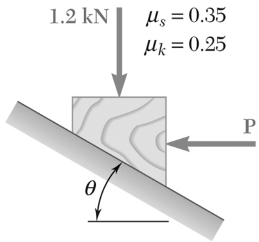
CHAPTER 8

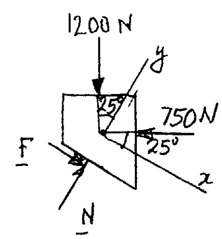
PROBLEM 8.1



Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25°$ and $P = 750$ N.

SOLUTION

<u>Assume equilibrium</u>:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (1200\text{ N})\sin 25° - (750\text{ N})\cos 25° = 0$$

$$F = +172.6\text{ N} \qquad \mathbf{F} = 172.6\text{ N} \searrow$$

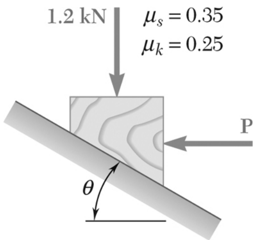
$$+\nearrow \Sigma F_y = 0: \quad N - (1200\text{ N})\cos 25° - (750\text{ N})\sin 25° = 0$$

$$N = 1404.5\text{ N}$$

Maximum friction force: $F_m = \mu_s N = 0.35(1404.5\text{ N}) = 491.6\text{ N}$

Since $F < F_m$, <u>block is in equilibrium</u> ◀

Friction force: $\mathbf{F} = 172.6\text{ N} \searrow 25.0°$ ◀

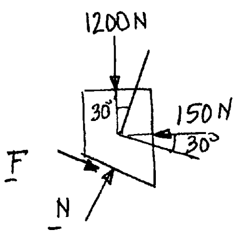


PROBLEM 8.2

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 30°$ and $P = 150$ N.

SOLUTION

Assume equilibrium:



$\searrow^{+} \Sigma F_x = 0$: $F + (1200 \text{ N})\sin 30° - (150 \text{ N})\cos 30° = 0$

$F = -470.1$ N $\qquad$ $\mathbf{F} = 470.1 \text{ N} \nwarrow$

$+\nearrow \Sigma F_y = 0$: $N - (1200 \text{ N})\cos 30° - (150 \text{ N})\sin 30° = 0$

$N = 1114.2$ N

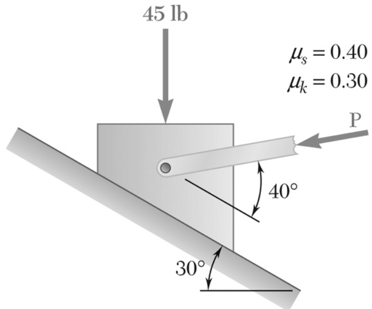
(a) Maximum friction force:

$$F_m = \mu_s N$$
$$= 0.35(1114.2 \text{ N})$$
$$= 390.0 \text{ N}$$

Since **F** is $\nwarrow$ and $F > F_m$, $\qquad$ block moves down ◀

(*b*) Actual friction force: $F = F_k = \mu_k N = 0.25(1114.2 \text{ N}) = 279$ N $\qquad$ $\mathbf{F} = 279 \text{ N} \nwarrow 30.0°$ ◀

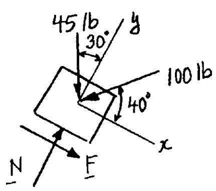
PROBLEM 8.3



Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 100$ lb.

SOLUTION

<u>Assume equilibrium</u>:



$$\searrow^{+} \Sigma F_x = 0: \quad F + (45 \text{ lb})\sin 30° - (100 \text{ lb})\cos 40° = 0$$

$$F = +54.0 \text{ lb}$$

$$+\nearrow \Sigma F_y = 0: \quad N - (45 \text{ lb})\cos 30° - (100 \text{ lb})\sin 40° = 0$$

$$N = 103.2 \text{ lb}$$

(*a*) Maximum friction force:

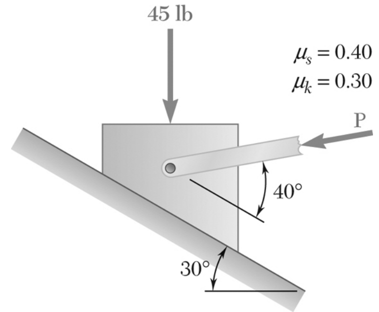
$$\begin{aligned} F_m &= \mu_s N \\ &= 0.40(103.2 \text{ lb}) \\ &= 41.30 \text{ lb} \end{aligned}$$

