DPCM and Delta Modulation

Differential Pulse Code Modulation

- The quantizers, that we studied so far, are memoryless, in the sense that quantization is done on a sample-by-sample basis. Each sample is quantized and encoded into n binary digits, regardless of any correlation with other samples.

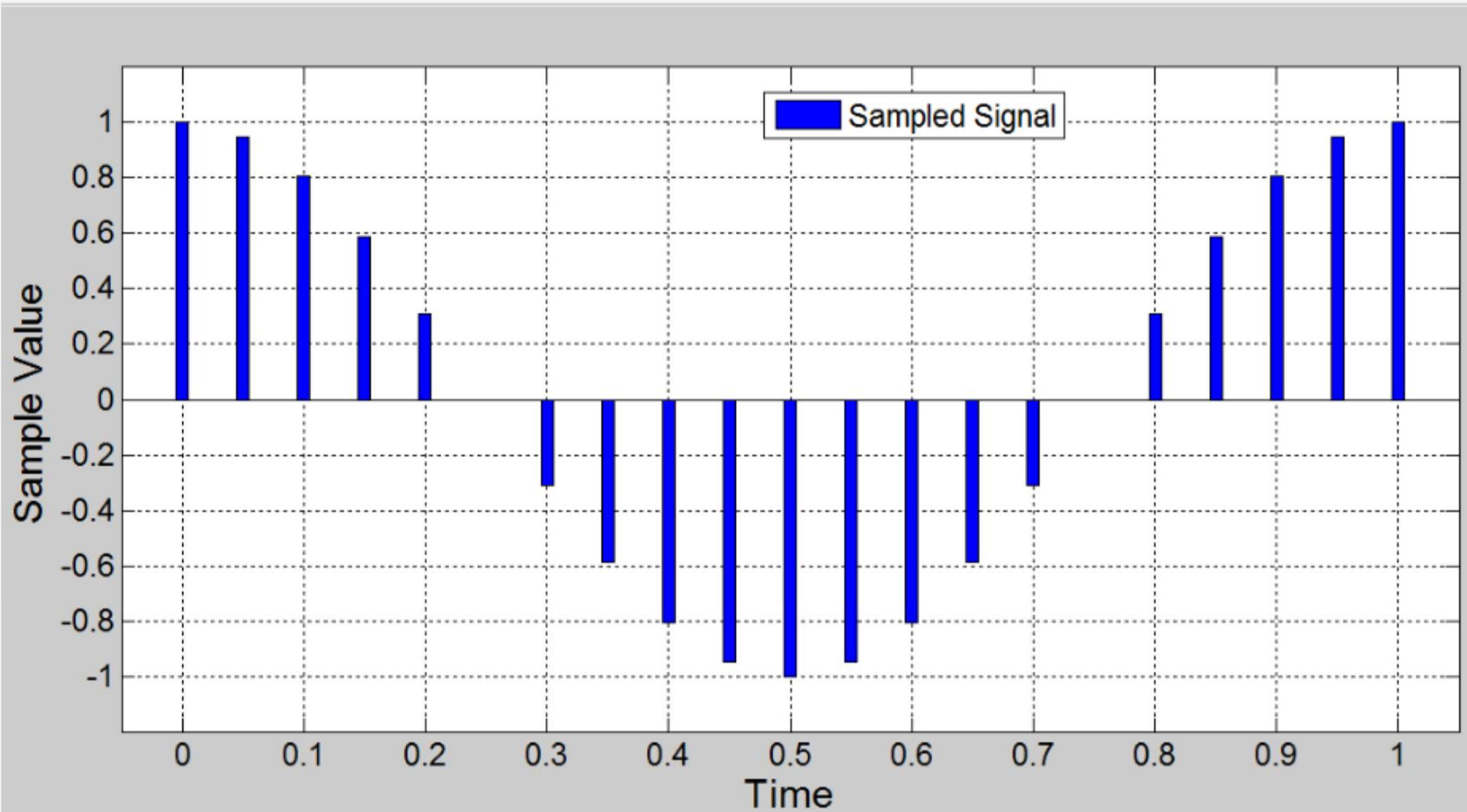
- A ***differential pulse-code modulation (DPCM)*** *quantizer* quantizes the difference between a sample and a predicted value of that sample. Here, correlation between successive samples is utilized.

- The prediction is based, in general, on past m samples of the signal. If successive samples are highly correlated, the predictor output will be very close to the next sample value, and hence the prediction error will be small.

- An error with a small variance further means that **fewer bits (r<n) are needed to represent the error**.

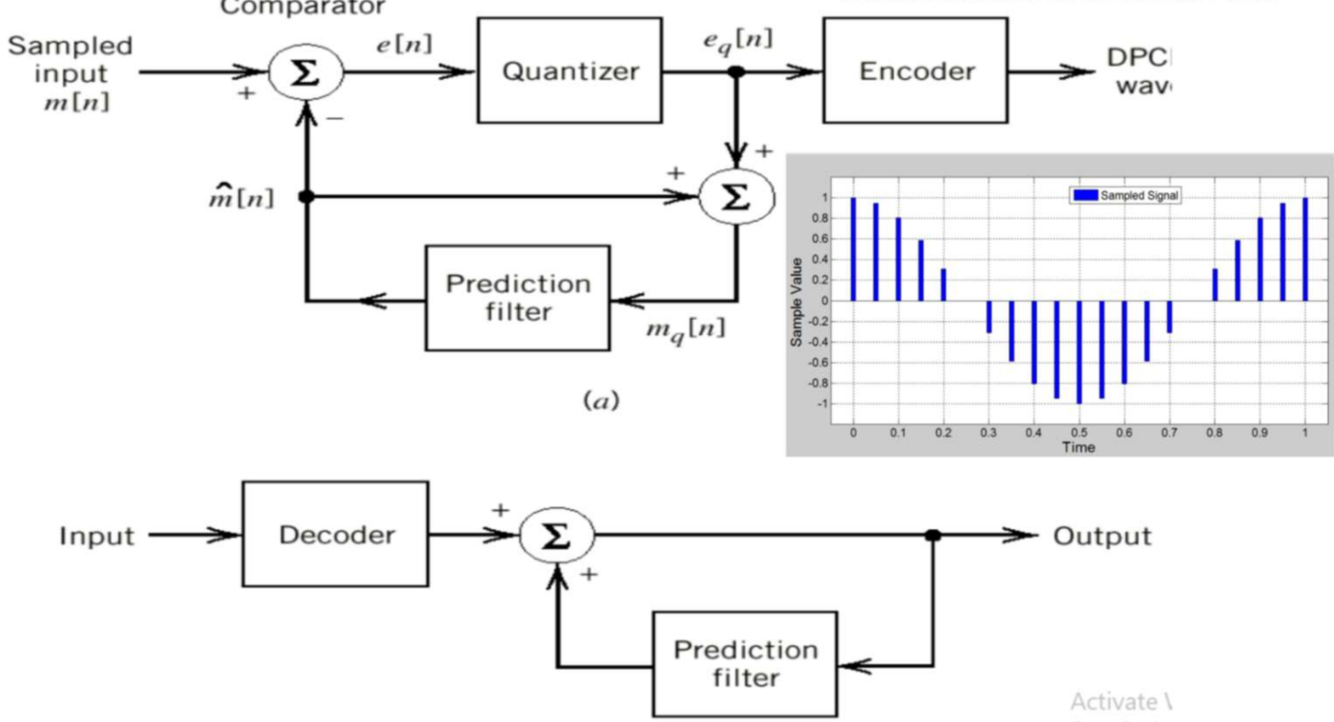
- At the receiver, a predictor similar to the one used at the transmitter is used to reconstruct the original waveform

