$$

$$P(f) = \frac{T_c}{2} \operatorname{sinc}\left(\frac{T_c}{2} f\right)$$

$$P(nf) = \frac{T_c}{2} \operatorname{sinc}\left(\frac{T_c}{2} \cdot n f_c\right)$$

$$\rightarrow g_n = f_c \cdot \frac{T_c}{2} \operatorname{sinc}\left(\frac{n}{2}\right)$$

$$\rightarrow \boxed{g_n = \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right)}$$



$$a_0 = g_0 \quad (\tfrac{1}{2}\operatorname{sinc}(0) = 1) \rightarrow \boxed{a_0 = \frac{1}{2}}$$

$b_n = 0 \rightarrow$ even function.

$$a_n = 2\operatorname{Re}\{g_n\} \Rightarrow 2 \cdot \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right)$$

$$\Rightarrow \frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}} = a_n$$

أي قيمة لـ even(n) يكون الجواب صفر.

$$a_1 = \frac{\sin\left(\frac{\pi}{2}\right)}{\pi/2} \Rightarrow \frac{2}{\pi} \quad , \quad a_2 = 0$$

$$a_3 = \frac{\sin\left(\frac{3\pi}{2}\right)}{3\pi/2} \Rightarrow \frac{-2}{3\pi} \quad , \quad a_4 = 0$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) \quad (b_n \rightarrow 0)$$

$$g(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_c t) - \frac{1}{3}\cos(3\omega_c t) + \text{———}\right)$$

$$V_2(t) = (m(t) + c(t))\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_c t) - \frac{1}{3}(3\omega_c t) + \dots\right)\right]$$

$$= \frac{1}{2} m(t) + \frac{2}{\pi} m(t)\cos(\omega_c t) + \frac{2}{\pi} m(t) \cdot \frac{-1}{3}\cos(3\omega_c t) + \dots$$

$$+ \frac{1}{2} c(t) + \frac{2}{\pi} c(t)\cos(\omega_c t) + \frac{2}{\pi} c(t) \cdot \frac{-1}{3}\cos(3\omega_c t) + \dots$$

$c(t) \rightarrow \cos(\omega_c t)$

A_c

$c(t)$

$$\Rightarrow y(t) = \frac{1}{2} A_c \cos(\omega_c t) + \frac{2}{\pi} m(t)\cos(\omega_c t)$$

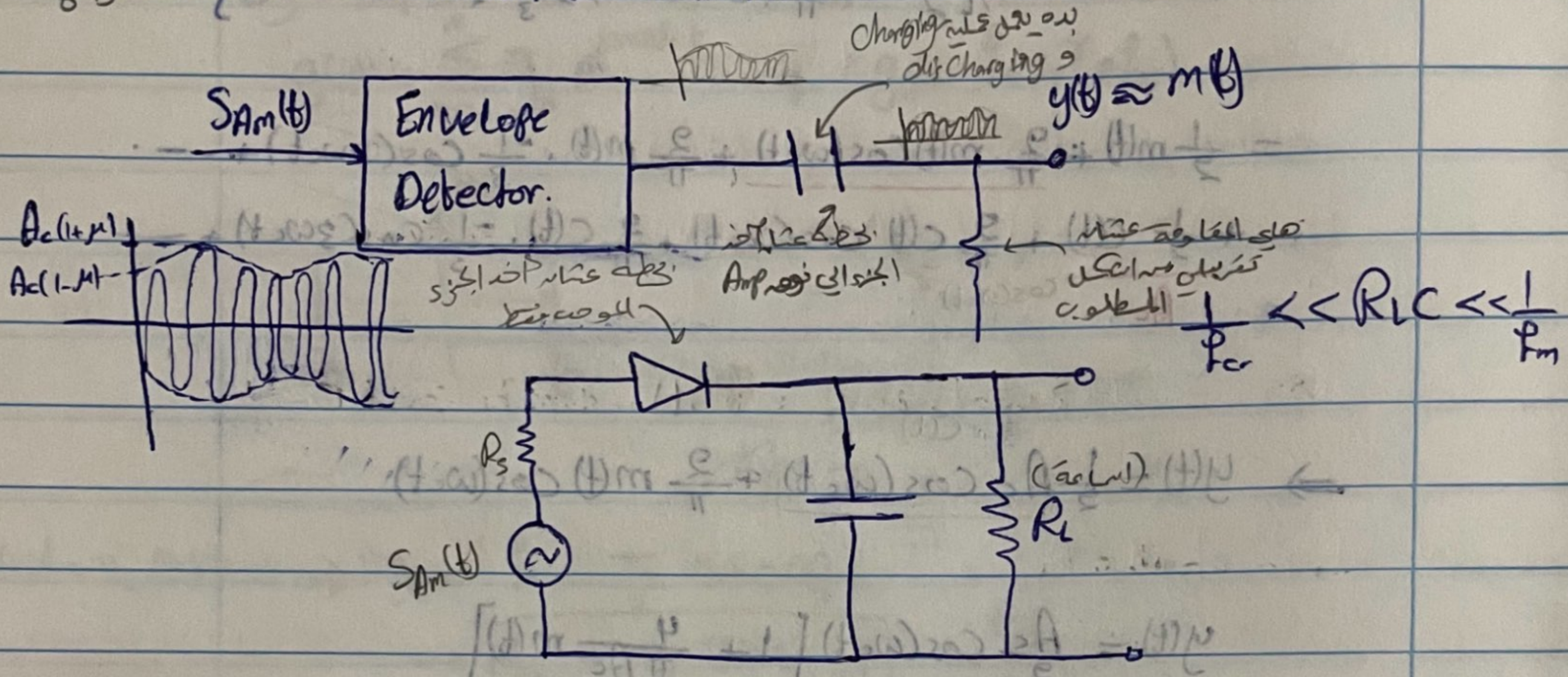
$$y(t) = \frac{A_c}{2}\cos(\omega_c t)\left[1 + \frac{4}{\pi A_c} m(t)\right]$$

Since $S_{AM}(t) = A_c(1 + k_a m(t))\cos(\omega_c t)$. ($\omega_c = 2\pi f_c$)

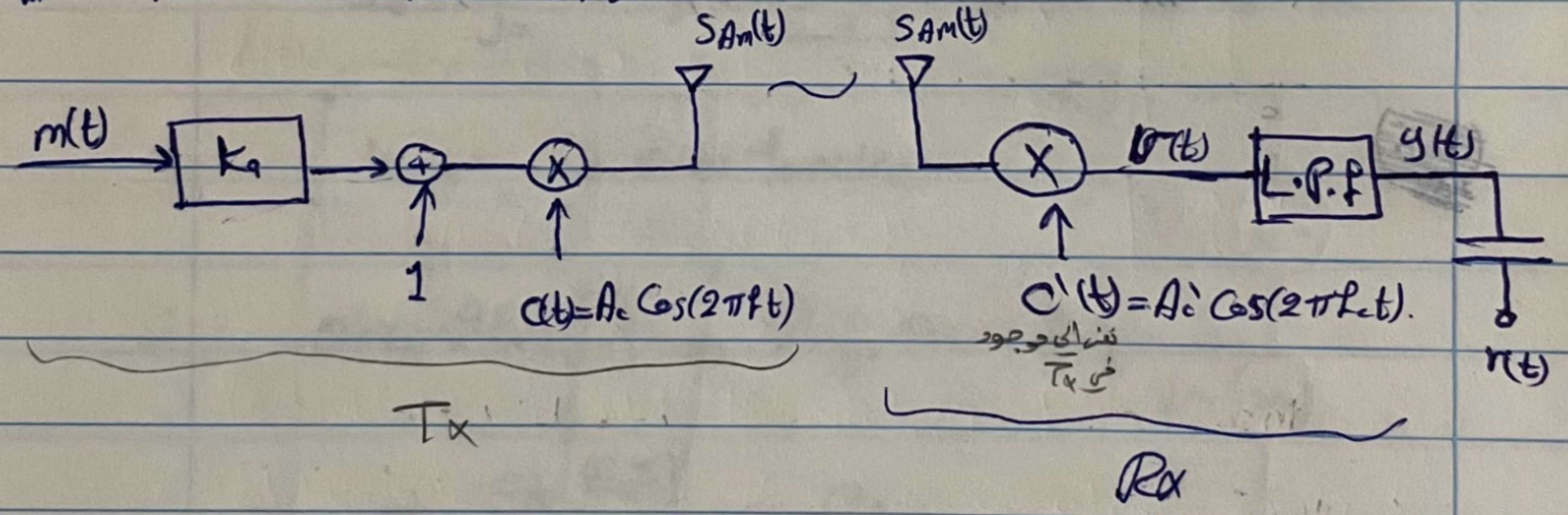
مشكلتين SAm ← الـ efficiency
بحجز $2f_m$ عشان أنقل f_m وحده

(L6) new

تعملية استرجاع للإشارة { Demodulation Normal AM.



$S_{Am}(t) = A_c(1 + k_a m(t))\cos(2\pi f t)$.



$$v(t) = S_{Am}(t) \cdot c'(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t) \cdot A_c'\cos(2\pi f_c t)$$

$$= A_c A_c'(1 + k_a m(t))\left[\frac{1}{2} + \frac{1}{2}\cos(4\pi f_c t)\right]$$

$$\Rightarrow v(t) = \frac{A_c A_c'}{2}(1 + k_a m(t)) + \frac{A_c A_c'}{2}(1 + k_a m(t))\cos(4\pi f_c t)$$

بدي ألغي هاي الإشارة ← بدي أدخل هاي الإشارة

بدخلهم على

L.P.F

$$\Rightarrow y(t) = \frac{1}{2}A_c A_c'(1 + k_a m(t)) \xrightarrow{\text{DC Value}} \dashv\vdash \rightarrow r(t) = \frac{1}{2}A_c A_c' k_a m(t)$$

عشان أتخلص من الـ Constant

حصلنا على نفس الـ m(t) ولكن الأمبليتيود بختلف.

Example The amplitude of a sinusoidal carrier is modulated by a single sinusoidal to obtain the amplitude modulated signal $S_{Am}(t) = 1.5\cos(900\pi t) + 3\cos(1000\pi t) + 1.5\cos(1100\pi t)$

(L.S.B = $1.5\cos(900\pi t)$, Carrier = $3\cos(1000\pi t)$, USB = $1.5\cos(1100\pi t)$)

Note:- $S_{Am}(t) = A_c(1 + K_a m(t))\cos(2\pi f_c t) = A_c\cos(\omega_c t) + K_a m(t)\cos(2\pi f_c t)$

[1] Evaluate the modulation index

By using $S_{Am}(t) = A_c\cos(2\pi f_c t) + K_a m(t)\cos(2\pi f_c t)$ and

$m(t) = A_m\cos(2\pi f_m t)$

$\rightarrow S_{Am} = A_c\cos(2\pi f_c t) + A_c K_a A_m \cos(2\pi f_m t)\cos(2\pi f_c t)$

متساوية

$= A_c\cos(2\pi f_c t) + \frac{A_c\mu}{2}\cos(2\pi(f_c+f_m)t) + \frac{A_c\mu}{2}\cos(2\pi(f_c-f_m)t)$

$\Rightarrow A_c = 3.$

$\frac{A_c\mu}{2} = 1.5$, $A_c = 3$

$\frac{3}{2}\mu = 1.5 \Rightarrow \boxed{\mu = 1}$ normal modulation

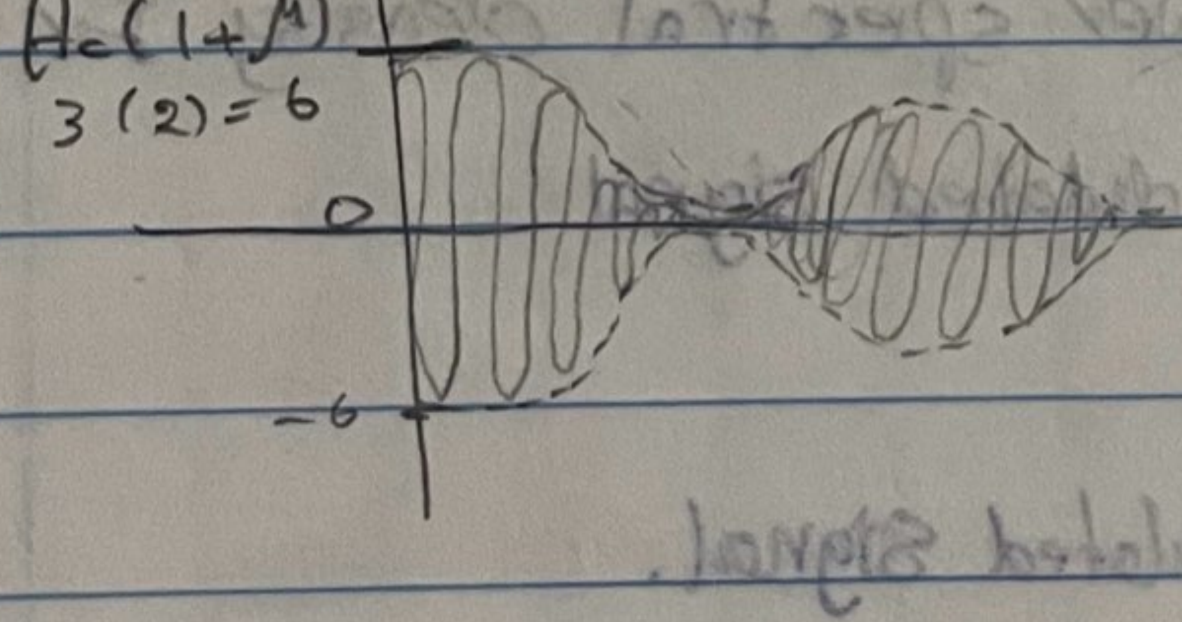
[2] Plot the modulated signal in time domain.

since $\mu = 1$, $A_{max} = A_c(\mu + 1)$

$= (3)(1+1) \Rightarrow A_{max} = 6$

$A_{min} = A_c(1-\mu)$

$= 3(1-1) \Rightarrow A_{min} = 0.$



3. Evaluate and Plot the spectrum of message & Carrier signal assume $k_a = 2$

From modulated signal given in our example:-

$$S_{AM}(t) = 1.5\cos(900\pi t) + 3\cos(1000\pi t) + 1.5\cos(1100\pi t)$$

$2\pi(f_c - f_m)$; carrier ; $\mu = k_a A_m$; $2\pi(f_c + f_m) = 1100\pi$

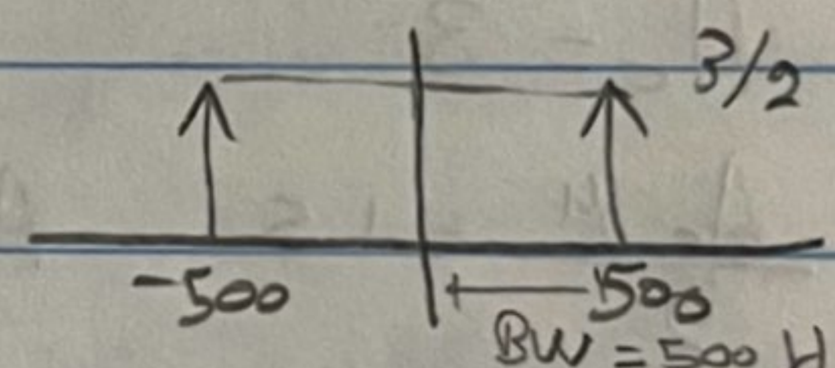
$500 + f_m = 550$; $f_c + f_m = 550$ $\Rightarrow$ $f_m = 50$

$$c(t) = A_c\cos(2\pi f_c t) = 3\cos(\overset{2(500)}{1000}\pi t)$$

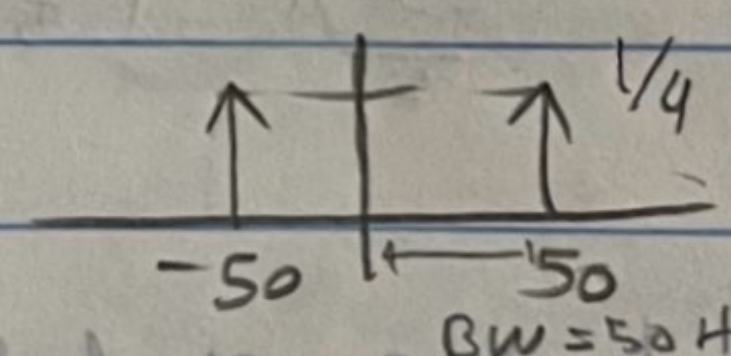
$$\mu = k_a A_m \Rightarrow A_m = \frac{1}{2}$$

$$m(t) = A_m\cos(2\pi f_m t) = \frac{1}{2}\cos(100\pi t)$$

$$C(f) = \frac{3}{2}\left(\delta(f-500) + \delta(f+500)\right)$$



$$m(f) = \frac{1}{4}\left(\delta(f-50) + \delta(f+50)\right)$$



4. Evaluate and Plot the spectrum of modulated signal
5. Evaluate and Plot the power spectral density for carrier, message and modulated signal
6. Evaluate the μ_t
7. Evaluate the BW of modulated signal.
8. Draw the demodulator Block diagram to recover $m(t)$.

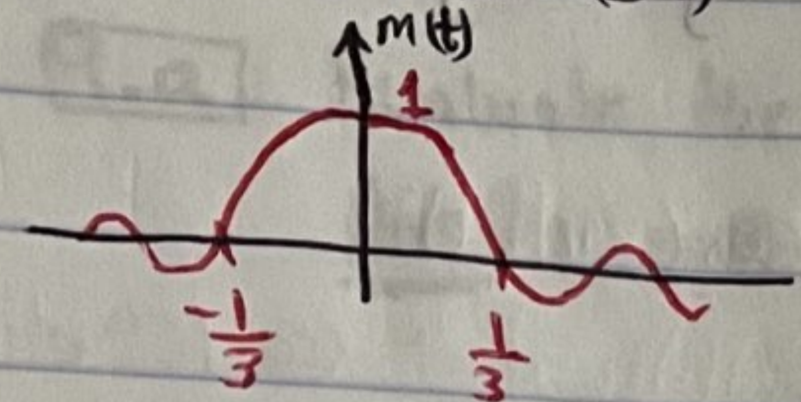
Examples on Normal Am.

(L7) [1]

Example [1] Consider the message signal $m(t) = \mathrm{sinc}^2(3t)$ and carrier signal, $c(t) = 3\cos(200\pi t)$ are applied to a modulator that generate Normal AM.

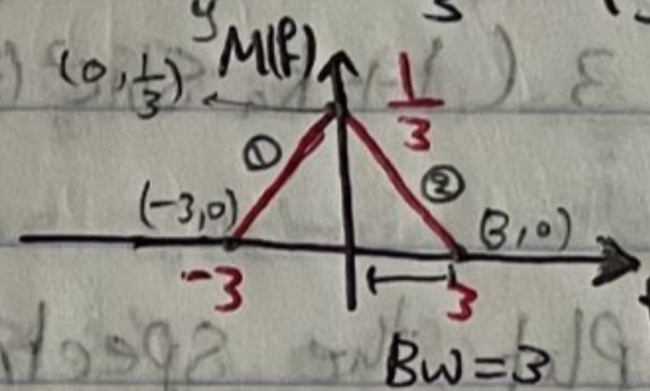
[1] sketch the message signal (modulating signal) in time domain.

Ans: $m(t) = \mathrm{sinc}^2(3t)$



[2] Evaluate & Plot the spectrum of the modulating signal

Ans: $m(t) = \mathrm{sinc}^2(3t) \xrightarrow{f} M(f) = \frac{1}{3}\Lambda\left(\frac{f}{3}\right)$



$y - y_0 = m(x - x_0)$

For ①:- $y - 0 = \frac{1}{9}(x+3)$

$\Rightarrow y = \frac{1}{9}x + \frac{1}{3} \Rightarrow M(f) = \frac{1}{9}f + \frac{1}{3}$, $-3 \leq f < 0$

For ②:- $y - \frac{1}{3} = \frac{-1}{9}(x - 0)$

$\rightarrow y = \frac{-1}{9}x + \frac{1}{3}$

$\Rightarrow M(f) = \frac{-1}{9}f + \frac{1}{3}$, $0 \leq f \leq 3$

[3] Evaluate the Energy spectral density and the total energy

Ans:

$$ESD \Rightarrow G_M(f) = |M(f)|^2$$

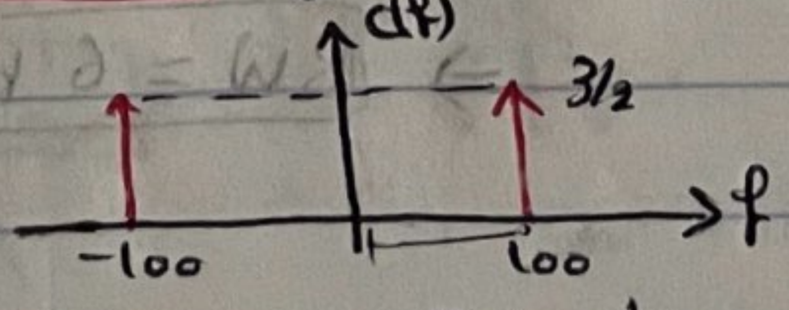
$$|M(f)|^2 = \begin{cases} \left(\frac{1}{9}f + \frac{1}{3}\right)^2, & -3 \leq f < 0 \\ \left(\frac{-1}{9}f + \frac{1}{3}\right)^2, & 0 \leq f < 3 \end{cases}$$

$$E = \int_{-3}^{3} |M(f)|^2\, df = \int_{-3}^{0} \left(\frac{1}{9}f + \frac{1}{3}\right)^2 df + \int_{0}^{3} \left(\frac{-1}{9}f + \frac{1}{3}\right)^2 df.$$

[4] Evaluate and Plot the spectrum of the carrier signal.

Ans: $c(t) = 3\cos(200\pi t) \xrightarrow{f} C(f) = \frac{3}{2}\delta(f - 100) + \frac{3}{2}\delta(f + 100).$

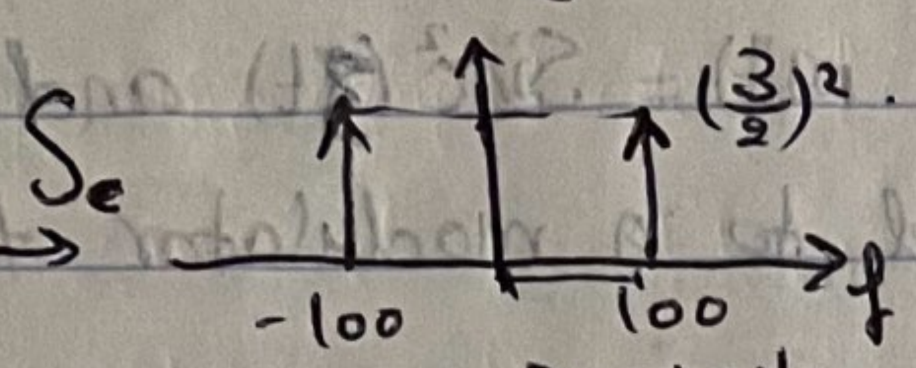
Periodic signal → Power signal.



5. Evaluate the average power for carrier signal.

Ans:

PSD → S_c



$\Rightarrow S_c(f) = (\frac{3}{2})^2 \delta(f-100) + (\frac{3}{2})^2 \delta(f+100)$.

$$P_{total} = (\frac{3}{2})^2 + (\frac{3}{2})^2 \Rightarrow 2 \cdot \frac{9}{4} \Rightarrow \boxed{P_{tot} = \frac{9}{2} W}$$

or we can evaluate the total power by using time domain.

Since $c(t) = 3\cos(200\pi t)$

$$P_{tot} = \frac{(3)^2}{2} \Rightarrow P_{tot} = \frac{9}{2} W.$$

6. Write the expression of the modulated signal in time domain.

Ans: Since we have normal AM.

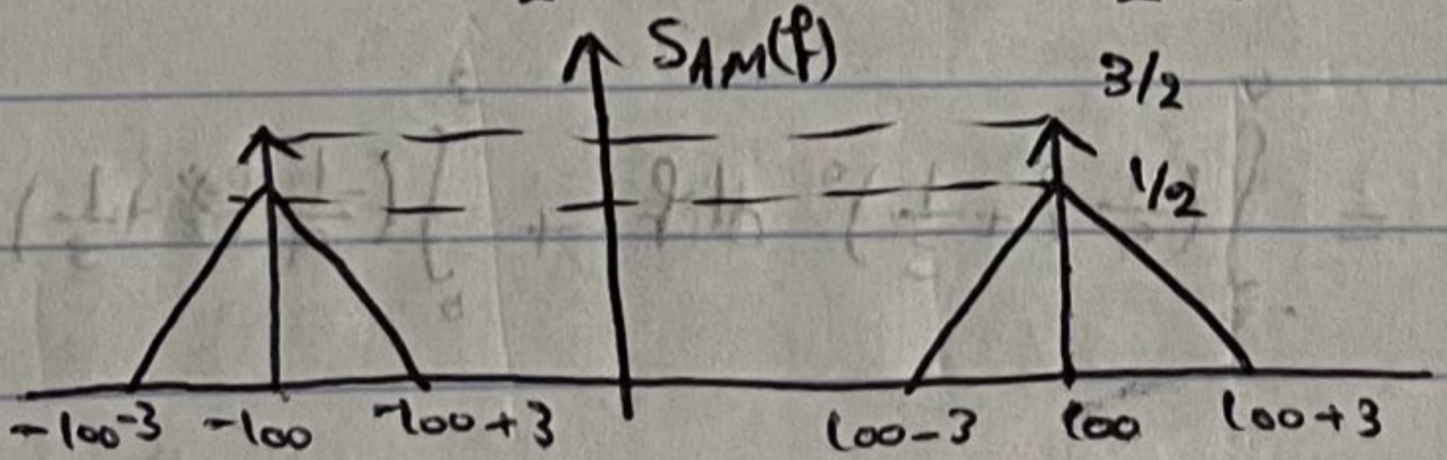
$$S_{AM}(t) = A_c(1 + K_a\, m(t)) \cos(2\pi f_c t).$$
$$= 3(1 + K_a \operatorname{sinc}^2(3t)) \cos(200\pi t).$$

7. Evaluate and plot the spectrum of the modulated signal; assume unity of the amplitude sensitivity ($K_a = 1$).

Ans:

$$S_{AM}(t) = 3(1 + \sin^2(3t)) \cos(200\pi t).$$
$$= 3\cos(200\pi t) + 3\operatorname{sinc}^2(3t)\cos(200\pi t)$$

$$S_{AM}(f) = \frac{3}{2}\delta(f-100) + \frac{3}{2}\delta(f+100) + \frac{3}{2}\cdot\frac{1}{3}\Lambda\left(\frac{1}{3}(f-100)\right) + \frac{3}{2}\cdot\frac{1}{3}\Lambda\left(\frac{1}{3}(f+100)\right)$$



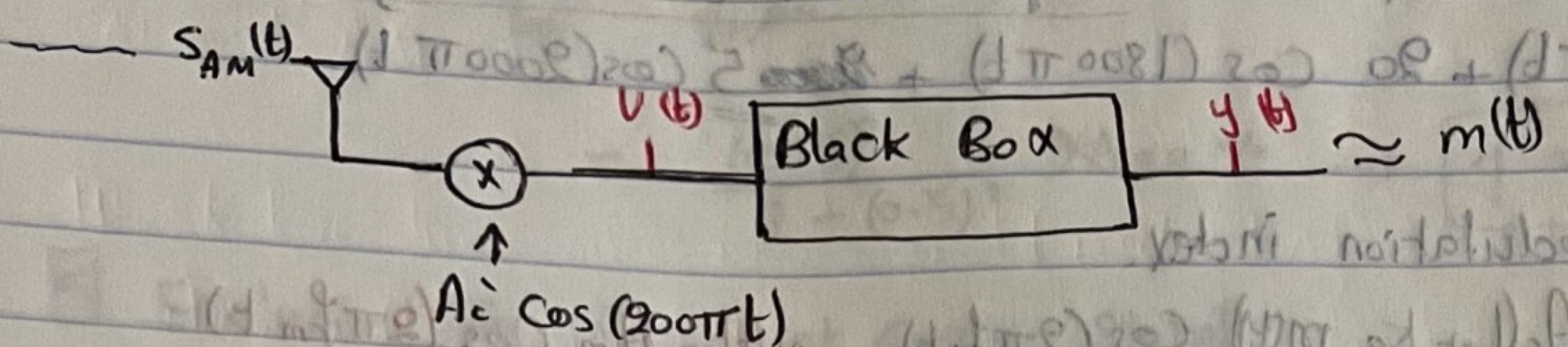
8. Evaluate the Bandwidth of modulated signal

Ans: $BW = (100+3) - (100-3)$

$$\Rightarrow \boxed{BW = 6 \text{ Hz}} = 2f_m.$$

9 Now, Consider the following demodulated scheme for normal AM.

Ans:



9.a Evaluate the signal v(t) in time domain. ($k_a = 1$).

Ans. $v(t) = S_{AM}(t) \cdot A_c' \cos(2\pi f_c t)$.

$= [3(1 + sinc^2(3t))\cos(200\pi t)] A_c' \cos(200\pi t)$.

$= 3A_c'(1 + sinc^2(3t))\cos^2(200\pi t)$

$= 3A_c'(1 + sinc^2(3t))[\frac{1}{2} + \frac{1}{2}\cos(400\pi t)]$

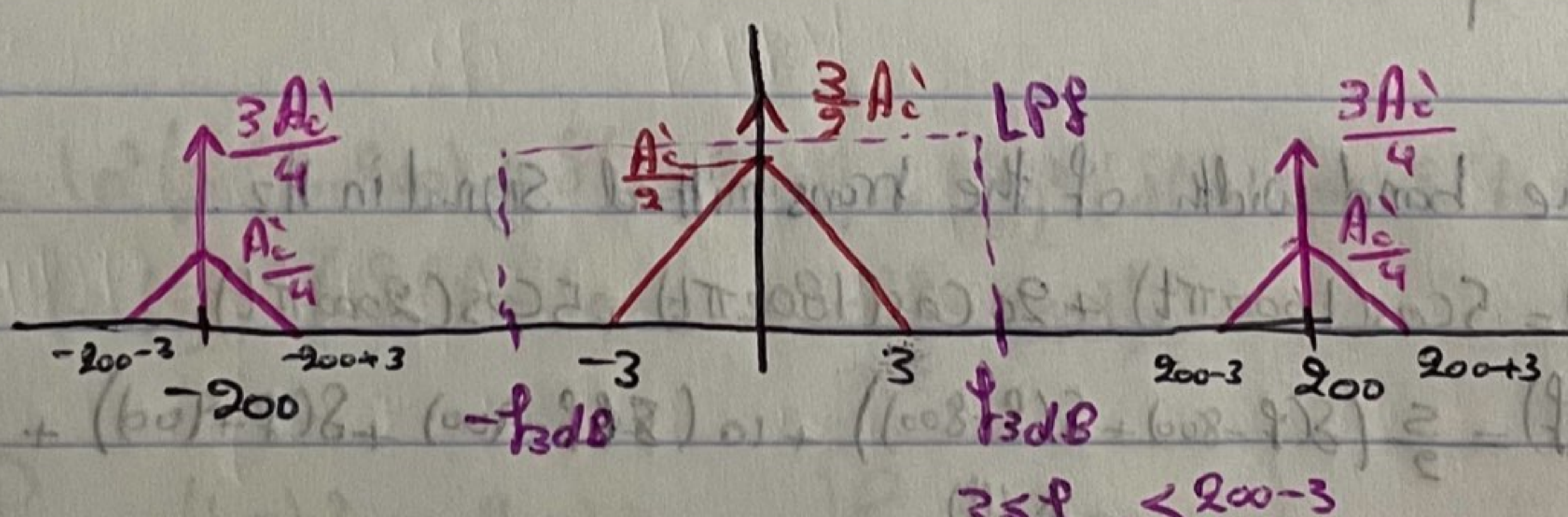
$v(t) = \frac{3}{2}A_c'(1 + sinc^2(3t)) + \frac{3}{2}A_c'(1 + sinc^2(3t))\cos(400\pi t)$.

$= \frac{3}{2}A_c' + \frac{3}{2}A_c' sinc^2(3t) + \frac{3}{2}A_c'\cos(400\pi t) + \frac{3}{2}A_c' sinc^2(3t)\cos(400\pi t)$

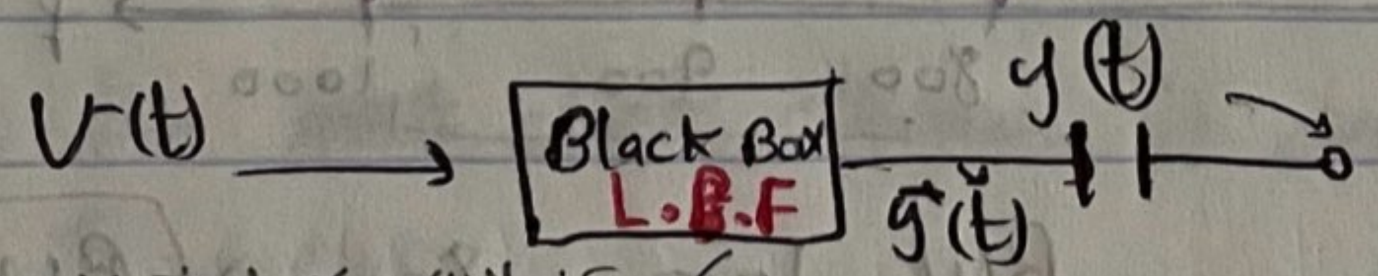
9.b Evaluate and plot the spectrum of the signal v(t)

$V(f) = \frac{3}{2}A_c'\delta(f) + \frac{3}{2}A_c' \cdot \frac{1}{3}\Lambda(\frac{f}{3}) + \frac{3}{2}A_c'\delta(f-200) + \frac{3}{2}A_c\delta(f+200)$

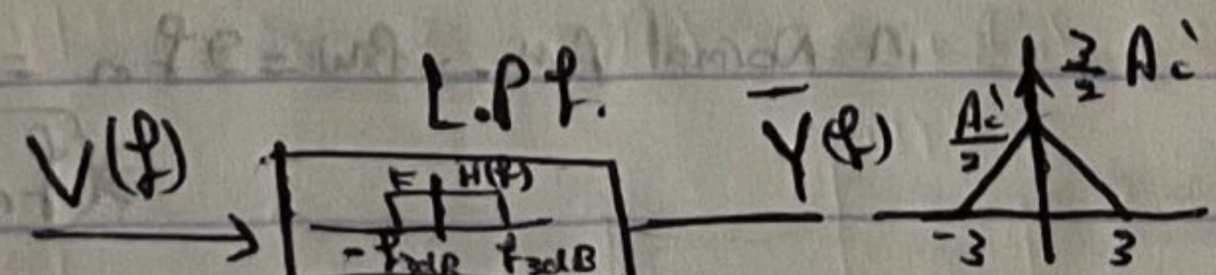
$+ \frac{3}{4}A_c' \cdot \frac{1}{3}\Lambda(\frac{f-200}{3}) + \frac{3}{4}A_c' \cdot \frac{1}{3}\Lambda(\frac{f+200}{3})$



To obtain: $y(t) \approx m(t)$



✓ لأنه الm(t) كان استخرنا عباحول الأوريجين



$\bar{Y}(f) = \frac{3}{2}A_c'\delta(f) + \frac{A_c'}{2}\Lambda(\frac{f}{3})$

$\bar{y}(t) = \frac{3}{2}A_c' + \frac{A_c'}{2} \cdot 3\, sinc^2(3t)$

DC value

✓ عشان أتخلص منها وأحصل على m(t) بحط Capacitor.

بوصلها خلف الـ Capacitor

$\Rightarrow y(t) = \frac{A_c'}{2} \cdot 3\, sinc(3t)$

$\Rightarrow$ if $k = \frac{1}{\frac{A_c'}{2} \cdot 3}$ $\Rightarrow y(t) \approx m(t)$

بدي أتخلص من هاي فبا خذ ال(k) مقلوبها

(L7) [2]

Example [9] The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal → Normal AM.

$$s(t) = 5\cos(1600\pi t) + 20\cos(1800\pi t) + 5\cos(2000\pi t)$$

[1] Evaluate the modulation index.

Ans: $S_{AM}(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t)$, $m(t) = A_m \cos(2\pi f_m t)$

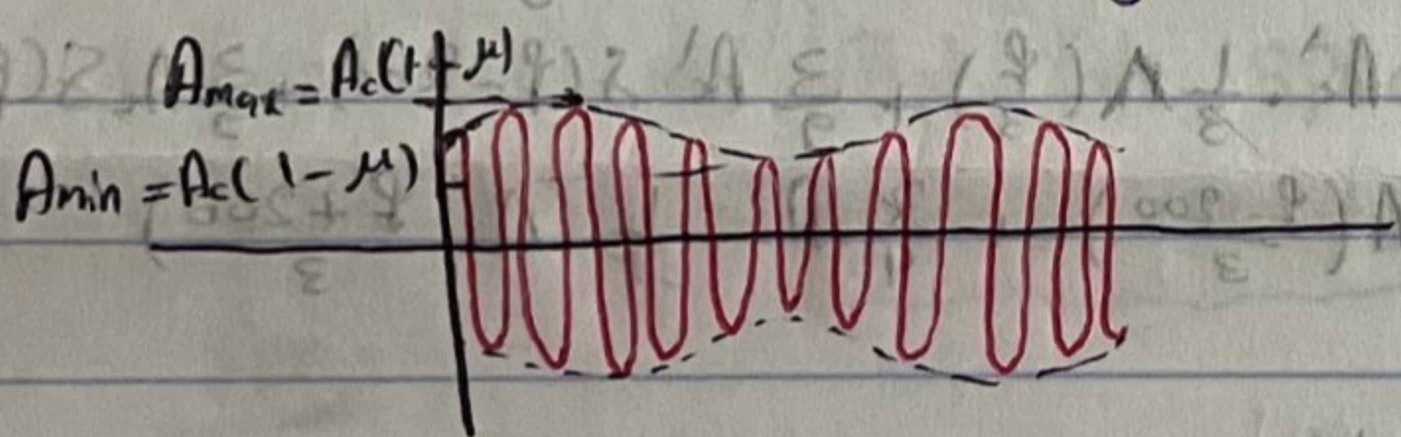
$$S_{AM}(t) = A_c \cos(2\pi f_c t) + A_c k_a A_m \cos(2\pi f_m t)\cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2}\cos(2\pi(f_c + f_m)t) + \frac{A_c \mu}{2}\cos(2\pi(f_c - f_m)t)$$

[we can conclude: $A_c = 20$ and $f_c = 900$

$5 = \frac{A_c \mu}{2}$ ($A_c = 20$) ⇒ $\mu = 0.5$ [under modulation].

[2] Sketch the modulated signal in time domain.



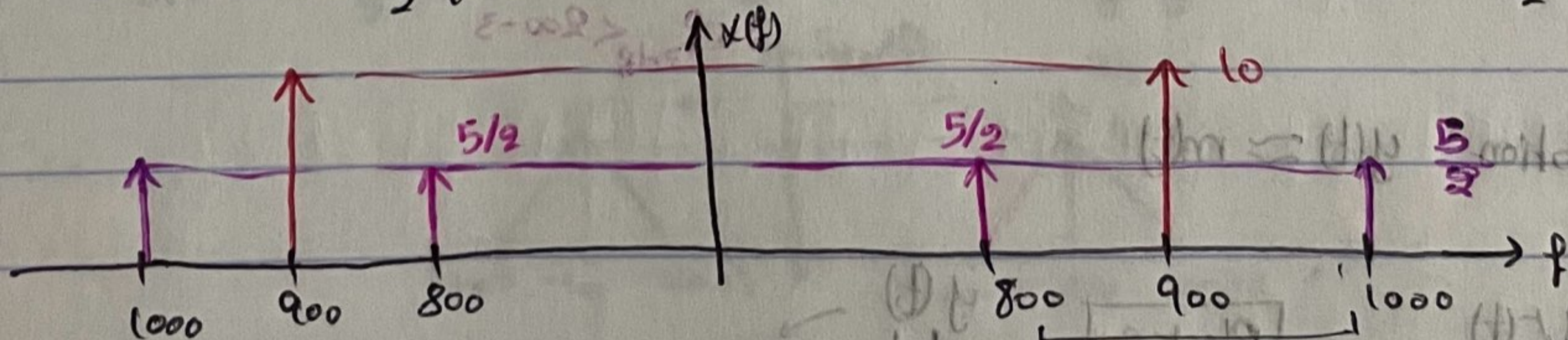
$A_{max} = A_c(1 + \mu) \Rightarrow 20(1+0.5) \Rightarrow A_{max} = 30$

$A_{min} = A_c(1 - \mu) \Rightarrow 20(1 - 0.5) \Rightarrow A_{min} = 10$

[3] find the band width of the transmitted signal in Hz.

Ans: $s(t) = 5\cos(1600\pi t) + 20\cos(1800\pi t) + 5\cos(2000\pi t)$.

$$\rightarrow S(f) = \frac{5}{2}(\delta(f-800) + \delta(f+800)) + 10(\delta(f-900) + \delta(f+900)) + \frac{5}{2}(\delta(f-1000) + \delta(f+1000))$$



$BW = 1000 - 800 \Rightarrow BW = 200\,Hz$

in Normal AM → $BW = 2f_m = 200$

$\Rightarrow f_m = 100$

4 Evaluate the efficiency of the modulated scheme

Ans: $\eta_{\%} = \frac{\mu^2}{2+\mu^2} \times 100\%$

$= \frac{(0.5)^2}{2+(0.5)^2} \times 100\% \Rightarrow \boxed{\eta = 11.11\%}$

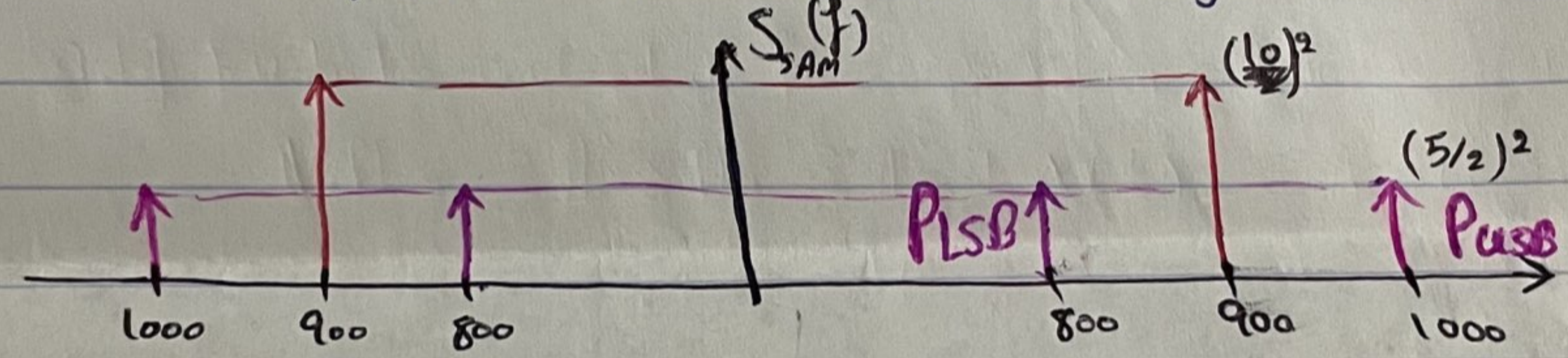
5 Evaluate ~~the~~ Power at Carrier signal.

Ans: $P_c = \frac{(20)^2}{2} \rightarrow P_c = 200\,W$.

6 Evaluate the P_{USB}. (Power at USB).

Ans: $P_{USB} = \frac{(5)^2}{2} \longrightarrow P_{USB} = 12.5\,W$

7 Evaluate and Plot the Power Spectral density of modulated signal.



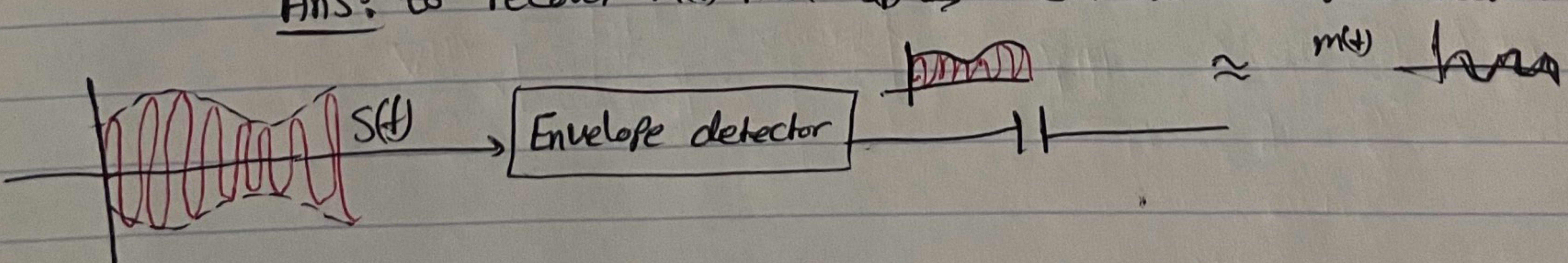
$$S_{S_{AM}}(f) = 10^2\left(\delta(f-900) + \delta(f+900)\right) + \left(\tfrac{5}{2}\right)^2\left(\delta(f-800) + \delta(f+800)\right) + \left(\tfrac{5}{2}\right)^2\left(\delta(f-1000) + \delta(f+1000)\right).$$

$P_c = (10)^2 + (10)^2 \Rightarrow \boxed{P_c = 200\,W}$

$P_{USB} = \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 \Rightarrow \boxed{P_c = 12.5\,W}$

8 Draw the ~~block~~ block diagram of demodulator used to recover $m(t)$ from $s(t)$

Ans: to recover $m(t)$ from $s(t) \Rightarrow$ we can use envelope detector.

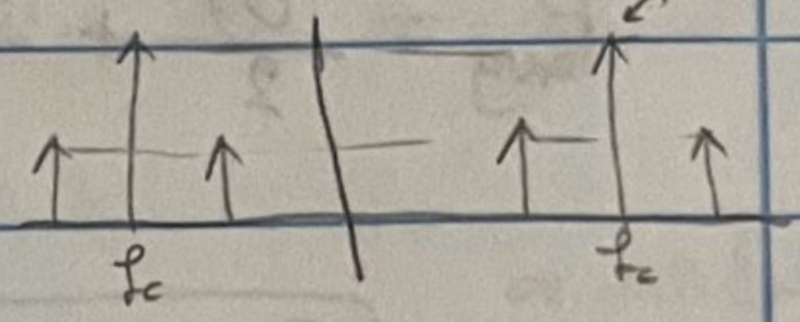


(L8)

Normal AM: DSB. Fc "Duble side-band full carrier".

modulated signal for DSB-FC

$S_{Am}(t) = A_c(1 + K_a m(t)) \cos(2\pi f_c t)$.

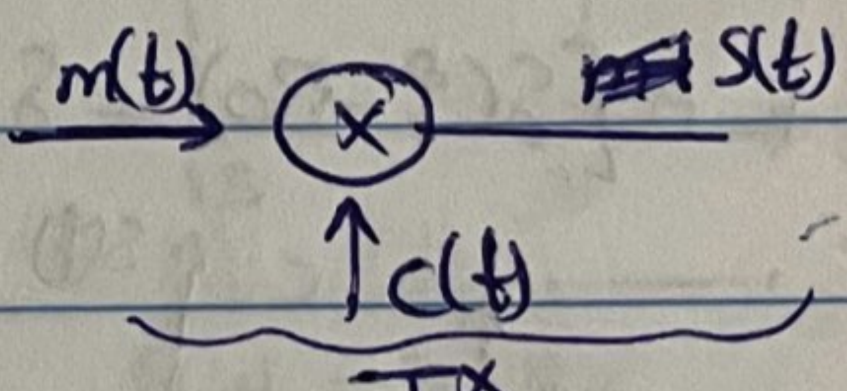


- To improve the efficiency → DSB-Fc → DSB-Sc

Carried مزوّع
suppressed Carrier

modulated signal for DSB-SC Can be expressed as

$S(t)_{DSB-SC} = m(t)\, c(t)$



Example| Consider the modulating signal m(t) is multiton which Can be expressed as

$m(t) = 2\cos(30\pi t) + 4\cos(600\pi t)$: base band signal

and the Carrier signal is given by.