We note that $F > F_m$. Thus, <u>block moves up</u> ◀

(*b*) Actual friction force:

$F = F_k = \mu_k N = 0.30(103.2 \text{ lb}) = 30.97$ lb, **F** $= 31.0$ lb $\searrow$ 30.0° ◀

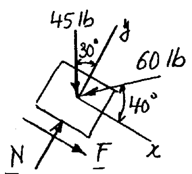
PROBLEM 8.4



Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 60$ lb.

SOLUTION

<u>Assume equilibrium</u>:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (45 \text{ lb})\sin 30° - (60 \text{ lb})\cos 40° = 0$$

$$F = +23.46 \text{ lb}$$

$$+\nearrow \Sigma F_y = 0: \quad N - (45 \text{ lb})\cos 30° - (60 \text{ lb})\sin 40° = 0$$

$$N = 77.54 \text{ lb}$$

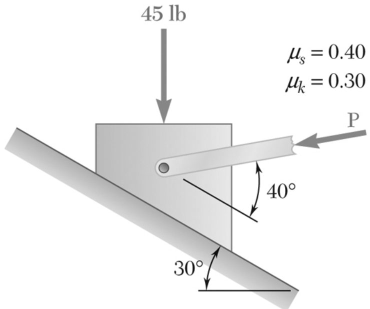
(*a*) Maximum friction force:

$$\begin{aligned} F_m &= \mu_s N \\ &= 0.40(77.54 \text{ lb}) \\ &= 31.02 \text{ lb} \end{aligned}$$

We check that $F < F_m$. Thus, <u>block is in equilibrium</u> ◀

(*b*) Thus $F = +23.46$ lb **F** = 23.5 lb ⤡ 30.0° ◀

Note: We have $F_k = \mu_k N = 0.30(77.54) = 23.26$ lb. Thus $F > F_k$. If block originally in motion, it will keep moving with $F_k = 23.26$ lb.



PROBLEM 8.5

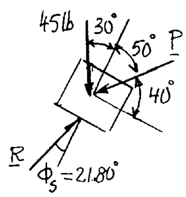
Determine the smallest value of P required to (*a*) start the block up the incline, (*b*) keep it moving up, (*c*) prevent it from moving down.

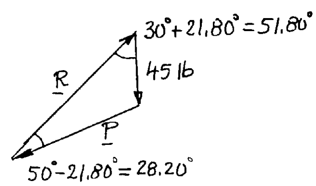
SOLUTION

(*a*) <u>To start block up the incline</u>:

$$\mu_s = 0.40$$

$$\phi_s = \tan^{-1} 0.40 = 21.80°$$





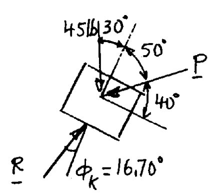
From force triangle:

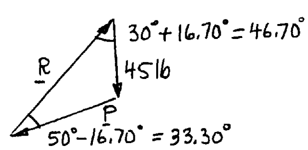
$$\frac{P}{\sin 51.80°} = \frac{45 \text{ lb}}{\sin 28.20°} \qquad P = 74.8 \text{ lb} ◀$$

(*b*) <u>To keep block moving up:</u>

$$\mu_k = 0.30$$

$$\phi_k = \tan^{-1} 0.30 = 16.70°$$

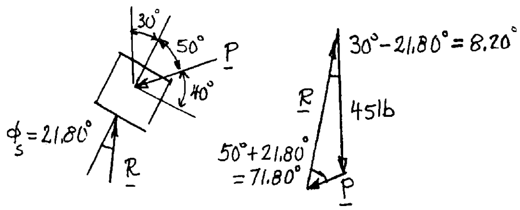




From force triangle:

$$\frac{P}{\sin 46.70°} = \frac{45 \text{ lb}}{\sin 33.30°} \qquad P = 59.7 \text{ lb} ◀$$