Differential Pulse Code Modulation



DPCM: Basic Operation

Encode each difference into r bits



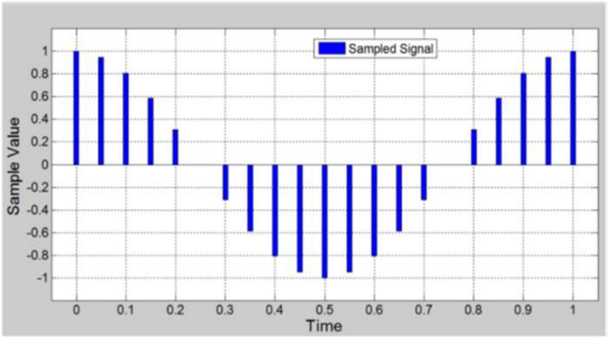
DPCM: Linear Prediction Filter

It is a discrete-time, finite-duration impulse response filter (FIR), which consists of three blocks:

1. Set of p (p: prediction order) unit-delay elements (z^{-1})

2. Set of multipliers with coefficients $w_1, w_2, \ldots w_p$

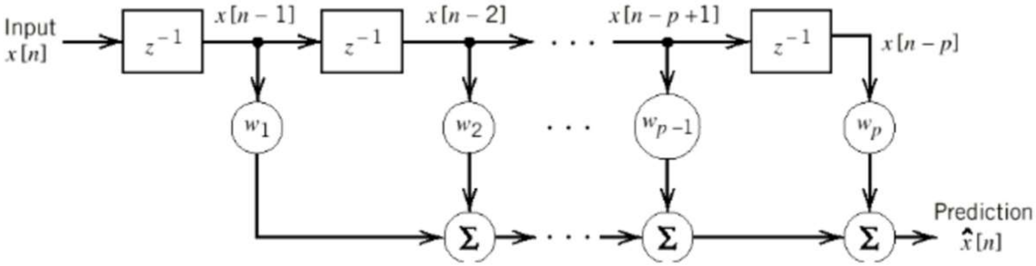
3. Set of adders ($\sum$)



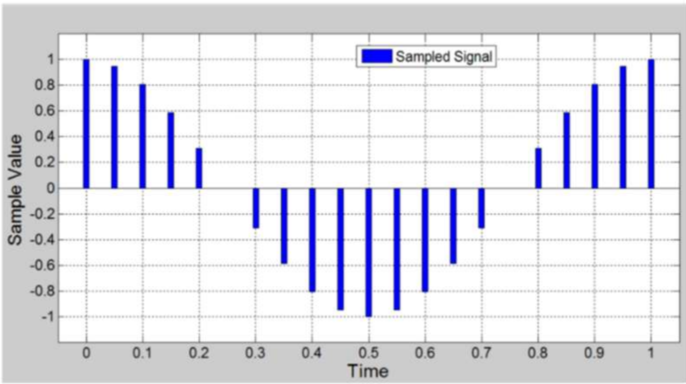
This filter expresses the predicted value of the sample at time (nT_s) as a linear combination of the past p samples of the signal.

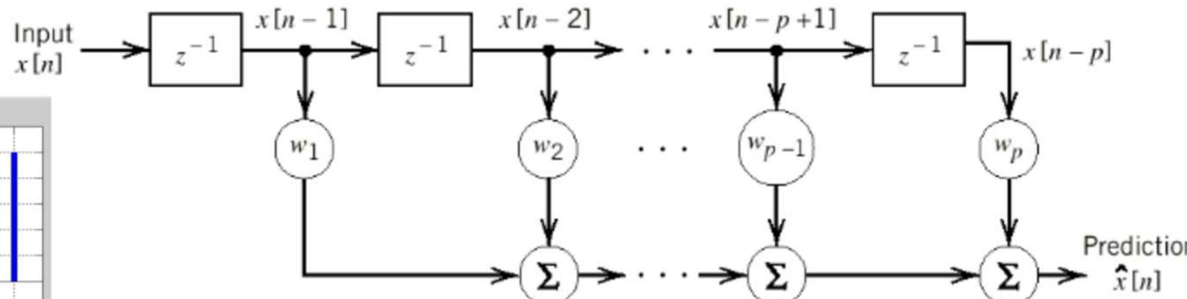
$$\hat{x}(n) = w_1 x(n-1) + w_2 x(n-2) + \cdots + w_p x(n-p)$$

The coefficients $w_1, w_2, \ldots, w_p$ are chosen so as to minimize the mean square error $E(x(n) - \hat{x}(n))^2$.



DPCM: Linear Prediction Filter





$$\epsilon = E((x(n) - \hat{x}(n))^2; \qquad \text{prediction error}$$

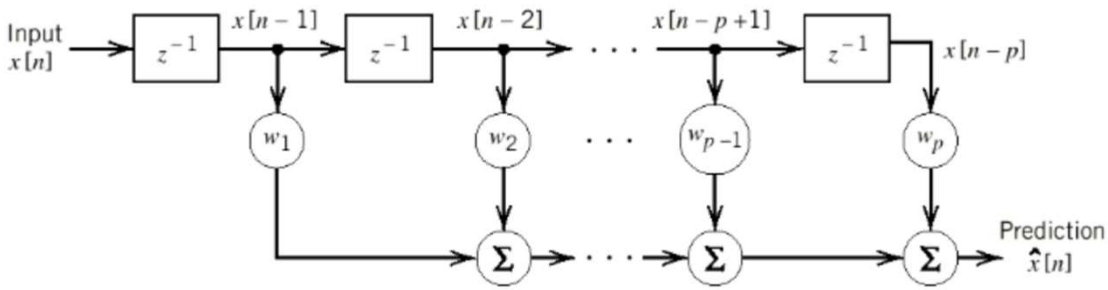
Substituting $\hat{x}(n) = w_1 x(n-1) + w_2 x(n-2) + \cdots + w_p x(n-p)$, the prediction error becomes:

$$\epsilon = E((x(n) - w_1 x(n-1) + w_2 x(n-2) + \cdots + w_p x(n-p))^2$$

Expanding ϵ and taking expectation of all terms, we get:

$$\epsilon = E(x(n)^2) - 2\sum_{i=1}^{p} w_i E[x(n)x(n-i)] + \sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j \; E[x(n-i)x(n-j)]$$

DPCM: Linear Prediction Filter



Recognize that: $R_x(i) = E[x(n)x(n-i)]$ is the autocorrelation function of $x(t)$.