$C(t) = 6\cos(600\pi t)$ → $f_c = 300$

where m(t) and c(t) are applied to DSB-SC modulater.

[1] write the expression of modulated signal in time domain.

Ans: modulated signal for DSB-SC express as

$S(t)_{DSB-SC} = m(t)\,c(t) = [2\cos(30\pi t) + 4\cos(60\pi t)][6\cos(600\pi t)]$

$= 12\cos(30\pi t)\cos(600\pi t) + 24\cos(60\pi t)\cos(600\pi t)$

$= 6[\cos(630\pi t) + \cos(570\pi t)] + 12[\cos(660\pi t) + \cos(540\pi t)]$

$\Rightarrow S(t)_{DSB-SC} = 6\cos(630\pi t) + 6\cos(570\pi t) + 12\cos(660\pi t) + 12\cos(540\pi t)$

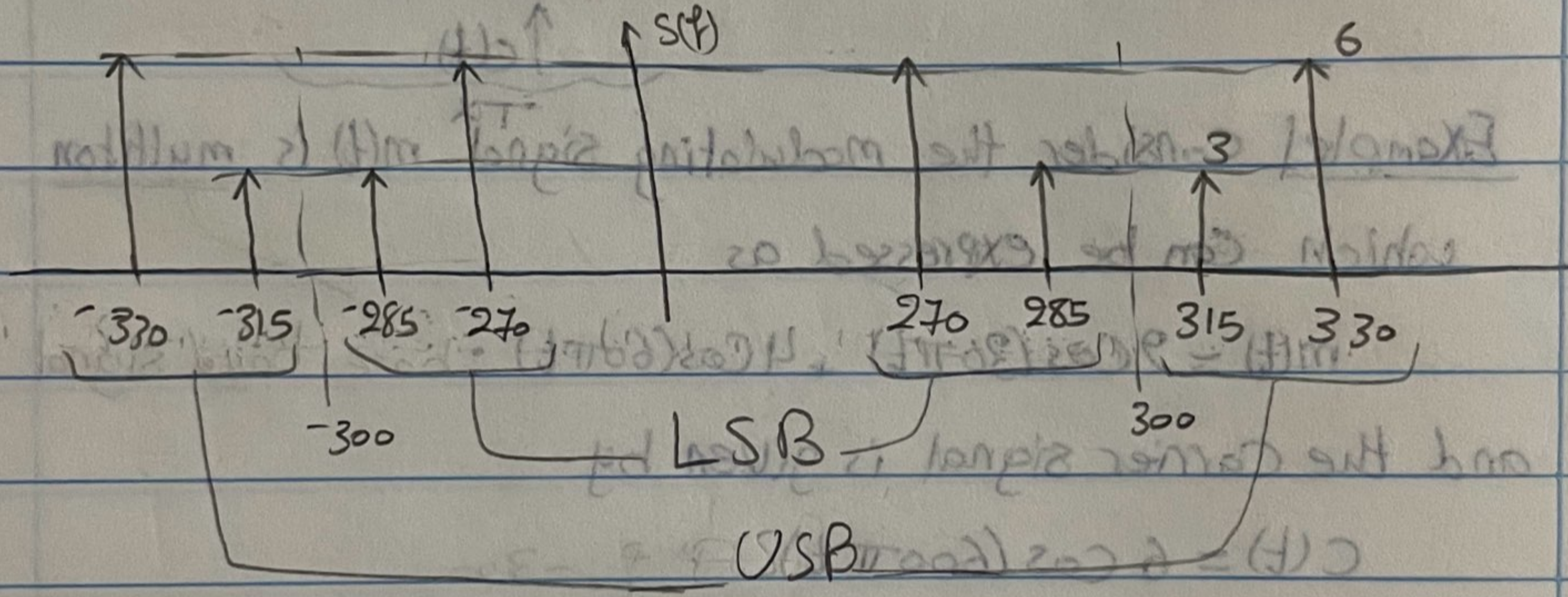
2 Evaluate the average power for modulated signal.

$$P_{avg} = \frac{(6)^2}{2} + \frac{(6)^2}{2} + \frac{(12)^2}{2} + \frac{(12)^2}{2}$$

$$\Rightarrow \boxed{P_{avg} = 180\,W}$$

3 Evaluate and plot the spectrum of modulated.

$$S_{DSB\text{-}SC}(f) = 3[\delta(f-315) + \delta(f+315)] + 3[\delta(f-285) + \delta(f+285)] + 6[\delta(f-330) + \delta(f+330)] + 6[\delta(f-270) + \delta(f+270)]$$



$$\eta = \frac{P_{sides}}{P_T} = \frac{P_{sides}}{P_{sides}} \times 100\% \Rightarrow \eta = 100\%$$

4 Evaluate the P_{USB}.

→ Time domain: $P_{USB} = \frac{6^2}{2} + \frac{12^2}{2}$

→ freq domain: $P_{USB} = 3^2 + 6^2 + 3^2 + 6^2$

Note:- $\eta_{\%\,DSB\text{-}SC} = 100\%$

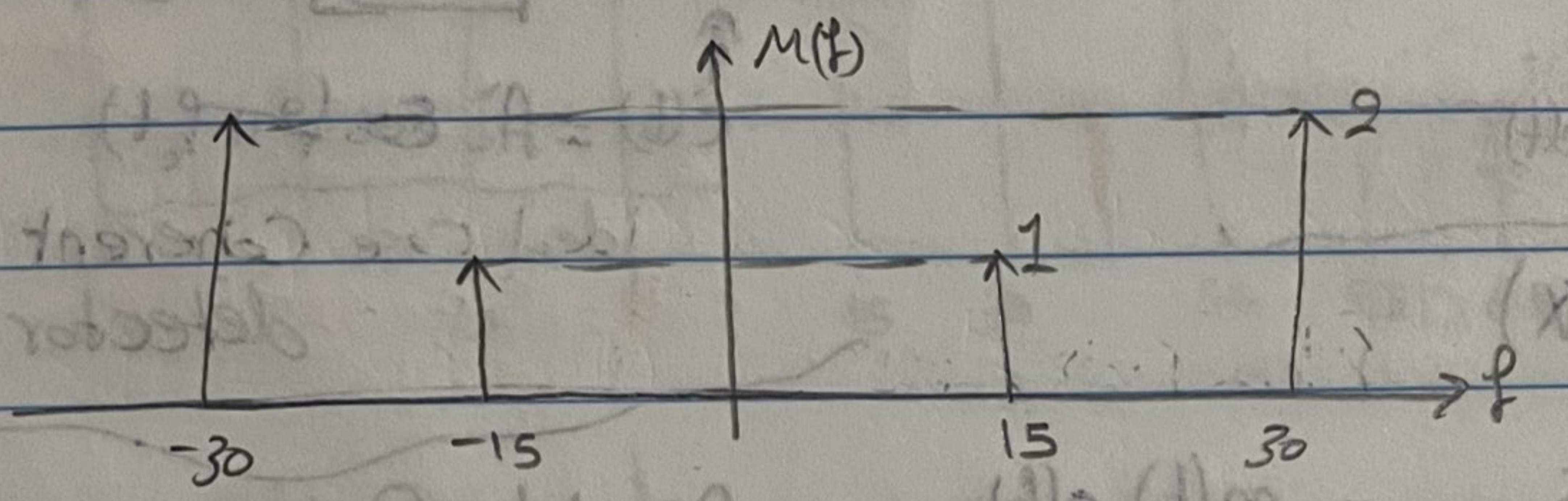
5. Evaluate the BW for modulating signal [message signal] & carrier signal.

- for modulating signal.

فتمحور حول الـ origin

$m(t) = 2\cos(30\pi t) + 4\cos(60\pi t)$ [base band signal]

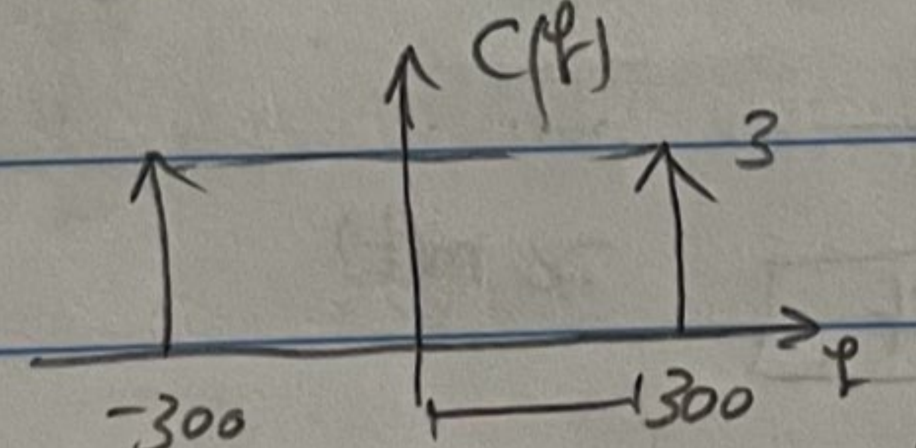
$M(f) = \delta(f-15) + \delta(f+15) + 2\delta(f-30) + 2\delta(f+30)$



BW = 30 Hz.

- for carrier signal.

$c(t) = 6\cos(600\pi t) \rightarrow C(f) = 3\delta(f-300) + 3\delta(f+300)$



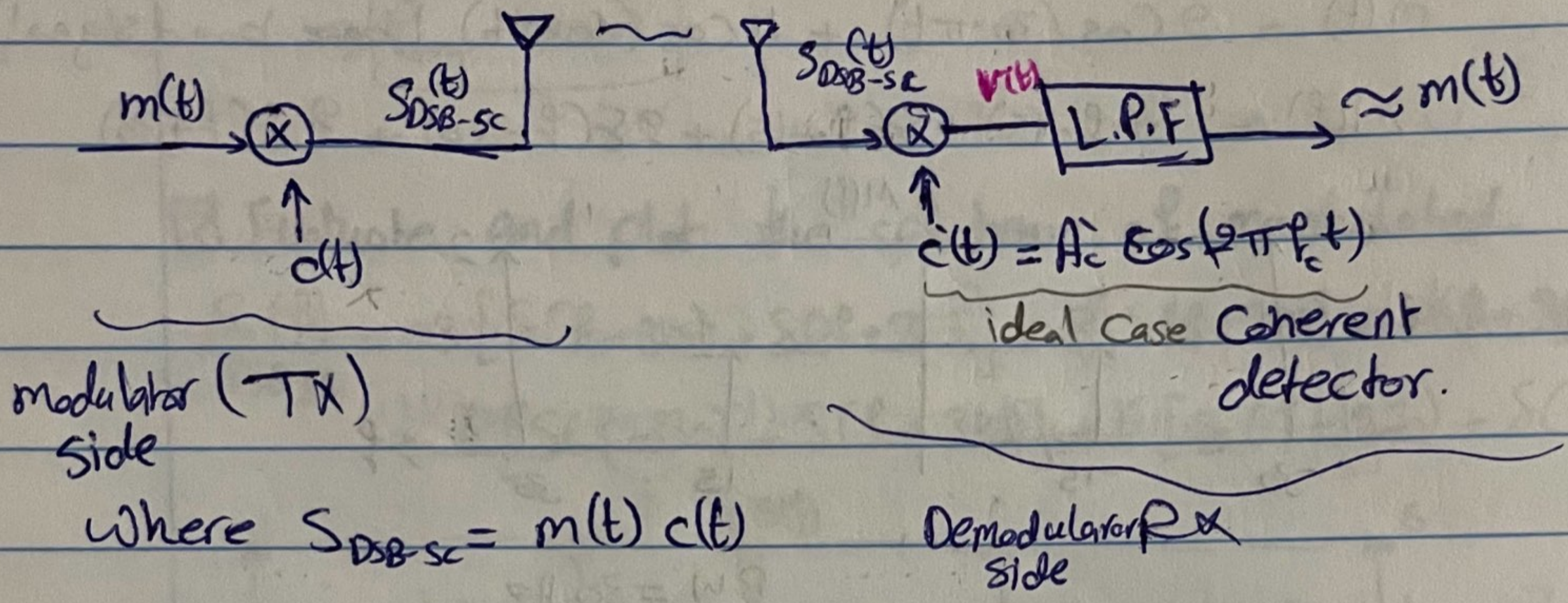
BW = 300 Hz.

6. Evaluate the BW of modulated signal.

$BW = 330 - 270 \rightarrow \boxed{BW = 60\,Hz} = 2f_{max}$ for modulated signal.

Demodulation Schemes (R_x) → Coherent ($f_c, \theta_c \rightarrow$ Constant) / Non Coherent ($f_c, \theta_c \rightarrow$ Non-Constant)

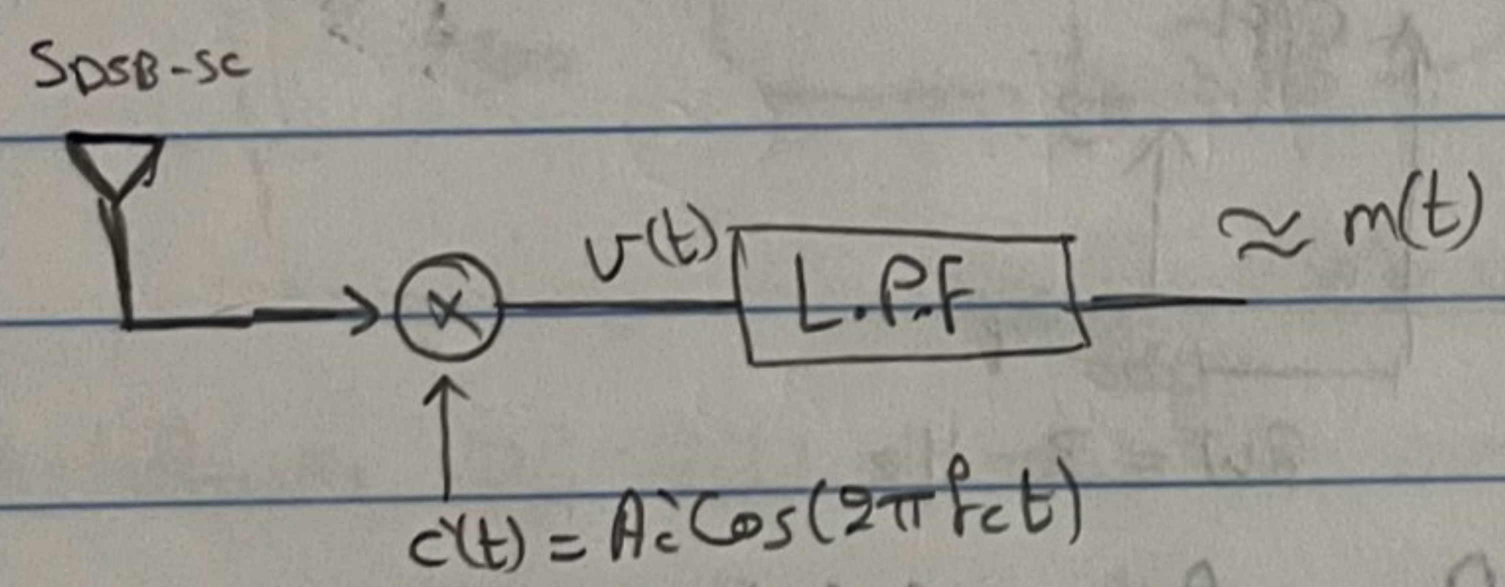
- Block diagram for DSB-SC modulator & demodulator.



where $S_{DSB-SC} = m(t)\, c(t)$

⇒

[?] Draw block diagram for coherent detector.



$$v(t) = S_{DSB-SC}(t)\, c'(t) = m(t)\, c(t)\, c'(t)$$

$$= m(t) . A_c \cos(2\pi f_c t) . A_c' \cos(2\pi f_c t)$$

$$= A_c A_c' m(t) \cos^2(2\pi f_c t)$$

$$= \frac{A_c A_c'}{2} m(t) \left[\frac{1}{2} + \frac{1}{2}\cos(4\pi f_c t)\right]$$

$$= \frac{1}{2} . 6 . A_c' \left[2\cos(30\pi t) + 4\cos(60\pi t)\right]\left[\frac{1}{2} + \frac{1}{2}\cos(1200\pi t)\right]$$

$$= 3A_c' \cos(30\pi t) + 6A_c' \cos(60\pi t) + 3A_c' \underbrace{\cos(30\pi t)\cos(1200\pi t)}_{\frac{1}{2}\cos(1230\pi t) + \frac{1}{2}\cos(1170\pi t)}$$

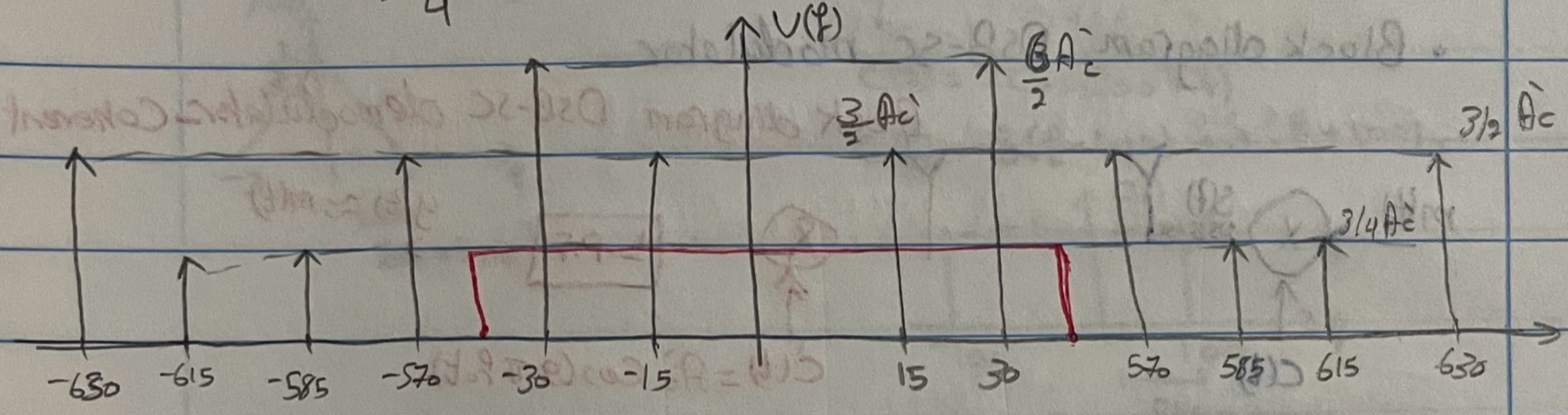
$$+ 6A_c' \underbrace{\cos(60\pi t)\cos(1200\pi t)}_{\frac{1}{2}\cos(1260\pi t) + \frac{1}{2}\cos(1140\pi t)}$$

⇒

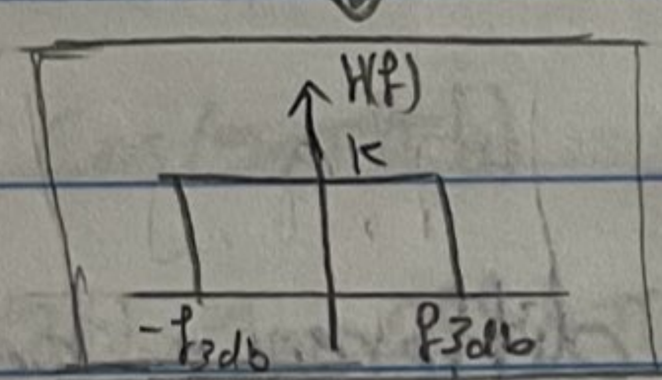
$$= \frac{3}{2}A_c\left[\delta(f-15)+\delta(f+15)\right]+\frac{6}{2}A_c\left[\delta(f-30)+\delta(f+30)\right]$$

$$+\frac{3A_c}{4}\left[\delta(f-615)+\delta(f+615)+\delta(f+585)+\delta(f-585)\right]$$

$$+\frac{3}{2}\frac{6A_c}{4}\left[\delta(f-630)+\delta(f+630)+\delta(f-570)+\delta(f+570)\right]$$



input

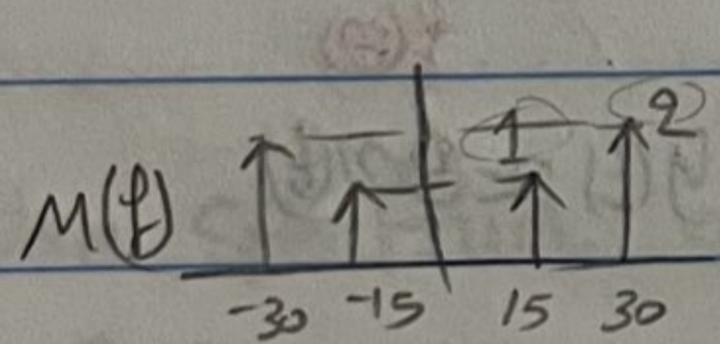


$30 \leq f_{3db} < 570$ Hz

$K = \frac{\text{Amp out}}{\text{Amp in}}$

$y(t) \approx m(t)$

$K_1 = \frac{1}{\frac{3}{2}A_c}$, $K_2 = \frac{2}{\frac{6}{2}A_c}$



output

$\Rightarrow K_1 = \frac{2}{3A_c}$, $K_2 = \frac{2}{3}A_c$

$\Rightarrow$ $k = \frac{2}{3}A_c$

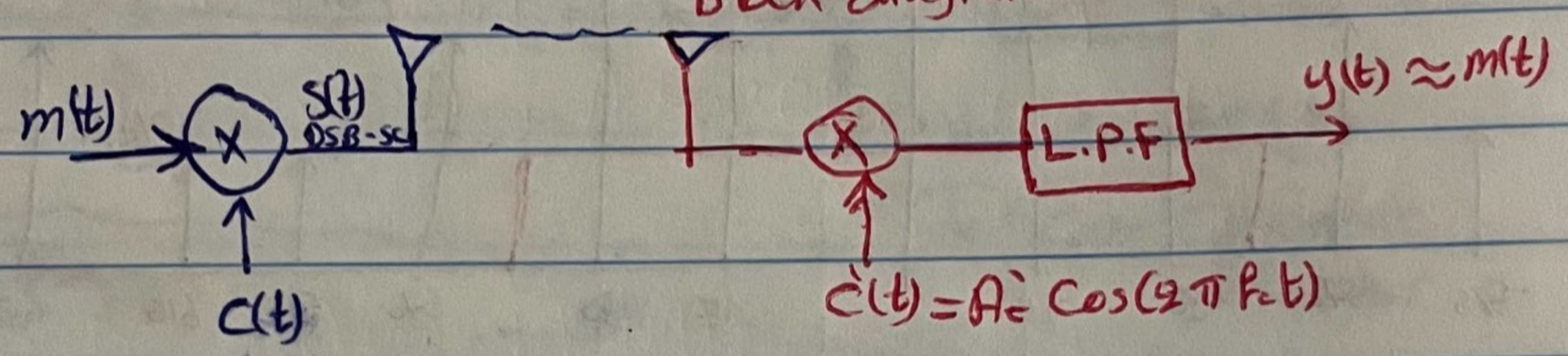
(L9)

Let message signal "modulating signal" $m(t) = \text{sinc}^2(3t)$ and carrier signal $c(t) = A_c \cos(2\pi f_c t)$, are applied on DSB-SC modulator.

$= 2\cos(200\pi t)$.

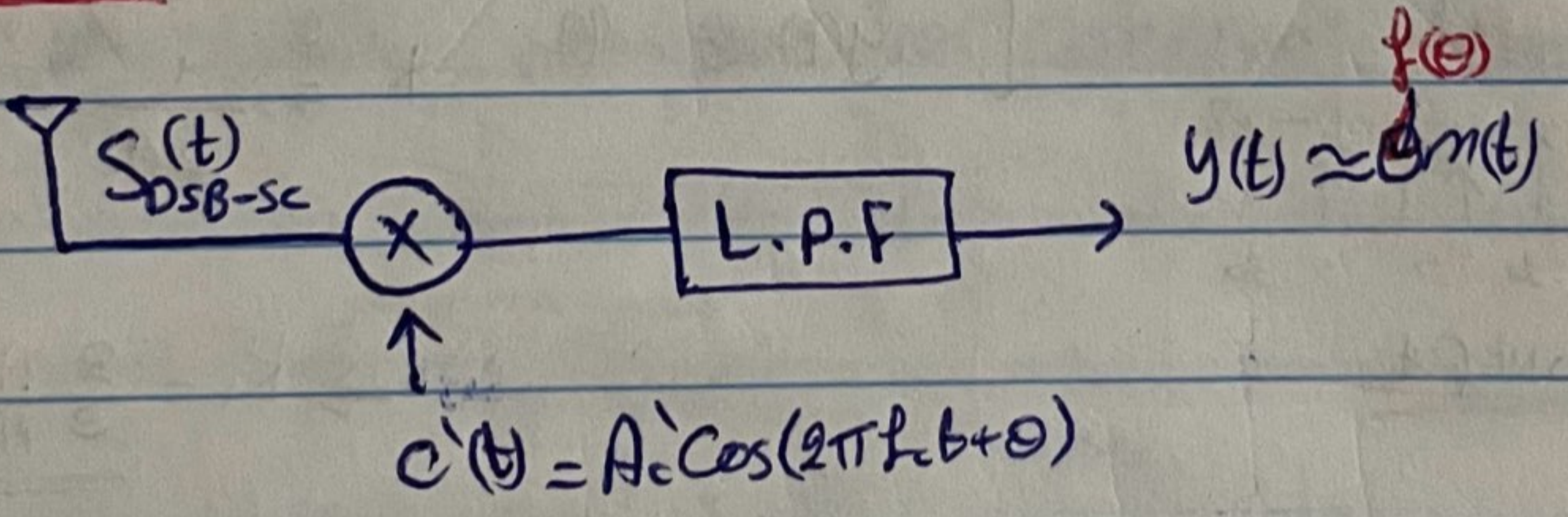
- Block diagram DSB-SC modulator

Block diagram DSB-SC demodulator - Coherent

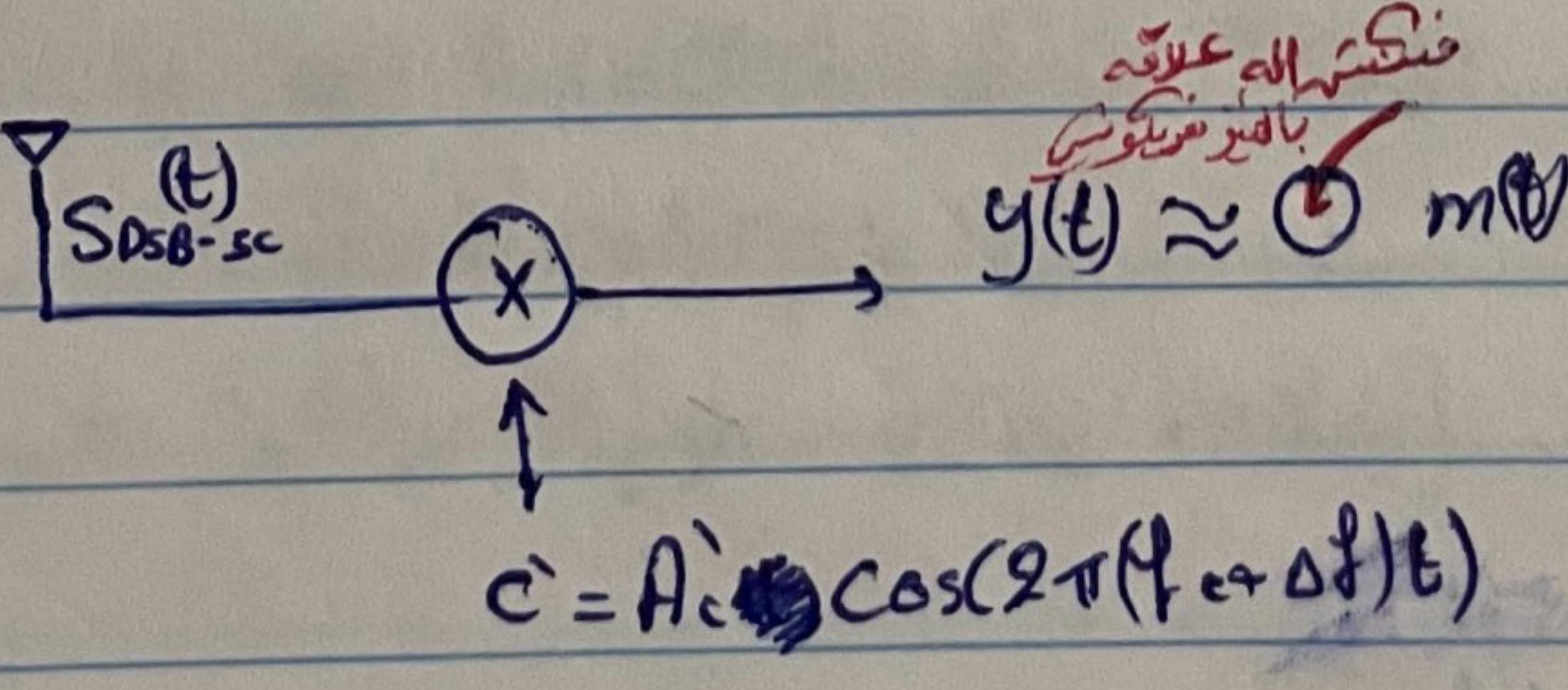


- Non-Coherent detection.

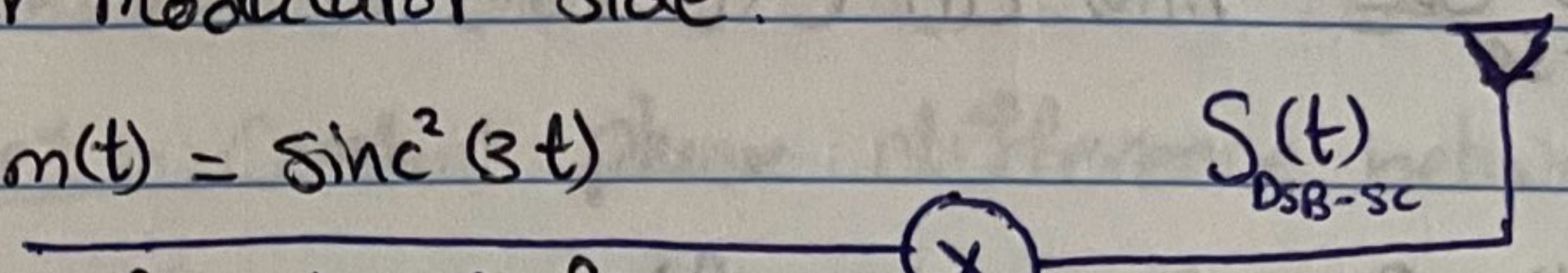
Case 1:- Constant phase difference between $c(t)$ and $c'(t)$



Case 2:- Constant frequency difference between $c(t)$ and $c'(t)$.

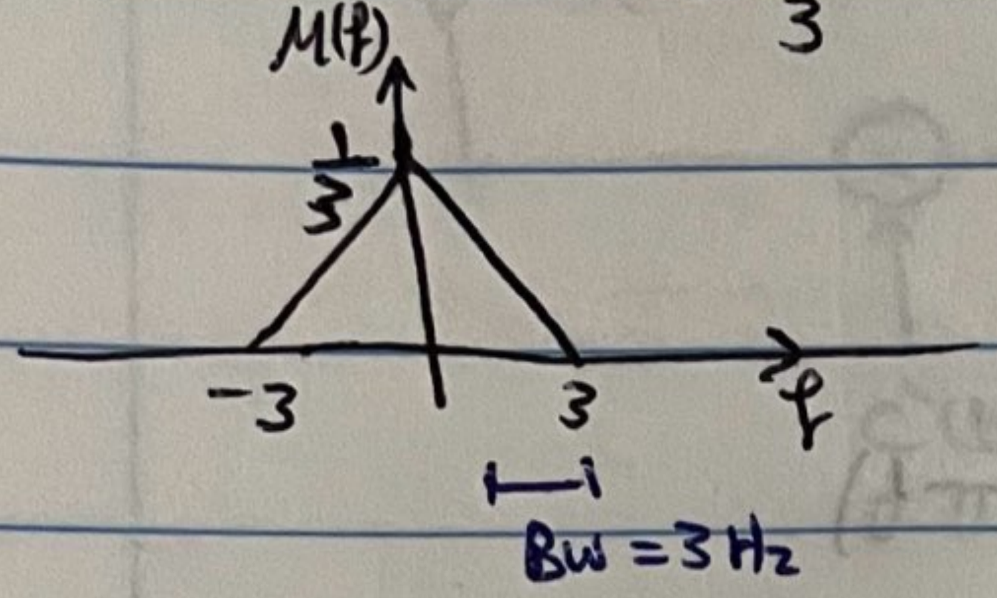


In our Modulator Side.



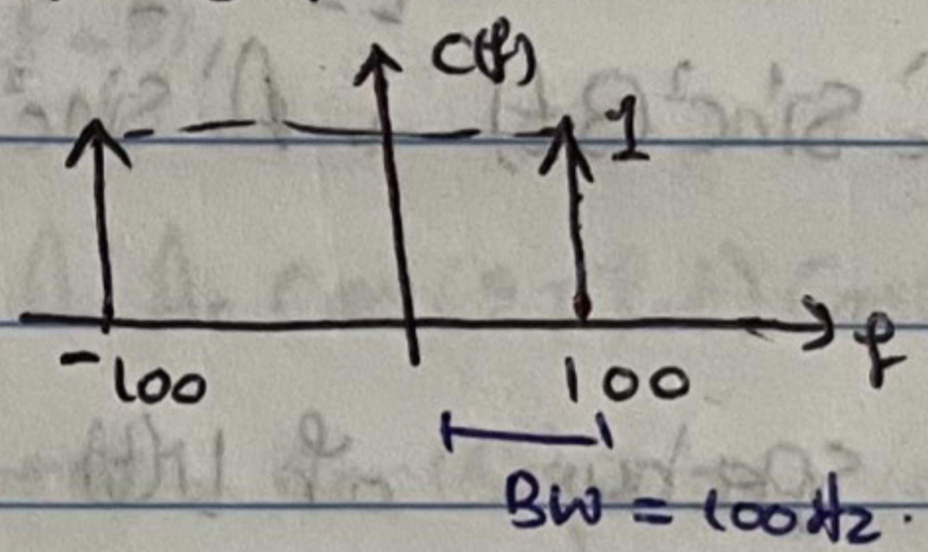
$m(t) = \text{sinc}^2(3t)$

$M(f) = \frac{1}{3}\Lambda(\frac{f}{3})$



$c(t) = 2\cos(200\pi t)$

$C(f) = \delta(f-100) + \delta(f+100)$.



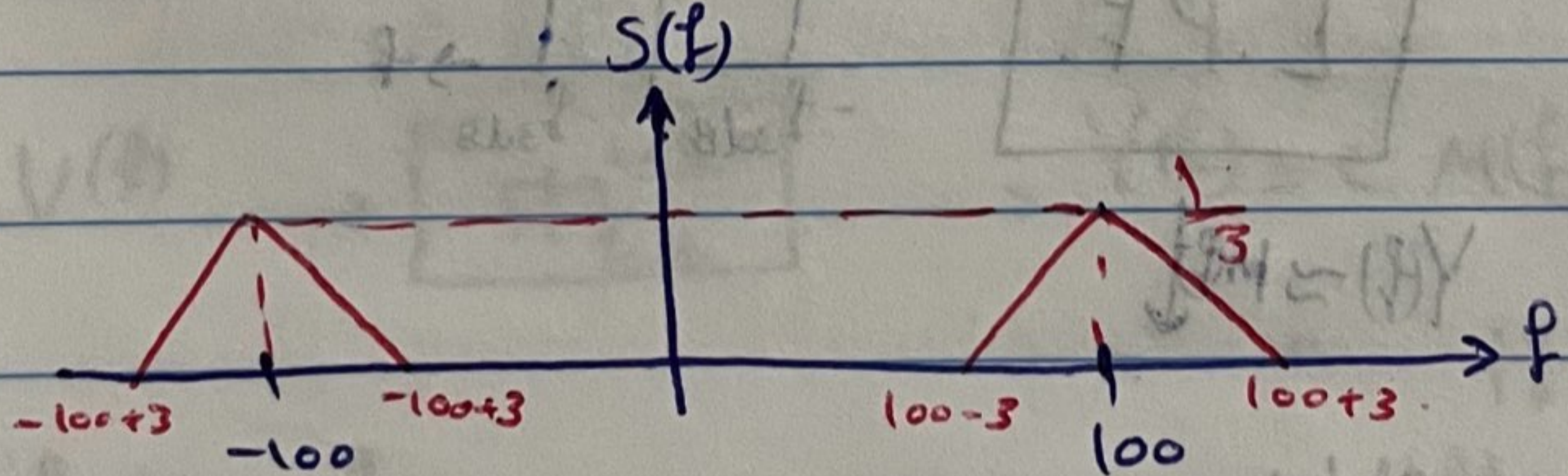
$$S_{DSB-SC}(t) = m(t)\,c(t)$$

$$= 2\,m(t)\cos(200\pi t)$$

$$= 2\,\text{sinc}^2(3t)\cos(200\pi t).$$

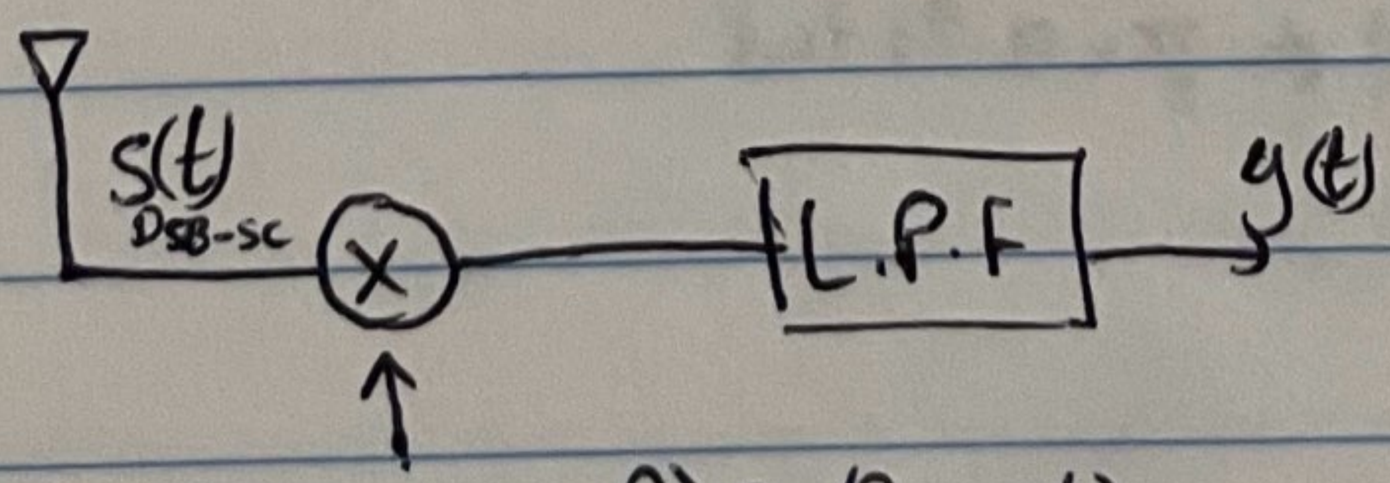
⇒ the Spectrum of modulated Signal can be give by.

$$S_{DSB-SC}(f) = \frac{2}{2}\cdot\frac{1}{3}\Lambda\left(\frac{1}{3}(f-100)\right) + \frac{2}{2}\cdot\frac{1}{3}\Lambda\left(\frac{1}{3}(f+100)\right)$$



- our Demodulator.

- Coherent Detector.



$c'(t) = A_c\cos(200\pi t)$

⇒

$v(t) = S_{DSB-SC}(t)\, c'(t) = m(t)\, c(t)\, c'(t)$

$= \text{sinc}^2(3t) \cdot 2\cos(200\pi t) \cdot A_c' \cos(200\pi t)$

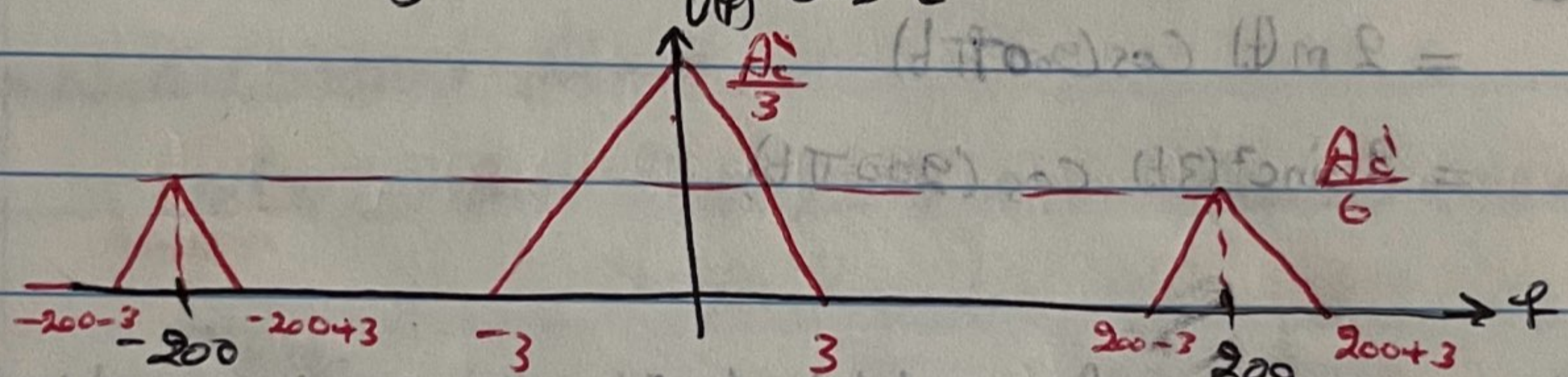
$= 2A_c' \, \text{sinc}^2(3t) \cos^2(200\pi t)$

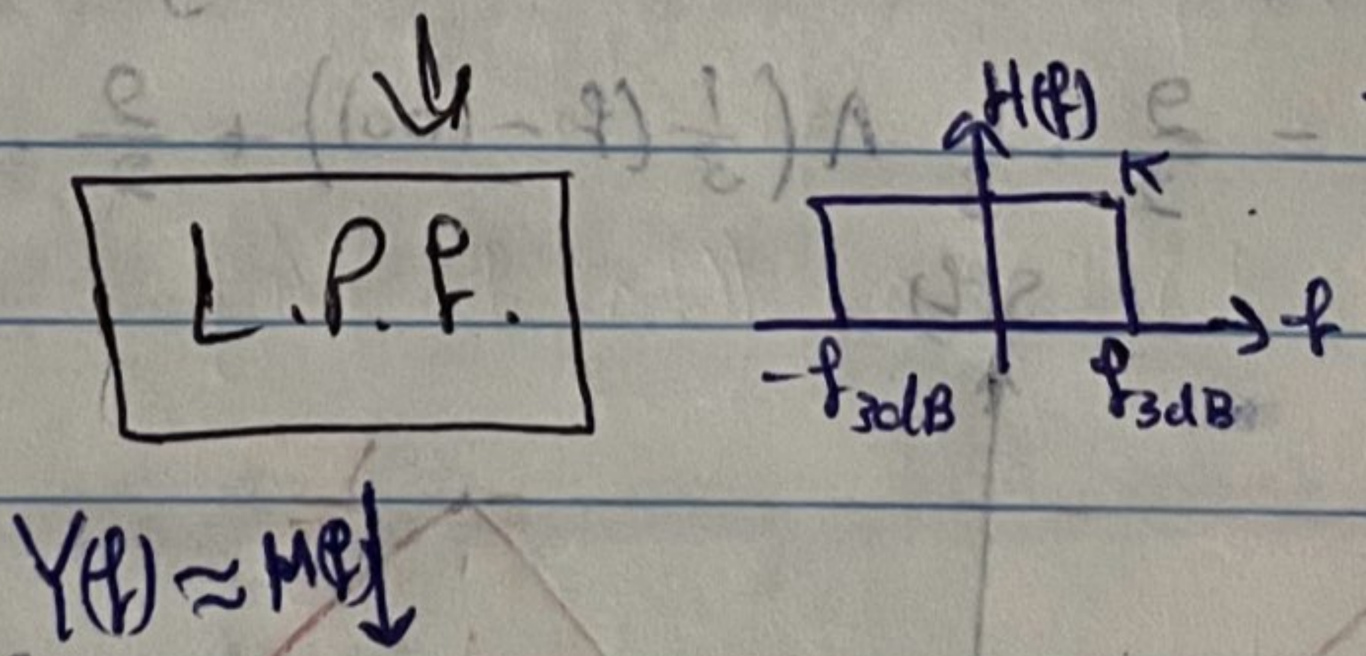
$= 2A_c' \, \text{sinc}^2(3t) \left[\frac{1}{2} + \frac{1}{2}\cos(400\pi t)\right]$

$= A_c' \, \text{sinc}^2(3t) + A_c' \, \text{sinc}^2(3t) \cos(400\pi t)$

- The spectrum of $v(t)$

$$V(f) = \frac{A_c'}{3} \Lambda\left(\frac{f}{3}\right) + \frac{A_c'}{3 \cdot 2}\left[\Lambda\left(\frac{1}{3}(f-200)\right) + \Lambda\left(\frac{1}{3}(f+200)\right)\right]$$





To Evaluate gain:

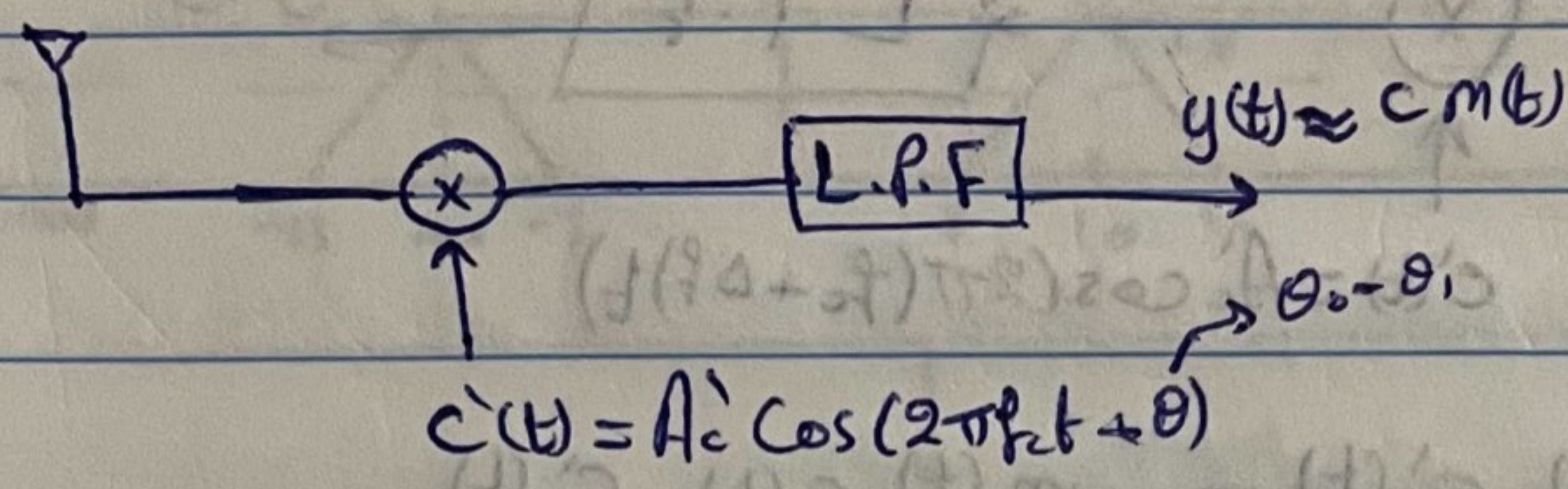
$\frac{A_c'}{3} \cdot k = \frac{1}{3} \Rightarrow \boxed{k = \frac{1}{A_c'}}$

$3 \leq f_{3dB} < 200 - 3$

[L(10)].