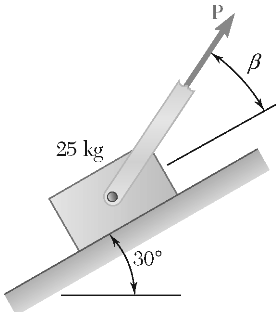
(*c*) <u>To prevent block from moving down</u>:



From force triangle:

$$\frac{P}{\sin 8.20°} = \frac{45 \text{ lb}}{\sin 71.80°}$$

$P = 6.76$ lb ◀

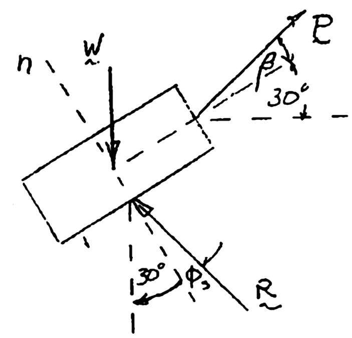


PROBLEM 8.6

Knowing that the coefficient of friction between the 25-kg block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of *P* required to start the block moving up the incline, (*b*) the corresponding value of β.

SOLUTION

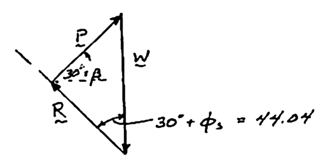
FBD block (Impending motion up)



$$W = mg$$
$$= (25\text{ kg})(9.81\text{ m/s}^2)$$
$$= 245.25\text{ N}$$

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.04°$$

(*a*) (*Note:* For minimum *P*, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$.)

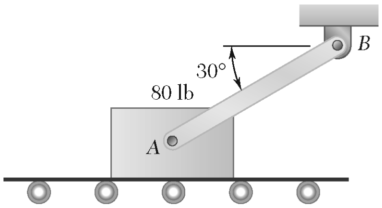


Then

$$P = W\sin(30° + \phi_s)$$
$$= (245.25\text{ N})\sin 44.04°$$

$P_{\min} = 170.5\text{ N}$ ◀

(*b*) We have $\beta = \phi_s$ $\beta = 14.04°$ ◀



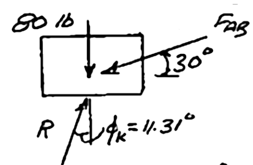
PROBLEM 8.7

The 80-lb block is attached to link *AB* and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force **P** that should be applied to the belt to maintain its motion (*a*) to the right, (*b*) to the left.

SOLUTION

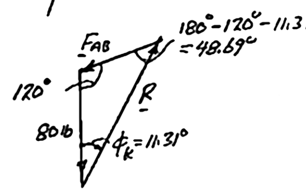
We note that link *AB* is a two-force member, since there is motion between belt and block $\mu_k = 0.20$ and $\phi_k = \tan^{-1} 0.20 = 11.31°$

(*a*) Belt moves to right



Free body: Block

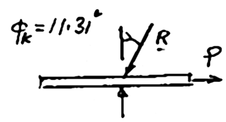
Force triangle:



$$\frac{R}{\sin 120°} = \frac{80 \text{ lb}}{\sin 48.69°}$$

$$R = 92.23 \text{ lb}$$

Free body: Belt

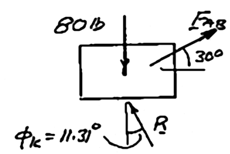


$$\xrightarrow{+} \Sigma F_x = 0: \quad P - (92.23 \text{ lb})\sin 11.31°$$

$$P = 18.089 \text{ lb}$$

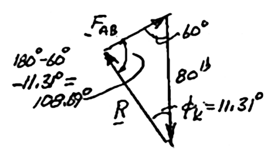
$\mathbf{P} = 18.09 \text{ lb} \longrightarrow$ ◀