Differentiating ϵ with respect to w_i, setting the derivative to zero, and solving, we get (assuming p=3)

$$\begin{bmatrix} R_x(0) & R_x(1) & R_x(2) \\ R_x(-1) & R_x(0) & R_x(1) \\ R_x(-2) & R_x(-1) & R_x(0) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} R_x(1) \\ R_x(2) \\ R_x(3) \end{bmatrix}$$

$$R(1) = R(T_s)$$
$$R(2) = R(2T_s)$$

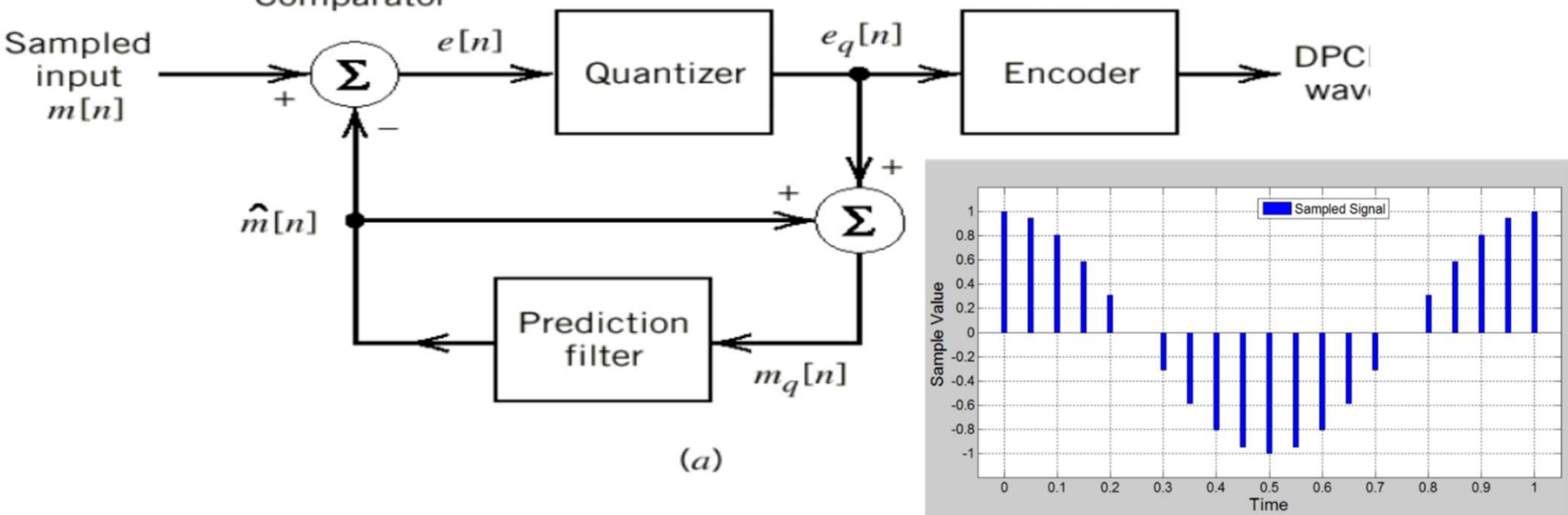
If p=1, the above equation reduces to

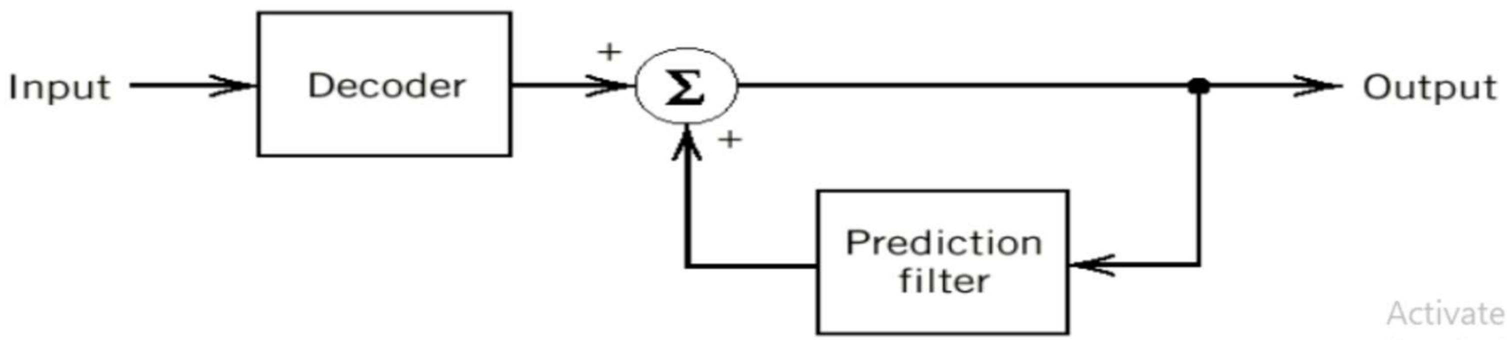
$$w_1 = R_x(1)/\, R_x(0)$$

Note that: $R_x(-1) = R_x(1)$, $R_x(-2) = R_x(2)$, $R_x(1) = R_x(T_s)$, $R_x(2) = R_x(2T_s)$, $R_x(3) = R_x(3T_s)$.