Non-Coherent detector.

Case1 :- Constant phase difference between c(t) & c'(t)

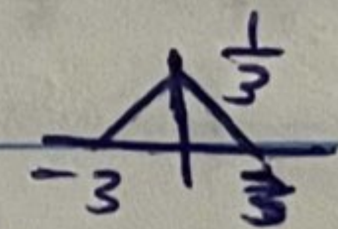


$$v(t) = m(t)\, c(t)\, c'(t) = m(t)\, A_c' A_c \cos(2\pi f_c t)\cos(2\pi f_c t + \theta).$$

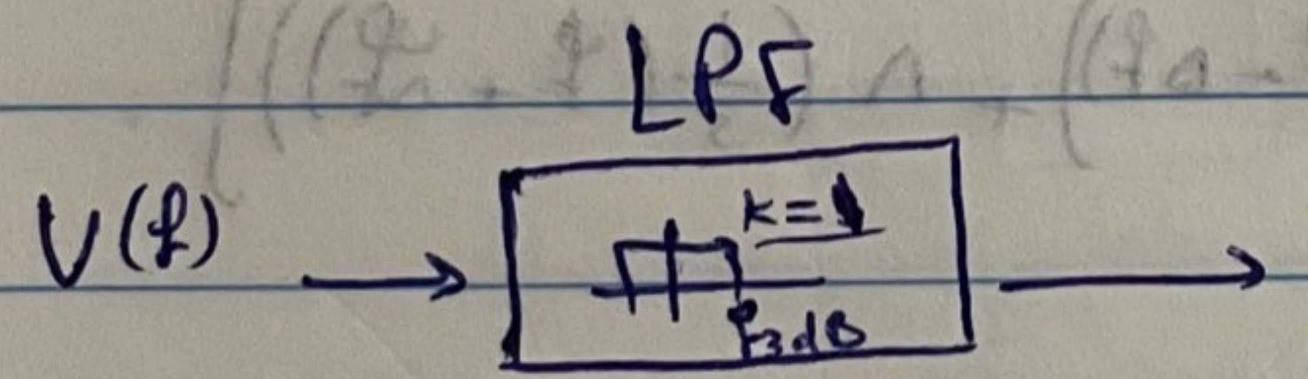
$$= m(t)\,\frac{A_c A_c'}{2}\left[\cos(4\pi f_c t + \theta) + \cos(\theta)\right].$$

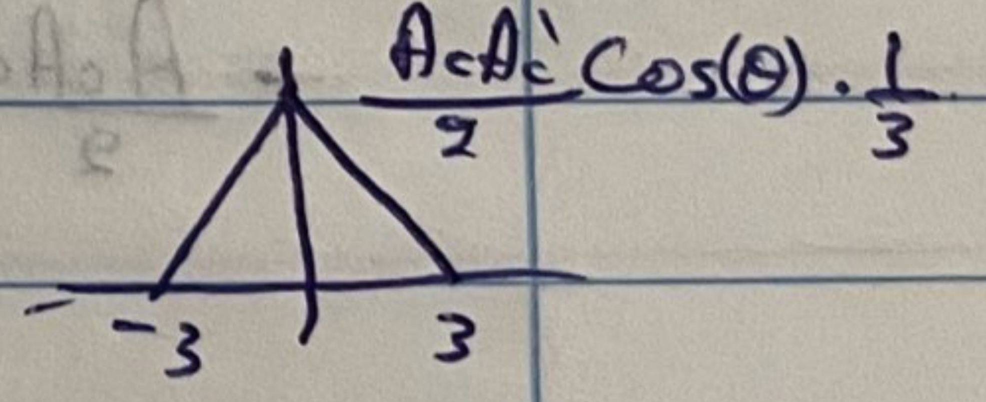
$$= \frac{A_c A_c'}{2} m(t)\cos(\theta) + \frac{A_c A_c'}{2} m(t)\cos(4\pi f_c t + \theta).$$

Since. $m(t) = \text{sinc}^2(3t)$



$$\Rightarrow v(t) = \frac{A_c A_c'}{2}\text{sinc}^2(3t)\cos(\theta) + \frac{A_c A_c'}{2}\text{sinc}^2(3t)\cos(4\pi f_c t + \theta).$$





if $k = \frac{2}{A_c A_c'}$

if k=1

$$y(t) = \frac{A_c A_c'}{2}\cos(\theta)\,\text{sinc}^2(3t)$$

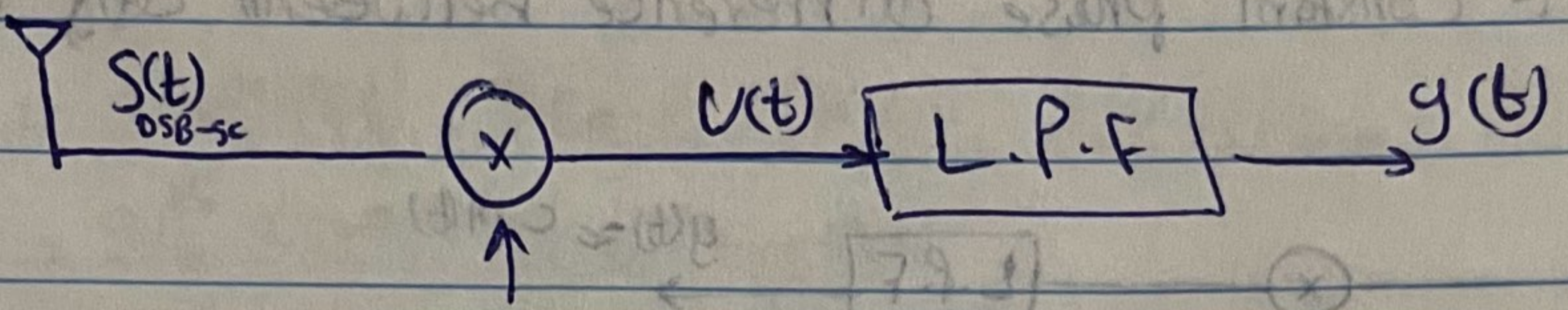
$$= \frac{A_c A_c'}{2}\cos(\theta)\, m(t)$$

$$\Rightarrow y(t) = \cos(\theta)\, m(t)$$

→ We have original m(t) → θ=0

but if $\theta = \frac{\pi}{2} \Rightarrow y(t) = 0$

Case2: Constant frequency difference between $c(t)$ & $c'(t)$



$$c'(t) = A_c' \cos(2\pi(f_c+\Delta f)t)$$

$$V(t) = \underset{DSB-SC}{S(t)}\, c'(t) = m(t)\, c(t)\, c'(t)$$

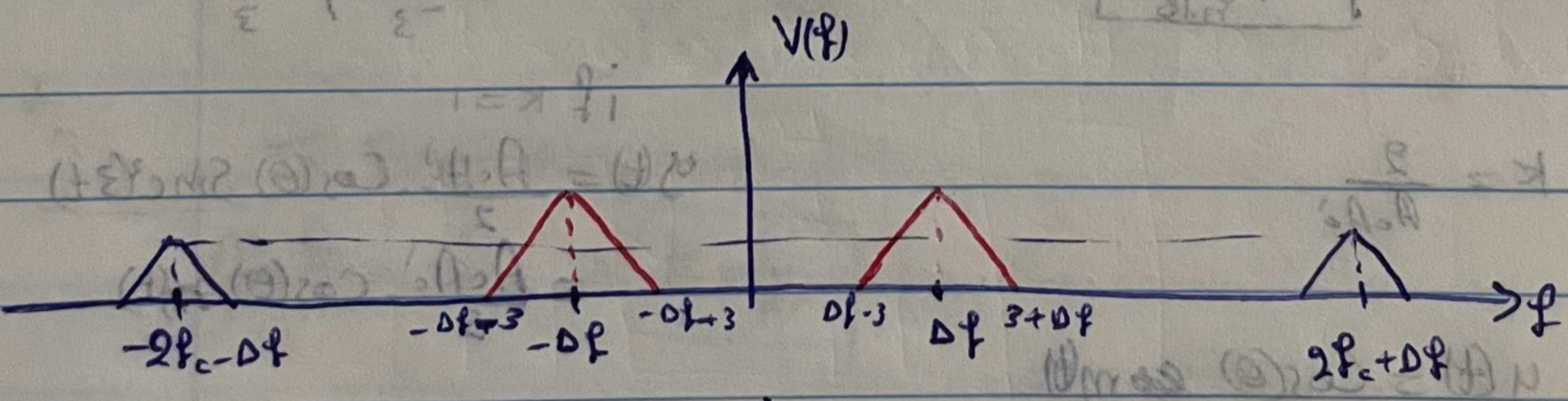
$$= m(t)\, A_c A_c' \cos(2\pi f_c t) \cos(2\pi(f_c+\Delta f)t)$$

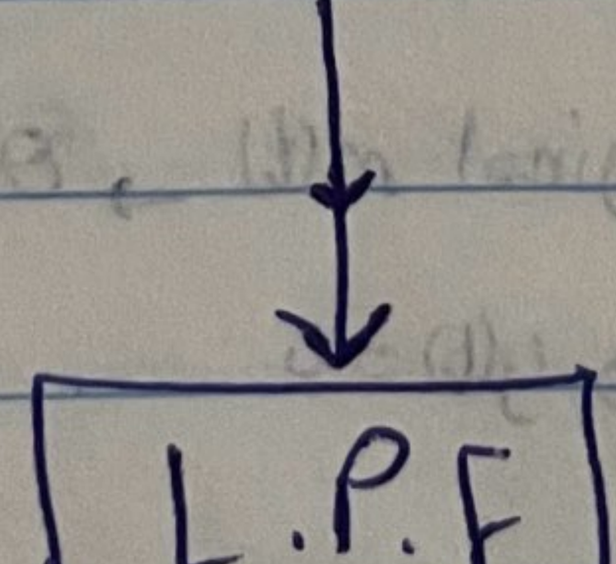
$$= \frac{A_c A_c'}{2} m(t) \left[\cos(4\pi f_c t + 2\pi \Delta f t) + \cos(2\pi \Delta f t)\right]$$

$$= \frac{A_c A_c'}{2} m(t) \cos(4\pi f_c t + 2\pi \Delta f t) + \frac{A_c A_c'}{2} m(t) \cos(2\pi \Delta f t)$$

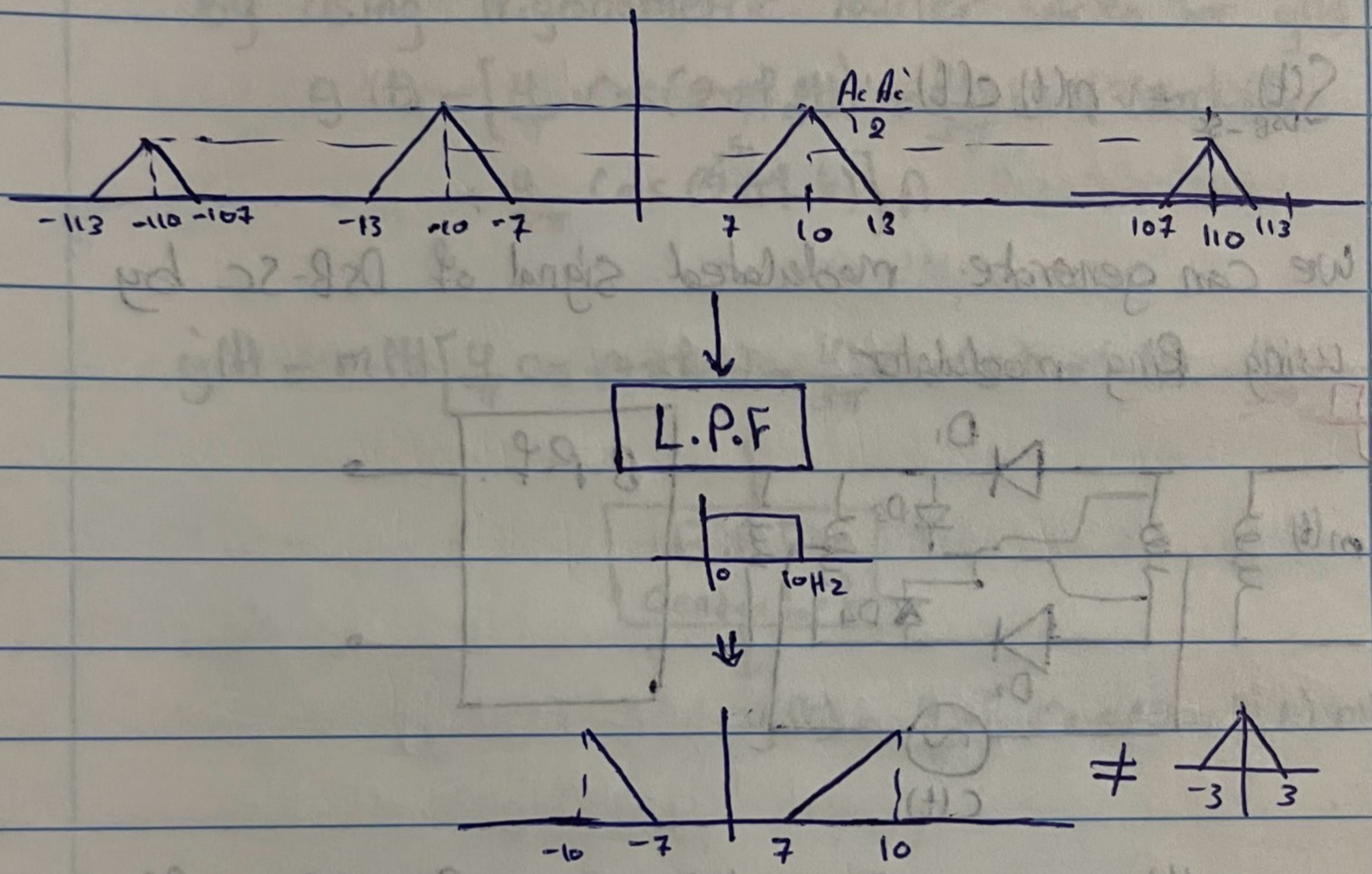
$$V(f) = \frac{A_c A_c'}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \left[\Lambda\left(\frac{1}{3}(f-(2f_c+\Delta f))\right) + \Lambda\left(\frac{1}{3}(f+(2f_c+\Delta f))\right)\right]$$

$$+ \frac{A_c A_c'}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \left[\Lambda\left(\frac{1}{3}(f-\Delta f)\right) + \Lambda\left(\frac{1}{3}(f+\Delta f)\right)\right].$$





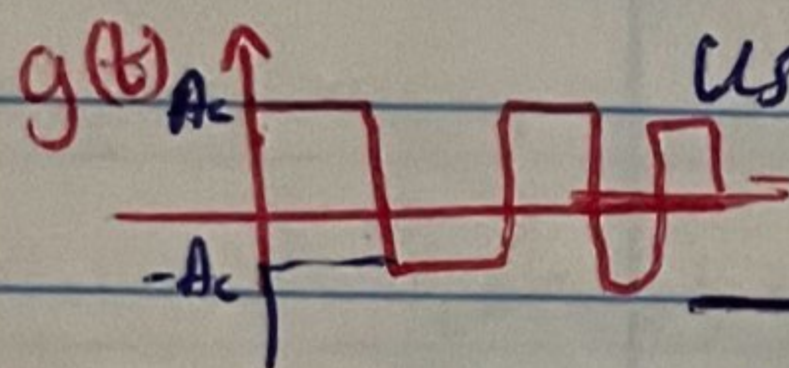
Now, Let $\Delta f = 10$ and $f_c = 50$

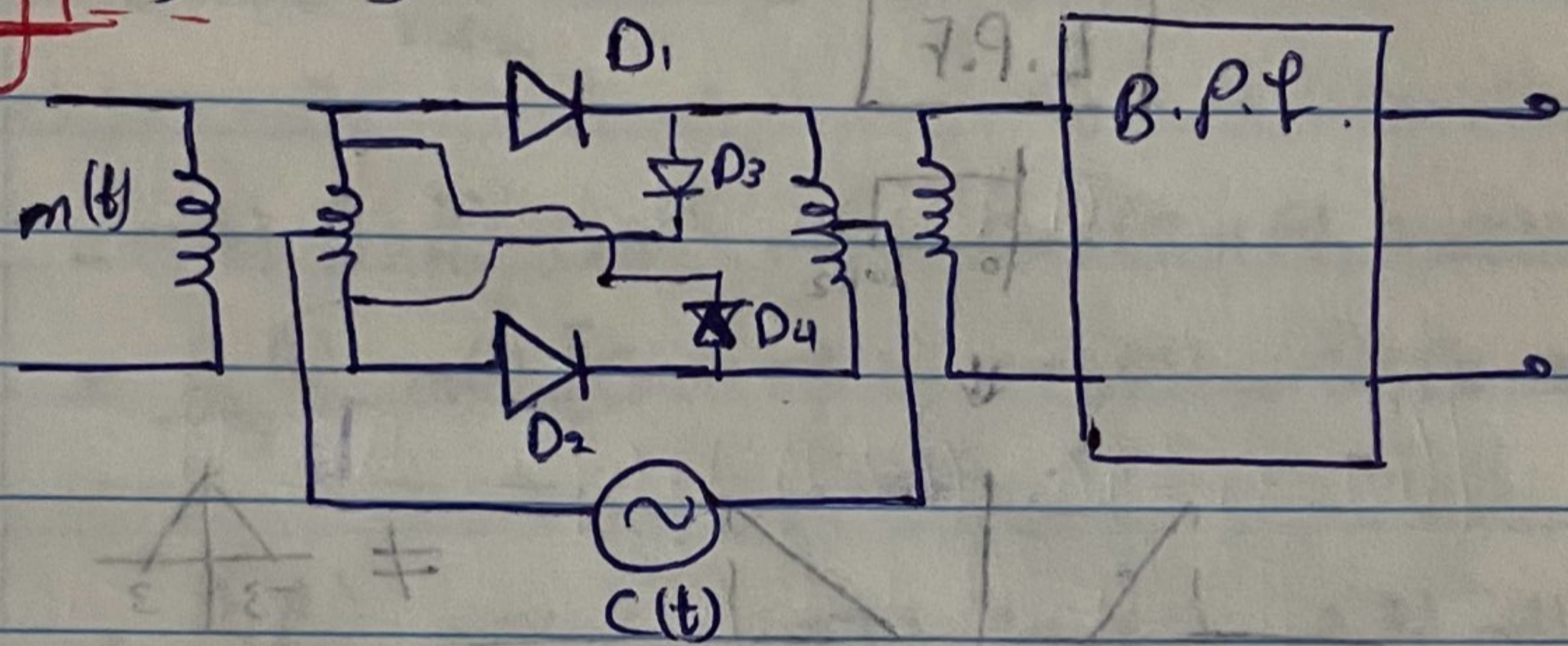


≡ Generat DSB-SC modulated Signal.

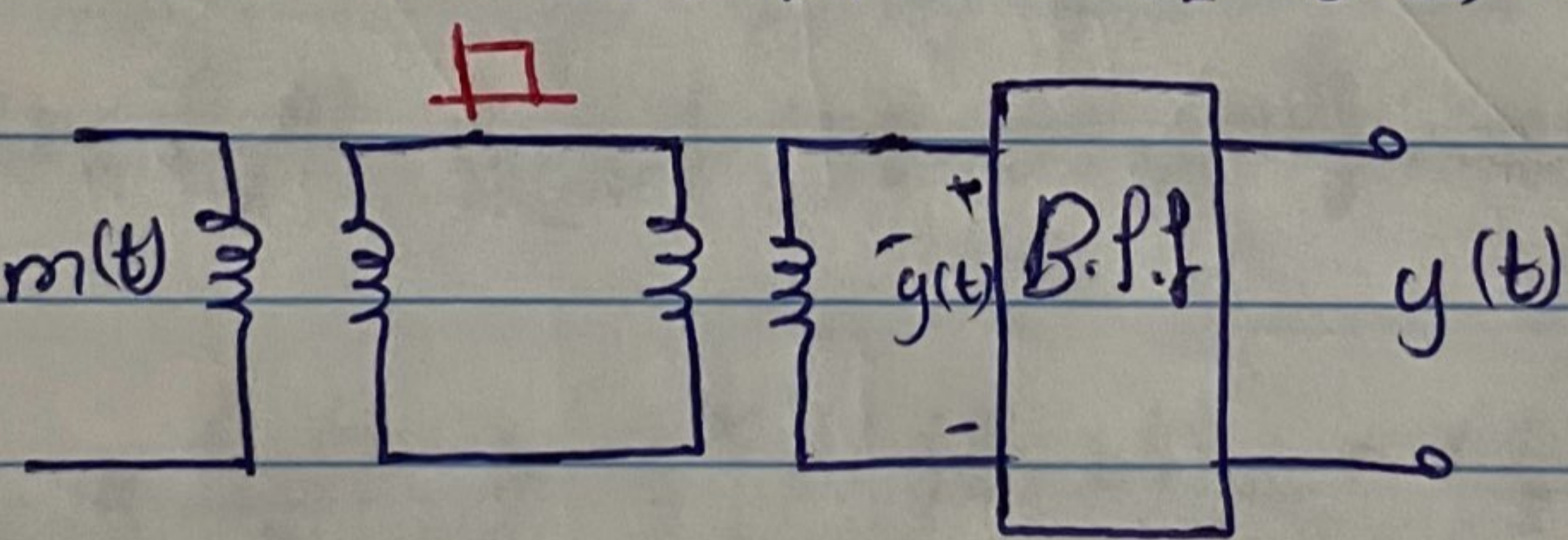
$$S(t)_{DSB-SC} = m(t)\,c(t)$$

We can generate modulated signal of DSB-SC by using Ring modulator.

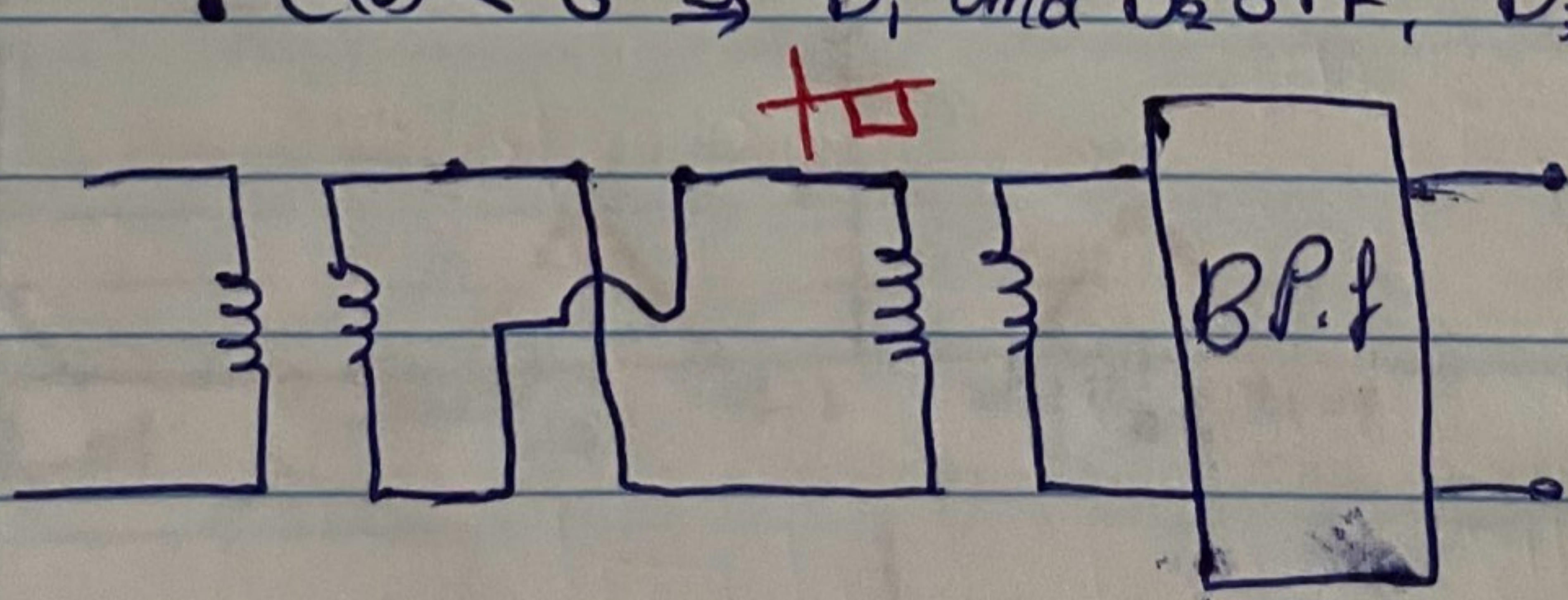




- $c(t) > 0 \rightarrow D_1$ and D_2 on, D_3 and D_4 off.



- $c(t) < 0 \Rightarrow D_1$ and D_2 off, D_3 and D_4 on



$y(t) = m(t)\, g(t)$

by using Trigonometric Fourier Series for $g(t)$

$$g(t) = \left[\frac{4}{\pi}\cos(2\pi f_c t) + \frac{4}{3\pi}\cos(3(2\pi f_c t)) + \frac{4}{5\pi}\cos(5(2\pi f_c t))\right] A_c$$

$$y(t) = m(t)\left[\frac{4}{\pi}\cos(2\pi f_c t) + \frac{4}{3\pi}\cos(3(2\pi f_c t)) + \frac{4}{5\pi}\cos(10\pi f_c t)\right] A_c$$

↓

B.P.F
Center $f_0 = f_c$

↓

$$y(t) = \frac{4}{\pi} A_c \cos(2\pi f_c t)\, m(t).$$

≡ SSB-SC :- Single side band.

In double side band, the modulated signal can be expressed as:

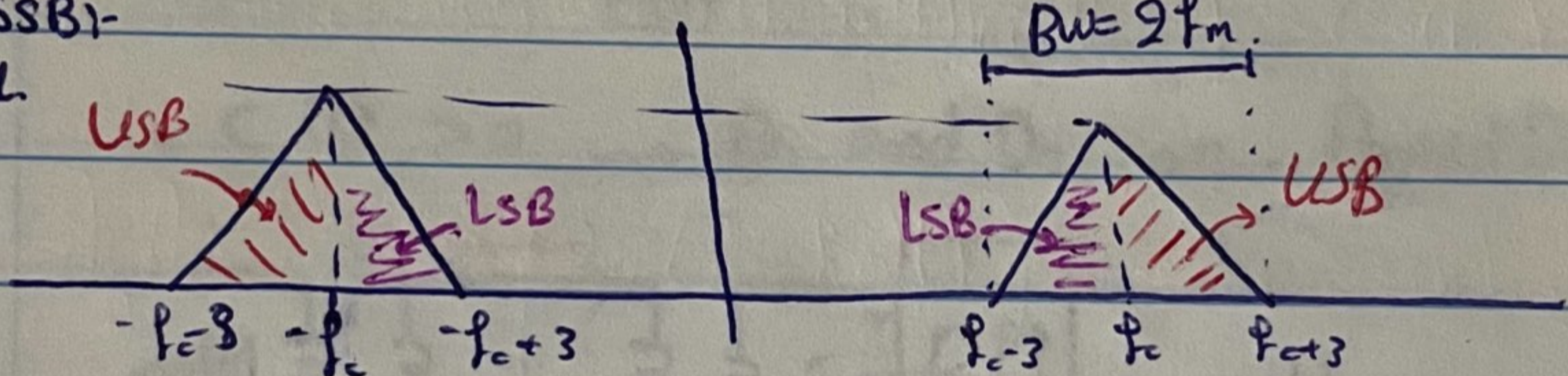
$$S_{DSB-SC}(t) = m(t)\, c(t)$$

Let $m(t) = \text{sinc}^2(3t)$

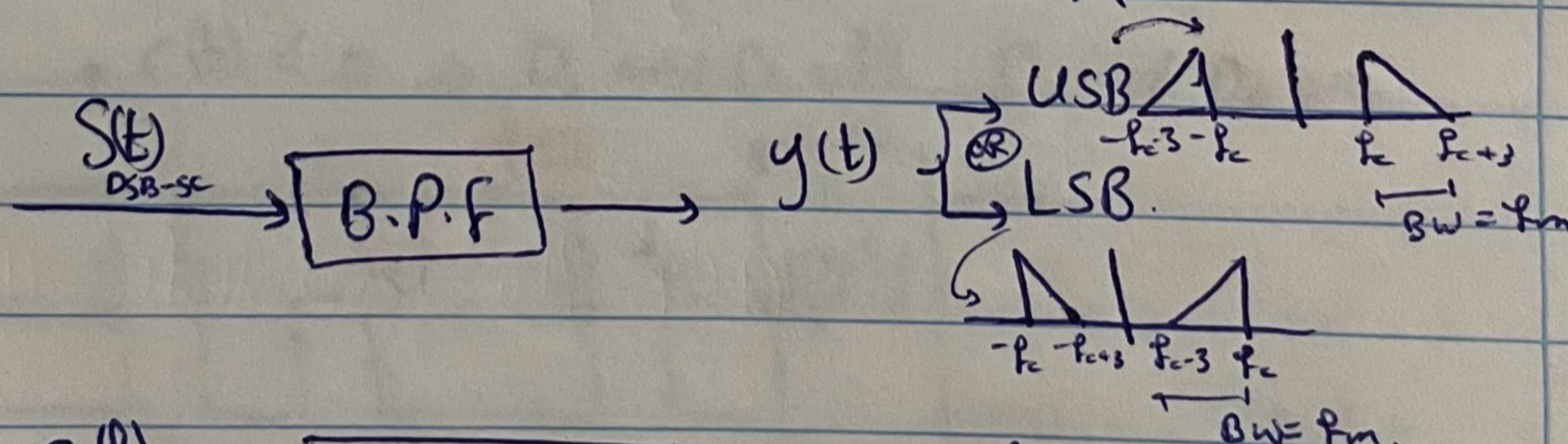
$$S_{DSB-SC}(t) = A_c\, \text{sinc}^2(3t) \cos(2\pi f_c t).$$

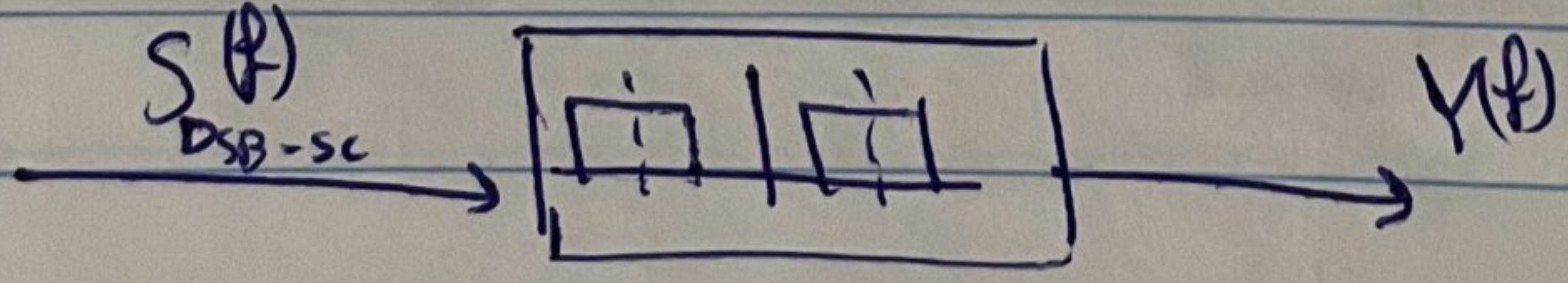
$$S_{DSB-SC}(f) = \frac{A_c}{2} \cdot \frac{1}{3}\left[\Lambda\left(\frac{1}{3}(f-f_c)\right) + \Lambda\left(\frac{1}{3}(f+f_c)\right)\right].$$

II Generation of SSB:- Filtering Method

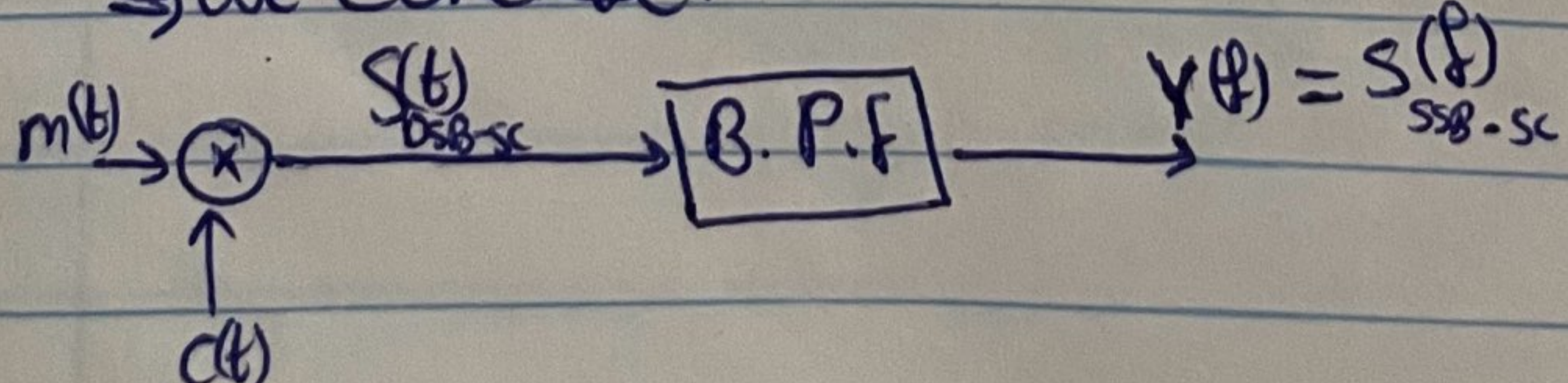


- To decrease the BW from 2fm to fm





⇒ we conclude.



2 Generation SSB signal : phase shift method.

SSB-SC modulated signal can be expressed as.

$$S_{SSB-SC}(t) = A_c\, m(t) \cos(\omega_c t) \pm A_c\, \hat{m}(t) \sin(\omega_c t).$$

$\hat{m}(t)$ → hilbert transform of m(t)

(+ → USB, − → LSB)

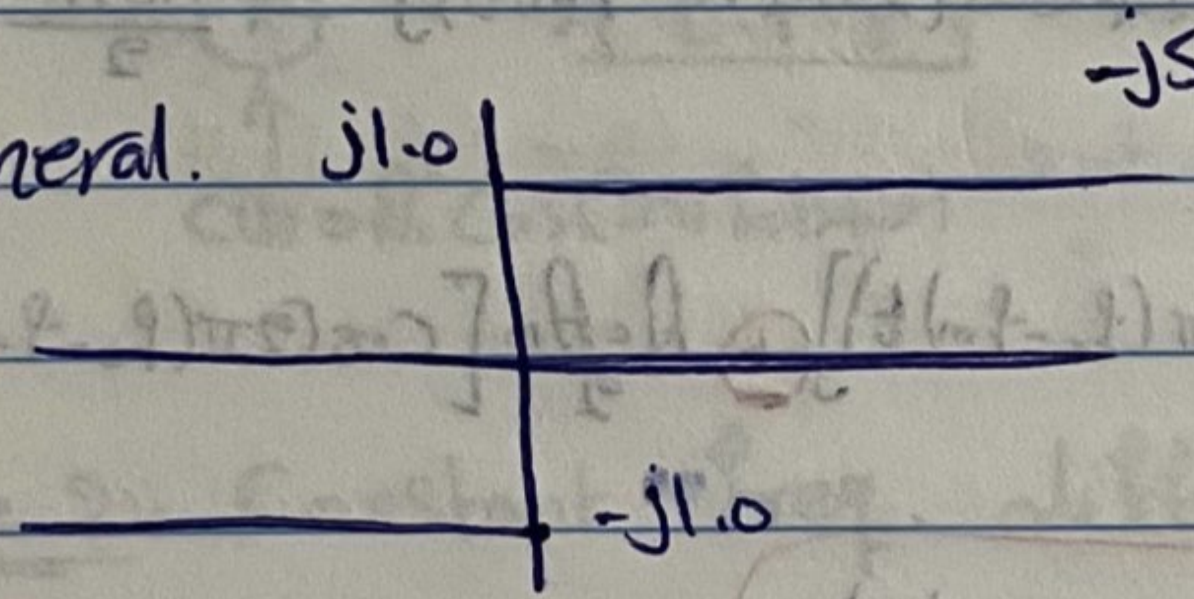
+ Sign ⇒ Lower Side band

− Sign ⇒ upper Side band

Now, Let $m(t) = A_m \cos(2\pi f_m t)$; $c(t) = A_c \cos(2\pi f_c t)$.

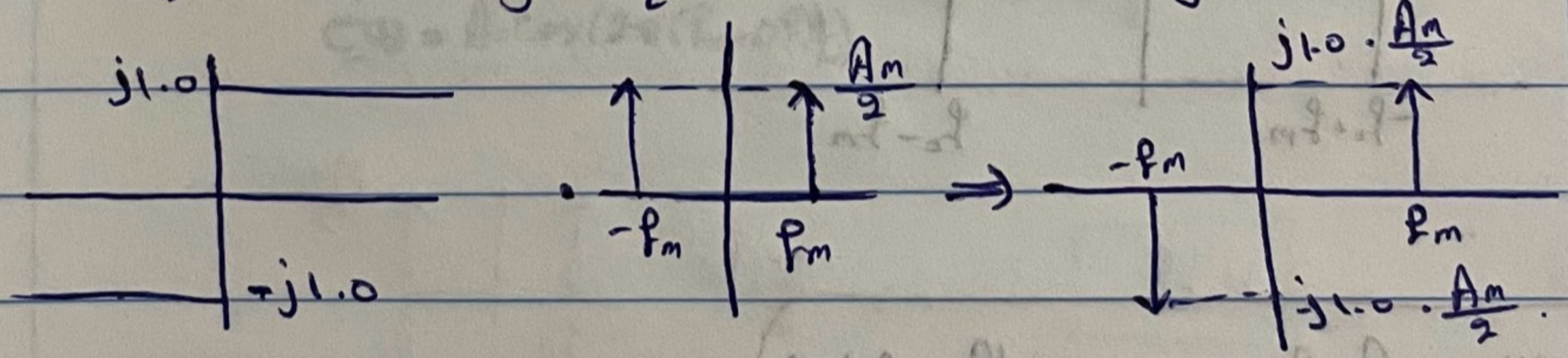
$$\hat{m}(t) = \frac{1}{\pi t} * m(t) \Rightarrow \hat{M}(f) = -j\,\text{sgn}(f)\, M(f).$$

In general.



Since $m(t) = A_m \cos(2\pi f_m t) \Rightarrow M(f) = \frac{A_m}{2}[\delta(f-f_m) + \delta(f+f_m)]$.

$$\Rightarrow \hat{M}(f) = -j\,\text{sgn}(f)[\delta(f-f_m) + \delta(f+f_m)].$$



$$\Rightarrow \hat{M}(f) = [-j\delta(f-f_m) + j\delta(f+f_m)] \cdot \frac{A_m}{2}.$$

$$= \frac{A_m}{j2}\delta(f-f_m) - \frac{A_m}{j2}\delta(f-f_m).$$

$$\hat{m}(t) = \frac{A_m}{j2}\left[e^{j2\pi f_m t} - e^{-j2\pi f_m t}\right] \Rightarrow \boxed{A_m \sin(2\pi f_m t)}$$

$\hat{m}(t)$

$$S_{SSB-SC}(t) = A_c \cdot A_m \cos(2\pi f_m t)\cos(2\pi f_c t) \pm A_c \cdot A_m \sin(2\pi f_m t)\sin(2\pi f_c t)$$

Remember:

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

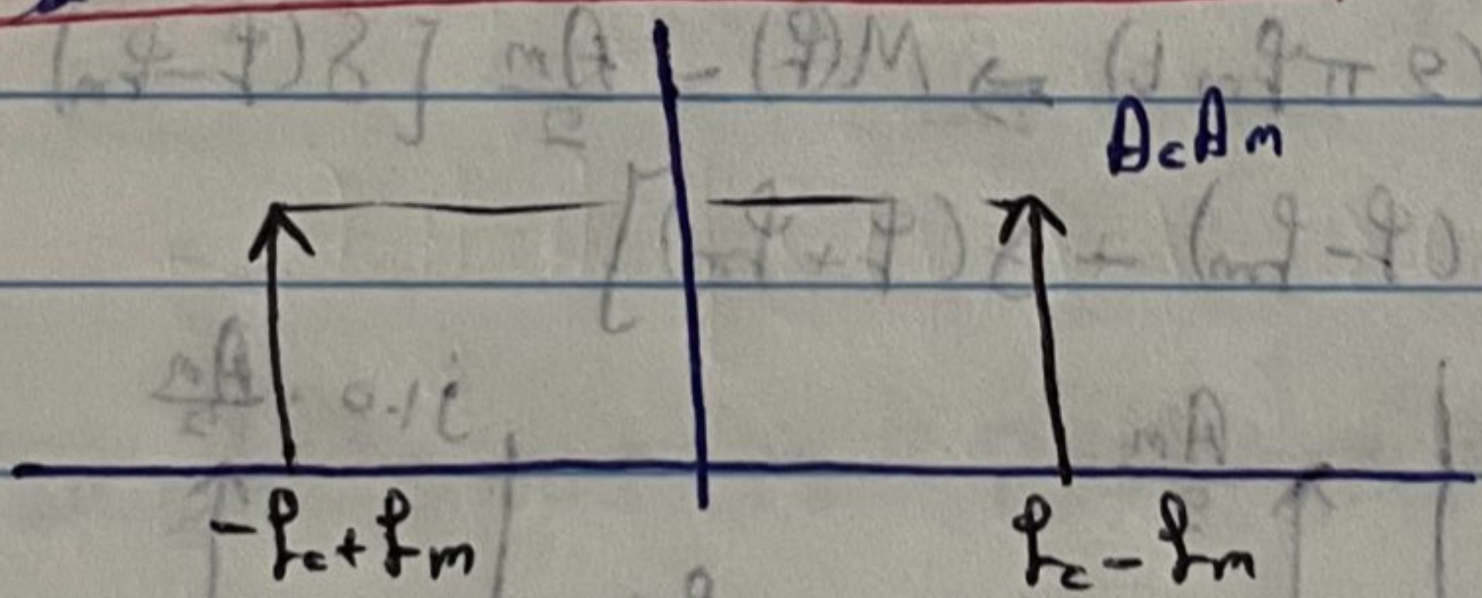
- $\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$
- $\sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$

$$= \frac{A_c A_m}{2}\left[\cos(2\pi(f_c+f_m)t) + \cos(2\pi(f_c-f_m)t)\right] \pm \frac{A_c A_m}{2}\left[\cos(2\pi(f_c-f_m)t) - \cos(2\pi(f_c+f_m)t)\right]$$

$$S_{LSB}(t) = \frac{A_c A_m}{2}\left[\cos(2\pi(f_c+f_m)t) + \cos(2\pi(f_c-f_m)t)\right] + \frac{A_c A_m}{2}\left[\cos(2\pi(f_c-f_m)t) - \cos(2\pi(f_c+f_m)t)\right]$$

⇒

- $$S_{LSB} = \frac{A_c A_m}{2} \cdot 2\cos(2\pi(f_c-f_m)t)$$

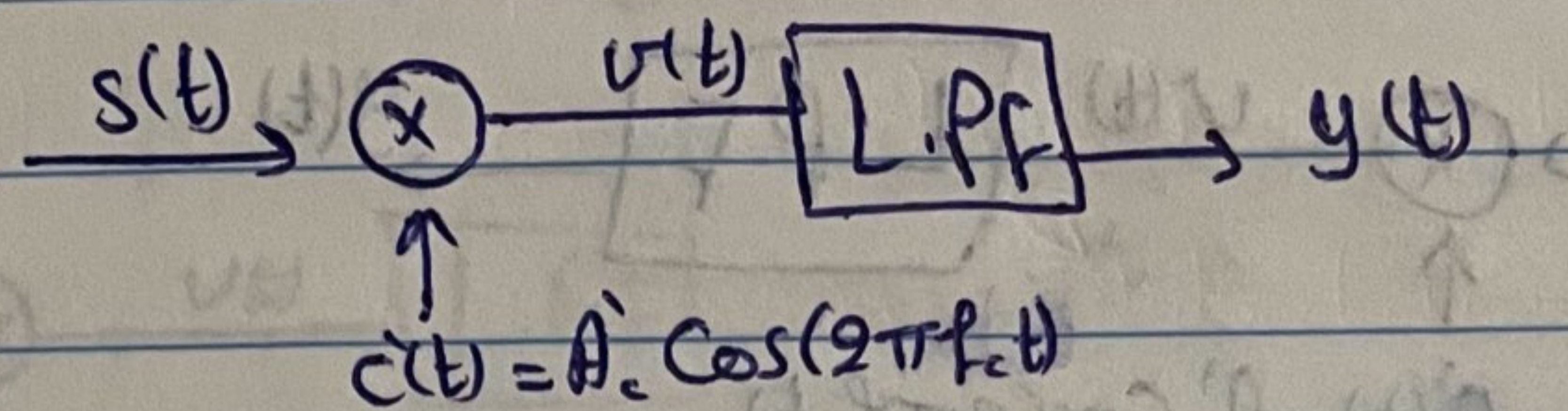


$$S_{USB}(t) = A_c A_m \cos(2\pi(f_c+f_m)t)$$

(L11)
1

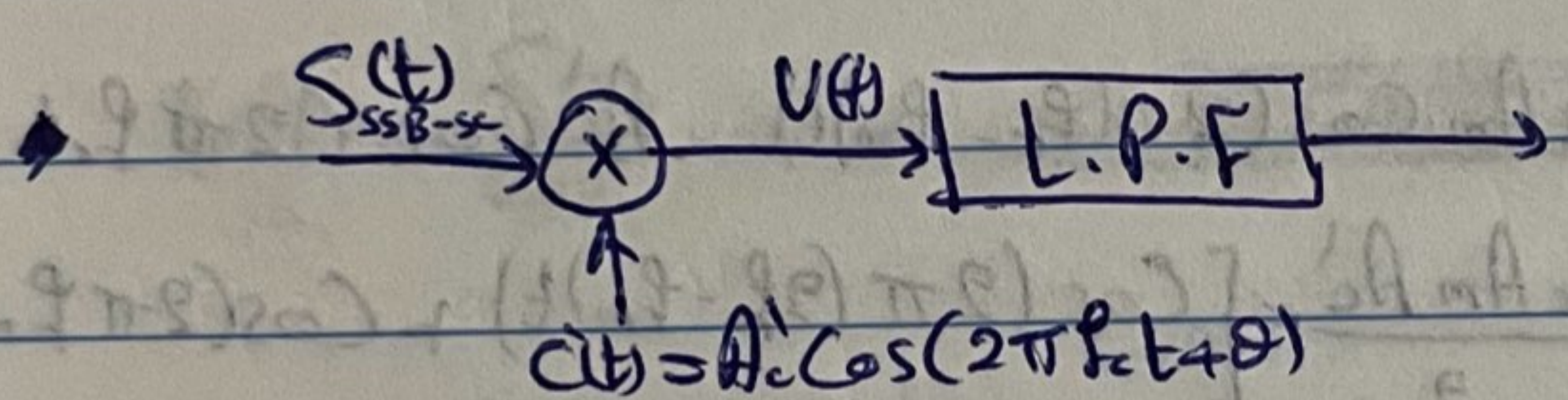
Demodulation of SSB-SC.