(*b*) Belt moves to left



Free body: Block

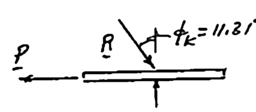
Force triangle:



$$\frac{R}{\sin 60°} = \frac{80 \text{ lb}}{\sin 108.69°}$$

$$R = 73.139 \text{ lb}$$

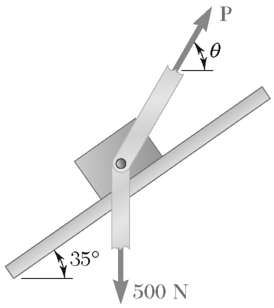
Free body: Belt



$$\xrightarrow{+} \Sigma F_x = 0: \quad (73.139 \text{ lb})\sin 11.31° - P = 0$$

$$P = 14.344 \text{ lb}$$

$\mathbf{P} = 14.34 \text{ lb} \longleftarrow$ ◀



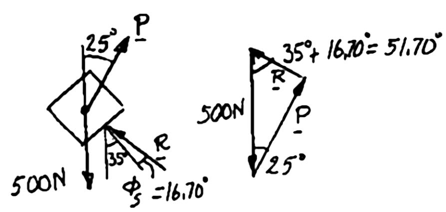
PROBLEM 8.8

The coefficients of friction between the block and the rail are $\mu_s = 0.30$ and $\mu_k = 0.25$. Knowing that $\theta = 65°$, determine the smallest value of P required (*a*) to start the block moving up the rail, (*b*) to keep it from moving down.

SOLUTION

(*a*) To start block up the rail:

$$\mu_s = 0.30$$
$$\phi_s = \tan^{-1} 0.30 = 16.70°$$

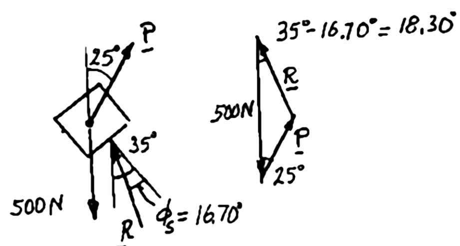


Force triangle:

$$\frac{P}{\sin 51.70°} = \frac{500 \text{ N}}{\sin (180° - 25° - 51.70°)}$$

$P = 403$ N ◀

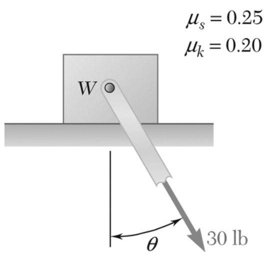
(*b*) To prevent block from moving down:



Force triangle:

$$\frac{P}{\sin 18.30°} = \frac{500 \text{ N}}{\sin (180° - 25° - 18.30°)}$$

$P = 229$ N ◀

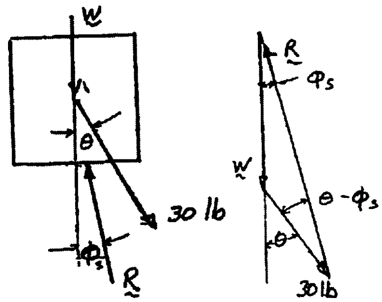


PROBLEM 8.9

Considering only values of θ less than 90°, determine the smallest value of θ required to start the block moving to the right when (*a*) $W = 75$ lb, (*b*) $W = 100$ lb.

SOLUTION

FBD block (Motion impending):



$$\phi_s = \tan^{-1}\mu_s = 14.036°$$

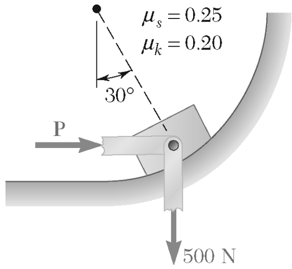
$$\frac{30\text{ lb}}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W \sin 14.036°}{30\text{ lb}}$$

or
$$\sin(\theta - 14.036°) = \frac{W}{123.695\text{ lb}}$$

(*a*) $W = 75$ lb: $\quad \theta = 14.036° + \sin^{-1}\dfrac{75\text{ lb}}{123.695\text{ lb}}$ $\qquad \theta = 51.4°$ ◀

(*b*) $W = 100$ lb: $\quad \theta = 14.036° + \sin^{-1}\dfrac{100\text{ lb}}{123.695\text{ lb}}$ $\qquad \theta = 68.0°$ ◀



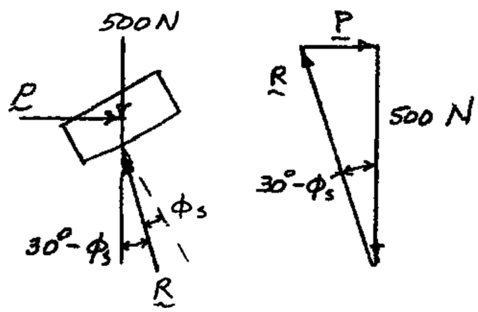
PROBLEM 8.10

Determine the range of values of P for which equilibrium of the block shown is maintained.