DPDM: Transmitter and Receiver

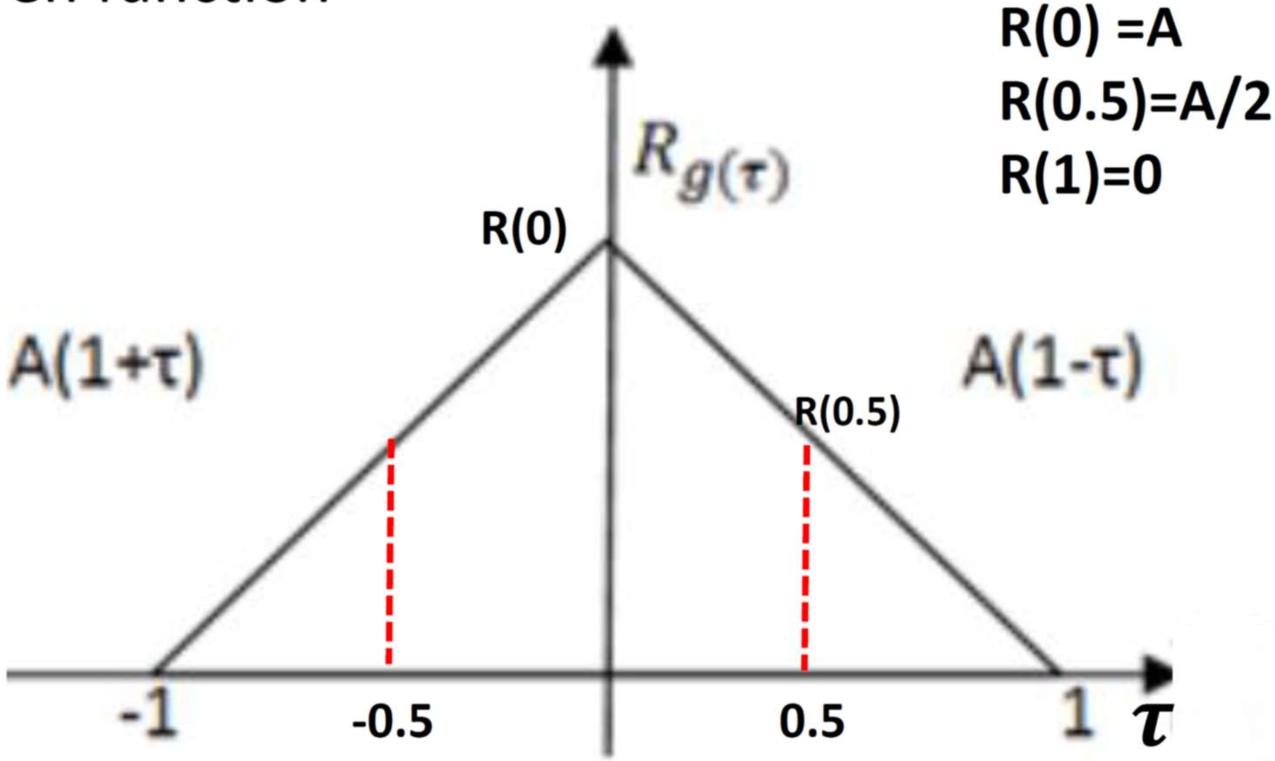
Encode each difference into r bits



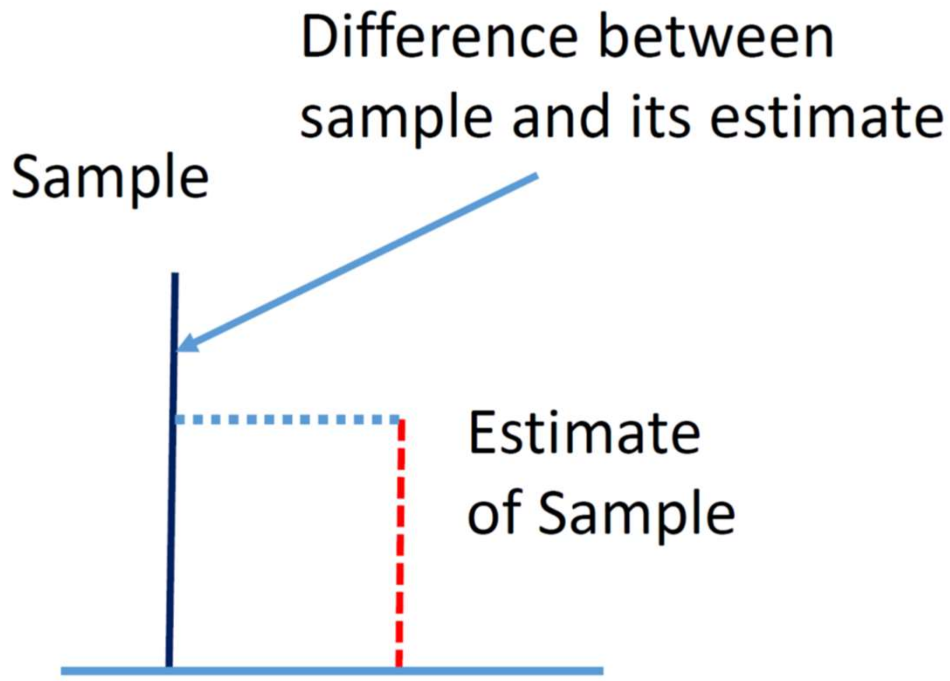


DPCM: Autocorrelation Function

R is an even function



DPCM: Concluding Summary

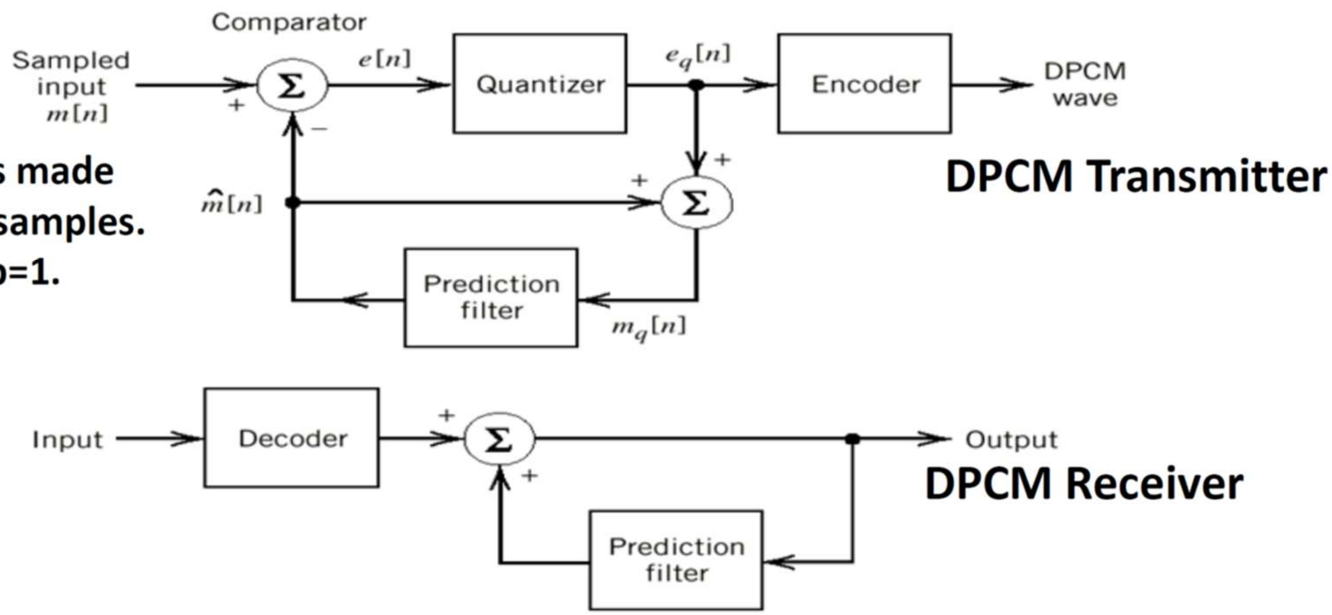


- **At transmitter**:
  - Samples are known
  - Estimate is known since estimate is a linear function of the samples.
- Transmit
- Difference= Sample – Estimate
- **At receiver**:
  - Receive Difference
  - Construct Estimate
- Sample = Estimate + Difference