- Coherent

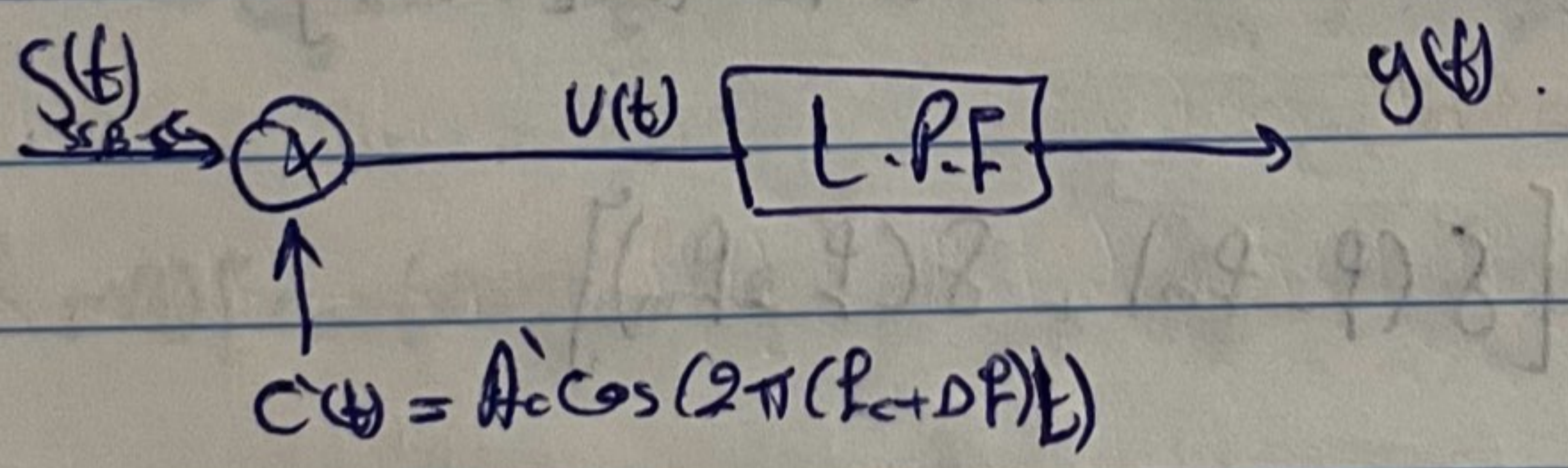


- Non-Coherent.

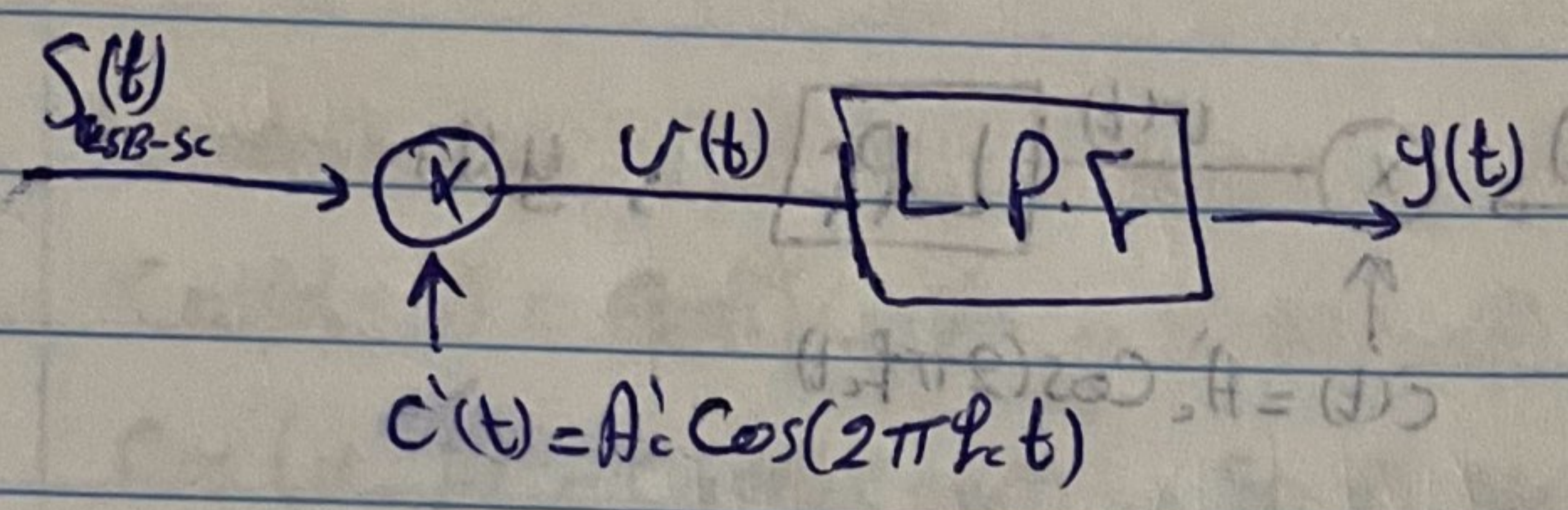
• Case 1: Constant phase difference between c(t) and c'(t)



• Case 2: Constant freq. difference between c(t) & c'(t).



Coherent Detector.



Let $m(t) = A_m \cos(2\pi f_m t)$.

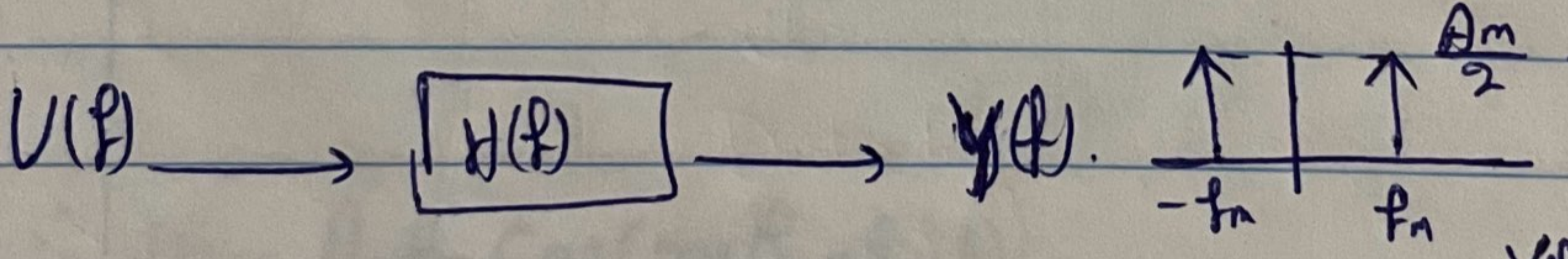
$$\Rightarrow S_{USB-SC}(t) = A_c A_m \cos(2\pi(f_c - f_m)t)$$

$$v(t) = S_{USB-SC}(t)\, c'(t) = A_c A_m \cos(2\pi(f_c - f_m)t) \cdot A_c' \cos(2\pi f_c t).$$

$$= \frac{A_c A_m A_c'}{2}\left[\cos(2\pi(2f_c - f_m)t) + \cos(2\pi f_m t)\right]$$

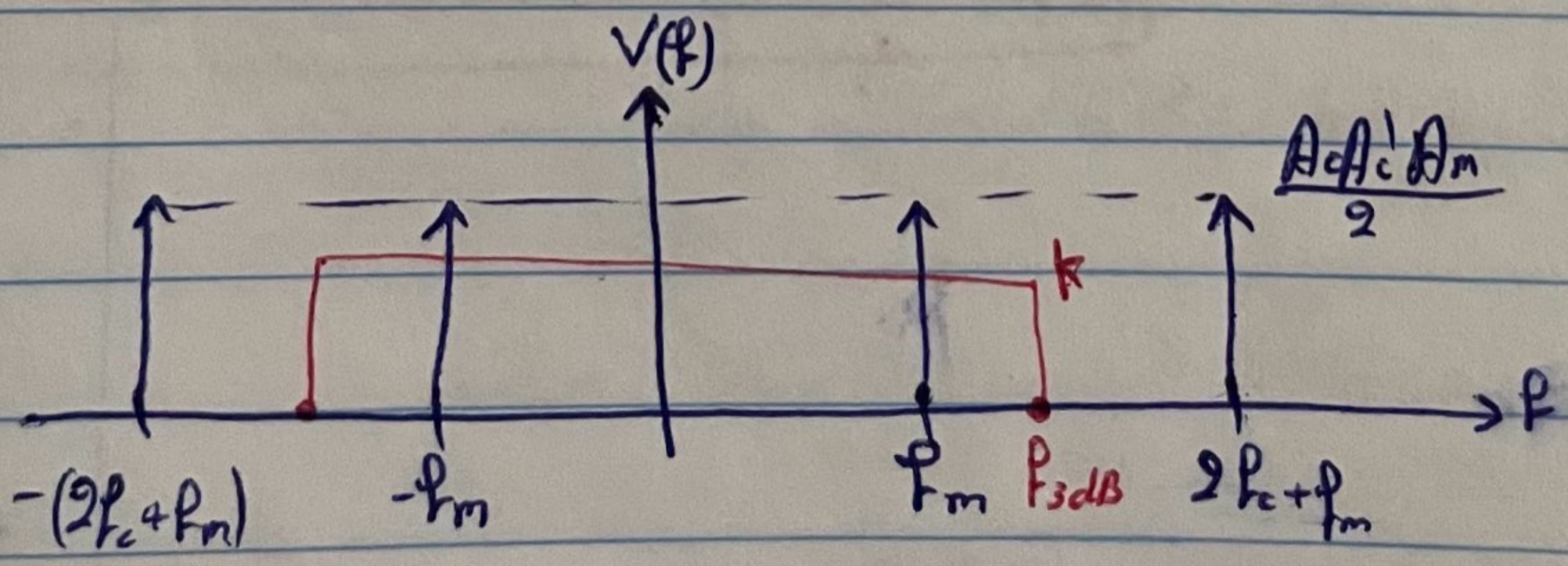
$$\rightarrow V(f) = \frac{A_c A_m A_c'}{4}\left[\delta(f - (2f_c - f_m)) + \delta(f + (2f_c - f_m))\right]$$

$$+ \frac{A_c A_m A_c'}{4}\left[\delta(f - f_m) + \delta(f + f_m)\right].$$



$$Y(f) = \frac{A_m}{2}\left[\delta(f - f_m) + \delta(f + f_m)\right]$$

$$y(t) = A_m \cos(2\pi f_m t).$$



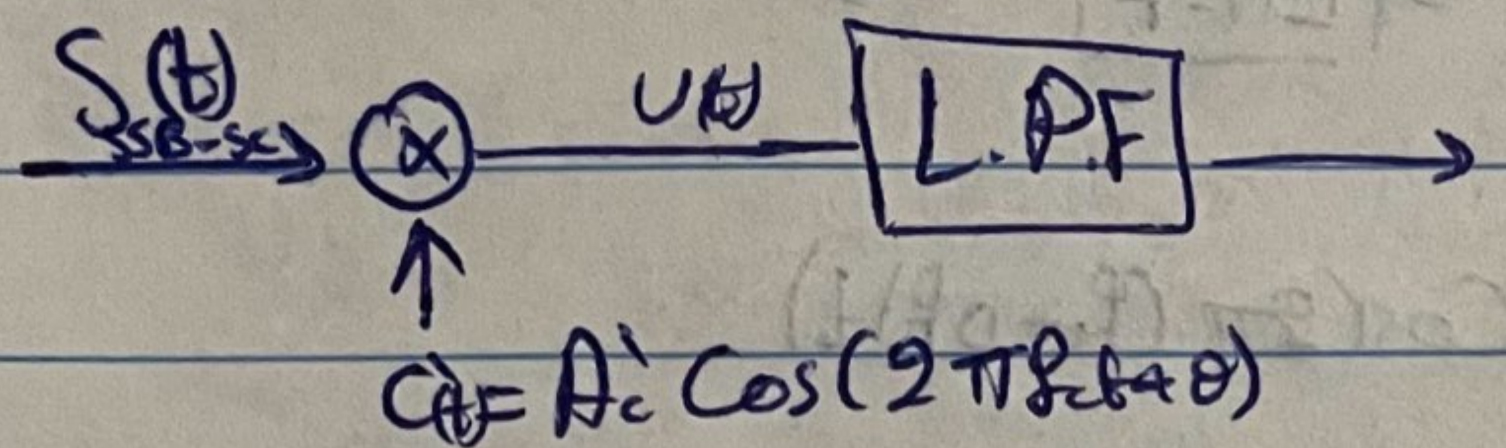
$$BW = f_m$$

$$f_m \leq f_{3dB} < 2f_c + f_m.$$

$$(k)\left(\frac{A_c A_c' A_m}{4\,2}\right) = \frac{A_m}{2} \rightarrow \boxed{k = \frac{2}{A_c A_c'}}$$

≡ Non - Coherent Detector.

- Case 1 :- phase constant difference between $c(t)$ & $c'(t)$.



$$S_{USB}(t) = A_c m(t) \cos(\omega_c t) - A_c \hat{m}(t) \sin(\omega_c t)$$

$$v(t) = S_{USB}(t) \cdot c'(t)$$

$$= [A_c m(t) \cos(\omega_c t) - A_c \hat{m}(t) \sin(\omega_c t)][A_c' \cos(\omega_c t + \theta)]$$

$$= A_c A_c' m(t) \cos(\omega_c t) \cos(\omega_c t + \theta) - A_c A_c' \hat{m}(t) \sin(\omega_c t) \cos(\omega_c t + \theta)$$

$$= \frac{A_c A_c'}{2} m(t) [\cos(2\omega_c t + \theta) + \cos(\theta)] - \frac{A_c A_c'}{2} \hat{m}(t) [\sin(2\omega_c t + \theta) + \sin(-\theta)].$$

- $\sin(\alpha)\cos(\beta) = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$.

$$v(t) = \frac{A_c A_c'}{2} m(t) [\cos(2\omega_c t + \theta) + \cos(\theta)] - \frac{A_c A_c'}{2} \hat{m}(t) [\sin(2\omega_c t + \theta) - \sin(\theta)].$$

↓

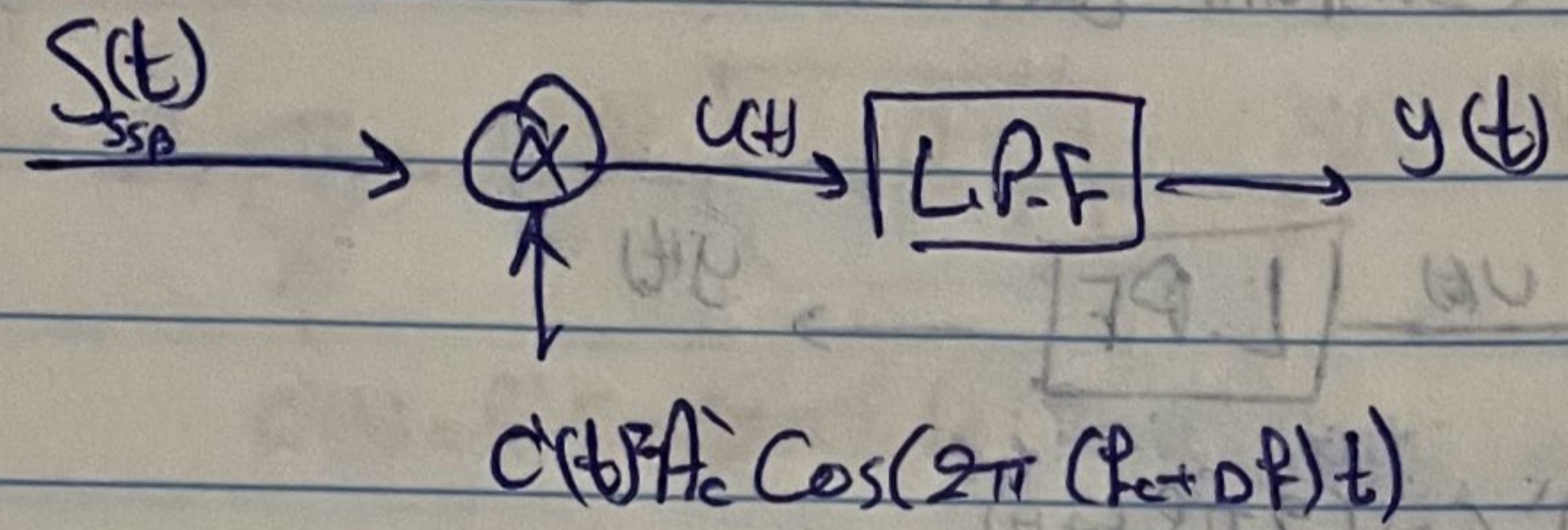
L.P.F.

↓

$$y(t) = \frac{A_c A_c'}{2} m(t) \cos(\theta) + \frac{A_c A_c'}{2} \hat{m}(t) \sin(\theta).$$

⇓ distortion

Case 2: Constant freq. difference between $c(t)$ & $c'(t)$



$$S_{USB}(t) = A_c\, m(t) \cos(\omega_c t) - A_c\, \hat{m}(t) \sin(\omega_c t)$$

$$v(t) = S_{USB}(t)\, c'(t) = \left[A_c\, m(t) \cos(\omega_c t) - A_c\, \hat{m}(t) \sin(\omega_c t)\right]\left[A'_c \cos(2\pi(f_c + \Delta f)t)\right]$$

$$= A_c A'_c\, m(t) \cos(\omega_c t) \cos(2\pi(f_c + \Delta f)t) - A_c A'_c\, \hat{m}(t) \sin(\omega_c t) \cos(2\pi(f_c + \Delta f)t)$$

$$v(t) = \frac{A_c A'_c}{2} m(t)\left[\cos(2\pi(2f_c + \Delta f)t) + \cos(2\pi \Delta f t)\right] + \frac{A_c A'_c}{2} \hat{m}(t)\left[\sin(2\pi(2f_c + \Delta f)t) - \sin(2\pi \Delta f t)\right]$$

⇓

L.P.F

$$y(t) = \frac{A_c A'_c}{2} m(t) \cos(2\pi \Delta f t) - \frac{A_c A'_c}{2} \hat{m}(t) \sin(2\pi \Delta f t)$$

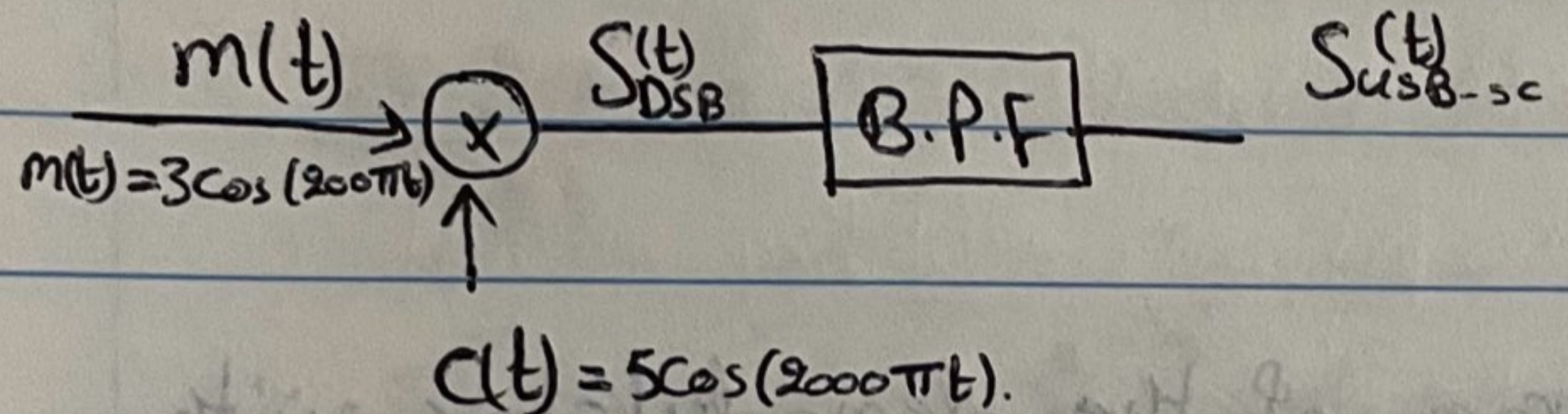
⇓

Distortion.

Example The message signal $m(t) = 3\cos(200\pi t)$ along with carrier signal $c(t) = 5\cos(2000\pi t)$ are applied to modulator that generates the <u>upper side band</u> suppressed carrier signal.

1) Draw block diagram of modulator.

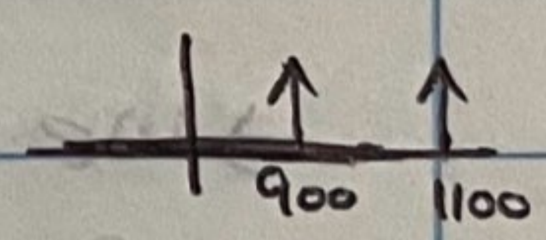
<u>Ans:</u> by using Filtering method.



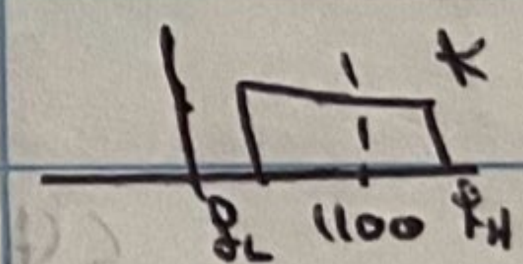
$S_{DSB}(t) = m(t)\, c(t) = 15\cos(200\pi t)\cos(2000\pi t)$

$= \frac{15}{2}\left[\cos(2200\pi t) + \cos(1800\pi t)\right]$

↑ USB ↑ LSB



B.P.F ⇒



$900 < f_L \leq 1100$

Center freq. $f_o = 1100$

after B.P.F.

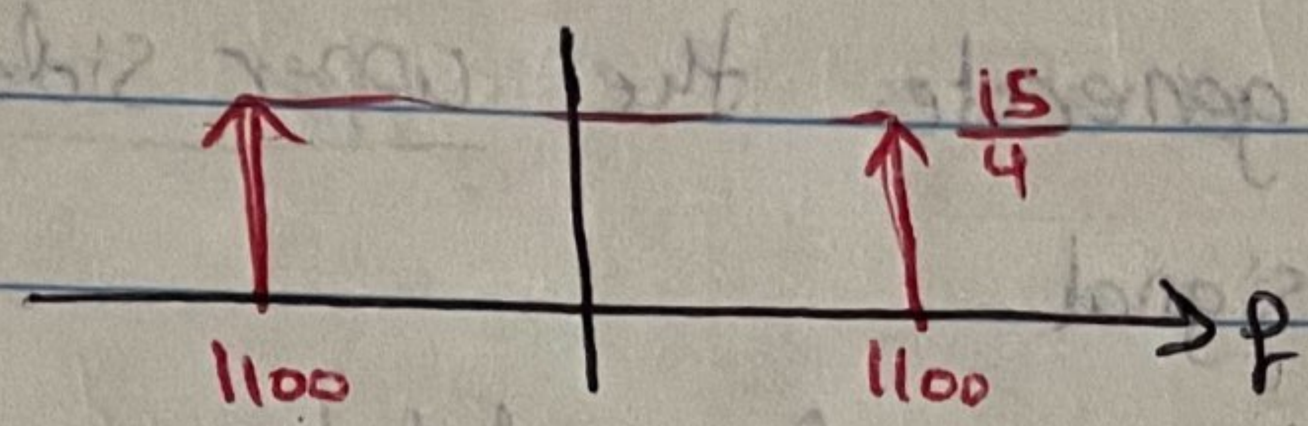
$\Rightarrow S_{USB}(t) = \frac{15}{2}\cos(2200\pi t)$.

2) write expression of modulation signal in time domain.

$S_{USB}(t) = \frac{15}{2}\cos(2200\pi t)$.

3) Evaluate and plot the spectrum of modulated signal.

$$S_{usb}(f) = \frac{15}{4}\left[\delta(f-1100) + \delta(f+1100)\right].$$



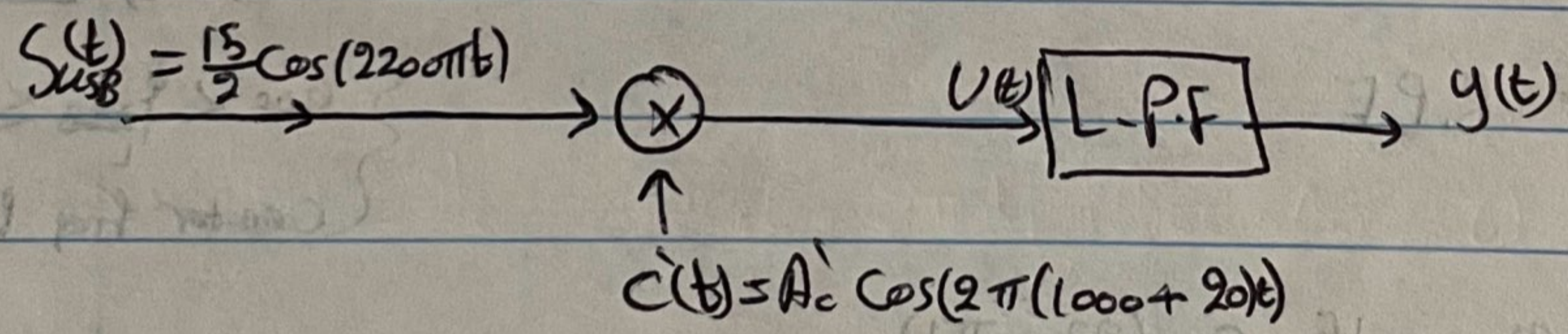
4) Evaluate the BW of modulated signal ← Pass band signal. $c(t) \to 1000$

(1100 − 1000 = 100)

$B = 100\,Hz.$

5) Draw block diagram of the demodulator with freq. distortion ($\Delta f = 20Hz$), specifying the details at each block.

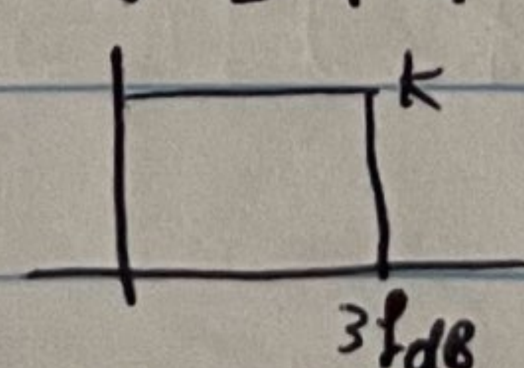
⇒ we have Non-Coherent detector with Constant freq. difference between $c(t)$ & $c'(t)$



$$v(t) = S_{usb}(t)\, c'(t) \Rightarrow \frac{15}{2}\cos(2200\pi t) \cdot A_c' \cos(2\pi(1000+20)t)$$

$$= \frac{15}{2} \cdot \frac{A_c'}{2}\left[\cos(2\pi(2f_c + \Delta f + f_m)t) + \cos(2\pi(f_m + \Delta f)t)\right]$$

↓ L.P.F $\quad \Delta f + 2f_c + f_m > f_{3dB} \geq f_m + \Delta f$



$$\Rightarrow y(t) = \frac{15}{4} A_c' \cos(2\pi(f_m + \Delta f)t) \neq m(t) \quad [\text{Distortion}]$$

L12

Continuous Communication system.

- Carrier signal.

$c(t) = A_c \cos(\phi(t))$

A_c → Amp, $\phi(t)$ → Angle.

$$\frac{\partial \phi_i(t)}{\partial t} = \omega_i$$

$$\frac{\partial \phi_i(t)}{\partial t} = 2\pi f_i \text{ [instantaneous freq.]}$$

$$\rightarrow f_i = \frac{1}{2\pi} \frac{\partial \phi_i(t)}{\partial t} \quad ①$$

In FM:-

$$f_i = f_c + K_f \, m(t) \quad ②$$

$$\frac{1}{2\pi} \frac{\partial \phi_i(t)}{dt} = f_c + K_f \, m(t).$$

$$\rightarrow \phi_i(t) = \int_0^t 2\pi f_c \, d\tau + 2\pi K_f \int_0^t m(\tau) \, d\tau$$

$$= 2\pi f_c t + 2\pi K_f \int_0^t m(\tau) \, d\tau$$

⇒

≡ FM modulated signal.

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right) \quad \text{— (3)}$$

where, k_f :- frequency sensitivity.

Now, Let $m(t) = A_m \cos(2\pi f_m t)$ — (4)

Remember:- $f_i = f_c + k_f\, m(t)$

$\rightarrow f_i - f_c = k_f\, m(t) \rightarrow \boxed{\Delta f = k_f\, m(t)}$ freq. deviation.

$\Delta f_{max} = k_f\, |m(t)|_{max}$

In our example.

$\Delta f_{max} = k_f . A_m$

By substituting (4) in (3).

$$\Rightarrow S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m \tau)\, d\tau\right).$$

$$= A_c \cos\left(2\pi f_c t + \frac{2\pi\, k_f A_m}{2\pi f_m} \sin(2\pi f_m t)\right)$$

($k_f A_m$ = Δf_{max})

$$= A_c \cos\left(2\pi f_c t + \frac{\Delta f_{max}}{f_m} \sin(2\pi f_m t)\right)$$

$\frac{\Delta f_{max}}{f_m}$ → β :- modulation index.

$$S_{FM}(t) = A_c \cos(\underbrace{2\pi f_c t}_{\alpha} + \underbrace{\beta \sin(2\pi f_m t)}_{\beta})$$

In general;

- $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
- $\sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$.

$\Rightarrow S_{FM}(t) = A_c\left[\cos(2\pi f_c t)\cos(\beta\sin(2\pi f_m t)) - \sin(2\pi f_c t)\sin(\beta\sin(2\pi f_m t))\right]$

- Remember, let $x(\theta) = \sin(\theta)$, if θ very small

$\Rightarrow \sin(\theta) \approx \theta$.

$y(\theta) = \cos(\theta)$, if θ very small

$\Rightarrow \cos(\theta) \approx 1$.

If $\beta \ll 1$. very small. (Narrow band).

$$\Rightarrow S_{FM}(t) = A_c\left[\cos(2\pi f_c t) - \sin(2\pi f_c t)\,\beta\sin(2\pi f_m t)\right].$$

$$= A_c\left[\cos(2\pi f_c t) - \frac{\beta}{2}\cos(2\pi(f_c-f_m)t) + \frac{\beta}{2}\cos(2\pi(f_c+f_m)t)\right]$$

if we compare $S_{FM}(t)$ in narrow band with $S_{AM}(t)$, we Can Conclued.

$$S_{FM}(t) = A_c\cos(2\pi f_c t) - \frac{A_c\beta}{2}\cos(2\pi(f_c-f_m)t) + \frac{A_c\beta}{2}\cos(2\pi(f_c+f_m)t).$$

and.

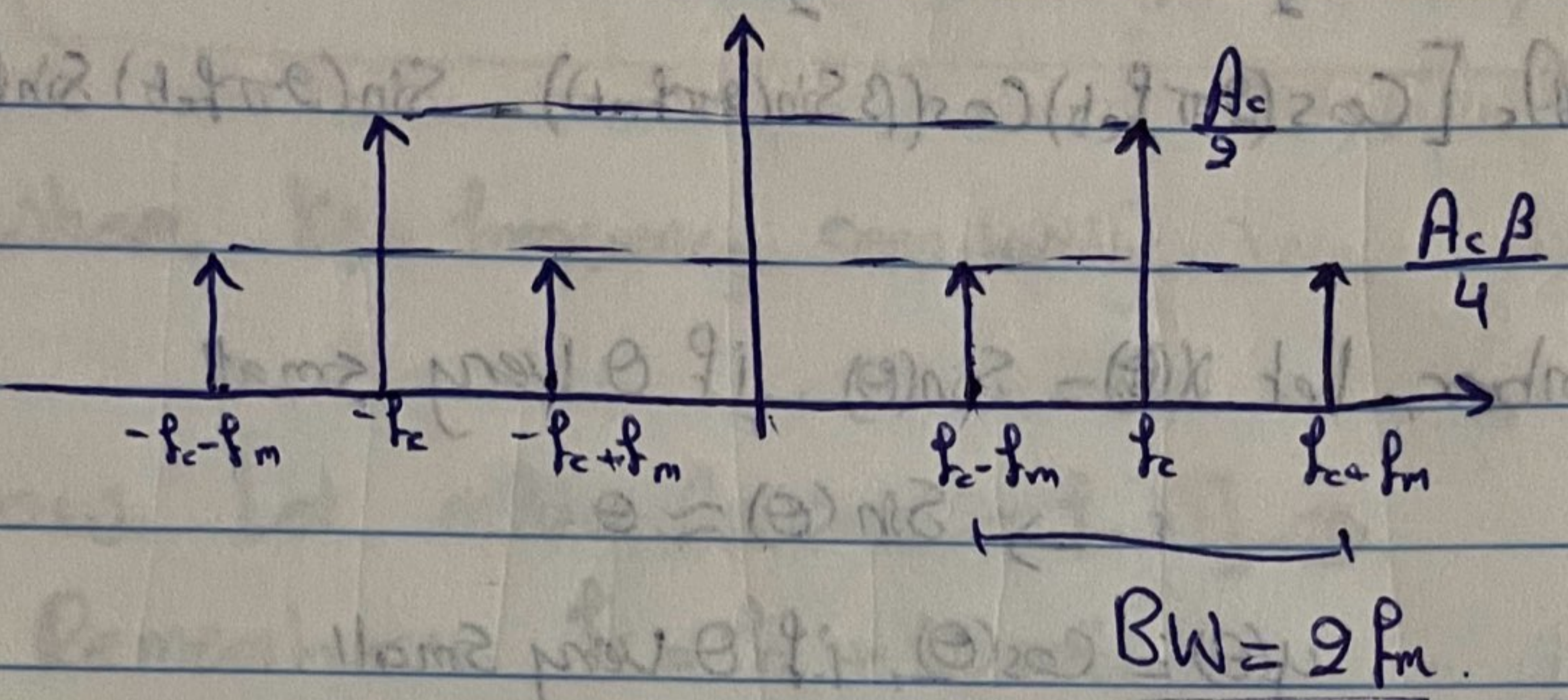
$$S_{AM}(t) = A_c\cos(2\pi f_c t) + \frac{A_c\mu}{2}\cos(2\pi(f_c+f_m)t) + \frac{A_c\mu}{2}\cos(2\pi(f_c-f_m)t).$$

(μ annotated: $K_a A_m$)

$\Rightarrow$

In FM :-

$P_c = \frac{A_c^2}{2}$, $P_{sides} = \frac{\left(\frac{A_c \beta}{2}\right)^2}{2} \cdot 2$, $BW = 2 f_m$



Now, if $\beta \gg 1$. "wide band"

$S_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ ← periodic signal.

$= \mathrm{Re}\{A_c e^{j(2\pi f_c t)} \cdot e^{j\beta \sin(2\pi f_m t)}\}$ — ⑤

(the term $e^{j\beta \sin(2\pi f_m t)}$ → periodic signal)

$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$, $C_n = \frac{1}{T_m} \int_{T_m} e^{j\beta \sin(2\pi f_m t)} \cdot e^{-j2\pi n f_m t} \, dt$

$= \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$ — ⑥

$J_n(\beta)$: Bessel function.

- Some Properties of $J_n(x)$:
  1- $J_n(x) = (-1)^n J_{-n}(x)$
  2- $J_n(x) = (-1)^n J_n(-x)$
  3- $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$
  4- for small value of x
  $J_n(x) \cong \frac{x^n}{2^n n!}$, $J_0(x) = 1$
  $J_1(x) = \frac{x}{2}$

5- $\sum_{-\infty}^{\infty} (J_n(x))^2 = 1$, for all x.

$$\Rightarrow S_{FM}(t) = Re\{A_c e^{j2\pi f_c t} \cdot \sum_{n=\infty}^{\infty} J_n(\beta) e^{-j2\pi n f_m t}\}$$

$$= Re\{A_c \sum_{n=\infty}^{\infty} J_n(\beta) e^{j2\pi(f_c - n f_m)t}\}$$

$$S_{FM}(t) = A_c \sum_{n=\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c - n f_m)t)$$

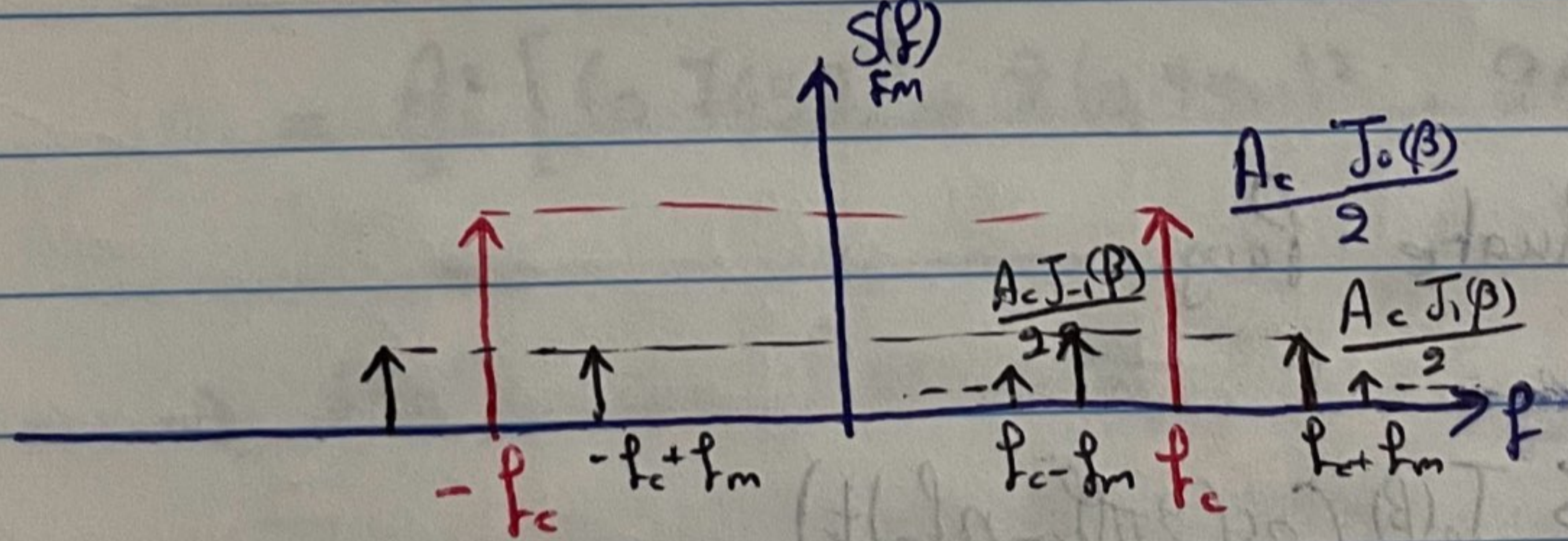
- To evaluate total Power.

$$P_T = \frac{A_c^2}{2} \sum_{n=\infty}^{\infty} J_n^2(\beta)^{\to 1} \Rightarrow P_T = \frac{A_c^2}{2}$$

Since.

$$S_{FM}(t) = A_c \sum_{n=\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c - n f_m)t)$$

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=\infty}^{\infty} J_n(\beta) [\delta(f - f_c + n f_m) + \delta(f + f_c + n f_m)]$$



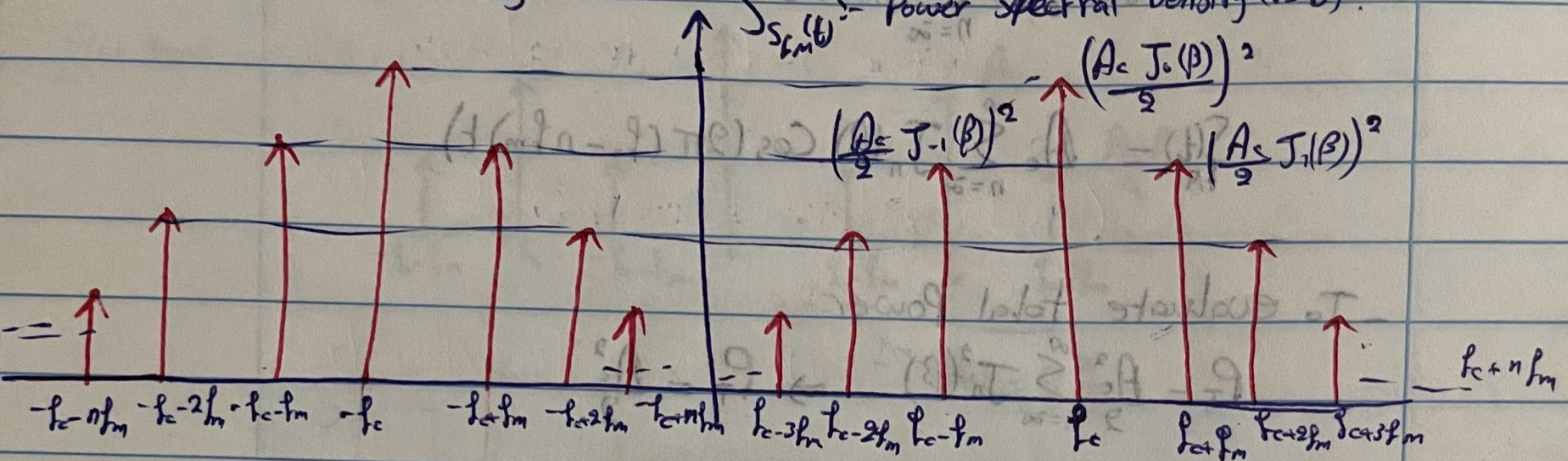
$$\boxed{BW = 2n f_m}$$

Carrier Power:-

$$P_c = 2 \cdot \left(\frac{A_c}{2} J_0(\beta)\right)^2 \Rightarrow P_c = \frac{A_c^2}{2} J_0^2(\beta)$$

To evaluate BW.

→ Evaluate 99% Power band width at an FM signal.



$$P_{avg} = \frac{A_c^2}{2} J_0^2(\beta) + \frac{A_c^2}{2} J_1^2(\beta) + \ldots$$

$$= \frac{A_c^2}{2}\left[J_0^2(\beta) + 2J_1^2(\beta) + \ldots\right]$$

We can evaluate P_{avg} in time domain.

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c - nf_m)t)$$

$$= \ldots + A_c J_{-1}(\beta)\cos(2\pi(f_c+f_m)t) + A_c J_0(\beta)\cos(2\pi f_c t) + A_c J_1(\beta)\cos(2\pi(f_c - f_m)t) + \ldots$$

$$P_{avg} = \ldots + \frac{A_c^2}{2} J_{-1}^2(\beta) + \frac{A_c^2}{2} J_0^2(\beta) + \frac{A_c^2}{2} J_1^2(\beta) + \ldots$$

$$P_{avg} = \frac{A_c^2}{2}\left[J_0^2(\beta) + 2J_1^2(\beta) + 2J_2^2(\beta) + \ldots\right] \times 100\%$$

بعرف أحل وباللحظة الي بوصل فيها للنسبة الي طالبها بوقف.

L13

if $\beta = 0.2$.

$$P_{avg} = \frac{A_c^2}{2}\left[J_0^2(0.2) + 2J_1^2(\beta) + \cdots\right].$$

$$= \frac{A_c^2}{2}\left[\underbrace{(0.99)^2 + 2(0.00498)^2}_{0.99}\right] \times 100\%$$

$\Rightarrow n = 1 \Rightarrow BW = (2)(1) f_m$

$\Rightarrow BW = 2f_m$.

if $\beta = 1$.

$J_0(1) = 0.7652$, $J_1(1) = 0.4401$, $J_2(1) = 0.1149$

$J_3(1) = 0.0195$, $J_4(1) = 0.002477$.

$$P_{avg} = \frac{A_c^2}{2}\left[J_0^2(1) + 2J_1^2(1) + 2J_2^2(1) + \cdots\right].$$

$$= \frac{A_c^2}{2}\left[(0.7652)^2 + 2(0.4401)^2 + 2(0.1149)^2\right].$$

$\Rightarrow n = 2 \rightarrow BW = (2)(2) f_m$

$\Rightarrow BW = 4f_m$.

- at 98% Power band width ⇒ using Carson's Rule.

• $BW = 2(\beta + 1) f_m$ •

≡ Generation of an FM signal

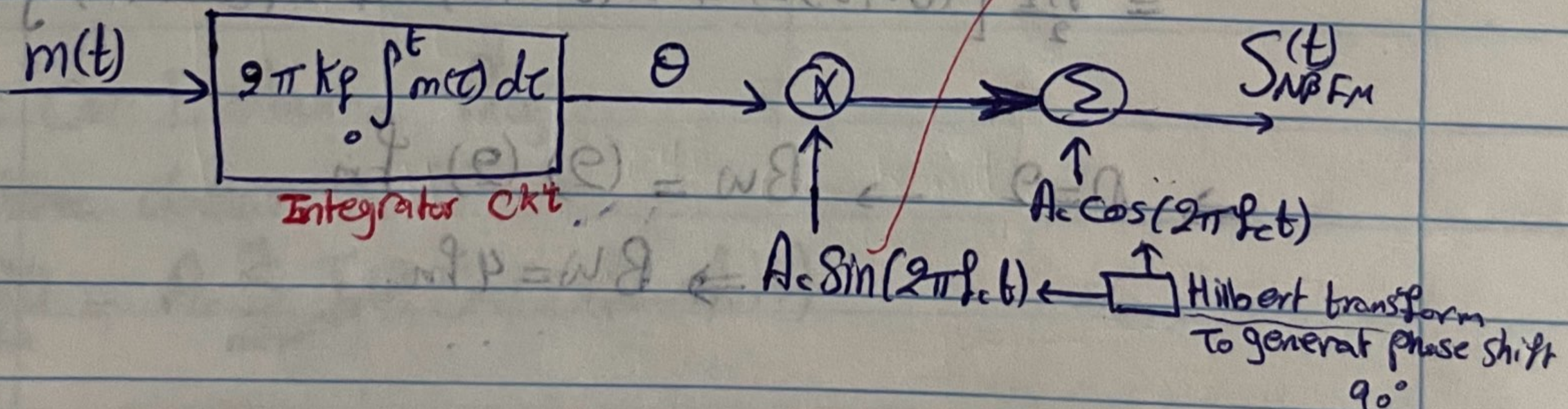
$$S_{FM}(t) = A_c \cos(2\pi f_c t + \underbrace{2\pi k_f \int_0^t m(\tau)\, d\tau}_{\theta})$$

• if $m(t) = A_m \cos(2\pi f_m t)$

↳ $S_{FM}(t) = A_c \cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t))$
$\quad - A_c \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))$

In NBFM.

$$S_{FM}(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_m t) \sin(2\pi f_c t)$$



In NBFM.

$$s_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

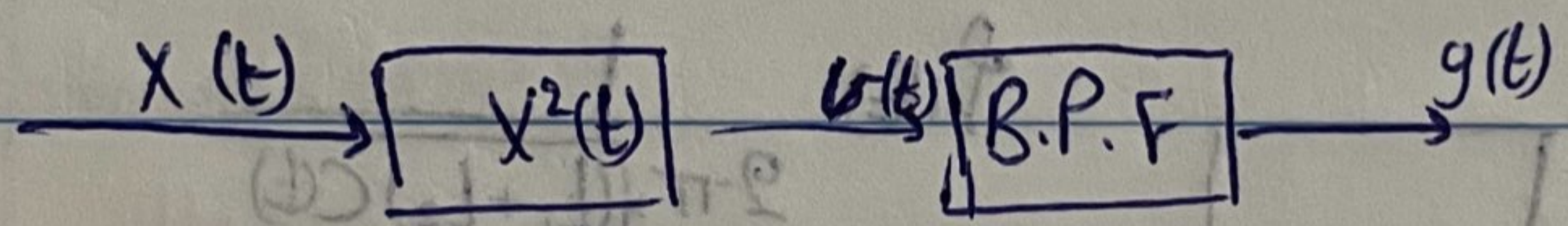
$$= A_c \cos(\underbrace{2\pi f_c' t + \beta' \sin(2\pi f_m t)}_{\text{NBFM}}) \rightarrow A_c \cos(2\pi f_c t + \underbrace{\beta \sin(2\pi f_m t)}_{\text{WBFM}})$$

$f_c = n f_c'$

$\beta = n\beta'$.