SOLUTION

FBD block:

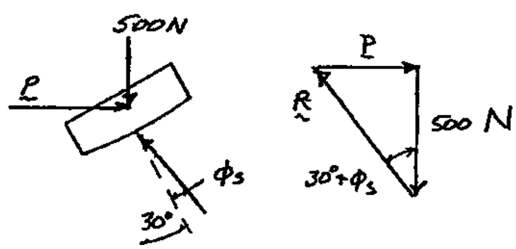
(Impending motion down):



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25$$

$$P = (500 \text{ N})\tan(30° - \tan^{-1} 0.25)$$
$$= 143.03 \text{ N}$$

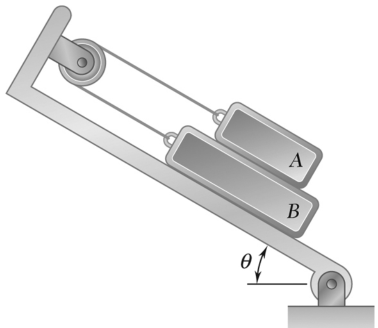
(Impending motion up):



$$P = (500 \text{ N})\tan(30° + \tan^{-1} 0.25)$$
$$= 483.46 \text{ N}$$

Equilibrium is maintained for $143.0 \text{ N} \le P \le 483 \text{ N}$ ◀

PROBLEM 8.11

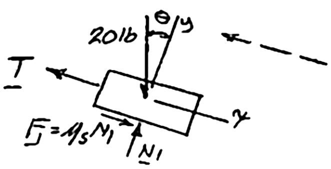


The 20-lb block A and the 30-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between the two blocks and zero between block B and the incline, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ between $A + B$

<u>Free body: Block *A*</u>



Impending motion:

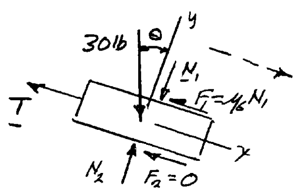
$$\Sigma F_y = 0: \quad N_1 = 20\cos\theta$$

$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 20\sin\theta - \mu_s N_1 = 0$$

$$T = 20\sin\theta + 0.15(20\cos\theta)$$

$$T = 20\sin\theta + 3\cos\theta \qquad (1)$$

<u>Free body: Block *B*</u>



Impending motion:

$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 30\sin\theta + \mu_s N_1 = 0$$

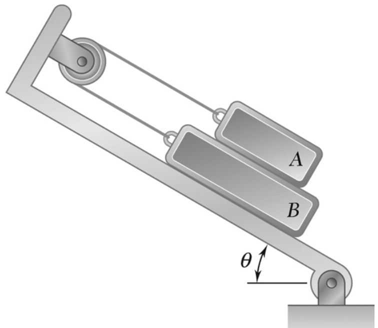
$$T = 30\sin\theta - \mu_s N_1$$

$$= 30\sin\theta - 0.15(20\cos\theta)$$

$$T = 30\sin\theta - 3\cos\theta \qquad (2)$$

Eq. (1)-Eq. (2): $\quad 20\sin\theta + 3\cos\theta - 30\sin\theta + 3\cos\theta = 0$

$$6\cos\theta = 10\sin\theta: \quad \tan\theta = \frac{6}{10}; \quad \theta = 30.96° \qquad \theta = 31.0° ◀$$



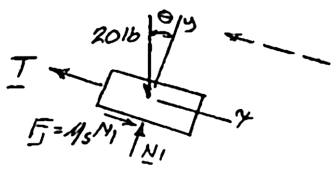
PROBLEM 8.12

The 20-lb block *A* and the 30-lb block *B* are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ at all surfaces.