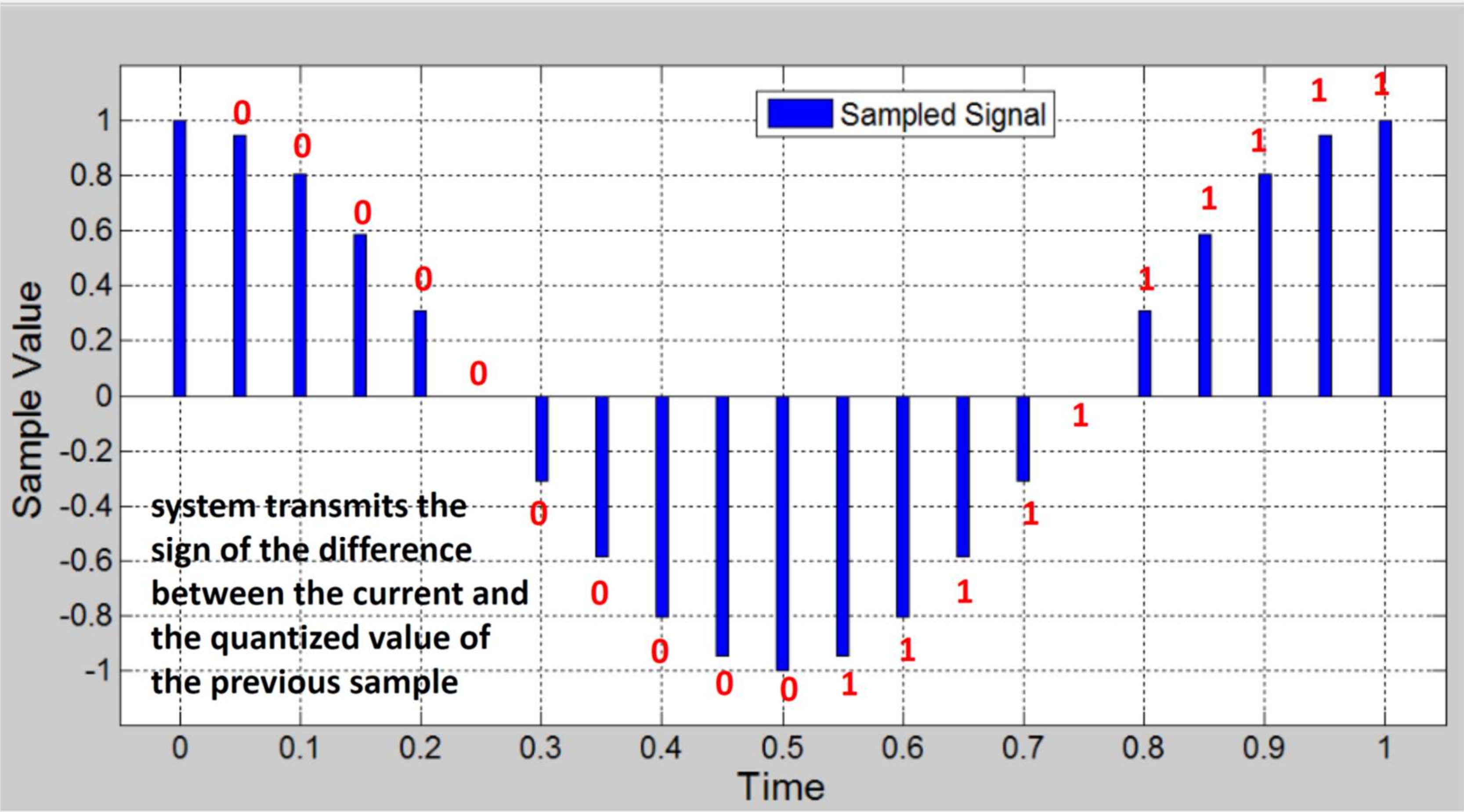
Delta Modulation

- **Remark:** Before you attend this lecture, please attend the previous one on DPCM.
- Delta modulation (DM), is a special case of Differential Pulse Code Modulation (DPCM).
- The order of the prediction filter in delta modulation is **p=1** and **represents only the quantized value of the previous sample**. The **number of quantization levels is two.**
- In this scheme, the system transmits the sign of the difference between the current sample and the quantized value of the previous sample. The **sign is represented by a single bit**.

Prediction in DPCM is made based on p previous samples. In delta modulation p=1.

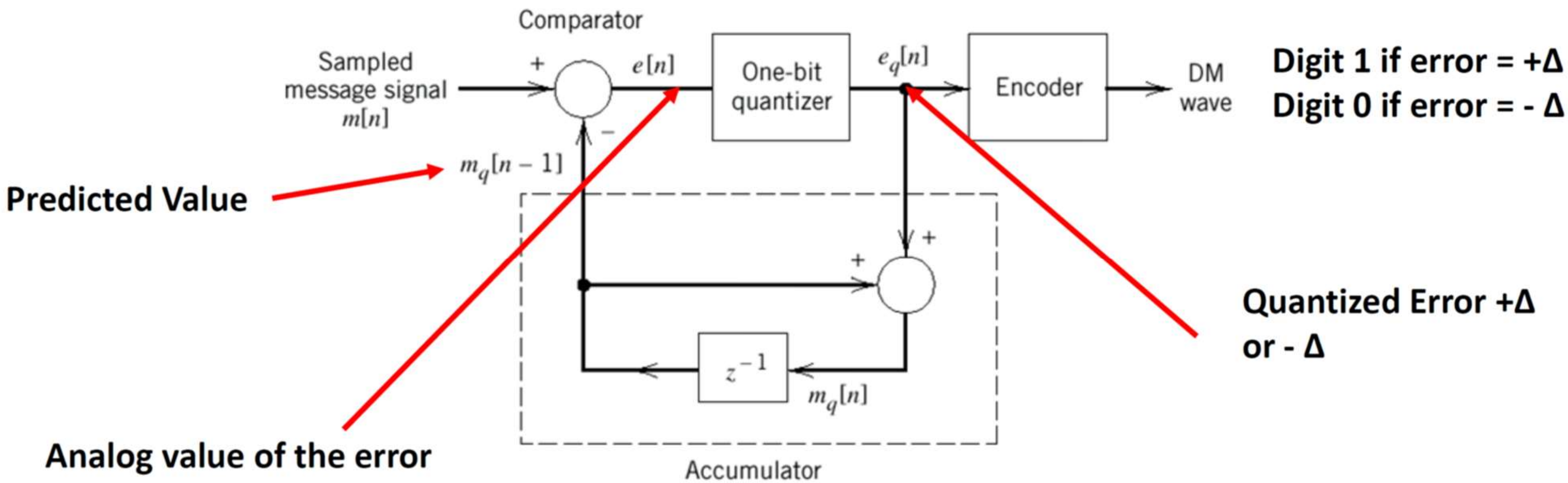


Delta Modulation: Basic Idea



Delta Modulation: The Transmitter Side

- The transmitter side consists of the comparator, the one bit quantizer, the encoder, and the accumulator.
- The accumulator (an integrator) adds the new quantized difference (**+Δ or -Δ**) to the old predicted value to generate the new predicted value.
- The output of the predictor is a staircase approximation of the message signal.