≡ Frequency Multiplier.

Example) The square Law device

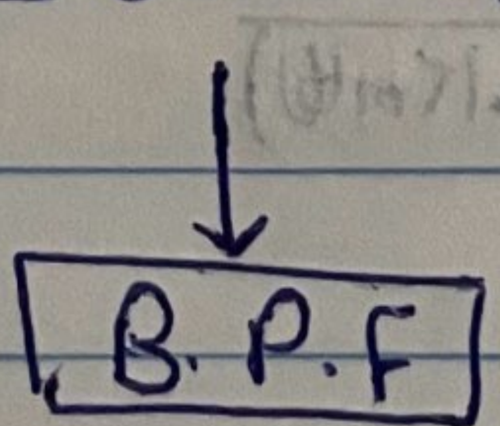


Let $x(t) = A_c \cos(\phi(t)) = A_c \cos(2\pi f_c' t + \beta' \sin(2\pi f_m t))$

$$v(t) = (A_c \cos(\phi(t)))^2 = A_c^2 \cos^2(\phi(t))$$

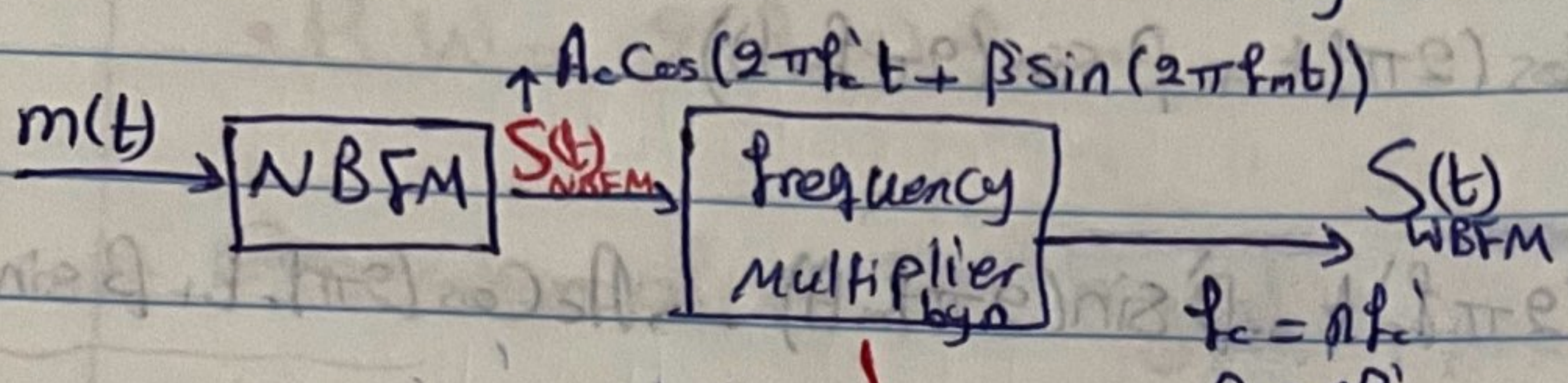
$$= \frac{A_c^2}{2} + \frac{A_c^2}{2} \cos(2\phi(t))$$

$$= \frac{A_c^2}{2} + \frac{A_c^2}{2} \cos(2\pi(2f_c')t + (2\beta')\sin(2\pi f_m t))$$



$$\Rightarrow y(t) = \frac{A_c^2}{2} \cos(2\pi(2f_c')t + 2\beta' \sin(2\pi f_m t))$$

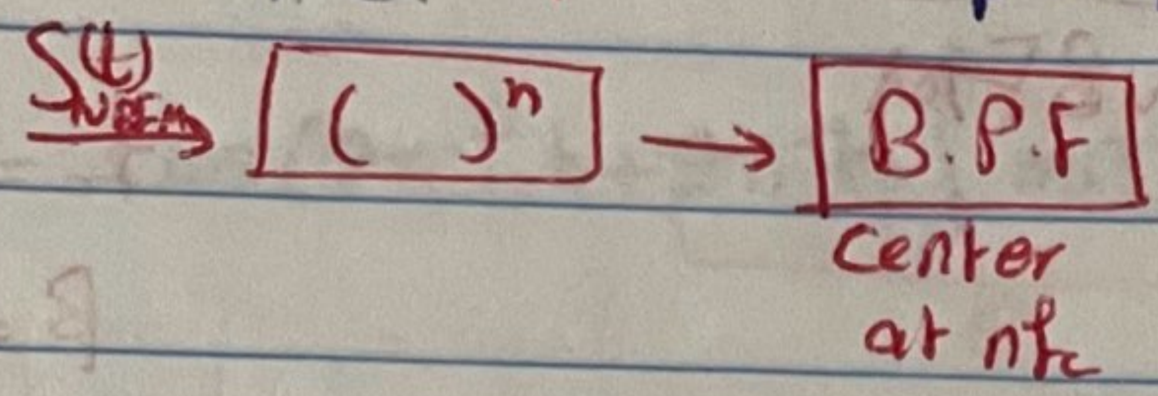
⇒ Indirect Method for Generating a wideband FM



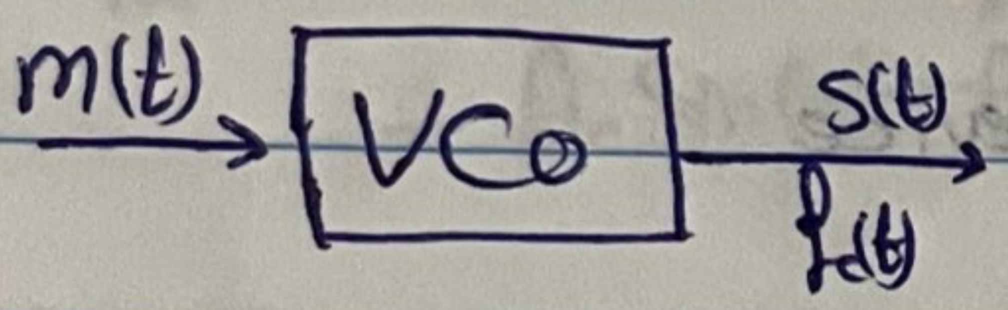
$f_c = n f_c'$

$\beta = n\beta'$

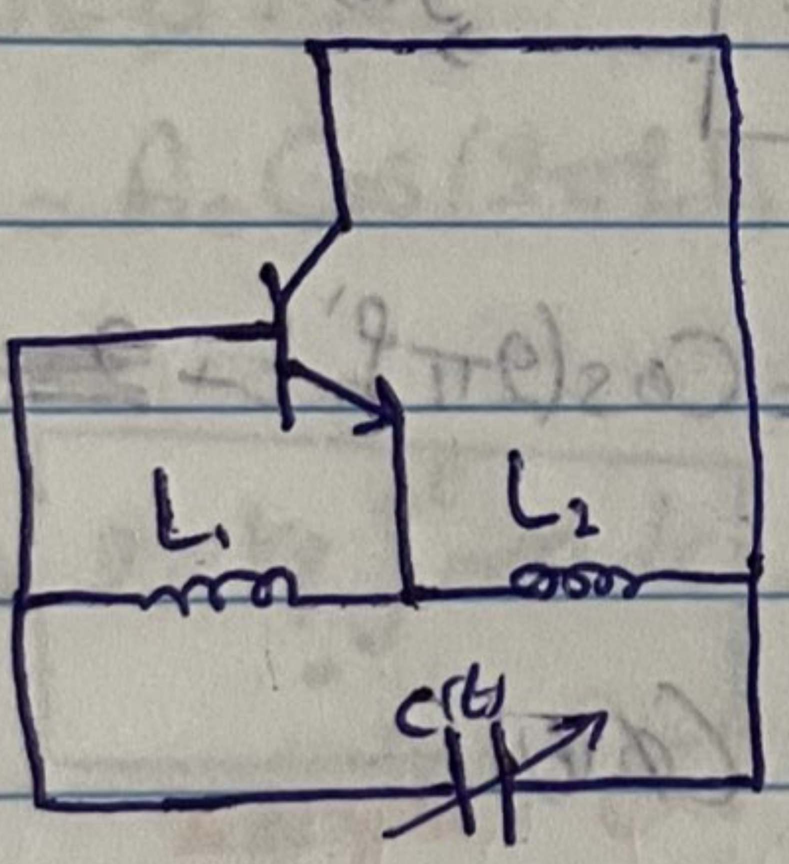
i.e.



≡ Direct method



$f_i(t) = f_c + K\, m(t)$



$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}}$$

$C(t) = C_0 - K\, m(t)$

$$\Rightarrow f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0 - K m(t))}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1 - \frac{Km(t)}{C_0}\right)}} = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}\cdot\sqrt{1 - \frac{K\,m(t)}{C_0}}}$$

⇒

$$f_c(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$$

$$\Rightarrow f_i(t) = f_i \cdot \left(1 - \frac{K\,m(t)}{C_0}\right)^{-1/2} ; \text{ if } \frac{K\,m(t)}{C_0} << 1$$

$$\Rightarrow (1-x)^{-1/2} = 1 + \frac{1}{2}x$$

$$f_i(t) = f_c\left(1 + \frac{K\,m(t)}{C_0}\right)$$

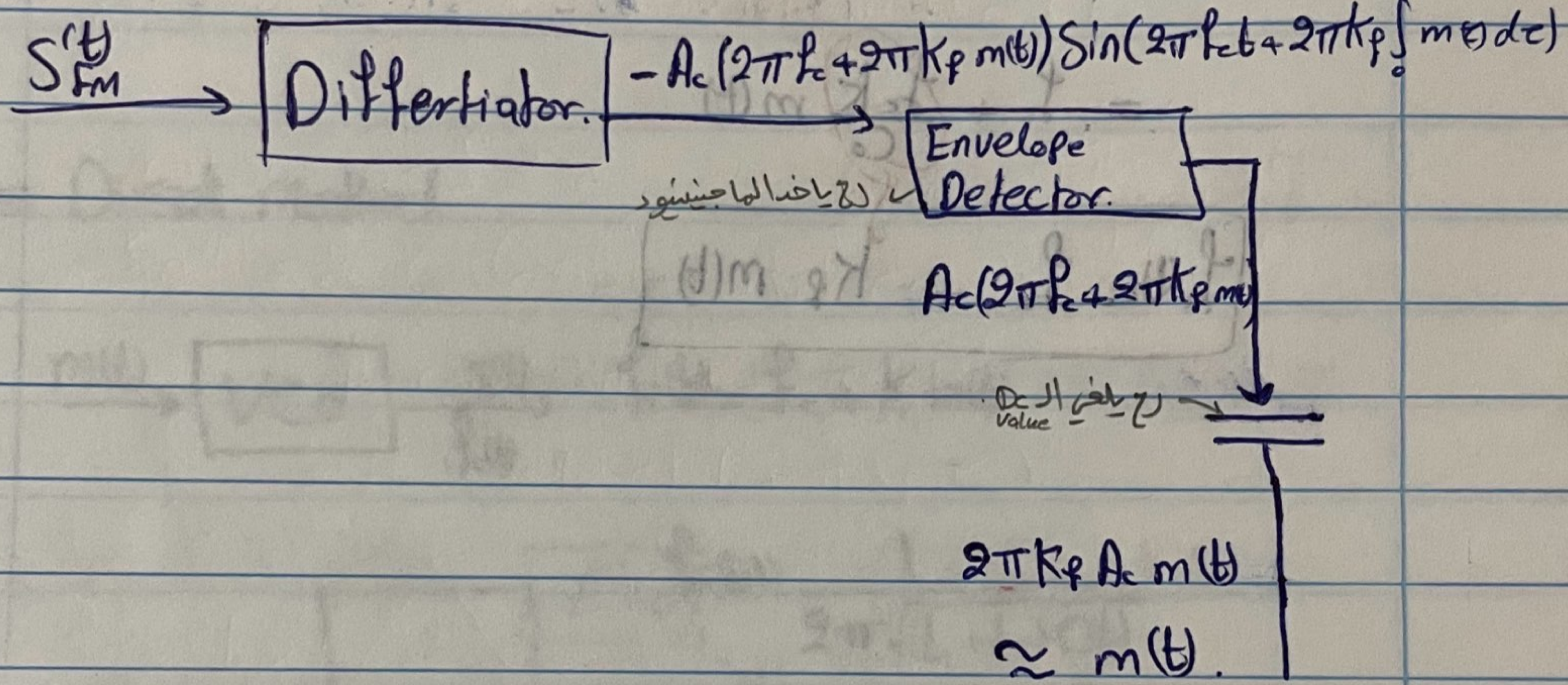
$$= f_c + \frac{f_c K}{C_0}\,m(t)$$

$$\boxed{f_i(t) = f_c + K_f\,m(t)}$$

Demodulation of the FM.

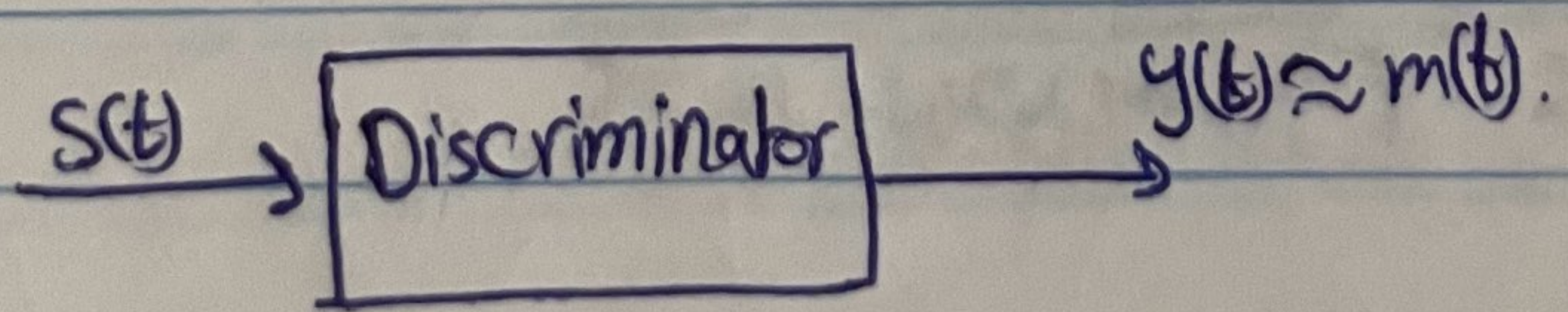
$$S_{FM}(t) = A_c (1 + k_f \int_0^t m(\tau)\, d\tau)$$

$$\rightarrow S_{FM}(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau)$$



To demodulate FM ⇒ we can use Differentiator followed by an envelope detector

- In general, to demodulate FM signal, we can use discriminator



(L14)

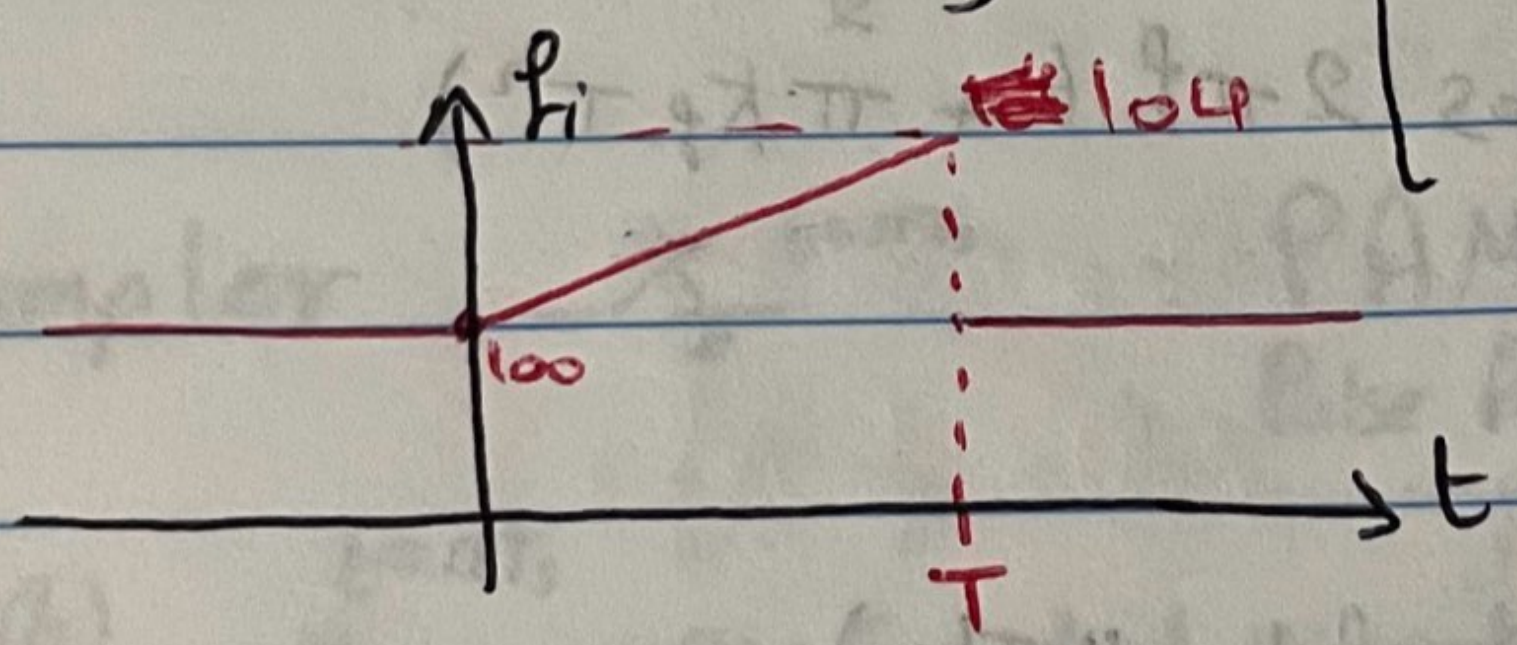
Example Consider the message signal g(t).

$$g(t) = \begin{cases} t, & 0 \le t \le T \\ 0, & o.w \end{cases} \quad ; \quad T = 2 \text{ sec}$$

g(t) is applied to an FM modulation with sensitivity $k_f = 2\,Hz/V$ to produce an FM signal s(t), The unmodulated carrier frequency is 100Hz (f_c), a) Find and plot the istantaneous freq. of s(t) versus time.

a) $f_i(t) = f_c + k_f\, m(t)$

$$= 100 + 2\, g(t) = \begin{cases} 100 + 2t, & 0 \le t \le T \\ 100, & o.w \end{cases}$$



b) find the peak freq. deviation of s(t).

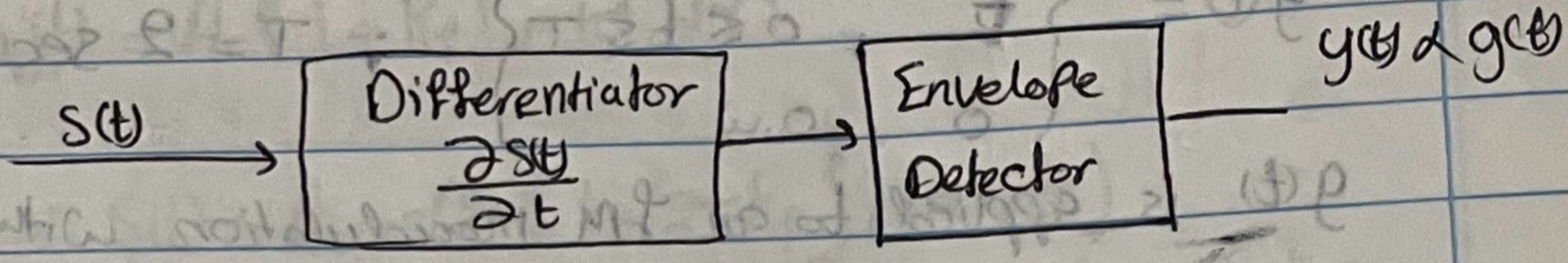
$\Delta f = f_i(t) - f_c = k_f\, m(t)$

$\rightarrow (\Delta f)_{max} = f_i(t)\big|_{max} - f_c$

$= 104 - 100 \Rightarrow 4\,Hz$

⇒

c) Suggest a method via which g(t) can be recovered from s(t).



$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

→ In general:
$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t \tau\, d\tau\right)$
$= A_c \cos\left(2\pi f_c t + 2\pi k_f \frac{t^2}{2}\right)$

when $0 \le t \le T$

$$\to S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^T \tau\, d\tau\right)$$

$$= A_c \cos\left(2\pi f_c t + \frac{2\pi k_f}{2} t^2 \Big|^T\right)$$

$$= A_c \cos\left(2\pi f_c t + \pi k_f T^2\right)$$

for $t > T$

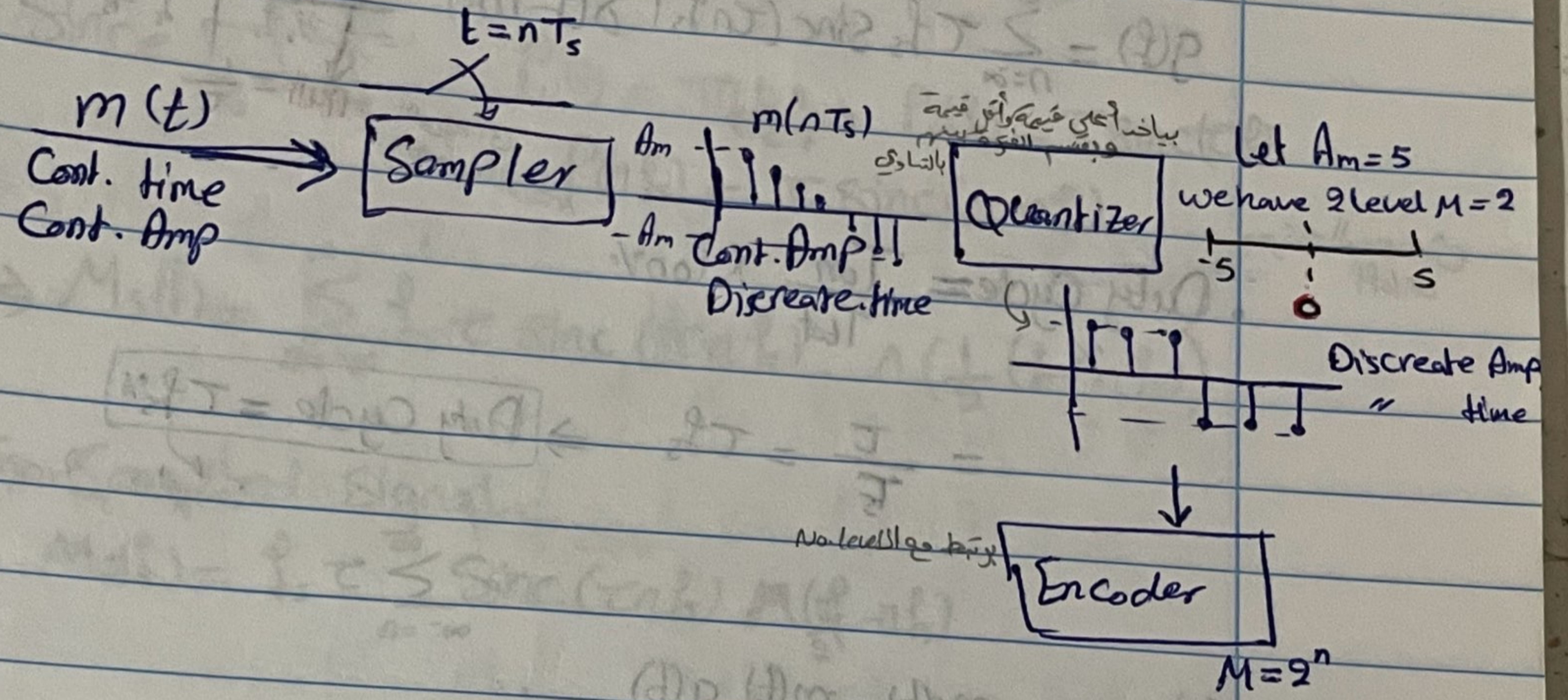
$$S_{FM}(t) = A_c \cos(2\pi f_c t + 4\pi k_f)$$

for $t < 0$

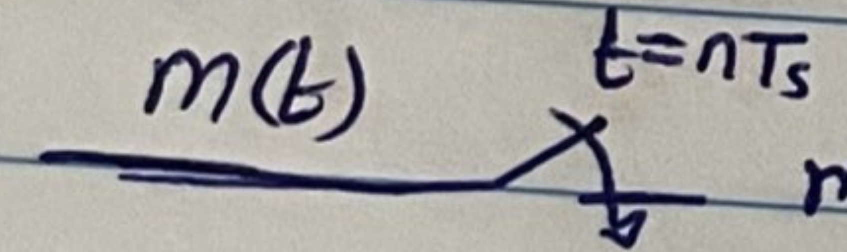
$$S_{FM}(t) = A_c \cos(2\pi f_c t).$$

Analog modulation ⁝ ADC ⁝ Digital modulation

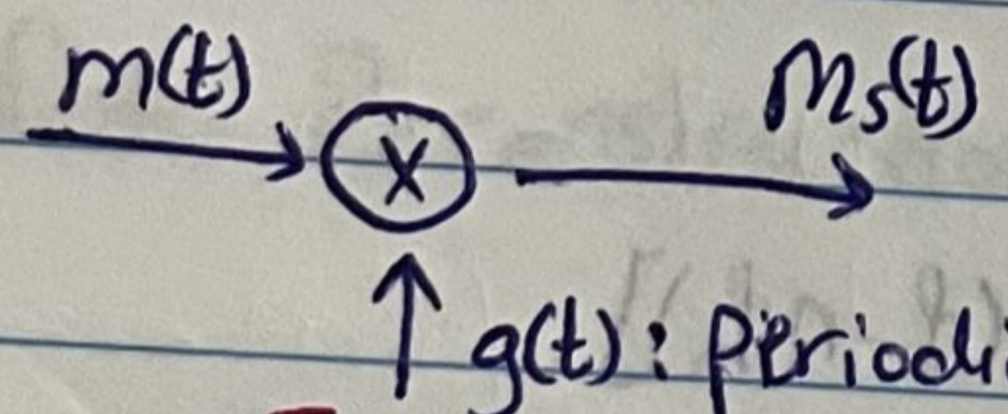
In ADC



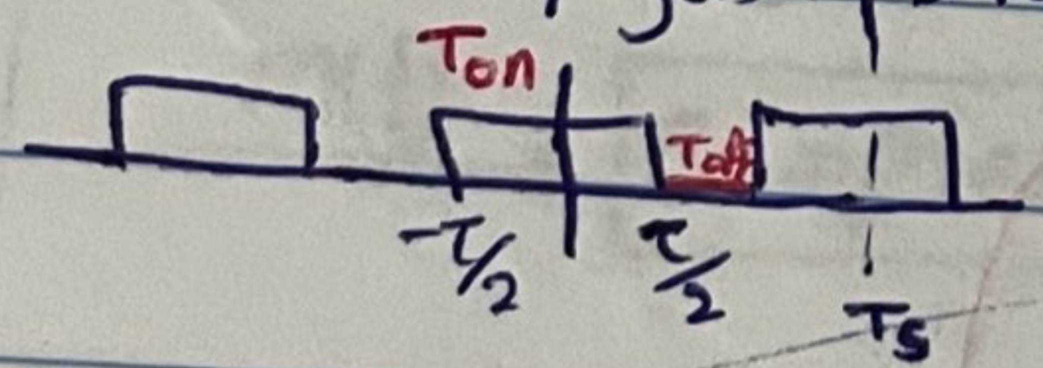
≡ Sampler ⁝ PAM
Pulse Amp. modulation.



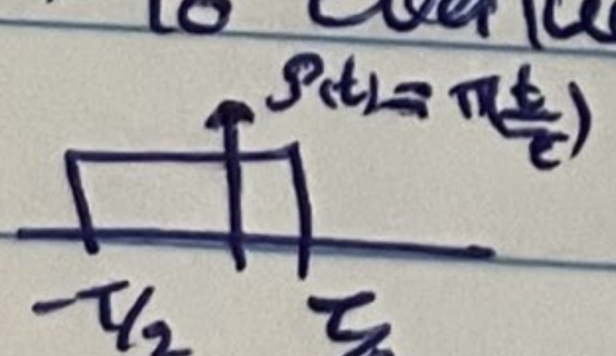
$m_s(t)$: Sampled signal.



$m_s(t) = m(t) \cdot g(t)$, since $g(t)$ is periodic signal $g(t) = \sum_{n=-\infty}^{\infty} g_n e^{j2\pi n f_s t}$



→ To evaluate g_n.



$p(t) = \Pi(\frac{t}{\tau}) \xrightarrow{\mathcal{F}} P(f) = \tau \, \text{sinc}(\tau f)$

$P(nf_s) = \tau \, \text{sinc}(\tau n f_s)$

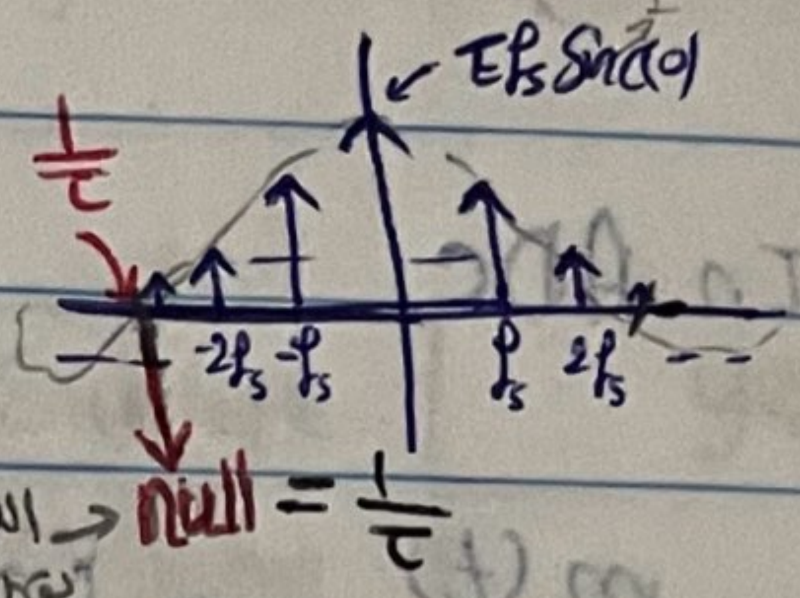
$\Rightarrow g_n = f_s P(nf_s) = \tau f_s \, \text{sinc}(\tau n f_s).$

→

$\Rightarrow$

$$g(t) = \sum_{n=-\infty}^{\infty} \tau f_s \operatorname{sinc}(\tau n f_s)\, e^{j2\pi n f_s t}$$

$$G(f) = \sum_{n=-\infty}^{\infty} \tau f_s \operatorname{sinc}(\tau n f_s)\, \delta(f - n f_s)$$



• Duty Cycle $= \frac{T_{on}}{T_{tot}} \times 100\%$

%

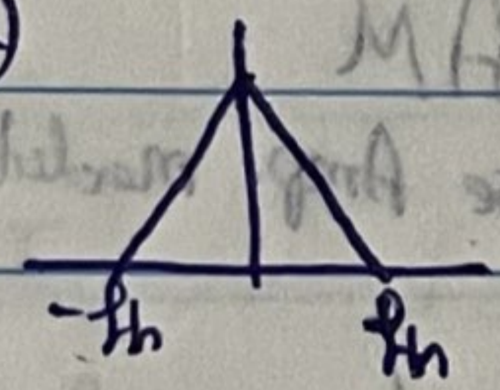
$$= \frac{\tau}{T_s} = \tau f_s \Rightarrow \boxed{\text{Duty Cycle} = \tau f_s \%}$$

Sampling freq

Since :- $m_s(t) = m(t)\, g(t)$,

$$M_s(f) = M(f) * G(f)$$

Let $M(f)$



$m(t) = \operatorname{sinc}(\tau f)$

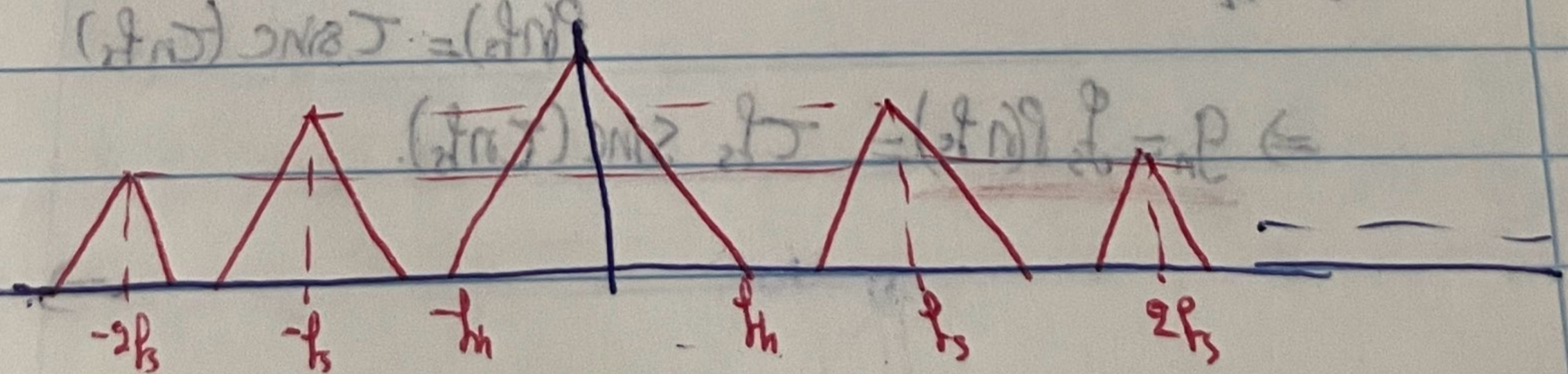
$M(f) = \frac{1}{\tau} \Lambda\left(\frac{f}{\tau}\right)$

$$M_s(f) = M(f) * \sum_{n=-\infty}^{\infty} \tau f_s \operatorname{sinc}(\tau n f_s)\, \delta(f - n f_s)$$

$$= \sum_{n=-\infty}^{\infty} \tau f_s \operatorname{sinc}(\tau n f_s) \left[M(f) * \delta(f - n f_s) \right]$$

$$\boxed{M_s(f) = \sum_{n=-\infty}^{\infty} \tau f_s \operatorname{sinc}(\tau n f_s)\, M(f - n f_s)}$$

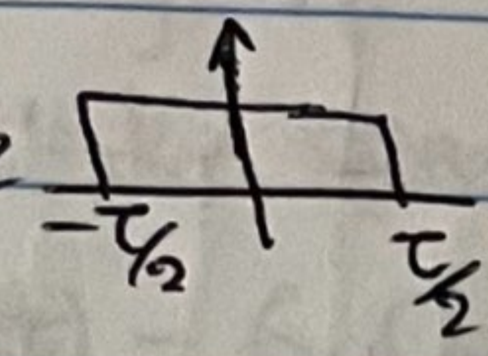
To Plot $M_s(f)$



L(15)

Now, Let $m(t) = 2\,\text{sinc}^2(2t)$

$$\Rightarrow M(f) = 2 \cdot \frac{1}{2} \Lambda\left(\frac{f}{2}\right) \Rightarrow M(f) = \Lambda\left(\frac{f}{2}\right)$$

Since 

$p_1(t) = \Pi\left(\frac{t}{\tau}\right) \Rightarrow P_1(f) = \tau\,\text{sinc}(\tau f)$

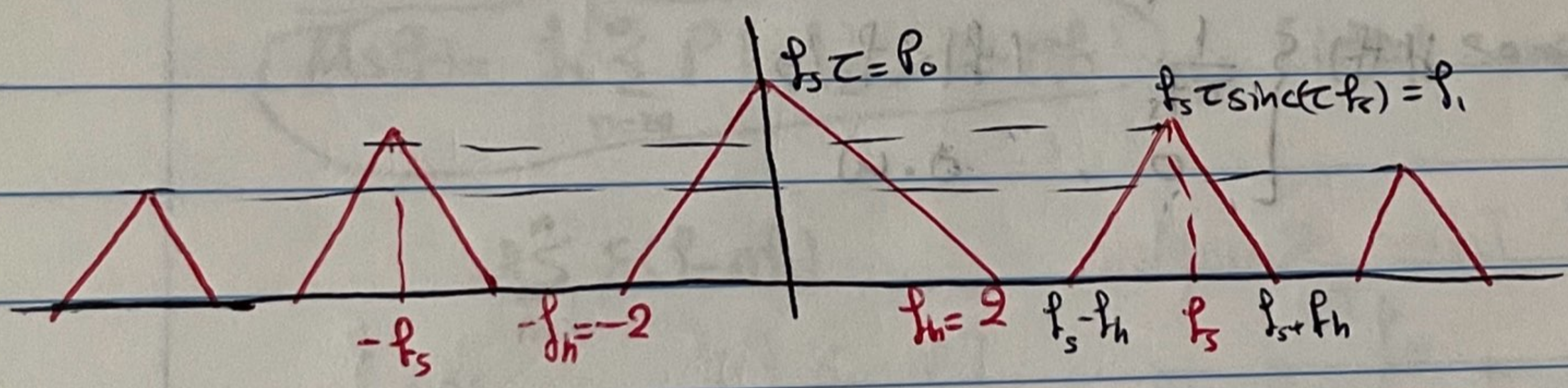
$P(nf_s) = \tau\,\text{sinc}(\tau n f_s)$

$$\Rightarrow M_s(f) = \sum_{n=-\infty}^{\infty} f_s \tau\,\text{sinc}(\tau n f_s) \cdot \Lambda\left(\frac{1}{2}(f - nf_s)\right)$$

In Sampled Signal.

ثابت

$$M_s(f) = f_s \tau \sum_{n=-\infty}^{\infty} \text{Sinc}(\tau n f_s)\, \Lambda\left(\frac{f - nf_s}{2}\right)$$



To recover data:-

$f_s - f_h \geq f_h \Rightarrow f_s \geq 2 f_h$

"Nyquist rate".

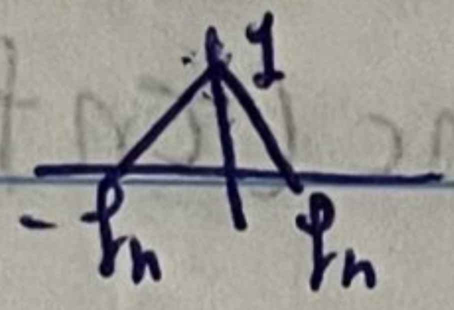
Data Reconstruction:-

$m(t) \rightarrow \boxed{\text{Sampler}} \xrightarrow{m_s(t)} \boxed{\text{L.P.F}} \rightarrow$

To recover message signal

$m_s(t) \rightarrow$ [$h(t)$] $\longrightarrow y(t) \approx m(t)$

$M_s(f) \rightarrow$ [$H(f)$] $\longrightarrow Y(f)$



لازم دائماً تحقق هذا الشرط

$$; H(f) \begin{cases} \frac{1}{f_o} & , |f| \le f_s/2 \\ 0 & , o.w. \end{cases}$$

$$Y(f) = M_s(f) H(f)$$

$$1 = (f_o)(1) k \Rightarrow k = \frac{1}{f_o} > \text{gain}$$

In our example:

$$H(f) \begin{cases} \frac{1}{\tau f_s} & , |f| \le f_s/2 \\ 0 & , o.w \end{cases}$$

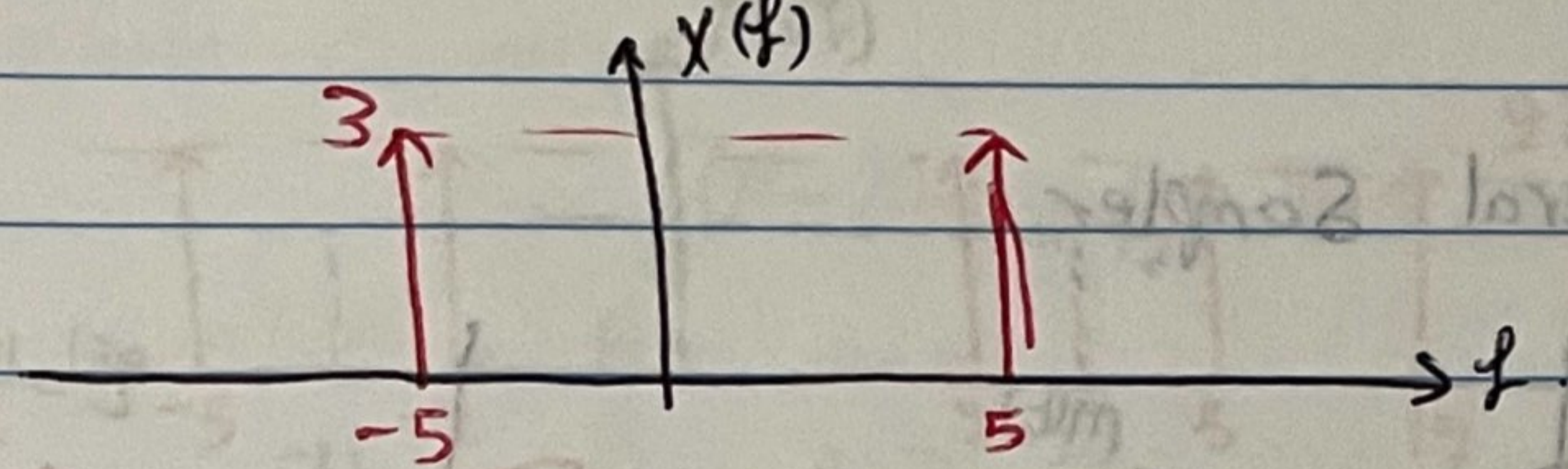
$2f_h = 10$

Example The signal $X(t) = 6\cos(10\pi t)$ ← Message signal or sampling signal.
Sampled at 7 Hz and 14 Hz, for each sampled frequency, مجموعة

a- Plot the spectrum of $X(t)$

$X(t) = 6\cos(10\pi t)$

$\Rightarrow X(f) = 3[\delta(f-5) + \delta(f+5)]$



b- Evaluate and plot the spectrum of sampled signal.

$M_s(f) = f_s \sum_{n=\infty} P(nf_s)\,\delta(f - nf_s)$ [ideal sampler].

$= f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$

~~$\Rightarrow M_s(f) = f_s \sum_{n=-\infty}^{\infty} [3\delta(f-5) + \delta(f+5)]$.~~

$M_s(f) = f_s \sum_{n=-\infty}^{\infty} \big[3[\delta(f-5-nf_s) + \delta(f+5-nf_s)]\big]$

at $f_s = 7$ Hz.

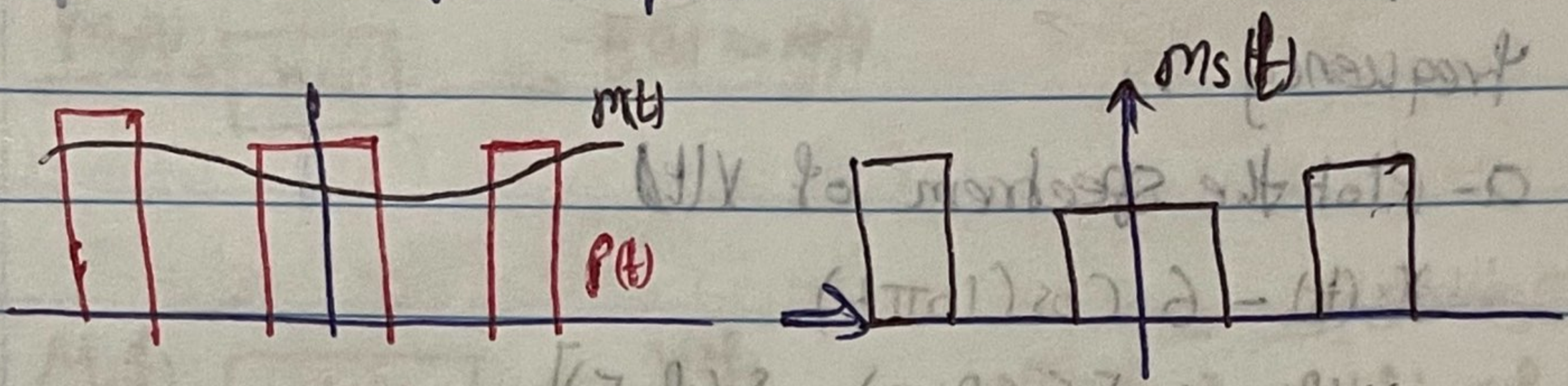
$M_s(f) = 7 \sum_{n=-\infty}^{\infty} [3\delta(f-5-7n) + 3\delta(f+5-7n)]$.

we can not recover the message signal.

since $f_s < 2f_h$.