Delta Modulation: The Transmitter Side

Let $m[n] = m(nT_s)$, $n = 0, \pm 1, \pm 2, \ldots$

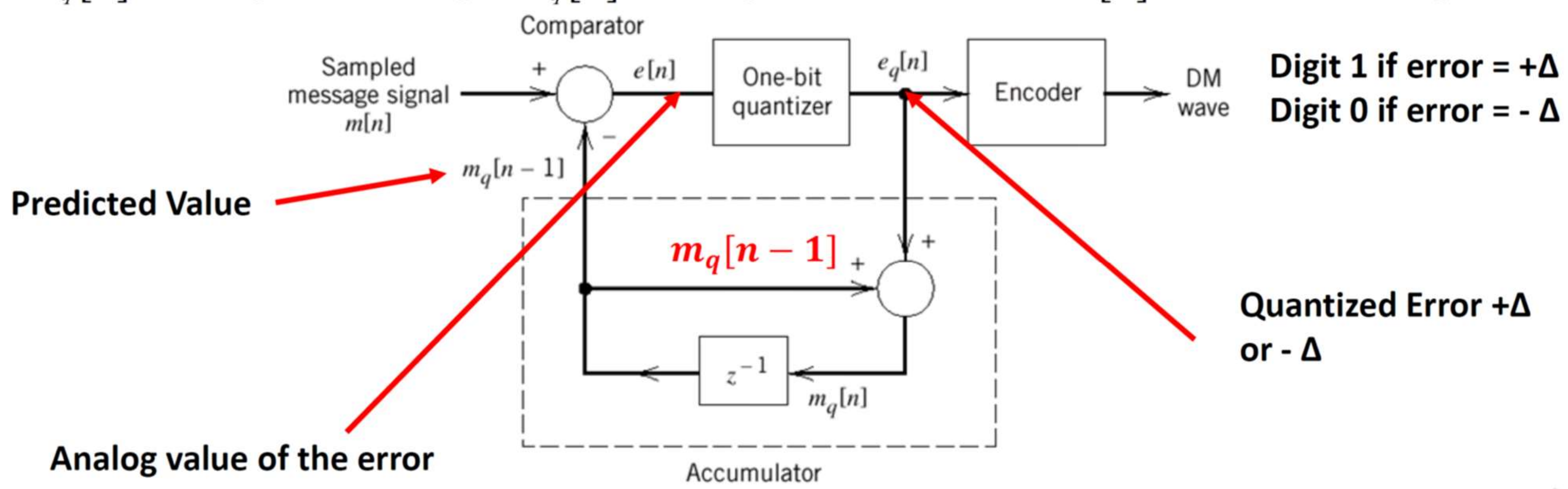
where T_s is the sampling period and $m(nT_s)$ is a sample of $m(t)$. The error signal is

$e[n] = m[n] - m_q[n-1]$

$e_q[n] = \Delta \operatorname{sgn}(e[n])$; quantized error

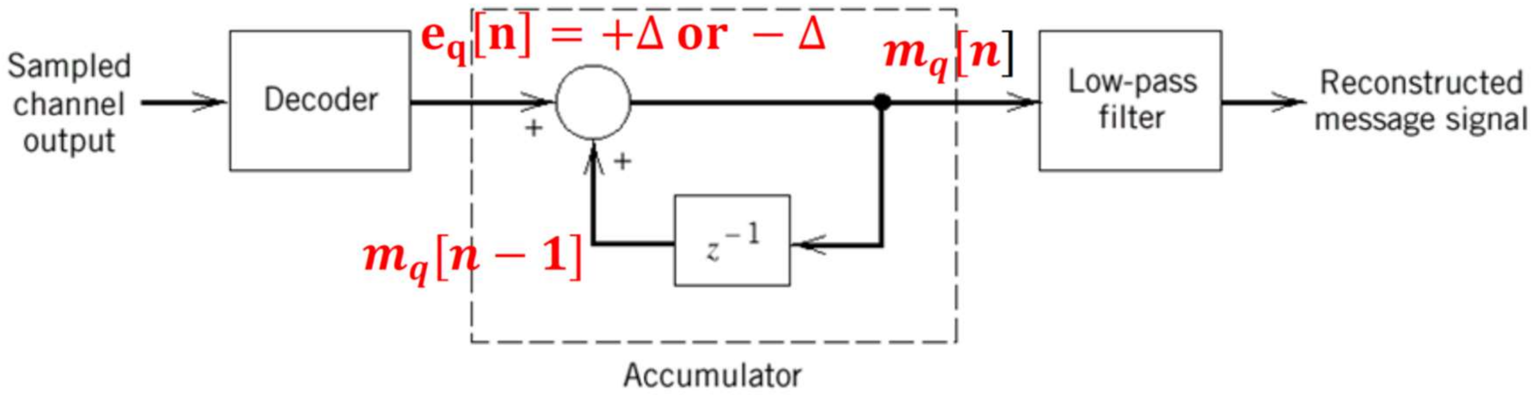
$m_q[n] = m_q[n-1] + e_q[n]$

where $m_q[n]$ is the quantizer output, $e_q[n]$ is the quantized version of $e[n]$, and Δ is the step size



Delta Modulation: The Receiver Part

- The receiver part consists of the decoder, the accumulator, and a low pass filter.
- The decoder interprets a zero as $-\Delta$ and one as $+\Delta$. These deltas represent the differences between current and previous samples.
- The accumulator regenerates the predicted staircase signal.
- The low pass filter smoothens the predicted signal by removing high frequency components.
- The reconstructed signal $m_q(t)$ is the same as the predicted signal used at the transmitter side

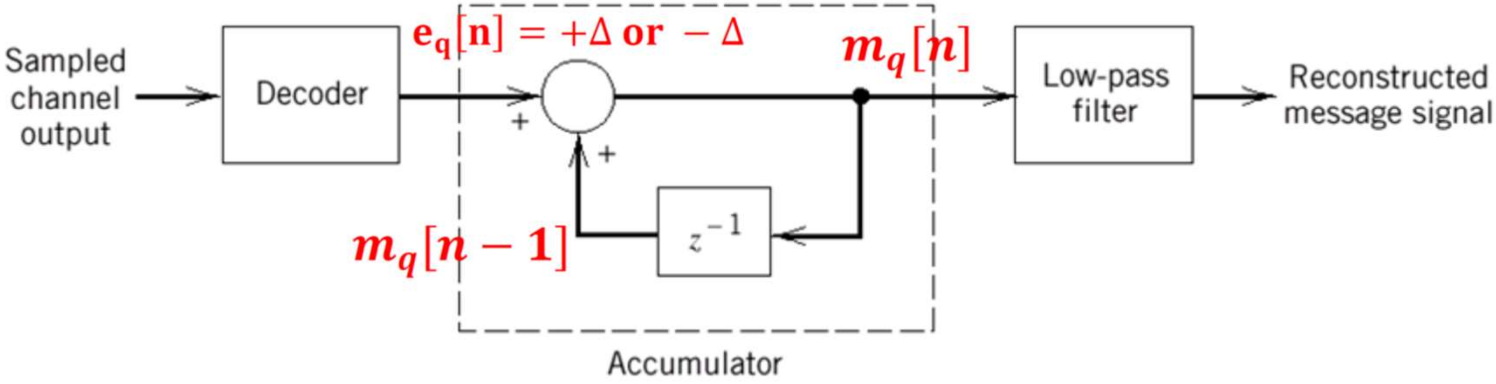


Delta Modulation: The Receiver Part

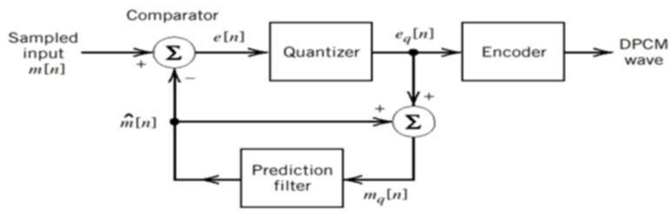
$$m_q[n] = m_q[n-1] + e_q[n]$$

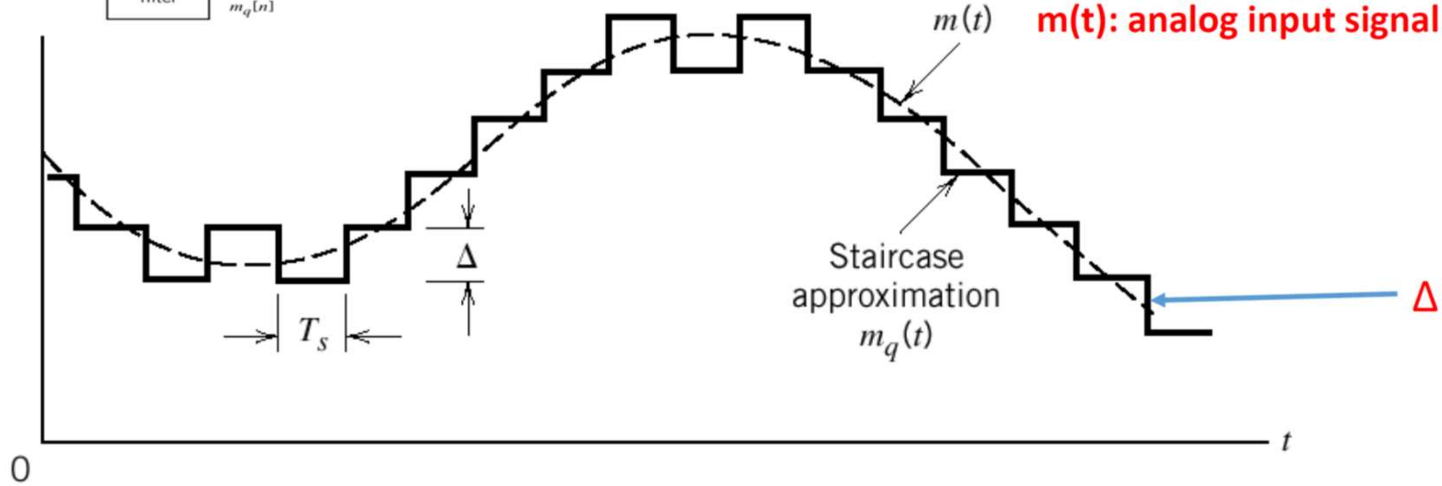
$$\Rightarrow m_q[n] = \Delta \sum_{i=1}^{n} \operatorname{sgn}(e[i])$$

$$= \sum_{i=1}^{n} e_q[i]$$



Delta Modulation: Basic Operation



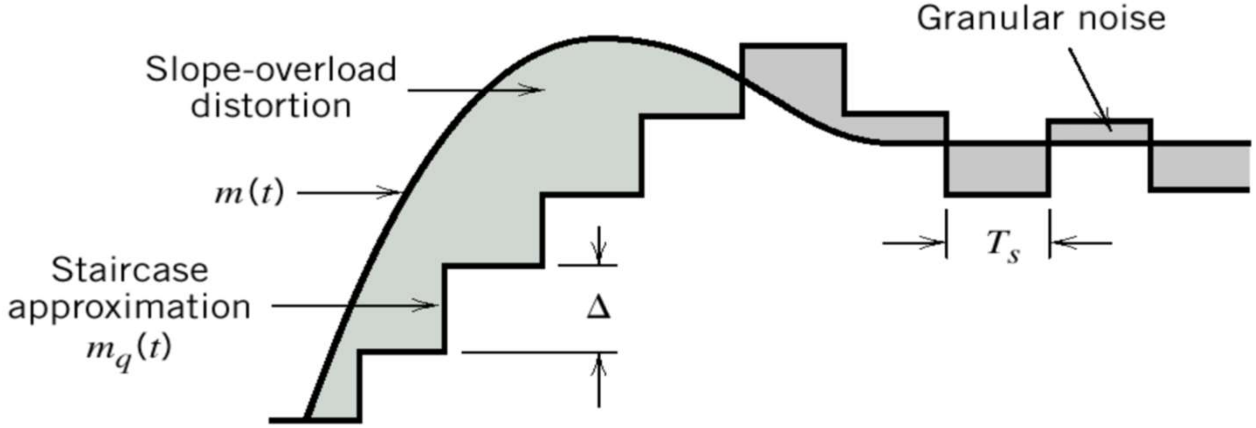


(a)

Binary sequence at modulator output: 0 0 1 0 1 1 1 1 1 0 1 0 0 0 0 0 0

Slope Overload Distortion and Granular Noise

- **Slope overload** distortion is due to the fact that the staircase approximation $m_q(t)$ can't follow closely the actual curve of the message signal $m(t)$. In contrast to slope-overload distortion, **granular noise** occurs when Δ is too large relative to the local slope characteristics of $m(t)$. granular noise is similar to quantization noise in PCM.
- It seems that a large Δ is needed for rapid variations of $m(t)$ to reduce the slope-overload distortion and a small Δ is needed for slowly varying $m(t)$ to reduce the granular noise. The optimum Δ can only be a compromise between the two cases.
- To satisfy both cases, an adaptive DM is needed, where the step size Δ can be adjusted in accordance with the input signal $m(t)$ (not to be covered in this lecture)



Slope Overload

Slope overload occurs when the signal changes at a rate faster than that of the predicted signal.
To avoid slope overload, we must have