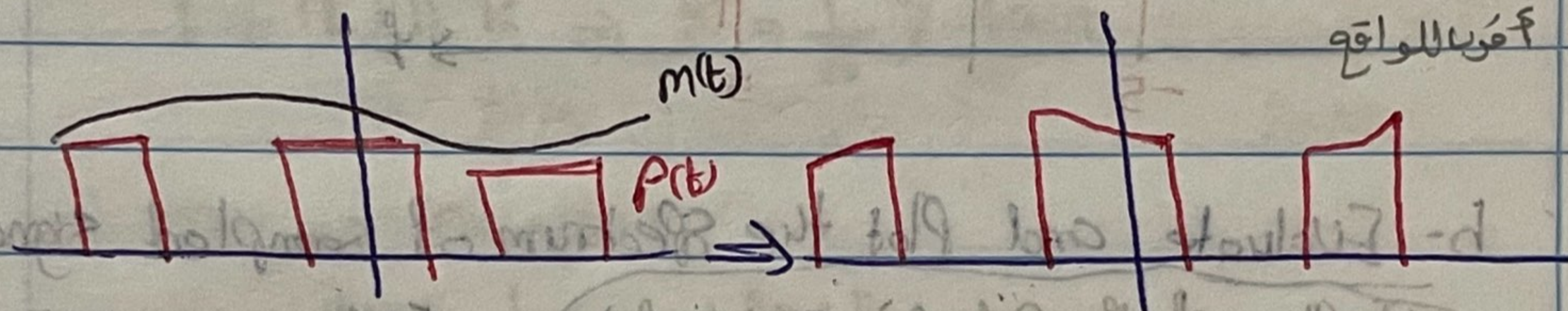
- Types of Sampler:-

1. Flat Top Sampler

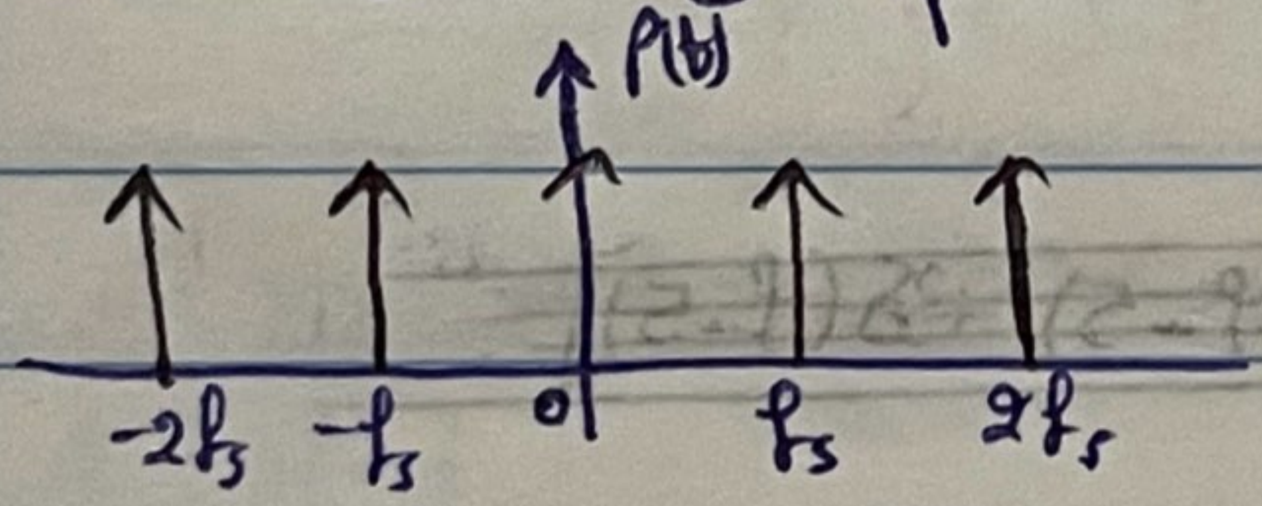


2. Natural Sampler.

أقرب للواقع



3. Ideal Sampler.

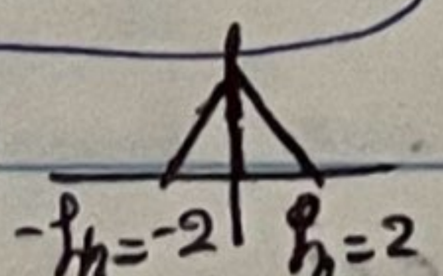


$$p(t) = \sum_{n=-\infty}^{\infty} P_n e^{j2\pi n f_s t} \quad ; \; P_n = f_s$$

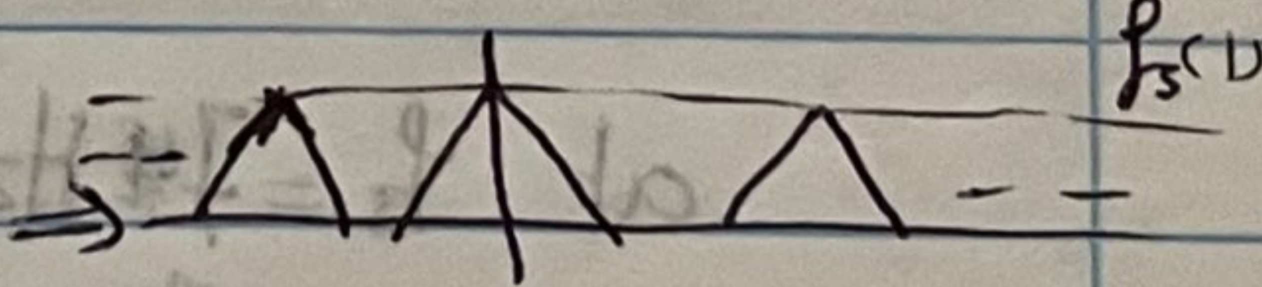
for ideal Sampler, the Sampled Signal.

$$M_s(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)$$

⇒ when $M(f) = \Lambda\left(\frac{f}{2}\right)$



⇒



$$H(f) = \begin{cases} \frac{1}{f_s}, & |f| \le f_s/2 \\ 0, & \text{o.w} \end{cases}$$

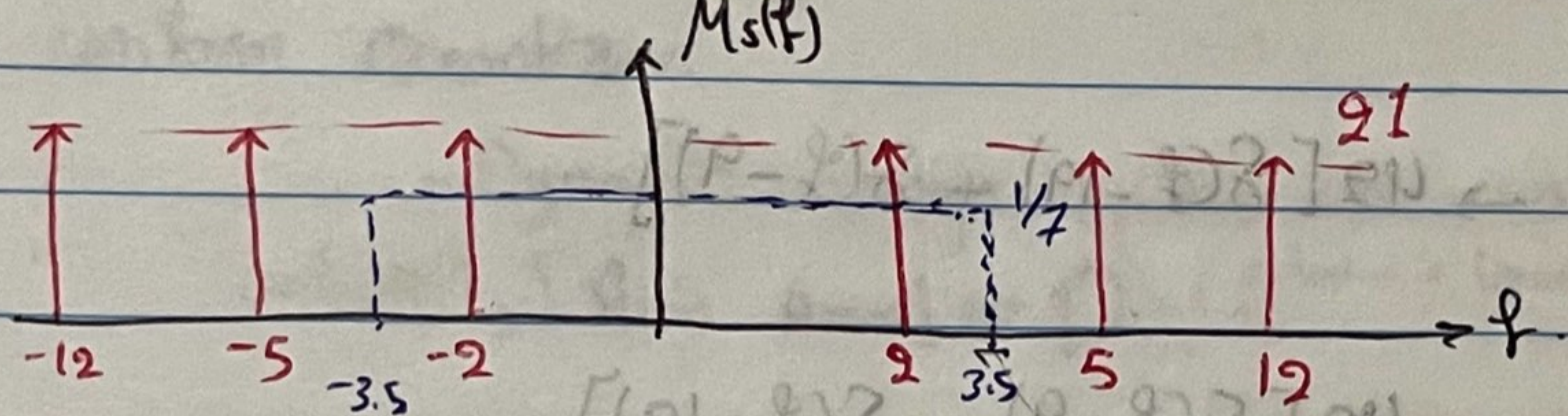
⇒

when

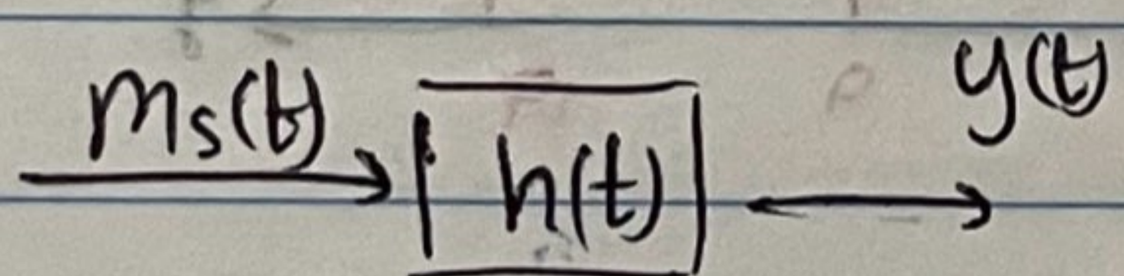
$n=0 \rightarrow M_s(f) = 21[\delta(f-5) + \delta(f+5)]$.

$n=1 \Rightarrow M_s(f) = 21[\delta(f-12) + \delta(f-2)]$.

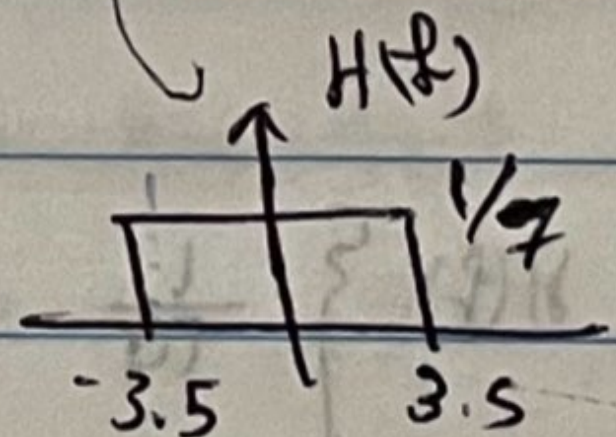
$n=-1 \Rightarrow M_s(f) = 21[\delta(f+2) + \delta(f+12)]$.

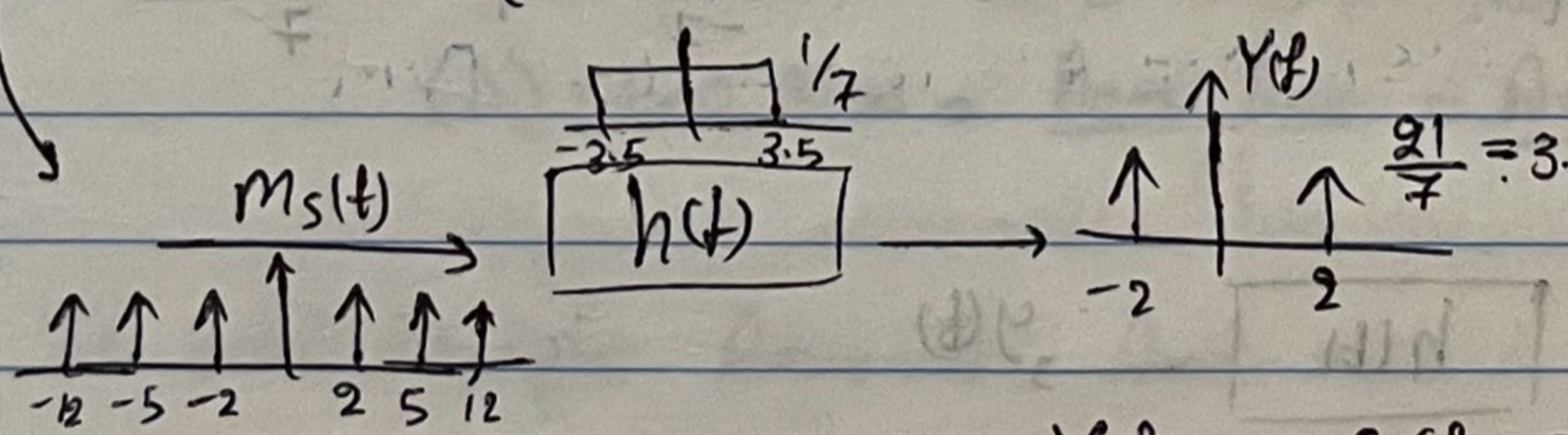


c) Plot the output of reconstruction filter.



$$H(f) = \begin{cases} \frac{1}{7}, & |f| \leq \frac{7}{2} \\ 0, & o.w \end{cases}$$





$Y(f) = 3\delta(f+2) + 3\delta(f-2)$

$y(t) = 6\cos(4\pi t) \not+ 6\cos(10\pi t)$

$f_s < 2f_m$.

⇒

when $f_s = 14$ Hz

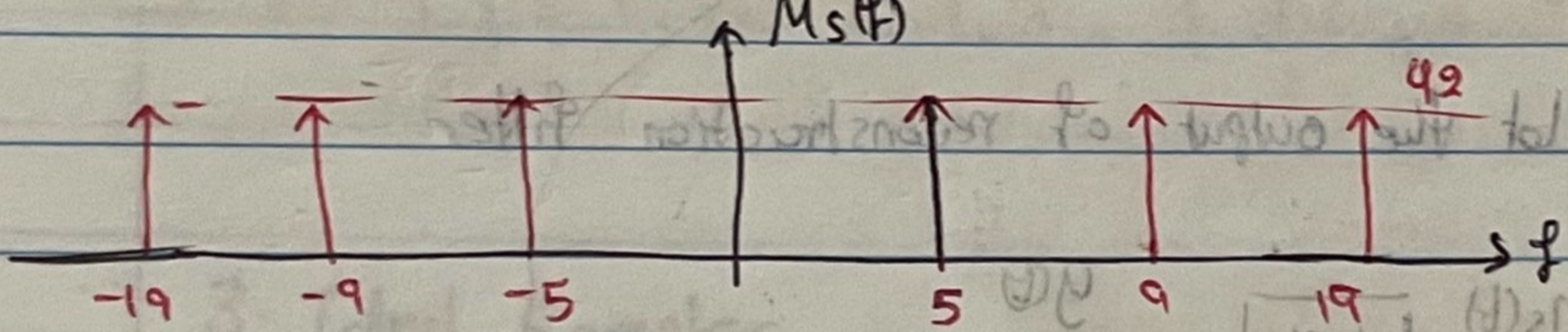
$$M_s(f) = 14 \sum_{n=-\infty}^{\infty} 3[\delta(f-5-14n) + \delta(f+5-14n)]$$

when

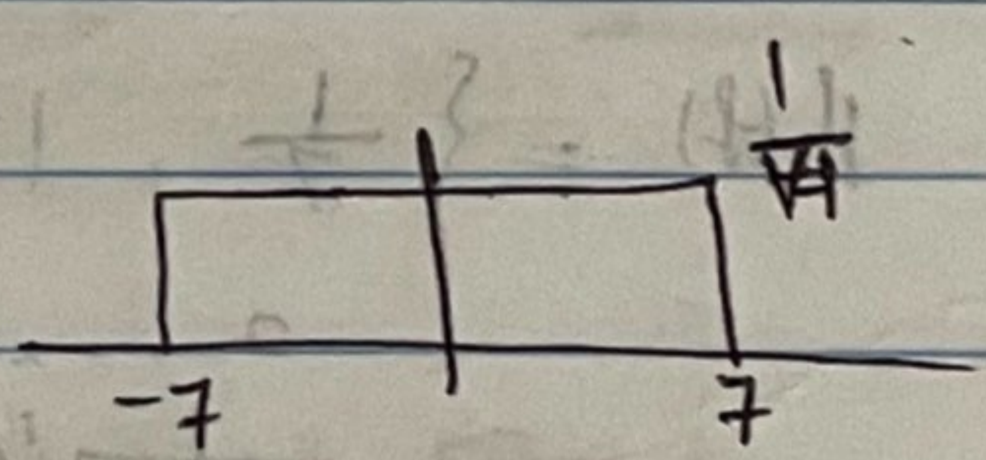
$n=0 \Rightarrow 42[\delta(f-5) + \delta(f+5)]$

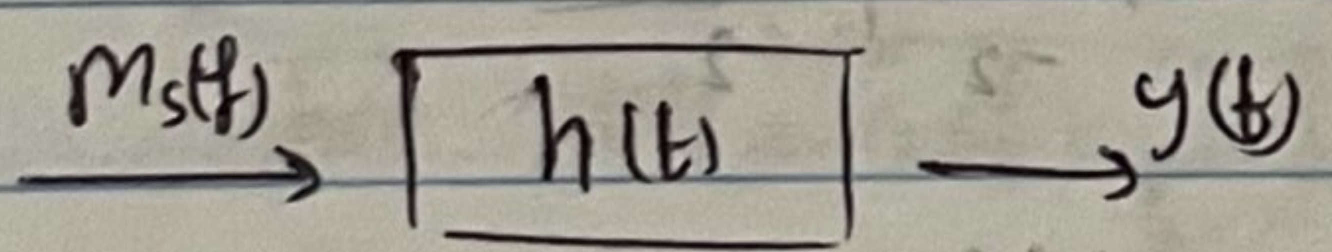
$n=1 \Rightarrow 42[\delta(f-19) + \delta(f-9)]$

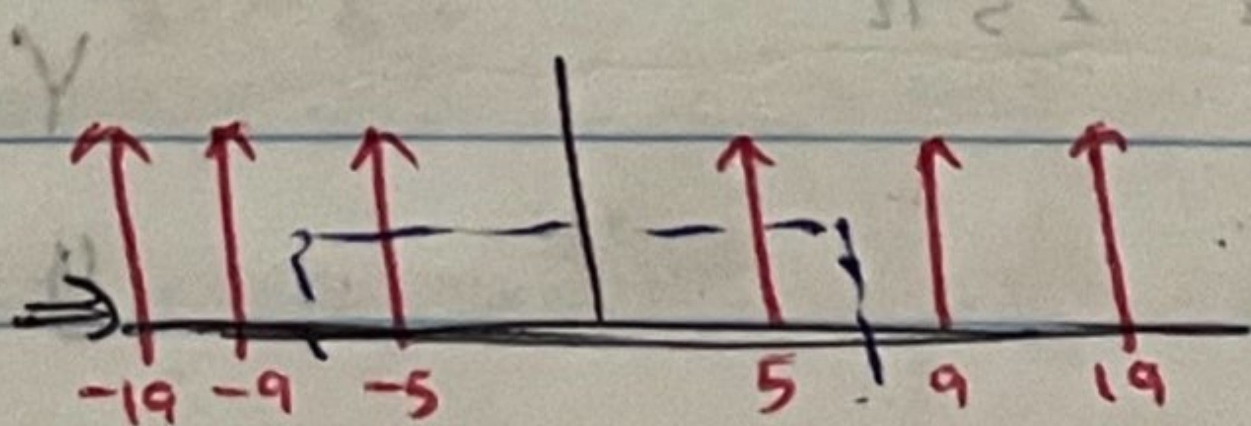
$n=-1 \Rightarrow 42[\delta(f+9) + \delta(f+19)]$



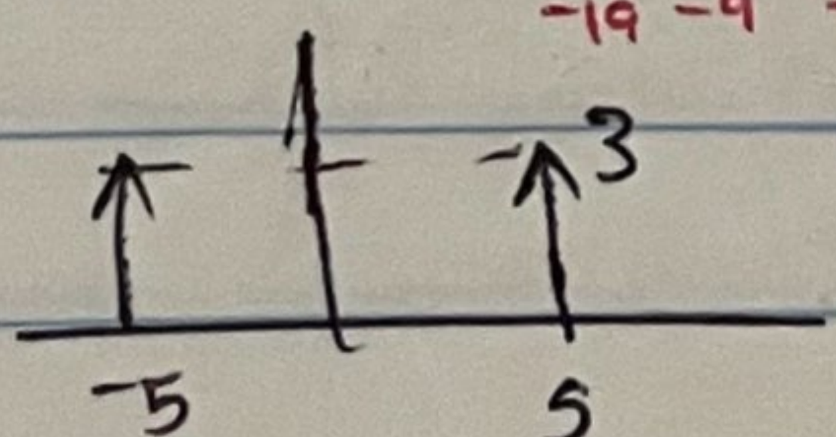
$H(f) = \begin{cases} \frac{1}{14}, & |f| \le \frac{14}{2} \\ 0, & o.w. \end{cases}$







$y(t) \Rightarrow$

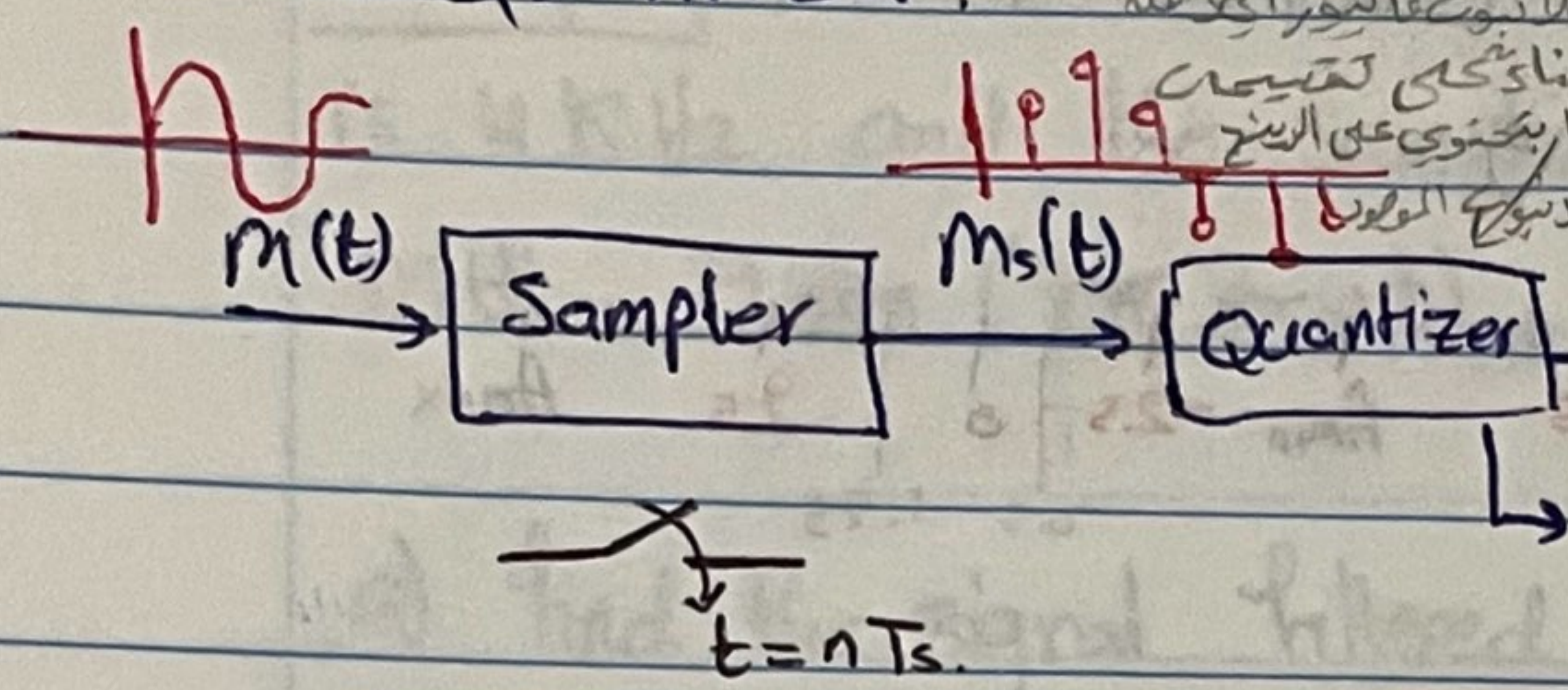


$y(t) = 3\cos(10\pi t)$

(L16)

≡ Quantizer.

تقريب اللانهائي للعدد الي اقله على بناءً على تقسيمه هو بيد خلها بيحتوي على الرينج كامل للانواع الموجودة



→ uniform Quantizer.

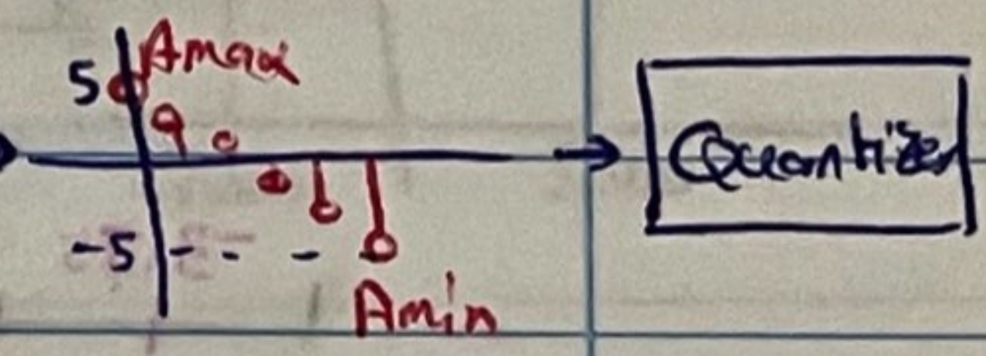
↳ non uniform Quantizer.

• uniform Quantizer.

- Dynamic range. هو الرينج الي نحصر عليه لحتى نقسم الحد لمستويات متساوية

مطلوب احدد ↳ [Amin, Amax] ≡ D

[Example] $X(t) = 5\cos(20\pi t)$ → Sampler →



→ D = [-5, 5] , 1 bit

- M Levels $= 2^n$ if Quantizer 1 bit

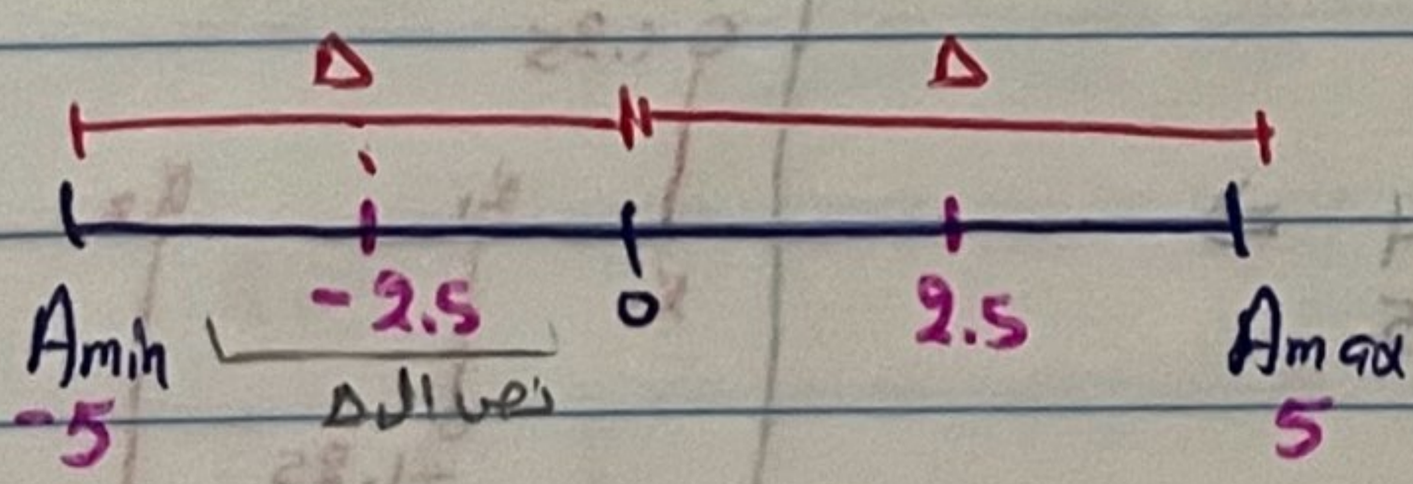
⇒ M Levels = 2

- Step Size. بده يا خذ الرينج كامل ويقسمه على حسب الـM

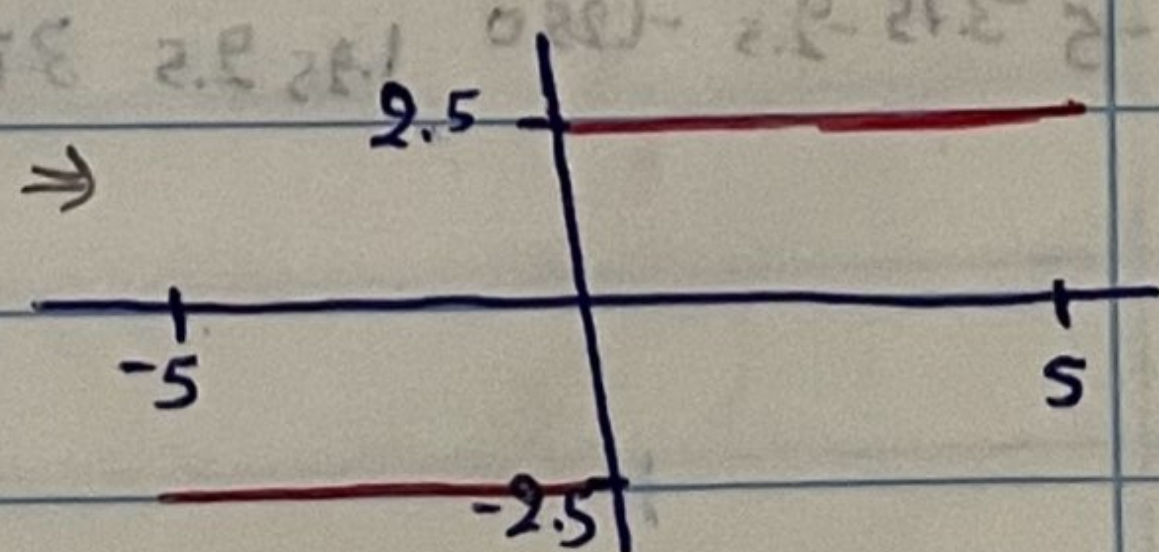
$$(\Delta)\ \text{Step size} = \frac{A_{max} - A_{min}}{M} = \frac{A_{max} - A_{min}}{2^n}$$

In our example $A_{max} = 5$, $A_{min} = -5$.

$$\Delta = \frac{5 - -5}{2^n} = \frac{10}{2} = 5$$



⇒



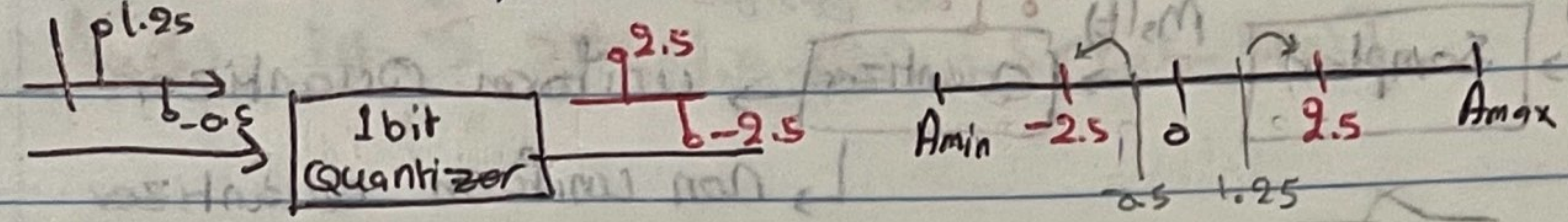
[mid rise type]

(L16)

Evaluate the output of quantizer for input value

$X_0 = 1.25, \quad X_1 = -0.5.$

التقريب يكون للعنصر المنخفض



$X_0 \rightarrow 2.5$

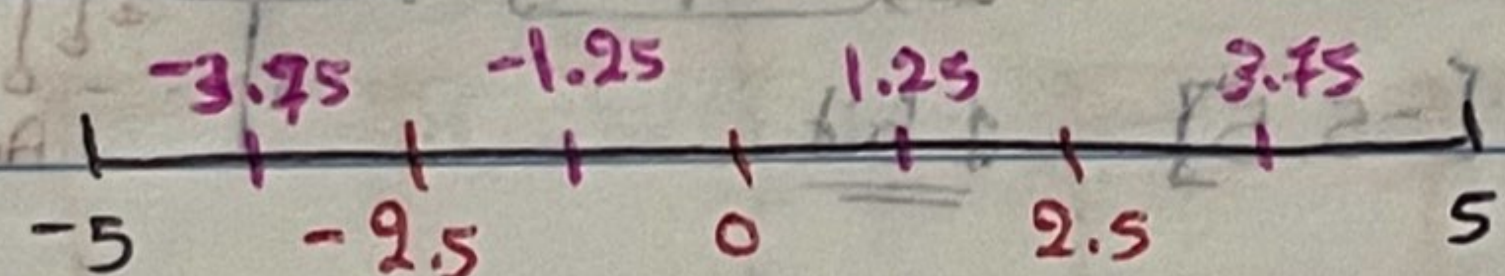
$X_1 \rightarrow -2.5.$

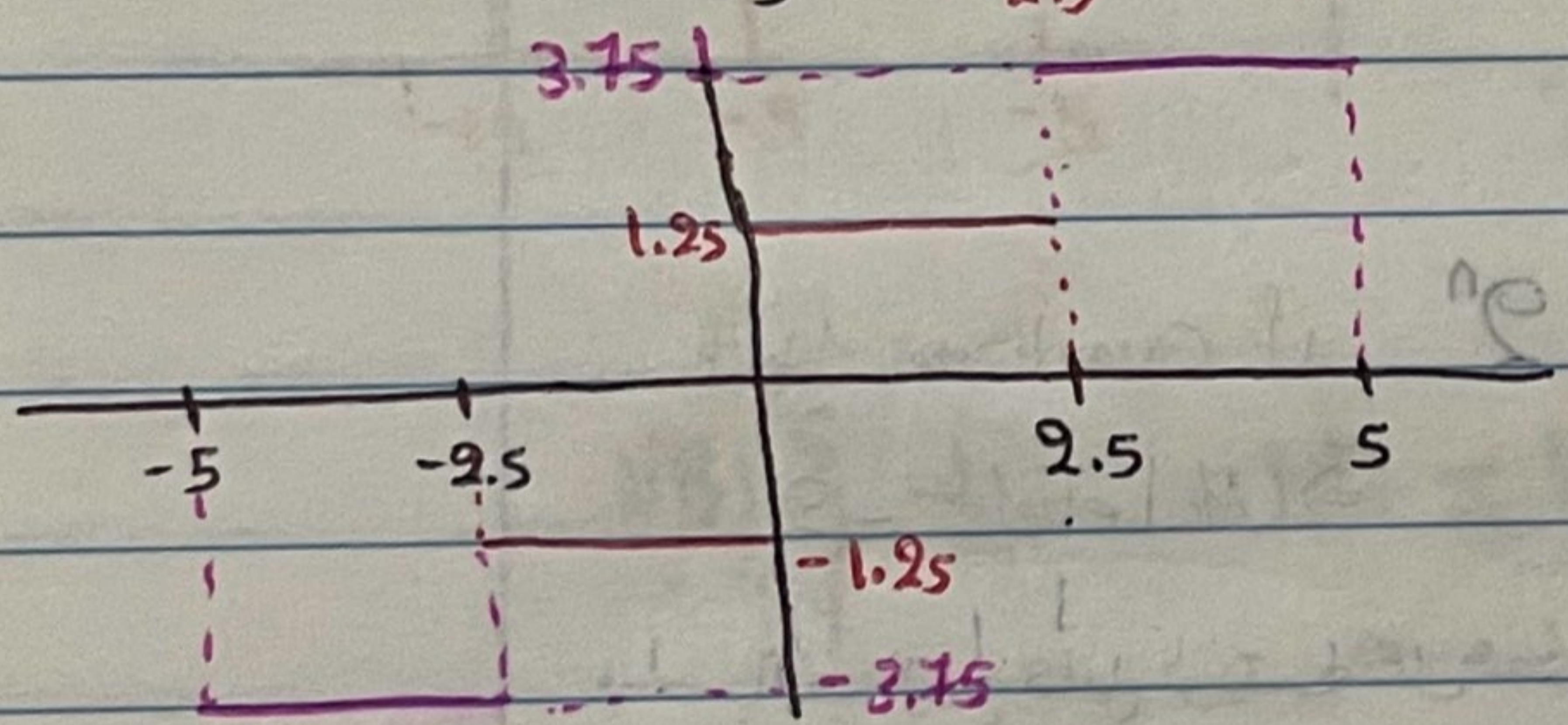
if we have 2bit quantizer.

$\Rightarrow M = 2^2 = 4$ levels.

Since $D = [-5, 5] \Rightarrow \Delta = \frac{5 - -5}{4} = \frac{10}{4} = 2.5.$

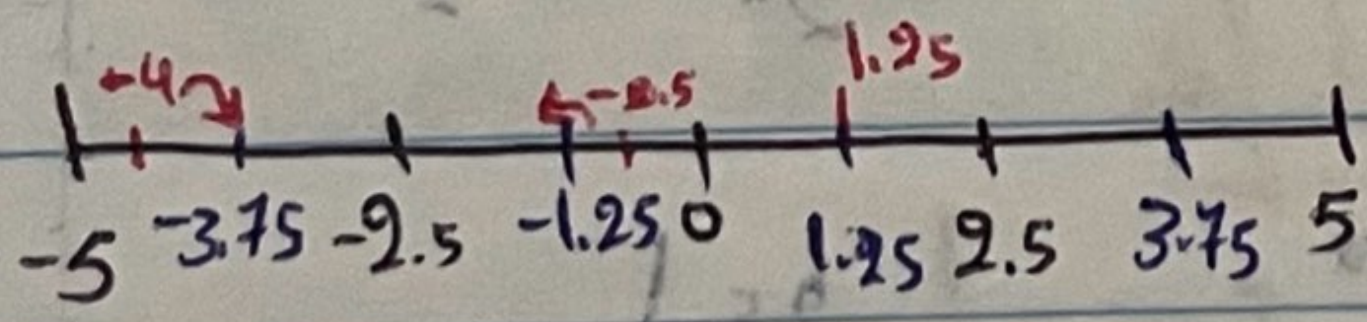
كل ما زاد عدد الـ levels يعني الايرور أقل وأفضل



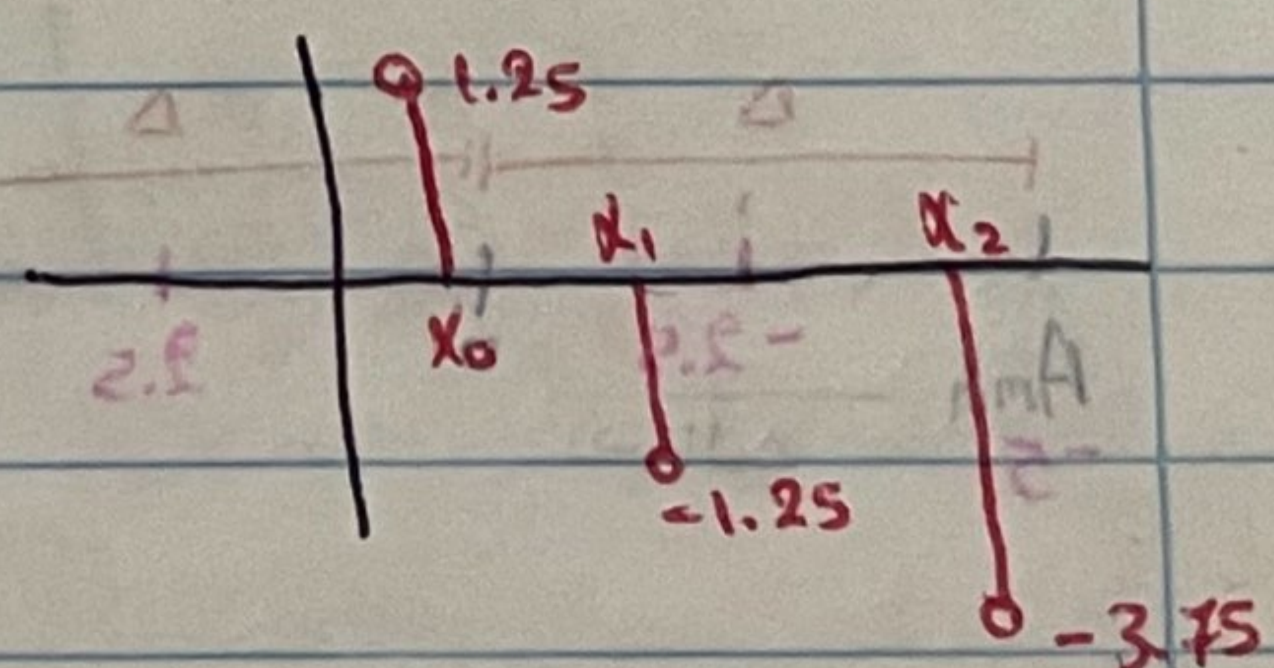


Evaluate the output of quantizer for input value.

$X_0 = 1.25, \quad X_1 = -0.5, \quad X_2 = -4.$



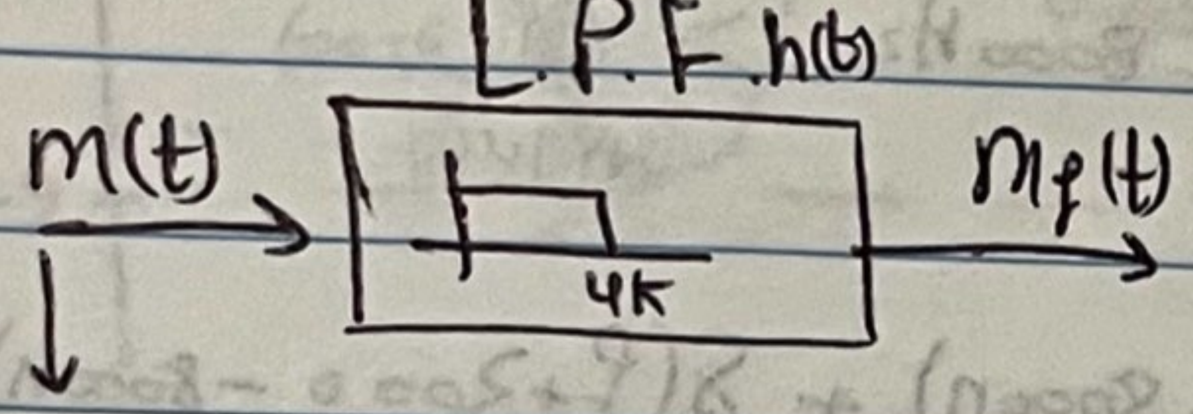
$\Rightarrow$



Example: Passband bandwidth of the low pass filter is 4 KHz and the sampling frequency (f_s) of 8 KHz.

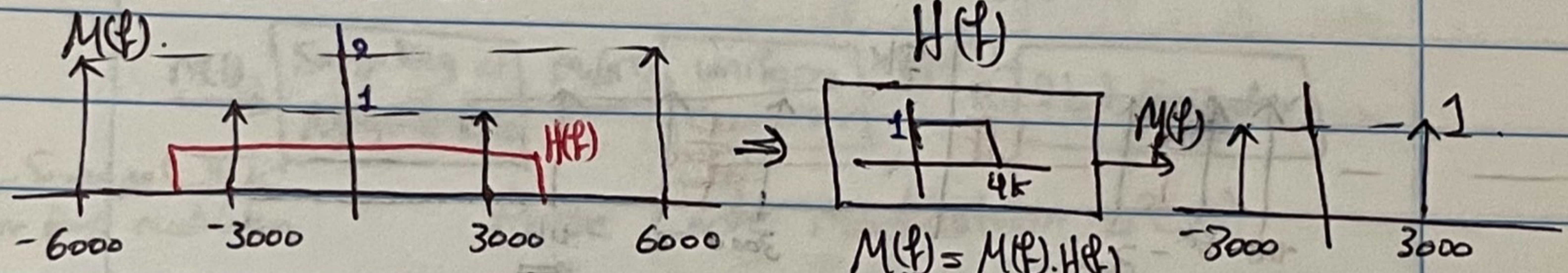
$m(t) = 2\cos(2\pi\,3000t) + 4\cos(2\pi\,6000t)$

a) find the signal filtered signal $m_f(t)$.



, $m_f(t) = m(t) * h(t)$.

$m(t) = 2\cos(2\pi 3000t) + 4\cos(2\pi 6000t)$.

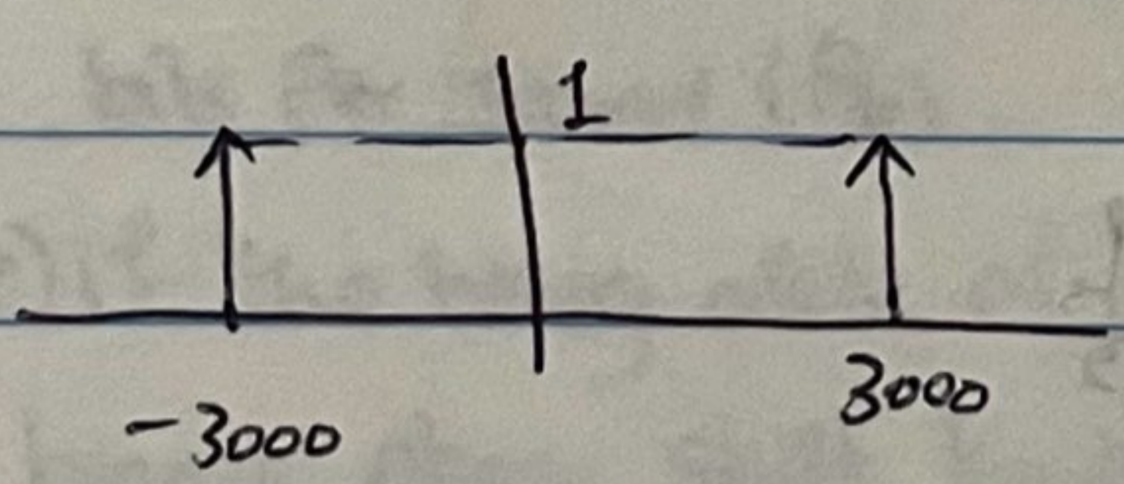


$M_f(f) = M(f) \cdot H(f)$

$M_f(f) = \delta(f-3000) + \delta(f+3000)$.

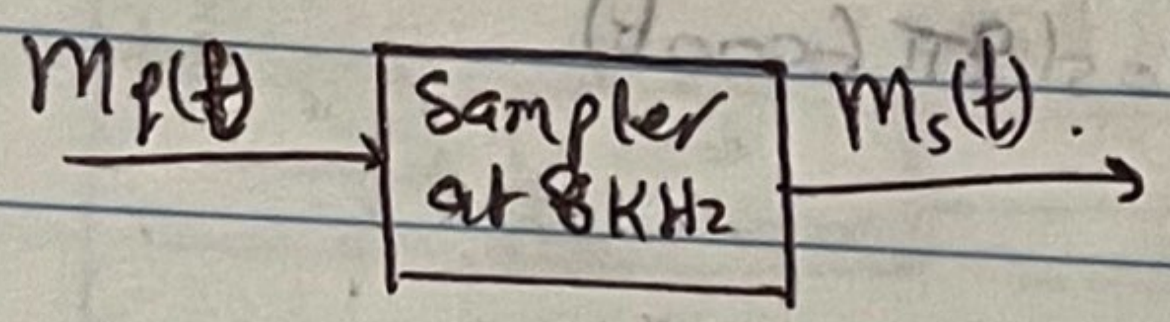
$m_f(t) = 2\cos(2\pi(3000)t)$

b) plot the amplitude spectrum of the filtered signal $m_f(t)$.



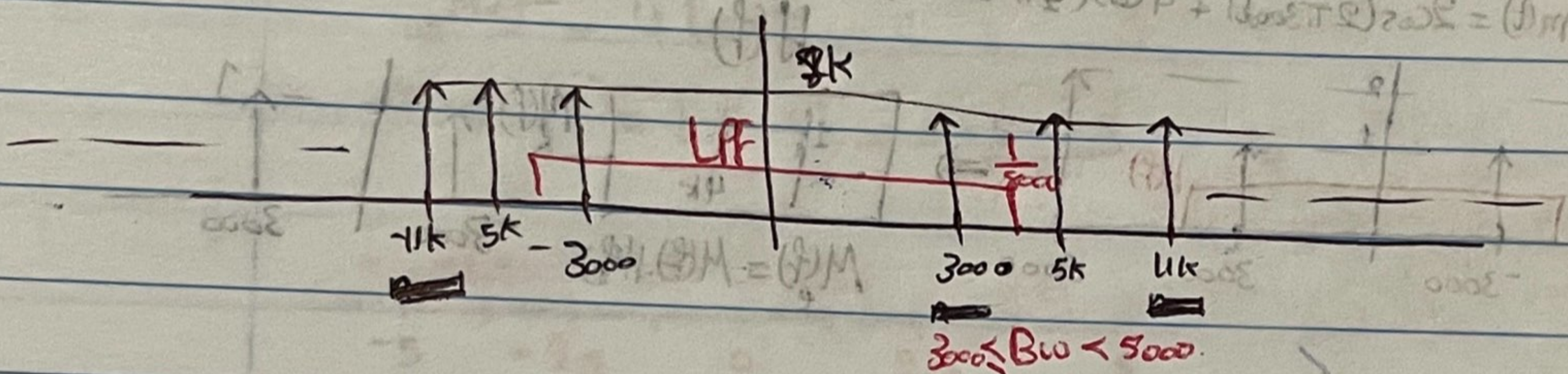
$\delta(f-3000) + \delta(f+3000)$.

c) Plot the amplitude spectrum of the sampled signal $M_s(t)$.

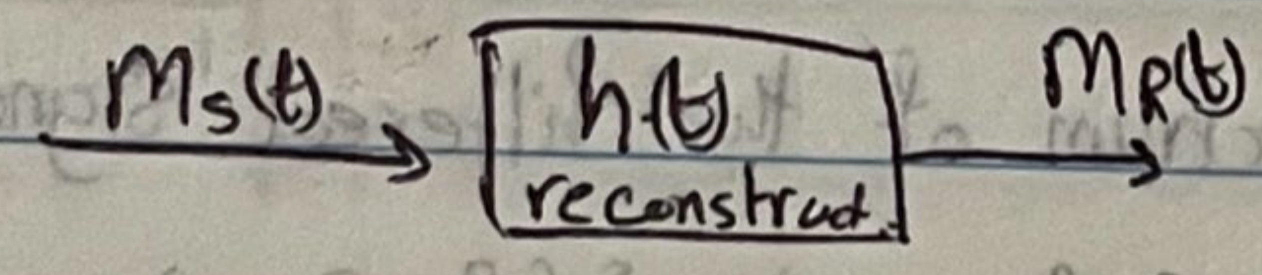


$M_s(f) = f_s \sum_{n=\infty}^{\infty} M_f(f - nf_s)$; $f_s = 8000$ Hz $\geq (2)(3000)$ NYquist.

$M_s(f) = 8000 \sum_{n=\infty} \delta(f - 3000 - 8000n) + \delta(f + 3000 - 8000n)$



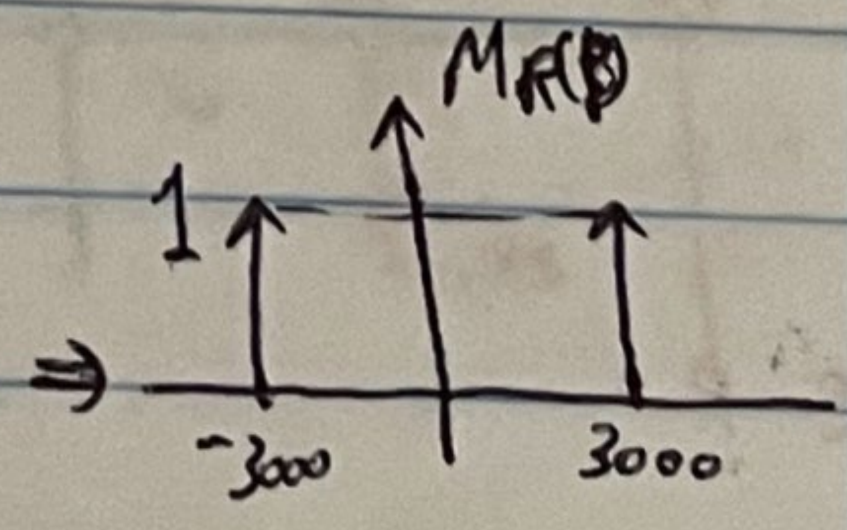
d) what is the type of the Reconstruction filter at the receiver?



$H(f) = \begin{cases} \frac{1}{f_s} = \frac{1}{8000} , & |f| < \frac{f_s}{2} \\ 0 , & o.w \end{cases}$

$= \begin{cases} \frac{1}{8000} , & |f| < 4000 \\ 0 , & o.w \end{cases}$

$\Rightarrow$

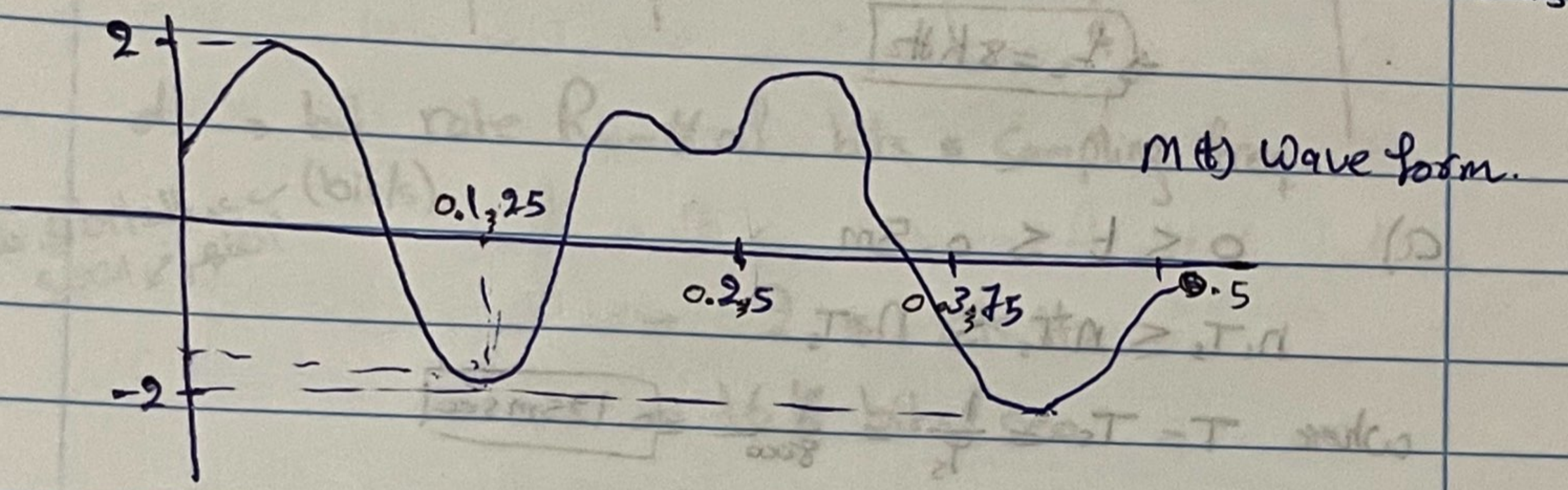


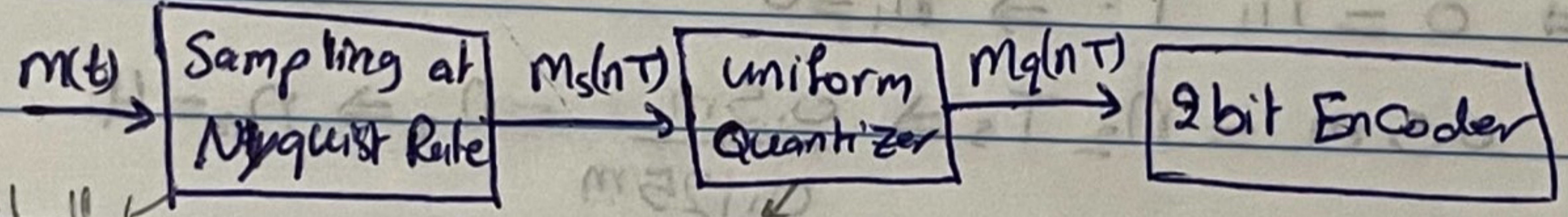
$M_R(f) = \delta(f - 3000) + \delta(f + 3000)$

$m_R(t) = 2\cos(2\pi 3000 t)$.

لو بدي أستخدم Sampler فريكونسي رح تكون ضعف هاي الفريكونسي (2)(4k)

Example ~~The~~ maximum freq. Component at 4K Hz. The dynamic range of the uniform quantizer is from −2 to 2 Volts.





البلس على كل اله تكرر نوعه
Pulse Amp. modulation

Pulse Code modulation (PCM).

a- Determine the Sampled signal value, $m_s(nT)$ for $0 \leq t \leq 0.5$ ms

b- Determine the quantizer signal value $m_q(nT)$ for $0 \leq t \leq 0.5$ ms.

c- Determine the binary signal value at the output of the 2 bit encoder for the quantized samples of Part b (m_b).

d) Determine the output bit rate at the quantizer output in bits per second (R_b)

e) if the binary data of Part c are transmitted using binary phase shift Keying. find the required transmission band width.

(L7)

$M_{level} = 2^{n}$ ← No. bit encoder
(count)

$0 \le t \le 0.5$

m(t) → [Sampling at Nyq rate] → $m_s(nT_s)$ (sampled signal) → [uniform Quantizer] → $m_q(nT)$ (quantized signal) → [2 bit encoder]

$f_s = 2 * 4\,KHz$

$\Rightarrow f_s = 8\,KHz$

$M = 2^2 = 4$

a) $0 \le t \le 0.5m$

$n_1 T_s \le nT_s \le n_2 T_s$

where T_s $T_s = \frac{1}{f_s} = \frac{1}{8000} = 0.125\,msec$

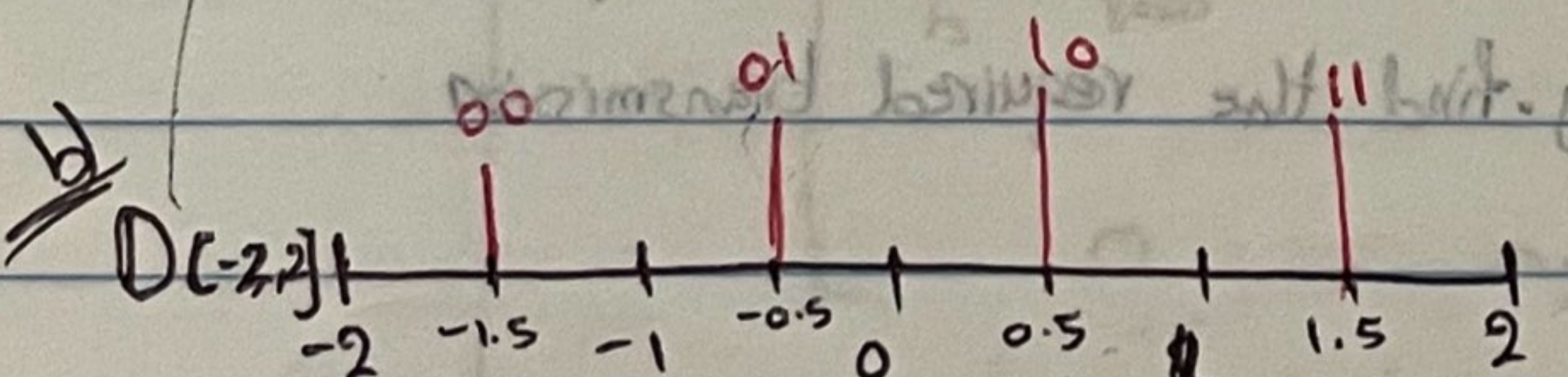
$0 = n_1 T \Rightarrow n_1 = 0$

$0.5m = n_2 T_s \Rightarrow \frac{0.5m}{0.125m} = n_2 \Rightarrow n_2 = 4$

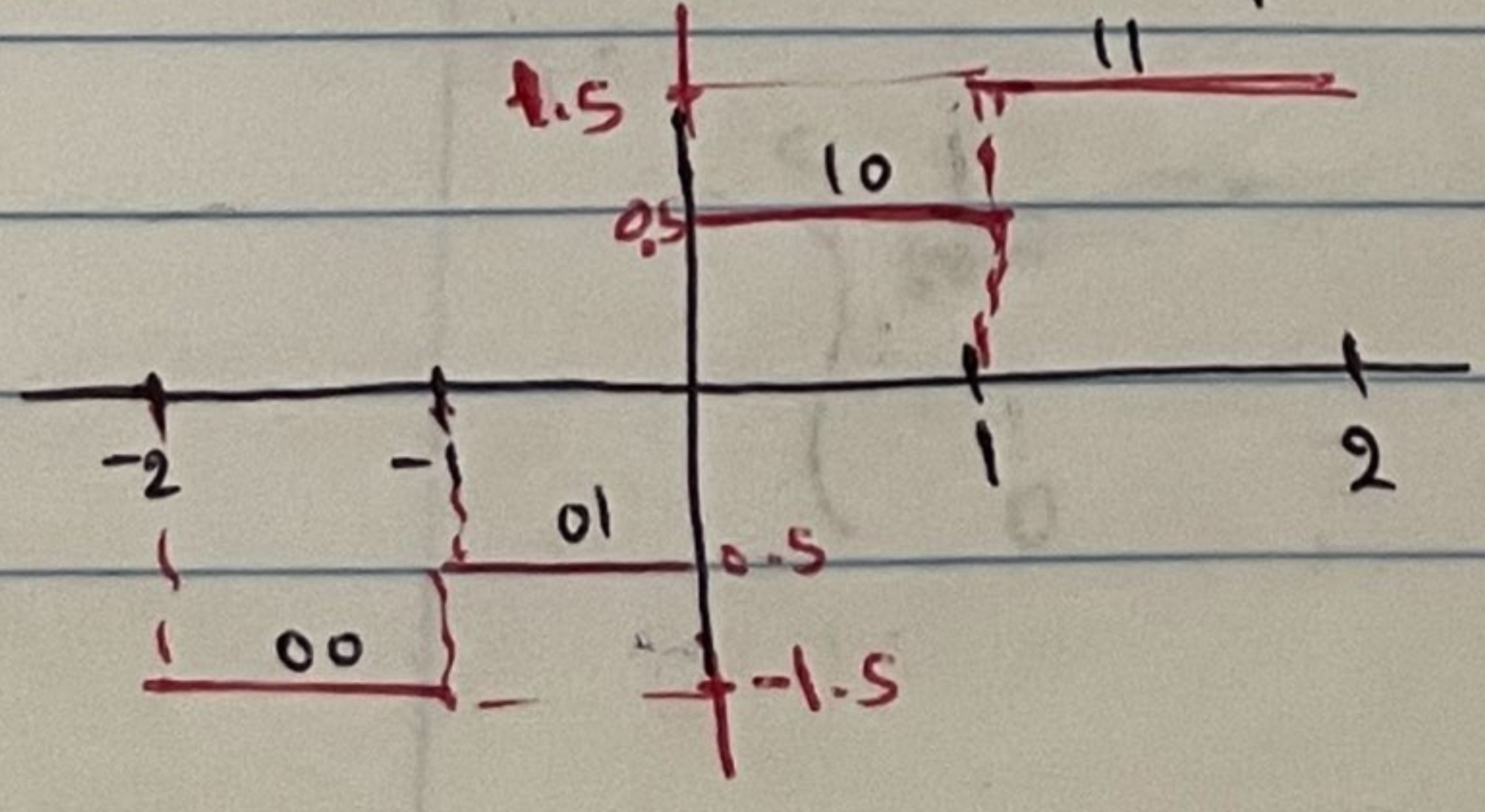
$\Rightarrow 0 \le nT \le 4$

	nT	0	0.125	0.25	0.375	0.5
	$m_s(nT)$	1.25	-1.45	0.9	-0.45	-0.2
b-	$m_q(nT)$	1.5	-1.5	0.5	-0.5	-0.5

b)



$\Delta = \frac{2 - -2}{M=4} = 1$



تكملة الجدول ←

c) Binary

1.5	-1.5	0.5	-0.5	-0.5
11	00	10	01	01

d) → bit rate R_b = # of bits * Sampling freq. (bit/s).

عدد الـ bits اللي بقدر انقلهم في الثانية ←

$= 2 * 8\,KHz$

$= 16\,K$ bits/sec.

F_s [Samples/sec]

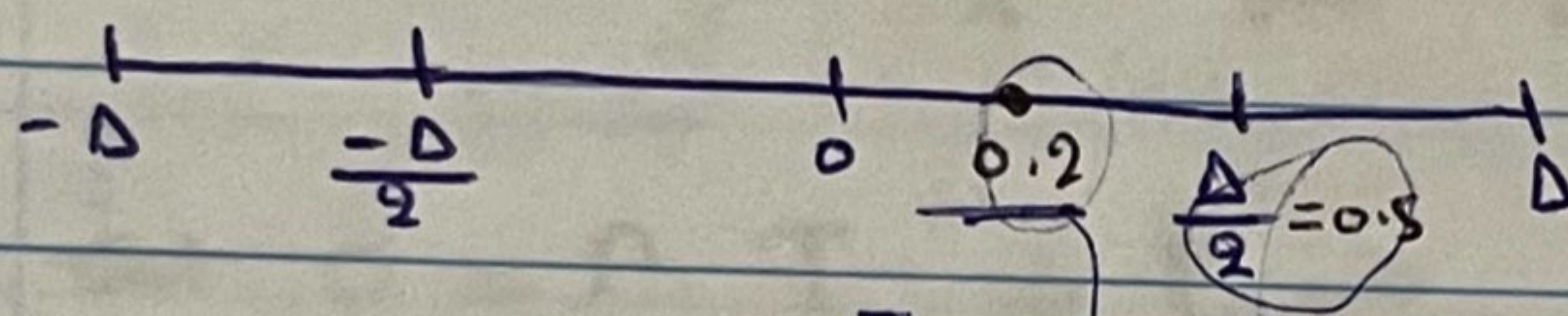
قيمة ال output الموجود عندي بالنسبة للايرور الي ممكن يصير

≡ SQNR: Signal quantization noice ratio.

$$SQNR = \frac{P_s}{D} \quad \begin{matrix} P_s \to \text{Power Rx} \\ D \to \text{noise} \end{matrix}$$

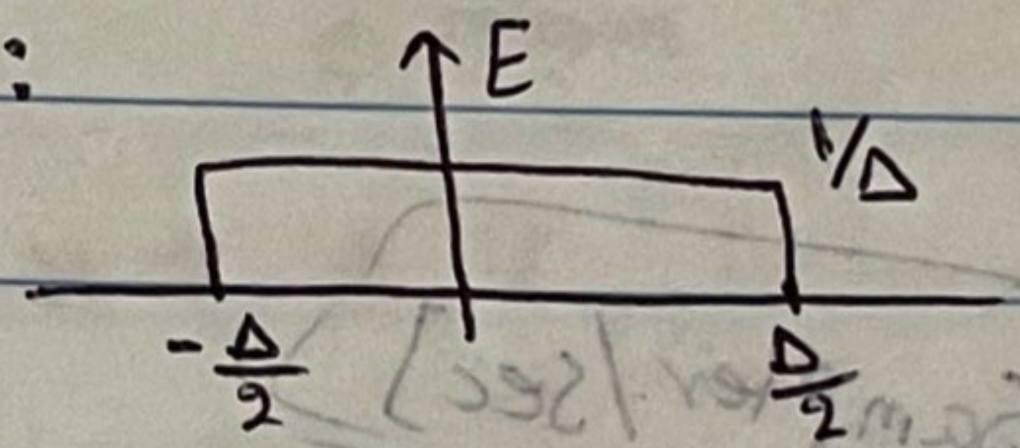
Let $X(t) = A\cos(\omega t) \Rightarrow P_s = \frac{A^2}{2}$

D:



$E = X_n - \frac{\Delta}{2}$ eg. → $E = 0.2 - 0.5 = 0.3$.

E:



$$\Rightarrow D = \frac{\Delta^2}{12} = \left(\frac{2A}{L = 2^n}\right)^2 / 12$$

- $SQNR = \frac{A^2/2}{\Delta^2/12}$,, $SQNR = \frac{3}{2}L^2 \to \frac{3}{2}2^{2n}$

In dB, $SQNR = 10\log\frac{P_s}{D} = 6.02n + 1.76$

Example | $X(t) = 4\cos(2\pi f_0 t)$ is applied to a uniform quantizer with L quantizer levels, and Dynamic range $(-4,4)$ V, find the minimum value of L that will achieve a signal to quantization noise ratio $SQNR_{dB} \geq 30$ dB.

$$SQNR_{dB} = 10 \log SQNR \Rightarrow SQNR = 10^{SQNR_{dB}/10}$$

$$30_{dB} = 10 \log SQNR \Rightarrow \boxed{SQNR = 1000}$$

$$SQNR = 1000 = \frac{P_s}{D} = \frac{A^2/2}{\Delta^2/12} = \frac{8}{\frac{(8/L)^2}{12}}$$

$$SQNR = \frac{8 * 12 * L^2}{64} \Rightarrow \frac{12}{8} L^2 \geq 1000$$

$$\Rightarrow L^2 \geq \frac{8000}{12} \Rightarrow L \geq 25.8$$

$$L = 26 \text{ Levels.}$$

OR

$$6.02n + 1.76 \geq 30 \text{ dB}$$

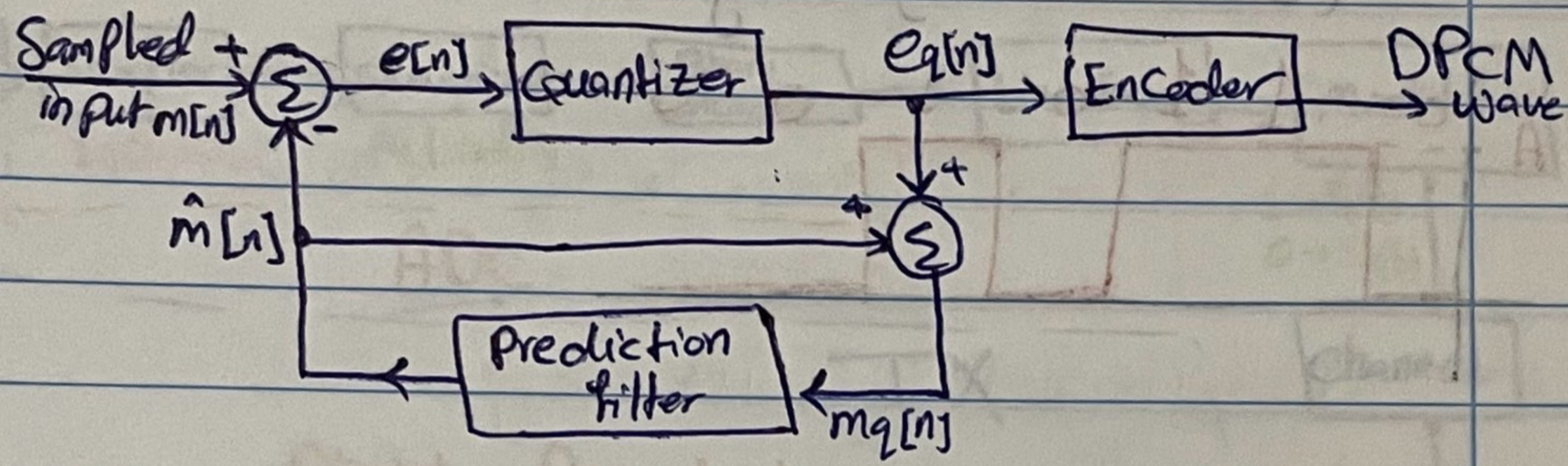
$$\frac{30 - 1.76}{6.02} = \frac{6.02n}{6.02}$$ بنطلع قيمة n ونعوض في $L = 2^n$

$$n = 4.69 \Rightarrow L = 2^n = 25.89$$

$$\Rightarrow L \approx 26 \text{ Level}$$

L(18) (25 min)

Differential Pulse Code Modulation (DPCM).



Liner prediction filter.

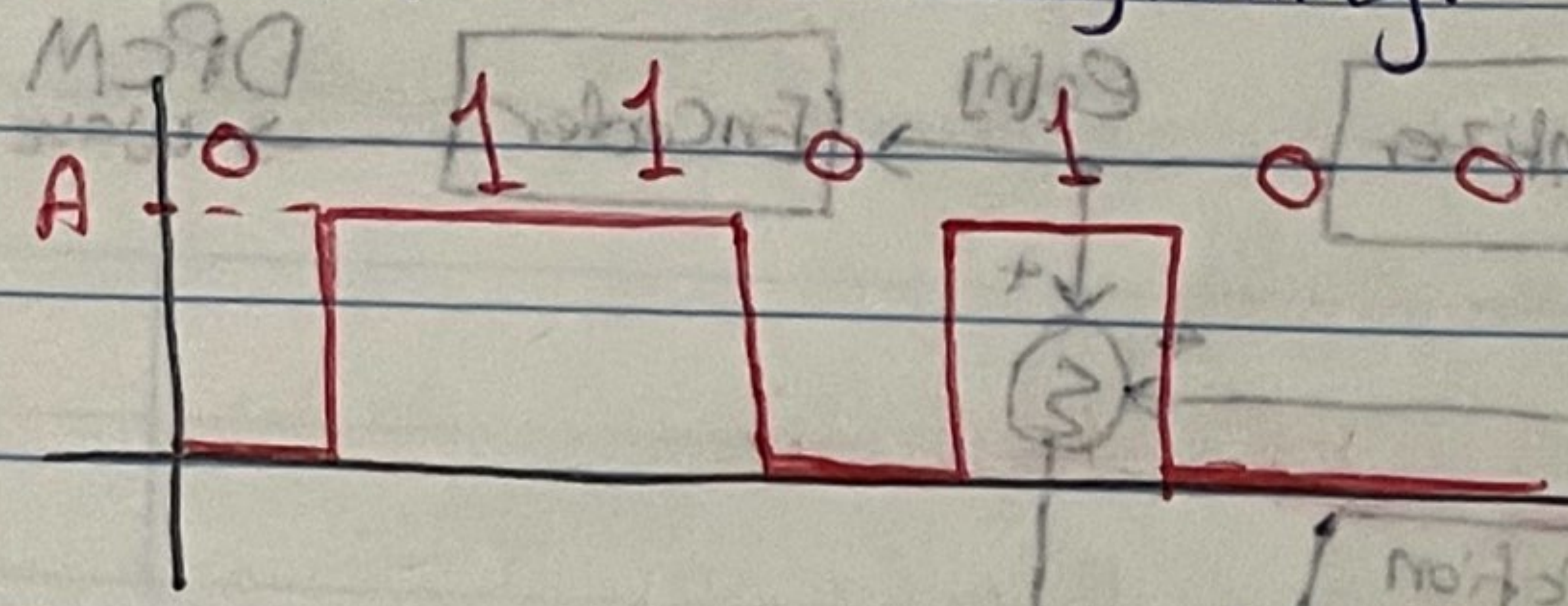
$$\hat{X}(n) = w_1 X(n-1) + w_2 X(n-2) + \cdots + w_p X(n-p)$$

$$\epsilon = E(X(n) - \hat{X}(n))^2$$ Prediction error.

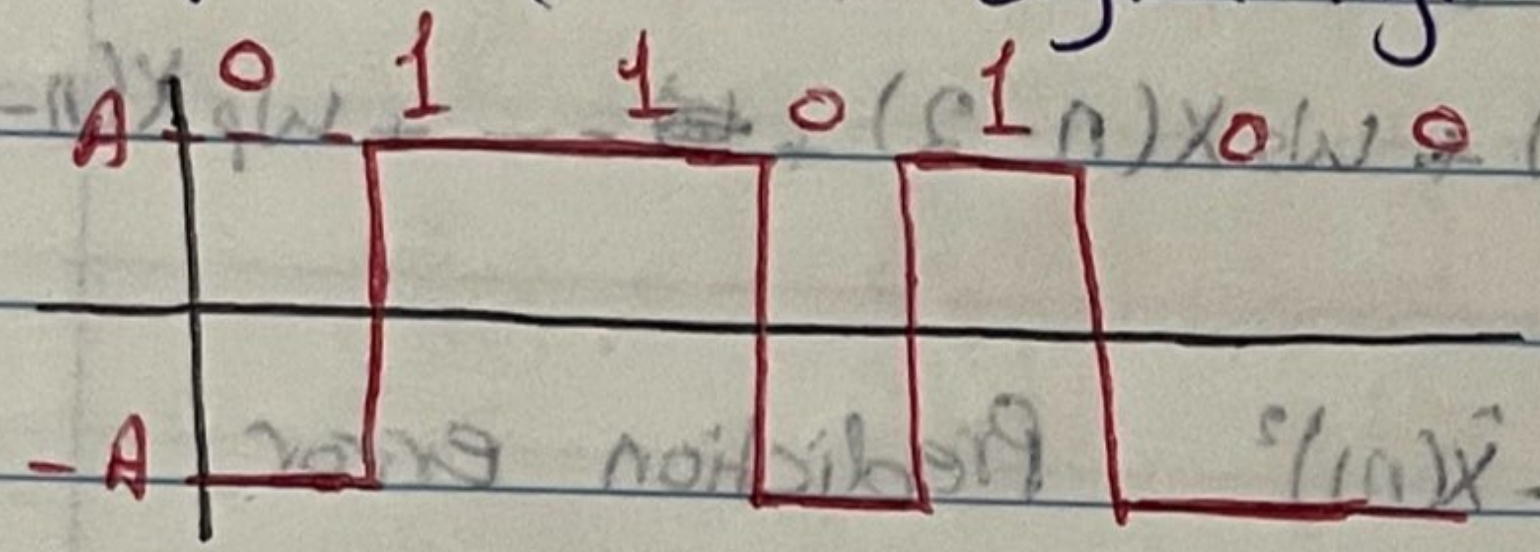
To avoid slope overload.

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

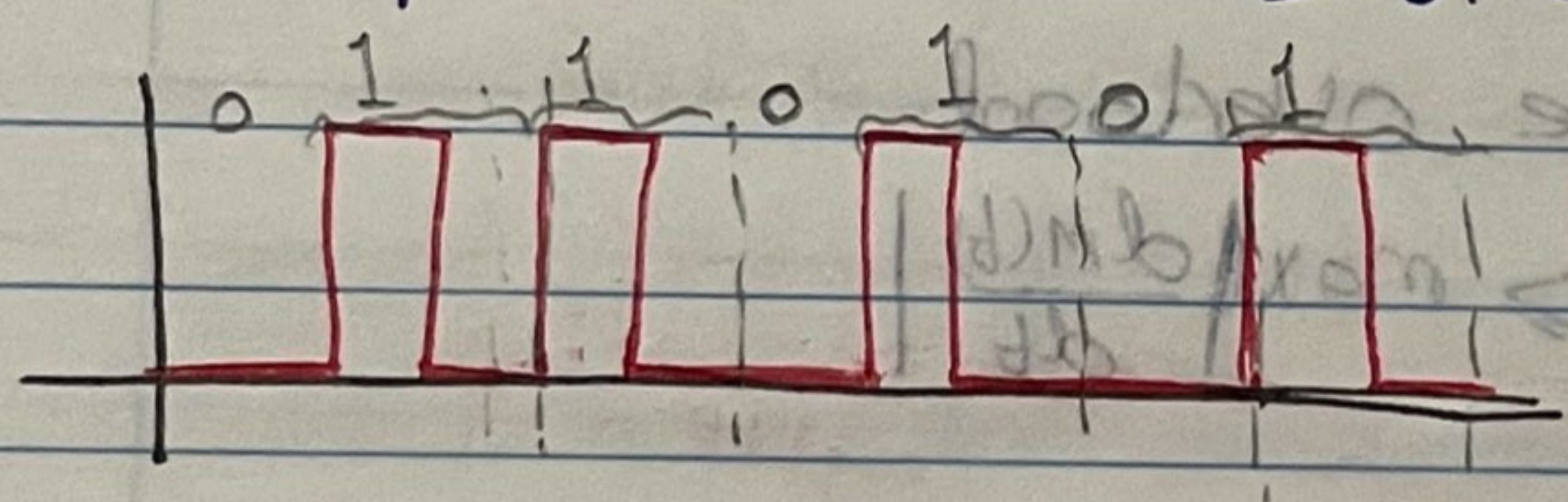
- unipolar nonreturn-to-zero (NRZ) (on-off) signaling.



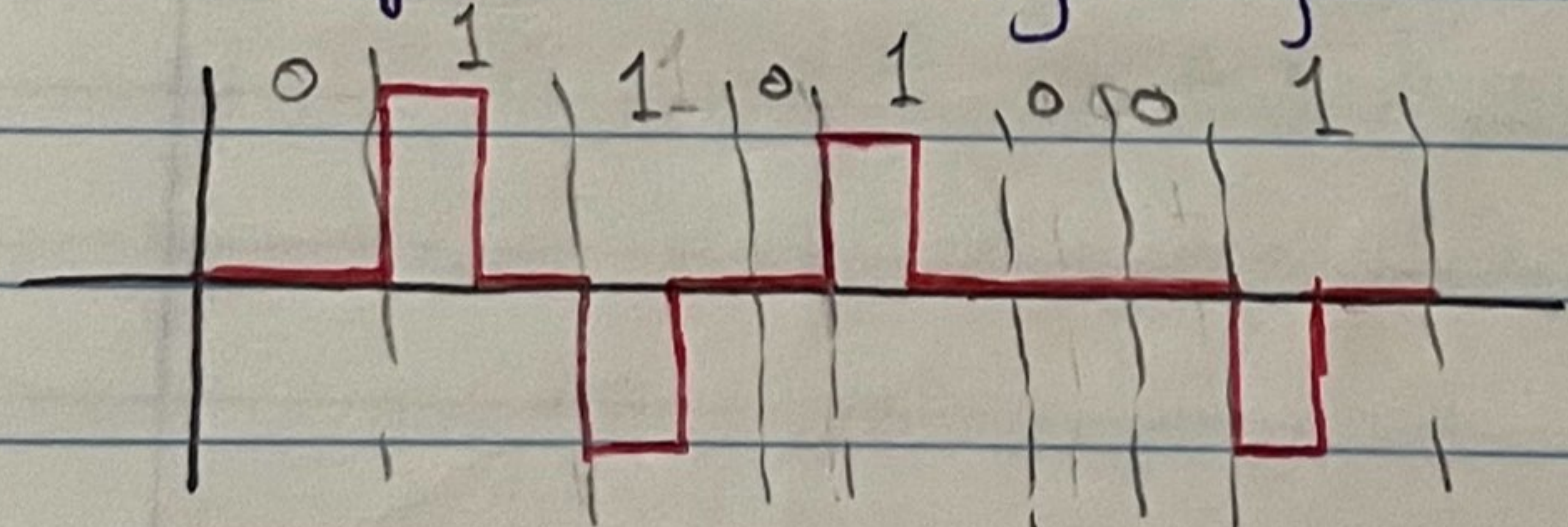
- Polar (NRZ) signaling.



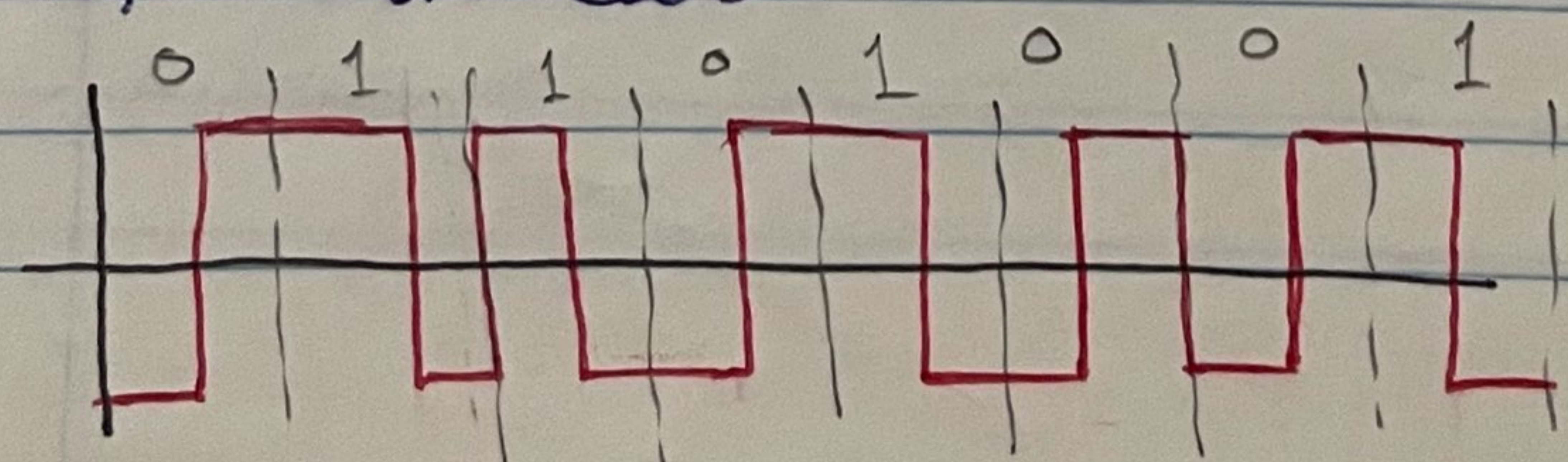
- unipolar return-to-zero (RZ) signaling



- Bipolar (RZ) Signaling

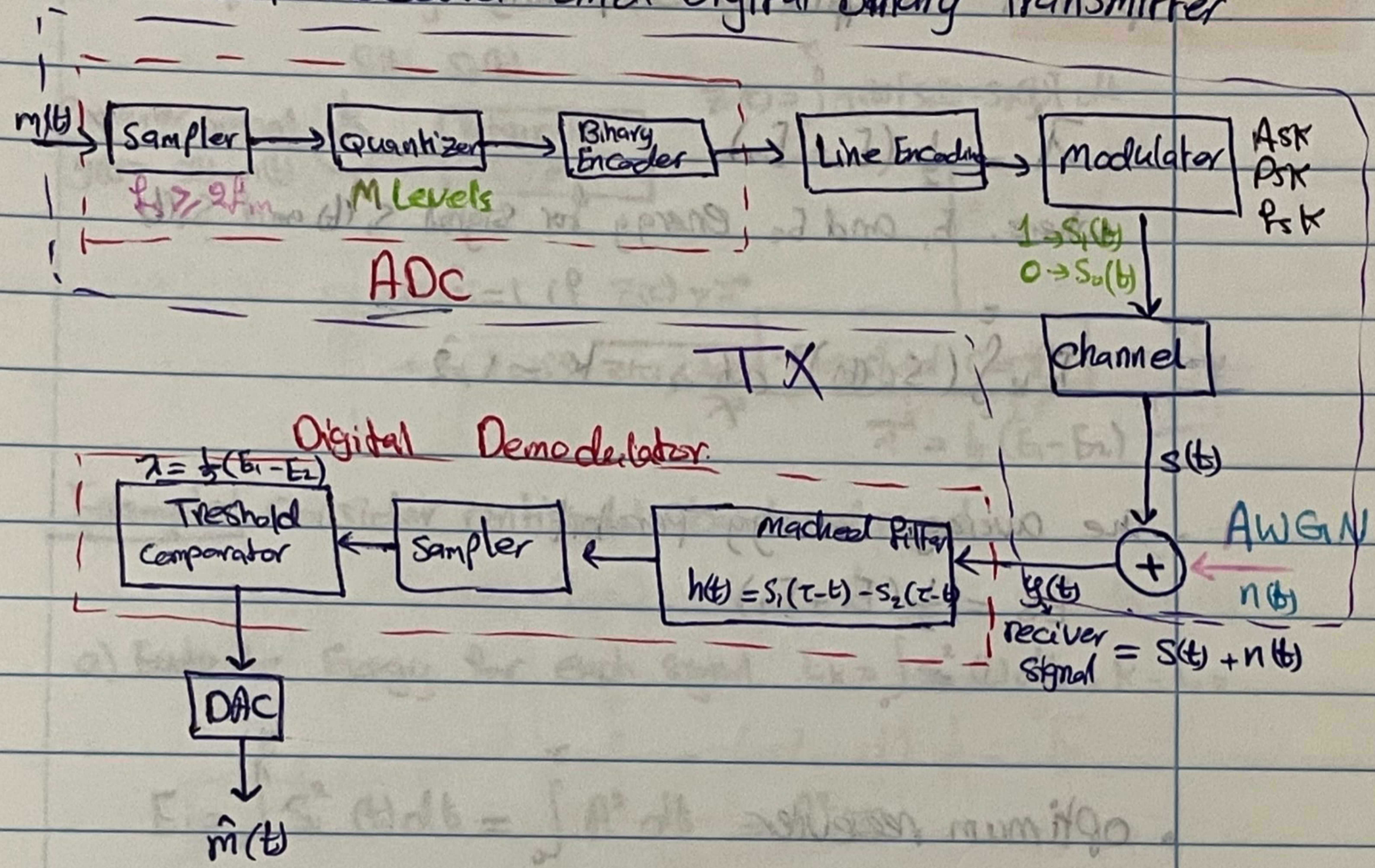


- Manchester Code.



(L19)

Optimum Receiver and Digital Binary Transmitter

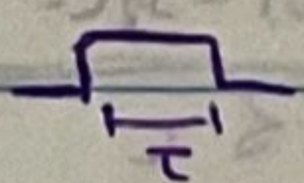


[Digital Communication system].

- Probability of error.

$$P_b^* \equiv P_e \equiv Q\left(\sqrt{\frac{\int^{\tau}(S_1(t) - S_2(t))^2\,dt}{2N_0}}\right)$$

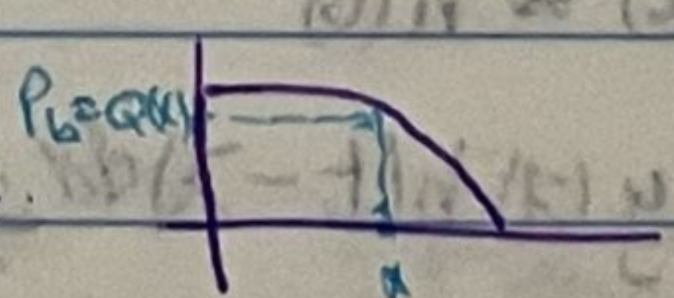
τ: bit duration



$\frac{1}{\tau}$: bit rate(R_b). or data rate.

N_0: thermal noise.

$Q(x)$: Q function



(P1_1)

- Optimum Threshold of Comparator.

$$\lambda^* = \frac{1}{2}(E_1 - E_2)$$

where, E_1 and E_2 energy for signal $s_1(t)$ and $s_2(t)$

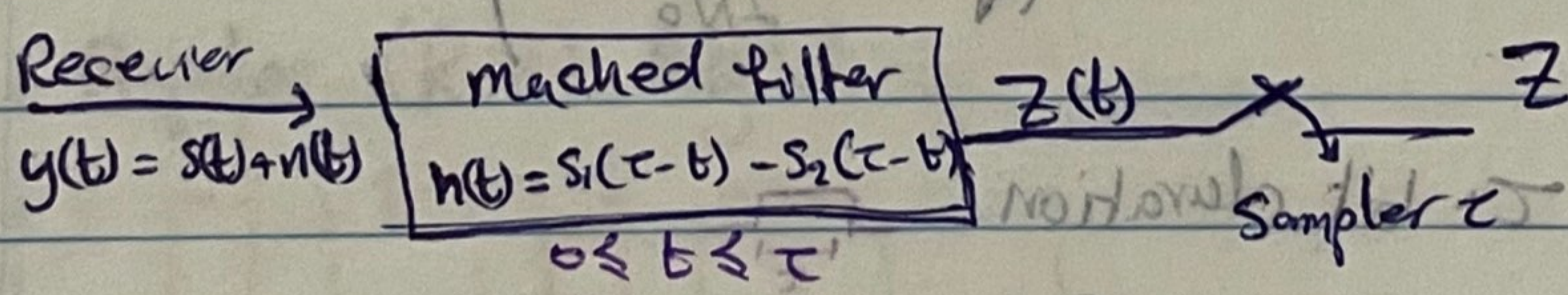
$$E_k = \int_0^{\tau} (s_k(t))^2 \, dt, \quad k = 1, 2$$

- The average energy per bit

$$E_b = \frac{1}{2}(E_1 + E_2)$$

- optimum receiver
  - ↳ mached filter
  - ↳ correlator

→ by mached filter



$$Z(\tau) = \int_0^{\tau} y(\lambda)\, h(\tau-\lambda)$$

$$Z(t) = y(t) * h(t)$$

$$Z(t) = \int_0^{t} y(\lambda)\, h(t-\lambda)\, d\lambda \rightarrow Z(\tau) = \int_0^{\tau} y(\lambda)\, h(\tau-\lambda)\, d\lambda$$

$$Z(\tau) = \int_0^{\tau} y(t)\,[s_1(t) - s_2(t)]\, dt$$

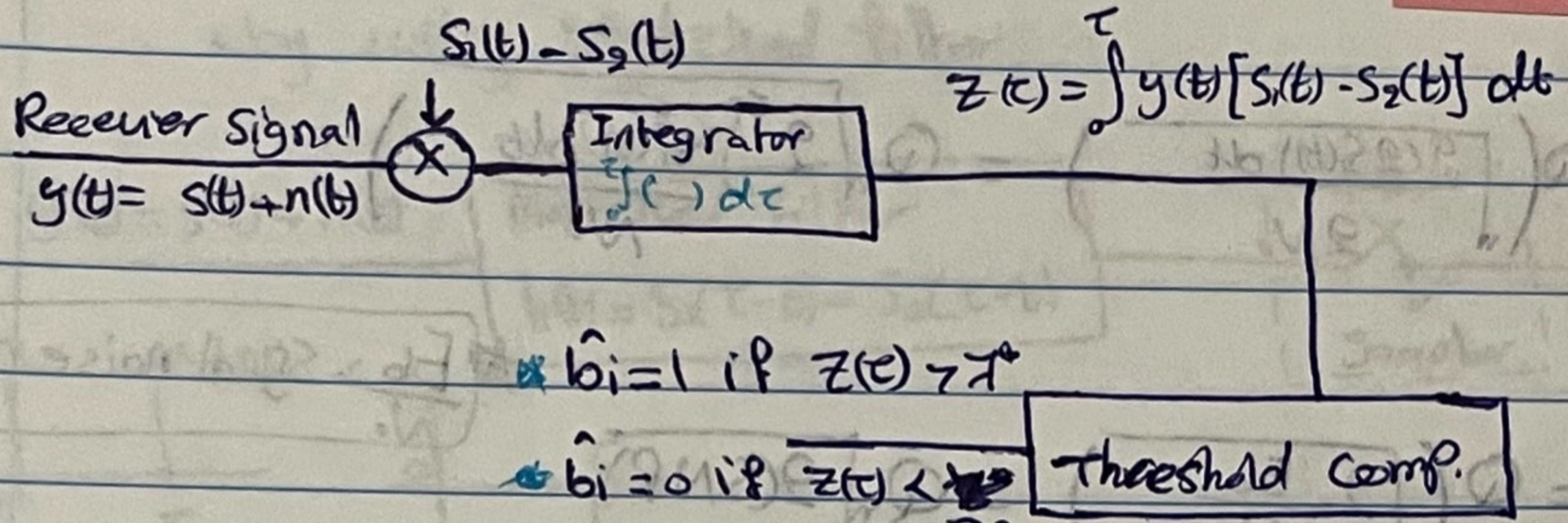
antipodal

$s_1(t) = A \,,\; 0 \le t \le \tau$

$s_2(t) = -A \,,\; 0 \le t \le \tau$

$\Rightarrow s_1(t) = -s_2(t)$

or equivalently:- [Correlator]



$\hat{b}_i = 1$ if $z(\tau) > \lambda^*$

$\hat{b}_i = 0$ if $z(\tau) < \lambda^*$

$\lambda^* = \frac{1}{2}(E_1 - E_2)$

Example) consider antipodal baseband signal

a) Evaluate Energy for each signal. $E_k = \int_0^\tau s_k^2(t)\,dt \,,\; k = 1, 0$

$$E_1 = \int_0^\tau s_1^2(t)\,dt = \int_0^\tau A^2\,dt = A^2\tau$$

$$E_2 = \int_0^\tau s_2^2(t)\,dt = \int_0^\tau (-A)^2\,dt = A^2\tau$$

b) Evaluate the average of energy (average energy per bit).

$$E_{Avg} = \frac{E_1 + E_2}{2} = \frac{A^2\tau + A^2\tau}{2}$$

$$E_{avg} = A^2\tau = E_b$$

c) Evaluate the P_b^*

$$P_b^* = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) - s_2(t))^2\,dt}{2N_o}}\right) = Q\left(\sqrt{\frac{\int_0^\tau (2s_1(t))^2\,dt}{2N_o}}\right)$$

$\Rightarrow$

Since we have antipodal signal $\Rightarrow S_1(t) = -S_2(t)$

$$= Q\left(\sqrt{\frac{\int (2S_1(t))\,dt}{2N_0}}\right) = Q\sqrt{\frac{2\int_0^T S_1^2(t)\,dt}{N_0}}$$

(Energy)

$$P_b^* = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) = Q\sqrt{2(SNR)}$$