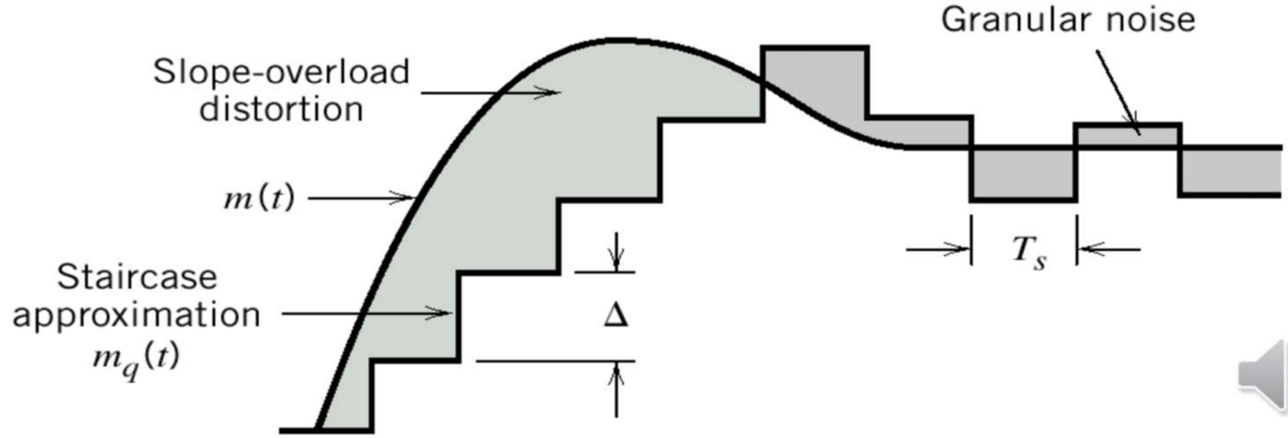
$$\frac{\Delta}{T_s} \geq max\left|\frac{dm(t)}{dt}\right|$$

When $\mathrm{m(t)} = A_m \cos(2\pi f_m t)$, the condition for avoiding slope overload becomes

$$\frac{\Delta}{T_s} \geq 2\pi A_m f_m \text{ ; OR } \quad \Delta \geq 2\pi T_s A_m f_m$$

As we can see, slope overload depends on three factors:

- Sampling frequency (larger sampling, reduces the effect)
- Message amplitude (larger amplitude, increases the effect)
- Message frequency (larger message frequency, increases the effect

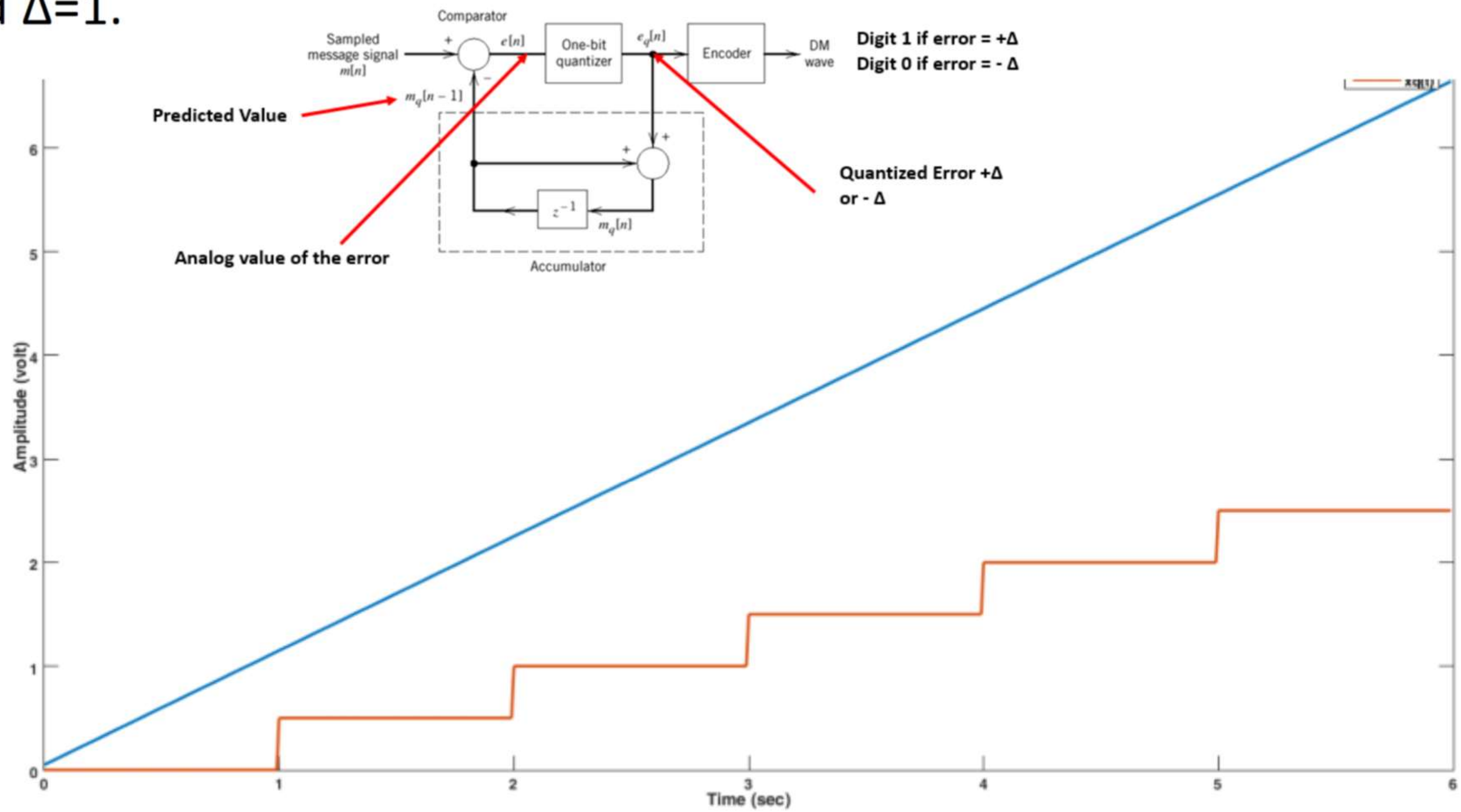


Adaptive Delta Modulation

- The step size in delta modulation affects the quality of the transmitted waveform (slope overload or granular noise).
- A larger step-size is needed in the steep slope of modulating signal
- a smaller step size is needed where the message has a small slope
- In adaptive delta modulation, the step size is adjusted via a feedback control signal so as to reduce both slope overload and granular noise effects.
- ADM quantizes the difference between the value of the current sample and the predicted value of the next sample. It uses a variable step height to predict the next values, for the faithful reproduction of the fast varying values.

Delta Modulation: Example

- Draw the output of the DM given that the input corresponds to $x(t) = 1.1t + 0.05$ when the input is sampled at t = 0, 1, 2, 3, 4, 5, … and Δ=1.



Delta Demodulation: Example

- Reconstruct a staircase signal at the receiver side of a delta demodulator with $\Delta = 0.1V$, when the received data sequence is 1 1 1 1 0 0 1 1 1 1 0 1 0 1 1 1.