$\frac{E_b}{N_0}$ = Signal noise ratio

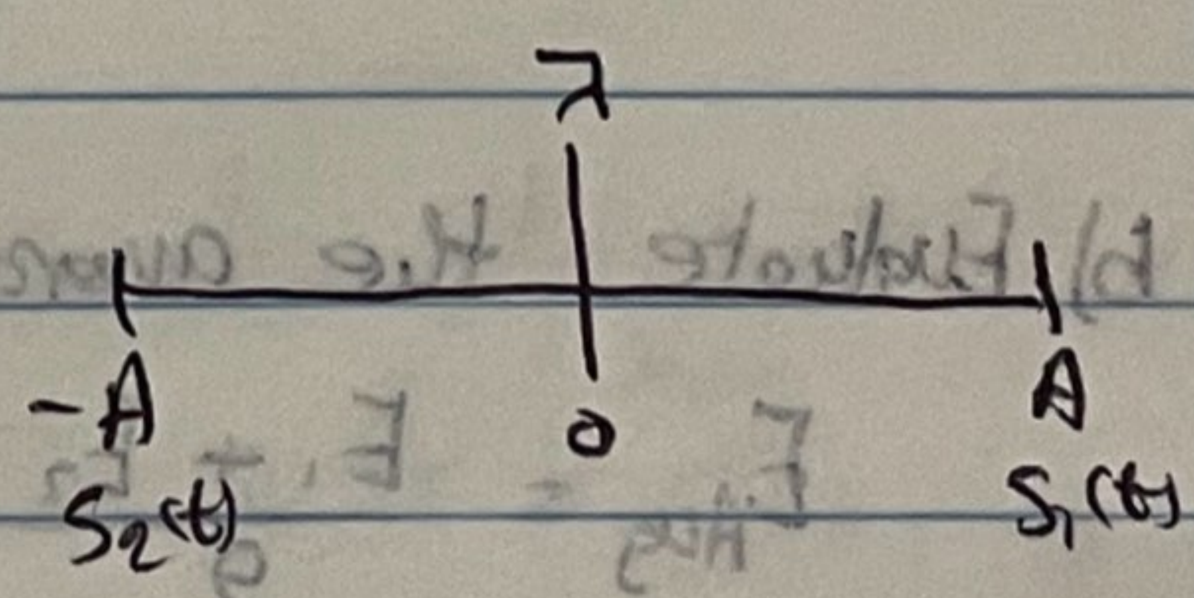
d) Evaluate the threshold Comparison.

$$\lambda^* = \frac{1}{2}(E_1 - E_2) \; ; \; E_1 = E_2$$

$\Rightarrow \lambda^* =$ zero

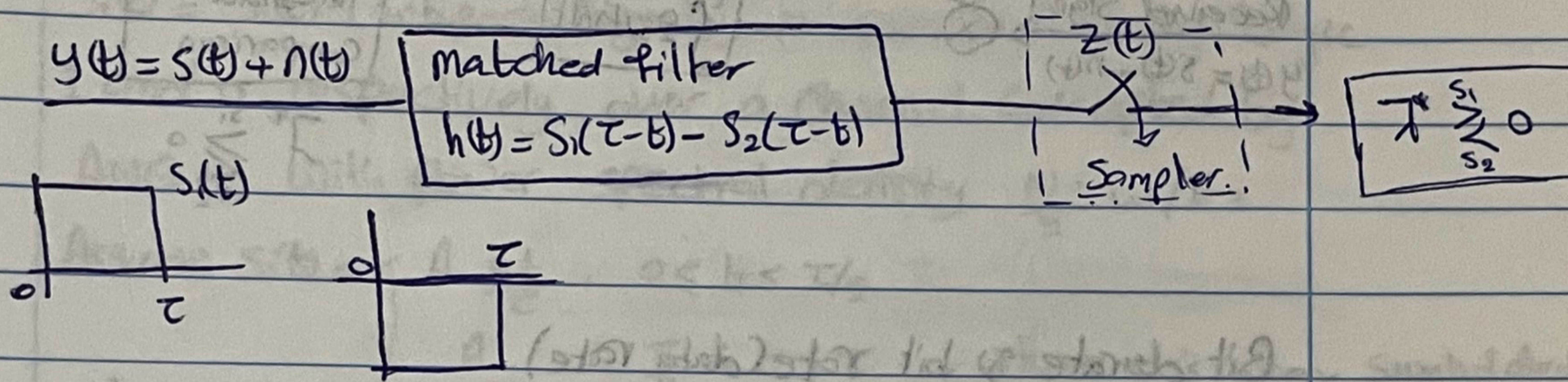
$\Rightarrow \hat{b}_i = 1 \Rightarrow Z(T) > 0$

$\hookrightarrow \hat{b}_i = 0 \Rightarrow Z(T) < 0$

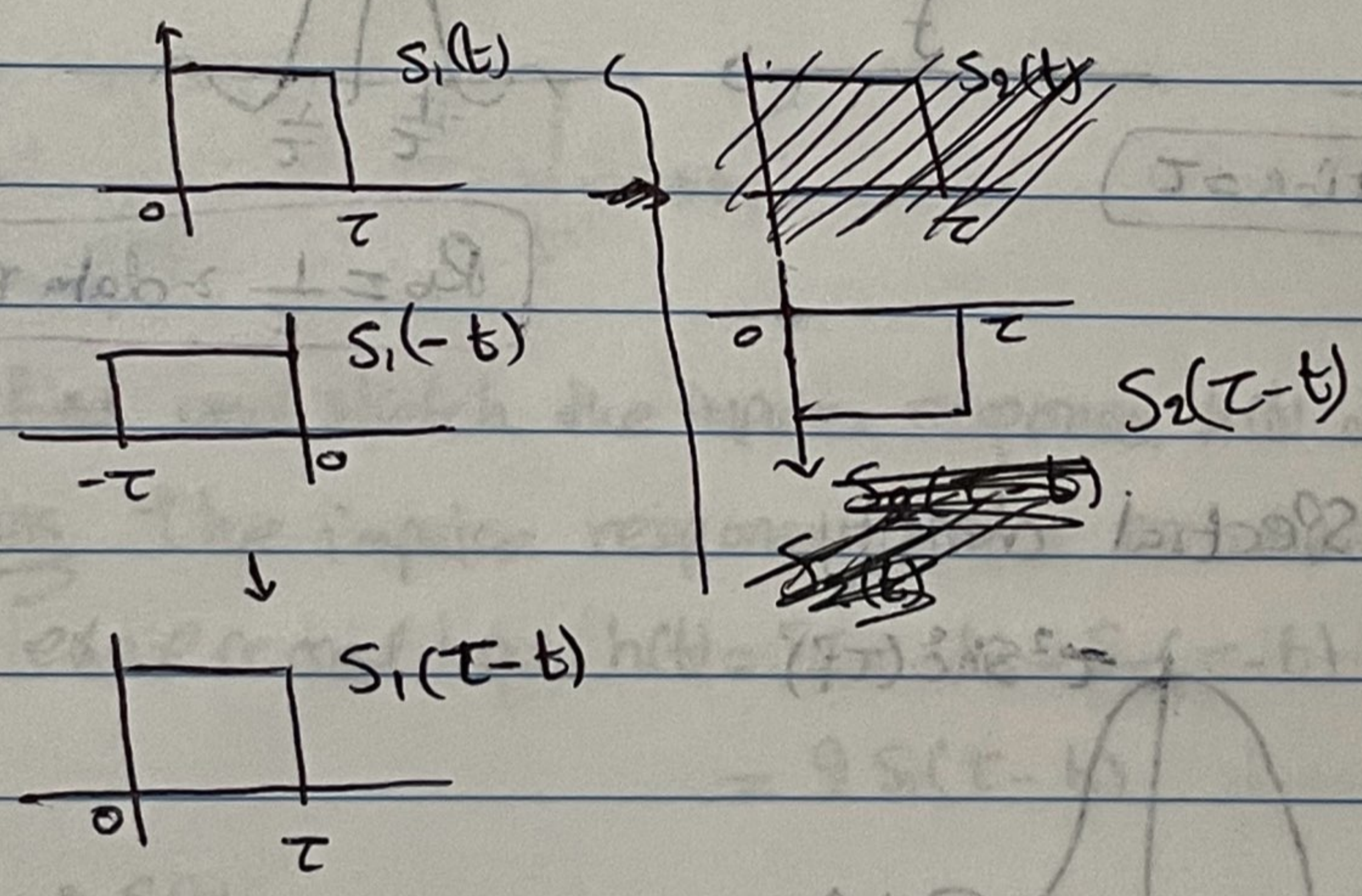


e) Implement optimum Receviter

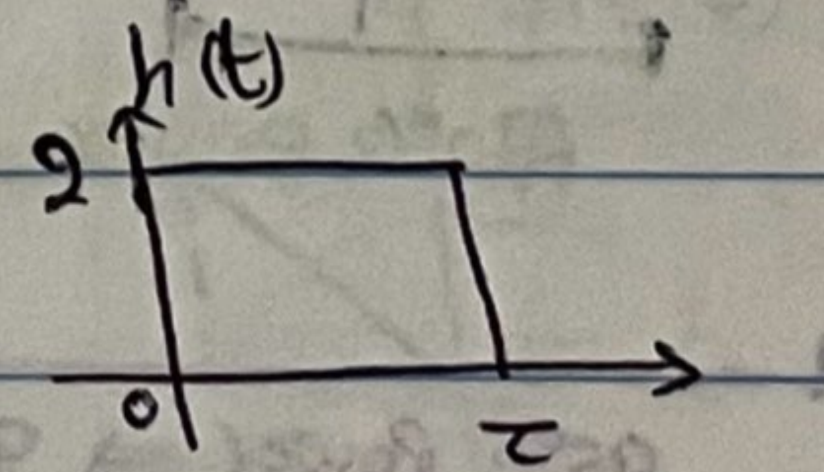
• by using matched filter.



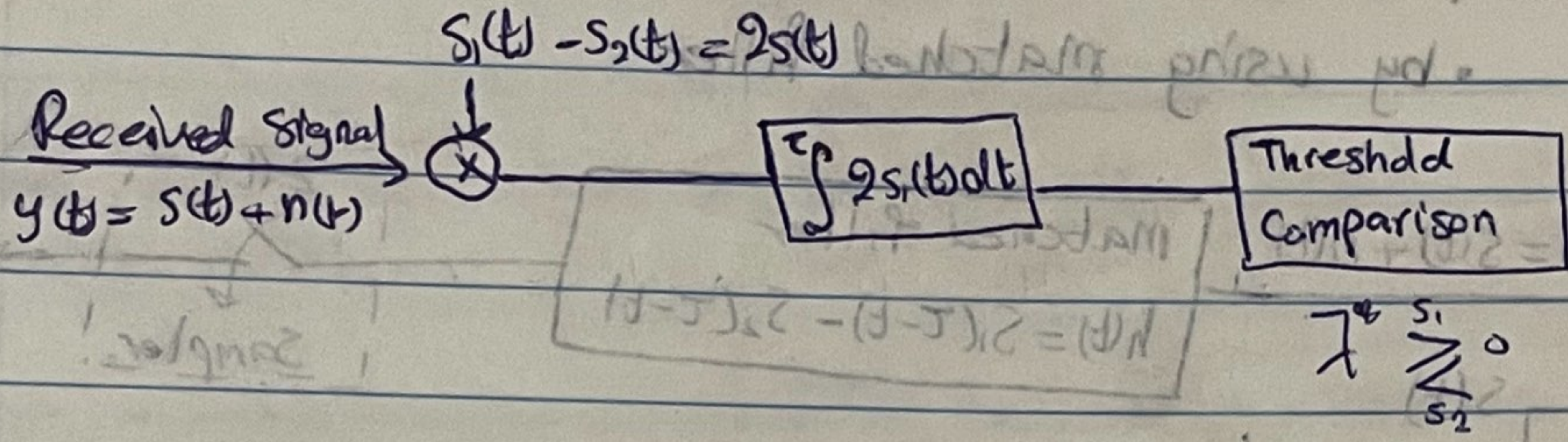
• Evaluate h(t).



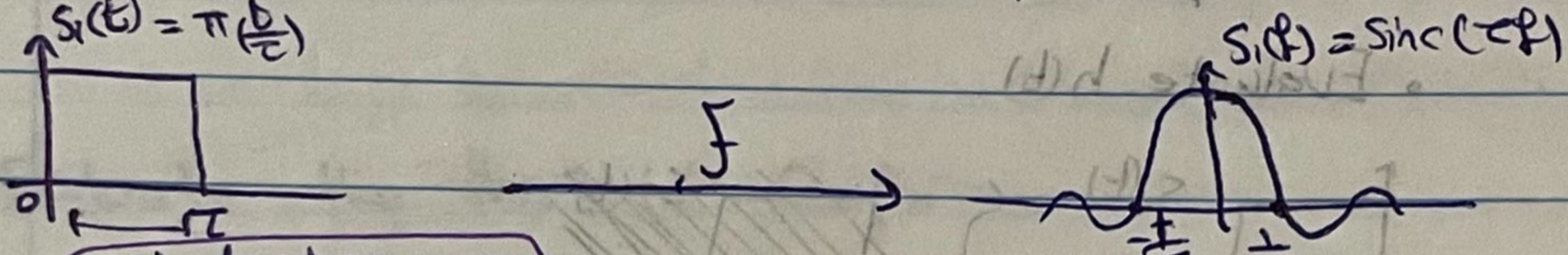
$S_1(\tau-t) - S_2(\tau-t) \Rightarrow$



• By using Correlator:



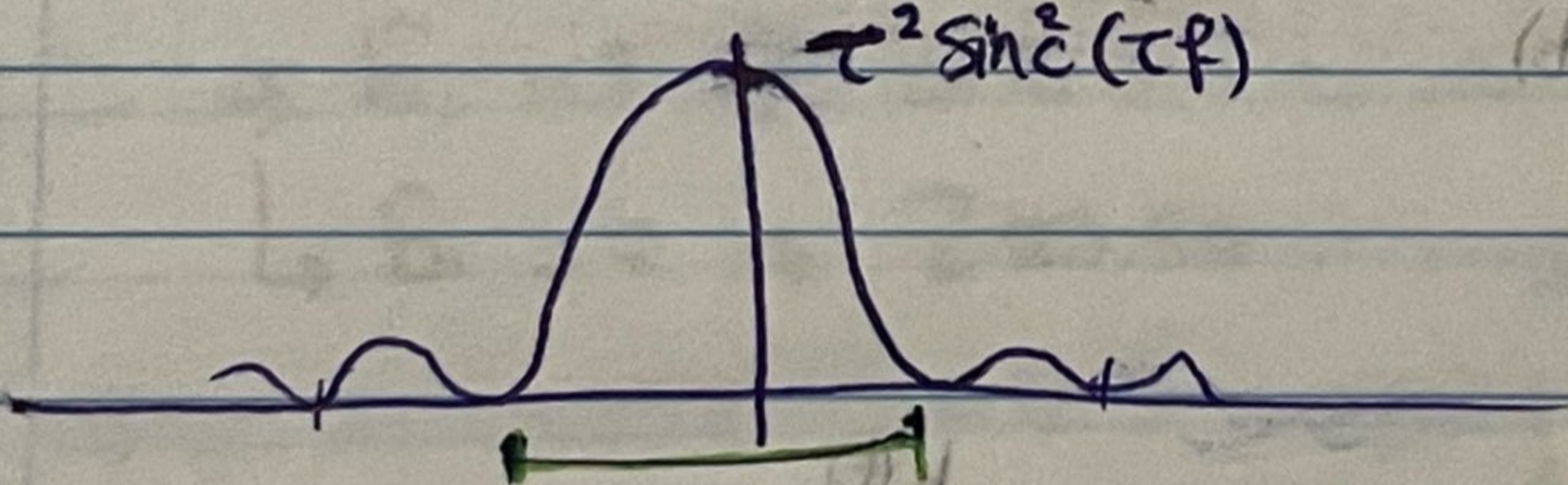
• Bit durate & bit rate (data rate)



• bit duration $= \tau$

$R_b = \frac{1}{\tau}$ → data rate.

Power spectral density.



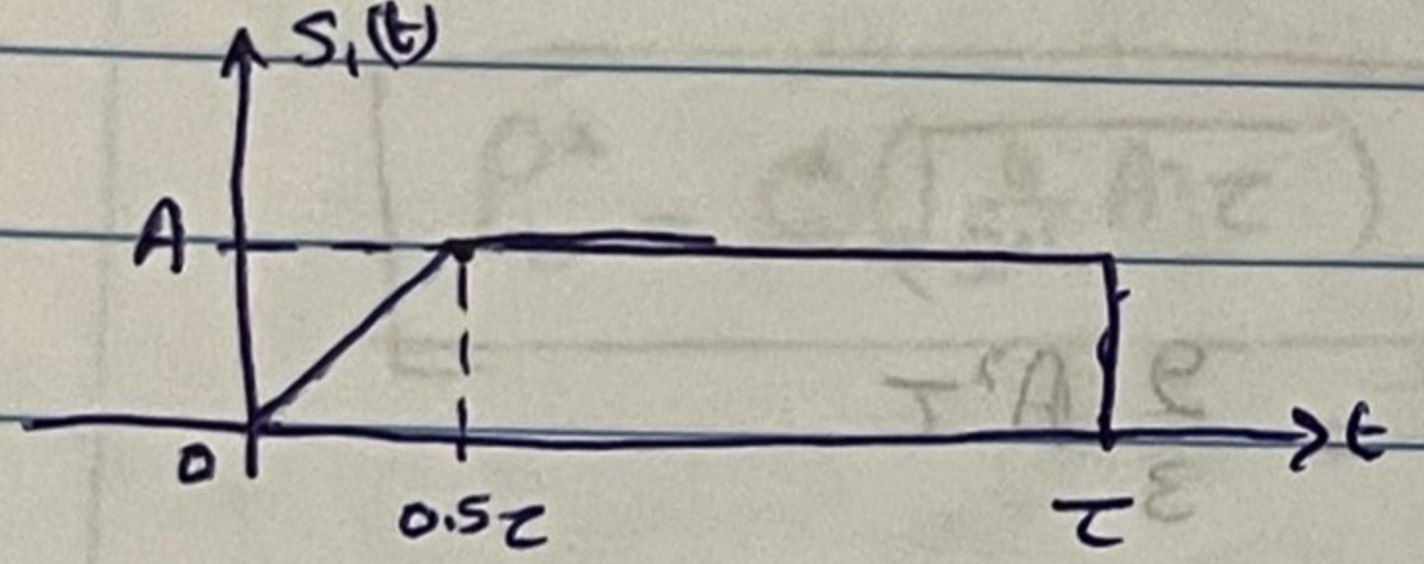
90% power → 90% bandwidth $= \frac{1}{\tau} = R_b$

95% power → 95% bandwidth $= \frac{2}{\tau} = 2R_b$

(L 20)

Example The binary digital communication signal scheme, employs the following two equally probable signals $S_1(t)$ and $S_2(t) = -S_1(t)$ [antipodal signals]. to represent binary Logic 1 and 0 respectively over a channel corrupted by AWGN with power spectral density $\frac{N_0}{2}$ W/Hz.

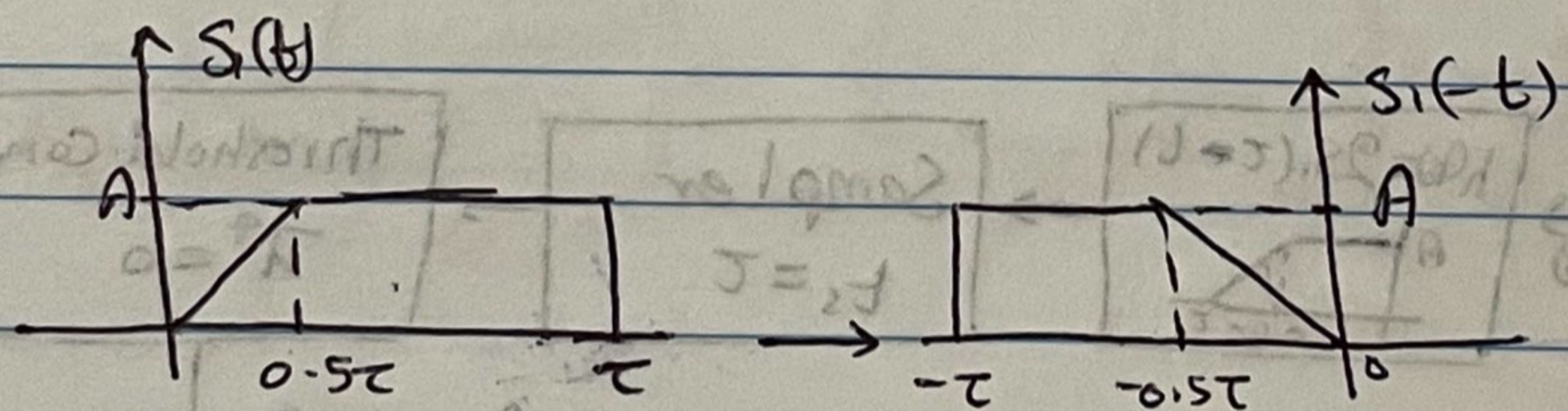
Assume $S_1(t) = \begin{cases} A \cdot \frac{2t}{\tau}, & 0 \le t \le \tau/2 \\ A, & \frac{\tau}{2} \le t \le \tau \end{cases}$, where τ is binary symbol duration

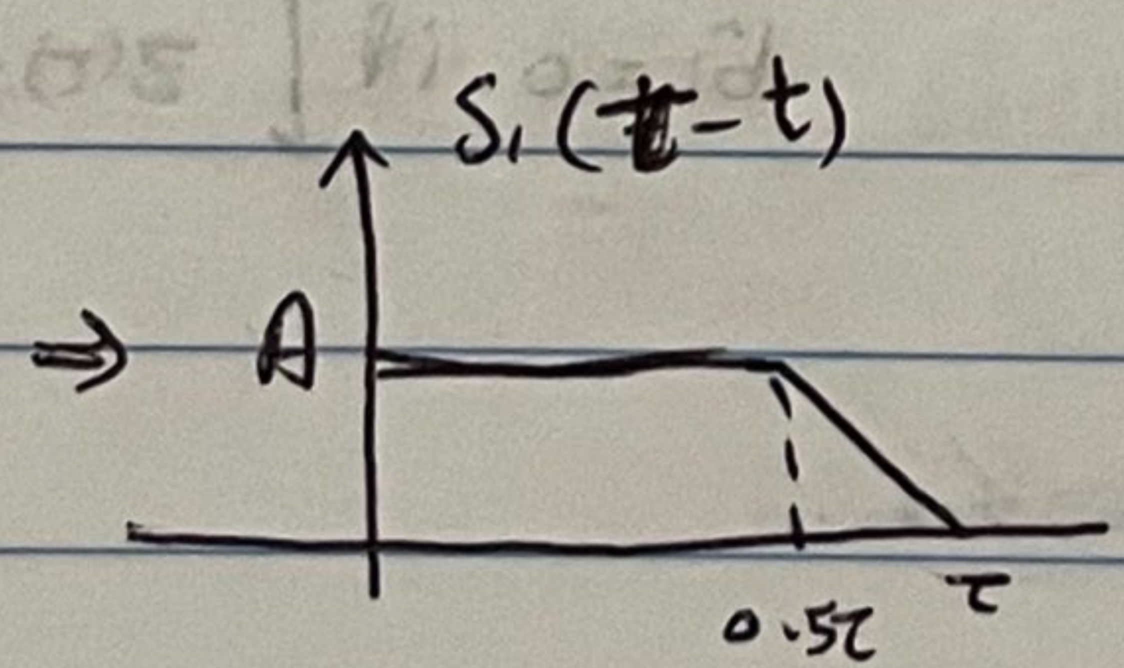


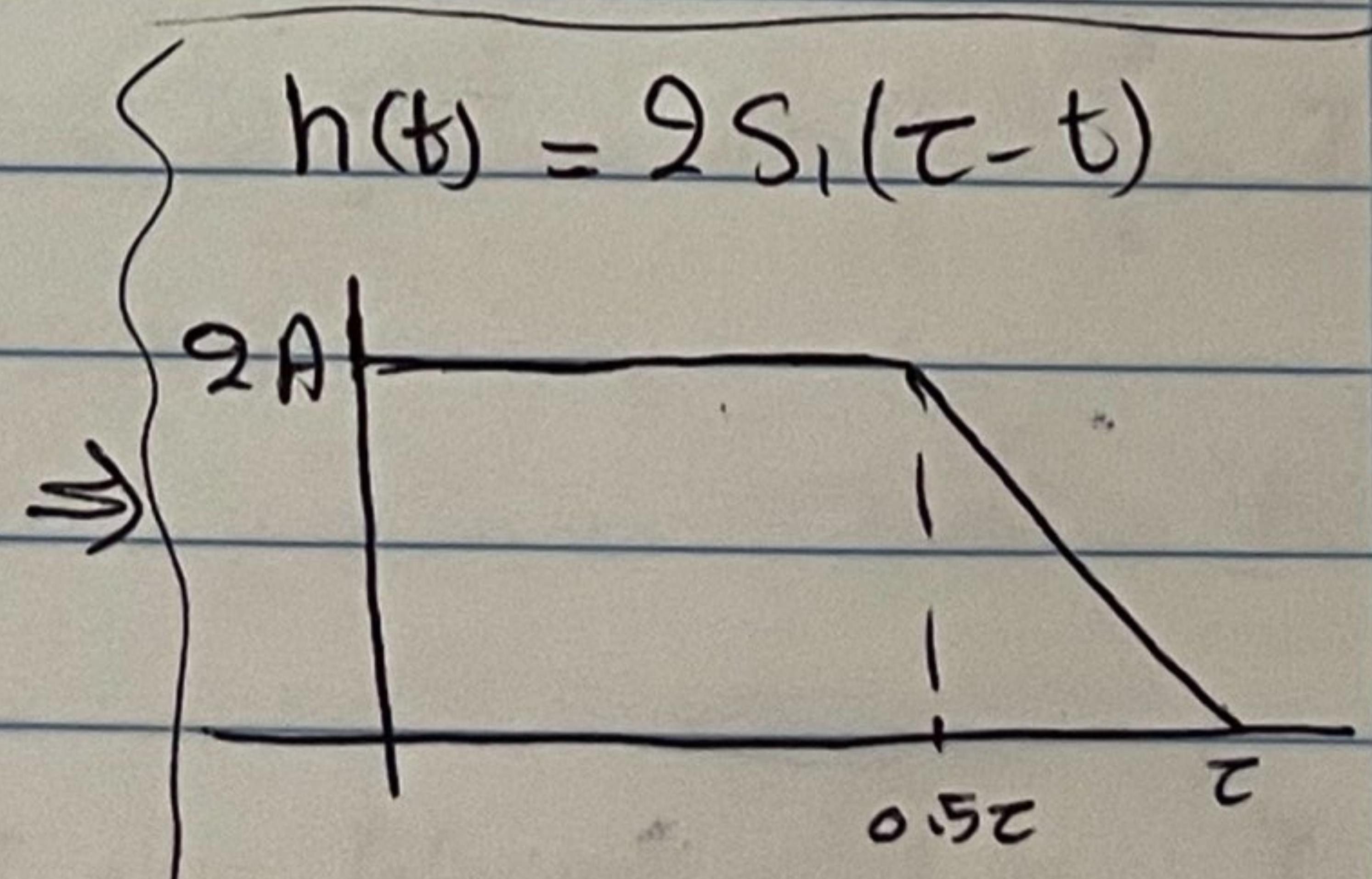
a) find and sketch the impulse response, h(t) of the matched filter

Ans the impulse response of the matched filter can be expressed as $h(t) = S_1(\tau - t) - S_2(\tau - t)$

$= 2S_1(\tau - t)$







b) Find the optimum threshold used at the receiver

$$\lambda^* = \frac{1}{2}(E_1 - E_2)$$

$$E_1 = \int_0^{\tau} s_1^2(t)\,dt = \int_0^{0.5\tau} \left(\frac{2A}{\tau}\right)^2 t^2\,dt + \int_{0.5\tau}^{\tau} A^2\,dt$$

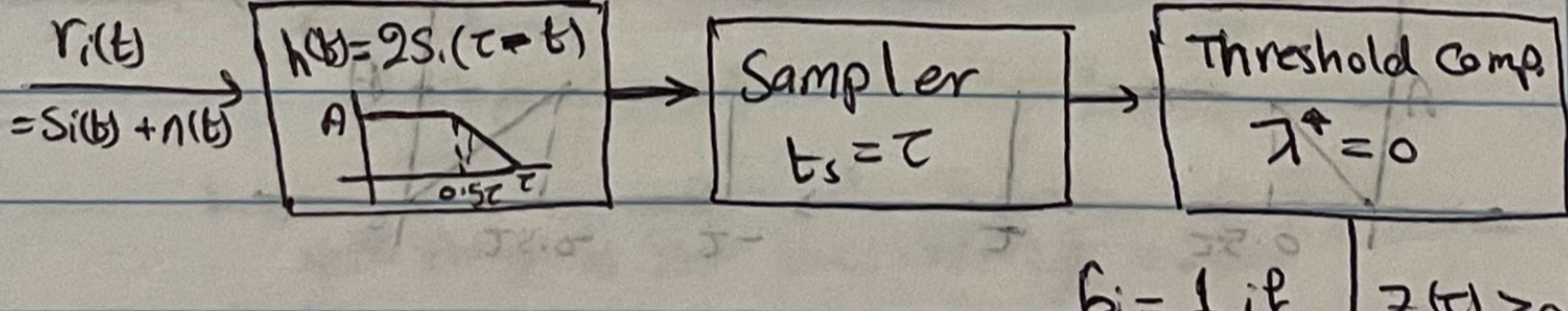
$$= \frac{4A^2}{\tau^2}\left(\frac{\tau}{2}\right)^2 + A^2 \frac{\tau}{2}$$

$$E_1 = \frac{2}{3}A^2\tau = E_2$$

$$E_b = \frac{1}{2}[E_1 + E_2] = E_1 = \frac{2}{3}A^2\tau$$

$$\Rightarrow \lambda^* = 0$$

c) sketch the optimum receiver highlightry its basic components.



$\hat{b}_i = 1$ if $z(\tau) > 0$

$\hat{b}_i = 0$ if $z(\tau) < 0$

d) Evaluate the minimum prob. of error.

$$P_b^* = Q\left(\sqrt{\frac{\int_0^T (s_1(t) - s_2(t))^2\, dt}{2N_0}}\right) = Q\left(\sqrt{\frac{\int (2s_1(t))^2\, dt}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{4\int_0^T s_1^2(t)\, dt}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_b^* = Q\left(\sqrt{\frac{2}{N_0} \cdot \frac{2}{3} A^2 \tau}\right)$$

$$\boxed{P_b^* = Q\left(\sqrt{\frac{4}{3N_0} A^2 \tau}\right)}$$

Binary Digital Bandpass Modulation

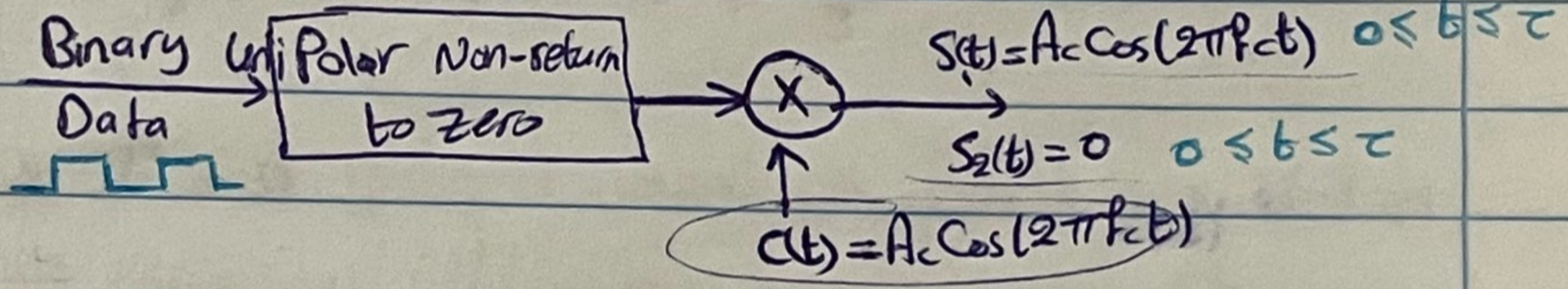
- ASK: Amplitude Shift Keying
- PSK: Phase Shift Keying
- FSK: freq 〃 〃

Amplitude Shift Keying (ASK):

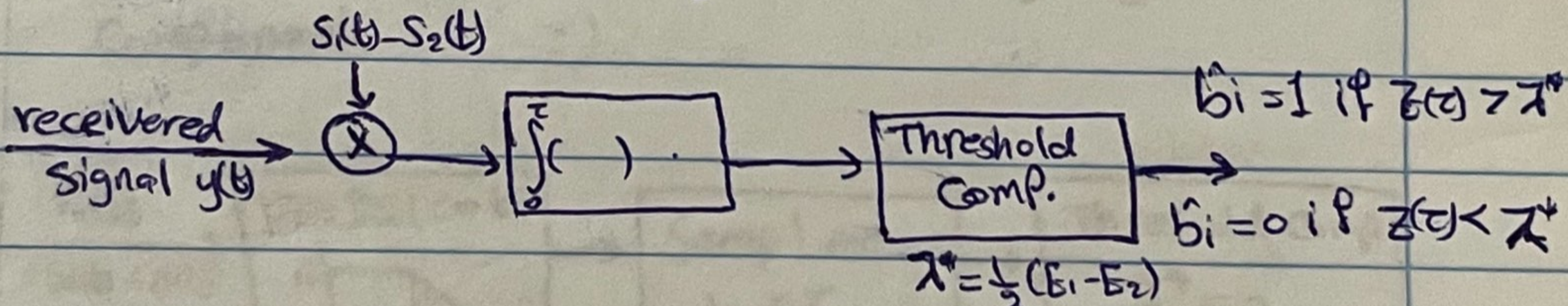
- Signal Representation

bit 1 → $S_1(t) = A_c \cos(2\pi f_c t)$ $0 \le t \le \tau$

bit 0 → $S_2(t) = 0$ $0 \le t \le \tau$



Optimum Receiver by using Correlator.



where

$$E_1 = \int_0^\tau S_1^2(t)\,dt = \int_0^\tau A_c^2 \cos^2(2\pi f_c t)\,dt = \int_0^\tau \left[\frac{A_c^2}{2} + \frac{A_c^2}{2}\cos(4\pi f_c t)\right] dt$$

$$E_1 = \frac{A_c^2}{2}\tau$$

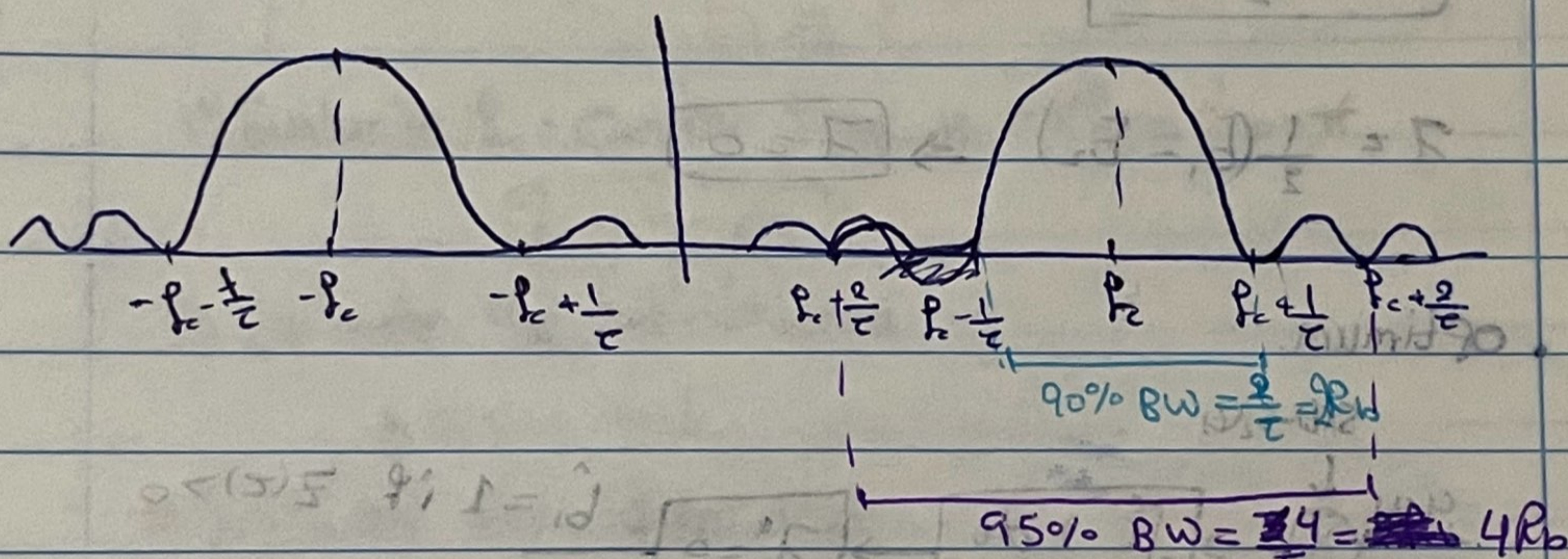
$$E_2 = 0 \text{ [No Signals]} \Rightarrow \lambda^* = \frac{1}{4} A_c^2 \tau = E_b$$

$$\Rightarrow P_b^* = Q\left(\sqrt{\frac{\int (S_1(t) - S_2(t))^2\, dt}{2N_0}}\right) = Q\left(\sqrt{\frac{\int^{\tau} S_1^2(t)\, dt}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{4E_1}{2N_0}}\right) = Q\left(\sqrt{\frac{4A^2\tau}{4N_0}}\right)$$

$$P_b^* = Q\left(\sqrt{\frac{E_b}{N_0}}\right).$$

- Power sprectral density.

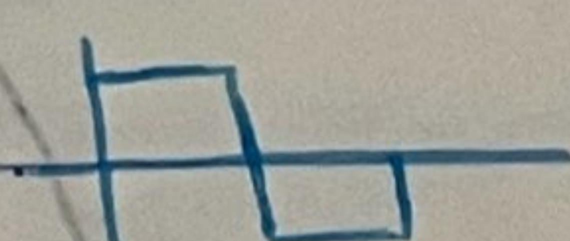


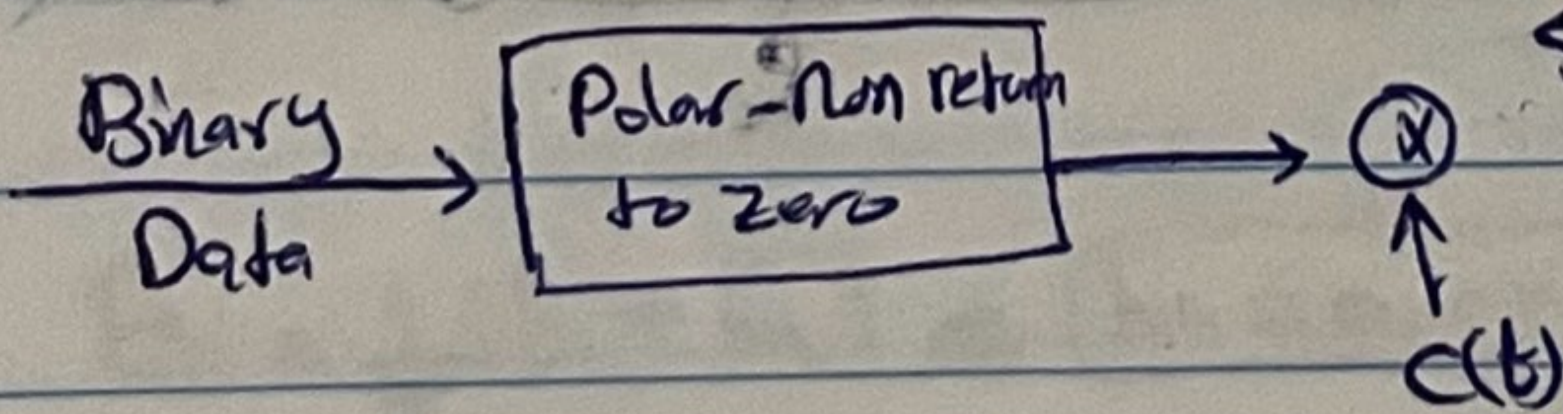
= Binary phase shift Keying (BPSK).

bit 1 $\Rightarrow S_1(t) = A\cos(2\pi f_c t)$

bit 0 $\Rightarrow S_2(t) = A\cos(2\pi f_c t + \pi) = -A\cos(2\pi f_c t)$.

$\Rightarrow S_1(t) = -S_2(t)$





$S_1(t) = A\cos(2\pi f_c t) \quad 0 \le t \le \tau$

$S_2(t) = -A\cos(2\pi f_c t) \quad 0 \le t \le \tau$

- $E_1 = \int_0^{\tau} s_1^2(t)\, dt = \int_0^{\tau} A^2 \cos^2(2\pi f_c t)\, dt$

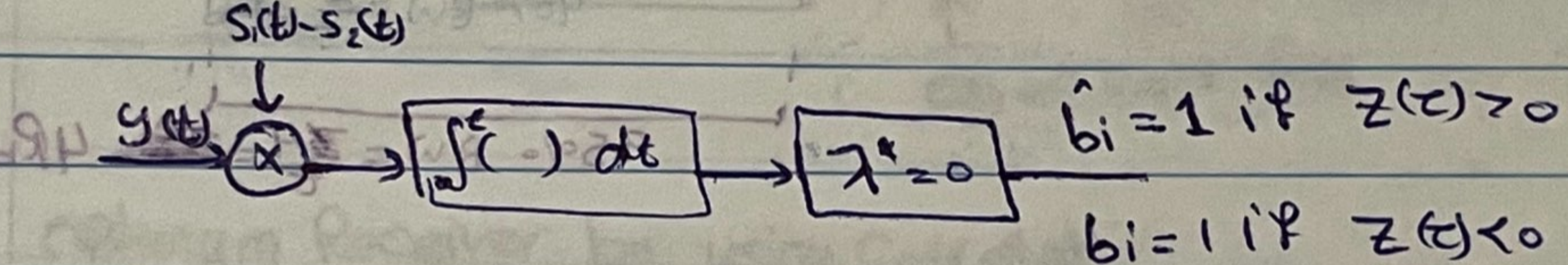
$$\boxed{E_1 = \frac{A^2 \tau}{2}}$$

- $E_2 = \int_0^{\tau} s_2^2(t)\, dt = \int_0^{\tau} A^2 \cos(2\pi f_c t)\, dt$

$$\boxed{E_2 = \frac{A^2 \tau}{2}}$$

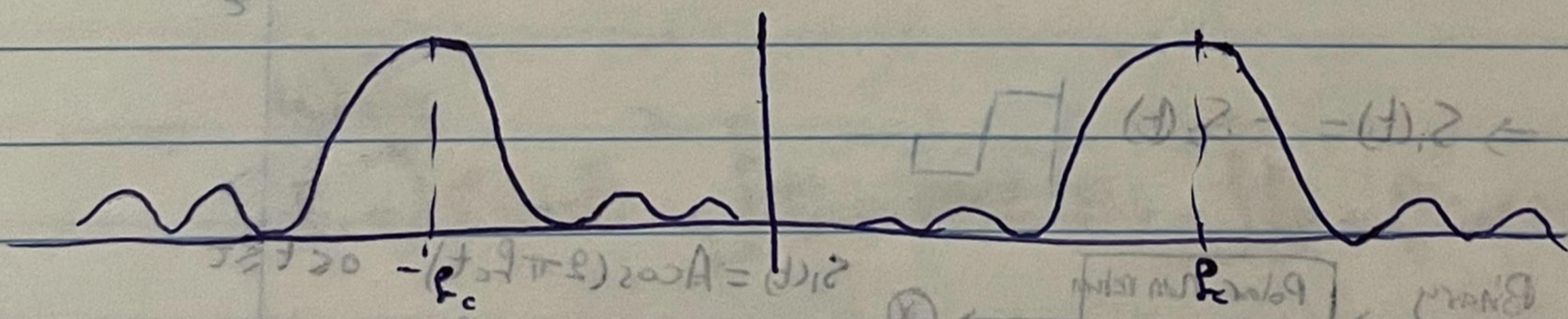
$$\lambda = \frac{1}{2}(E_1 - E_2) \Rightarrow \boxed{\lambda = 0}$$

- optimum.



- $P_b^* = Q\left(\sqrt{\frac{A^2 \tau}{N_0}}\right) \Rightarrow P_b^* = Q\sqrt{\frac{2E_b}{N_0}}$

- Power Spectral density



90% BW = $2R_b$

95% BW = $4R_b$

Example Binary Phase Shift Keying modulation scheme, employ the following two equally probable signals $S_1(t)$ and $s_2(t)$. to represent binary logic 1 and 0 respectively, over a channel corrupted by AWGN with power spectral density $\frac{N_o}{2} = 0.5 \times 10^{-4}$ W/Hz.

$$S_1(t) = \begin{cases} 2\cos(2000\pi t), & 0 \le t \le 0.001 \\ 0, & \text{o.w.} \end{cases}$$

1) Evaluate f_c: Carrier freq. $\Rightarrow 2\pi f_c = 2000\pi$

$$\boxed{f_c = 1000}$$

2) Evaluate the bit duration.

$$\Rightarrow \tau = 0.001$$

3) Evaluate the data rate.

$$R_b = \frac{1}{\tau} = \frac{1}{0.001} \Rightarrow \boxed{R_b = 1000}$$

4) Evaluate the avg energy per bit.

$$E_1 = \int_0^{\tau} S_1^2(t)\, dt = \int_0^{\tau} (2\cos(2000\pi t))^2\, dt$$

$$= \frac{(2)^2}{2}(0.001) \rightarrow E_1 = 0.002 = E_2$$

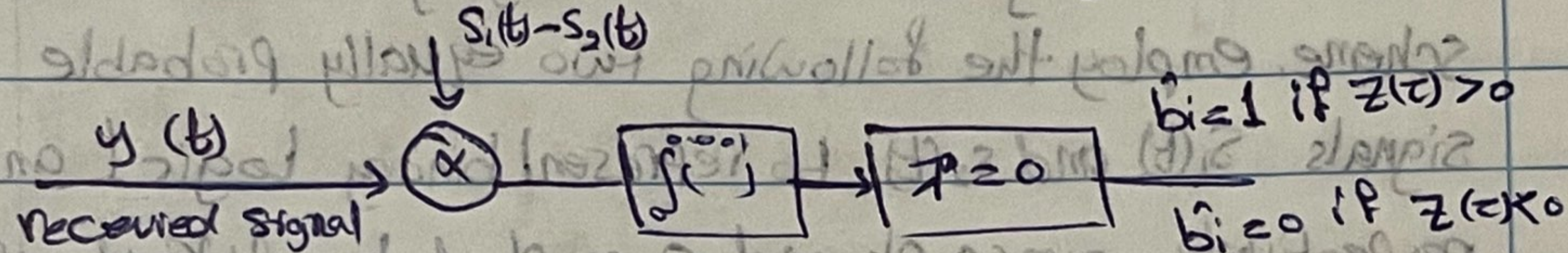
$$E_b = \frac{1}{2}(E_1 + E_2) \Rightarrow \boxed{E_b = 0.002}$$

5) Evaluate the optimum threshold.

$$\lambda^* = \frac{1}{2}(E_1 - E_2) \Rightarrow \lambda^* = 0$$

$\Rightarrow$

6) sketch the optimum receiver using ~~matched filter~~ Correlator.



7) Evaluate the minimum Prob. of error.

$$P_b^* = Q\left(\sqrt{\frac{\int_0^{0.001} 2 s_i^2(t)\, dt}{2N_0}}\right) = Q\left(\sqrt{\frac{2\int_0^{\tau} s_i^2(t)\, dt}{N_0}}\right)$$

$$P_b^* = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{(2)(0.002)}{N_0}}\right)$$

$$= Q\sqrt{\left(\frac{0.004}{10^{-4}}\right)} \approx Q(6.32).$$

Q-table بطلع قيمته من

* اذا ما كانت القيمة موجودة باخد القيمة الي اكبر منها والقيمة الي اقرب صفر وباخد الاقرب الهم