Free body: Block *A*



Impending motion:

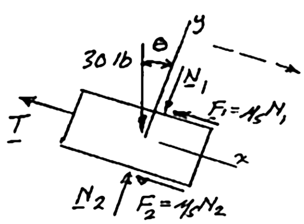
$$\Sigma F_y = 0: \quad N_1 = 20\cos\theta$$

$$+\nwarrow \Sigma F_x = 0: \quad T - 20\sin\theta - \mu_s N_1 = 0$$

$$T = 20\sin\theta + 0.15(20\cos\theta)$$

$$T = 20\sin\theta + 3\cos\theta \qquad (1)$$

Free body: Block *B*



Impending motion:

$$+\nearrow \Sigma F_y = 0: \quad N_2 - 30\cos\theta - N_1 = 0$$

$$N_2 = 30\cos\theta + 20\cos\theta = 50\cos\theta$$

$$F_2 = \mu_s N_2 = 0.15(50\cos\theta) = 6\cos\theta$$

$$+\nwarrow \Sigma F_x = 0: \quad T - 30\sin\theta + \mu_s N_1 + \mu_s N_2 = 0$$

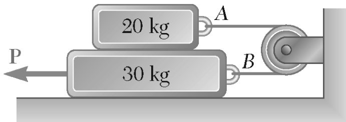
$$T = 30\sin\theta - 0.15(20\cos\theta) - 0.15(50\cos\theta)$$

$$T = 30\sin\theta - 3\cos\theta - 7.5\cos\theta \qquad (2)$$

Eq. (1) subtracted by Eq. (2): $\quad 20\sin\theta + 3\cos\theta - 30\sin\theta + 3\cos\theta + 7.5\cos\theta = 0$

$$13.5\cos\theta = 10\sin\theta, \quad \tan\theta = \frac{13.5°}{10}$$

$\theta = 53.5°$ ◀

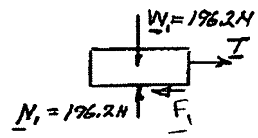


PROBLEM 8.13

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (a) is attached as shown, (b) is removed.

SOLUTION

(a) <u>Free body: 20-kg block</u>

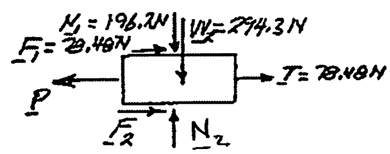


$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

<u>Free body: 30-kg block</u>



$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

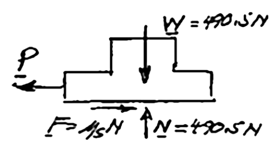
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 - T = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} + 78.48 \text{ N} = 353.2 \text{ N}$$

$\mathbf{P} = 353 \text{ N} \longleftarrow$ ◀

(b) <u>Free body: Both blocks</u>

<u>Blocks move together</u>



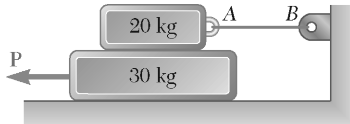
$$W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$\mathbf{P} = 196.2 \text{ N} \longleftarrow$ ◀

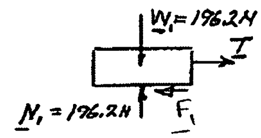
PROBLEM 8.14



The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (a) is attached as shown, (b) is removed.

SOLUTION

(a) Free body: 20-kg block

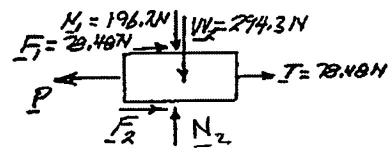


$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

Free body: 30-kg block



$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

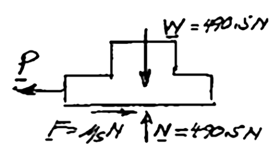
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} = 274.7 \text{ N}$$

$\mathbf{P} = 275 \text{ N} \longleftarrow$ ◀

(b) Free body: Both blocks

Blocks move together

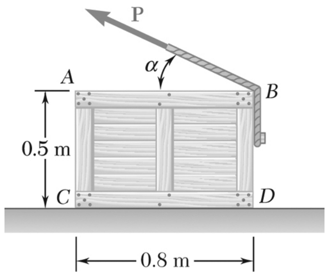


$$W = (50 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 490.5 \text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$\mathbf{P} = 196.2 \text{ N} \longleftarrow$ ◀



PROBLEM 8.15

A 40-kg packing crate must be moved to the left along the floor without tipping. Knowing that the coefficient of static friction between the crate and the floor is 0.35, determine (*a*) the largest allowable value of α, (*b*) the corresponding magnitude of the force **P**.

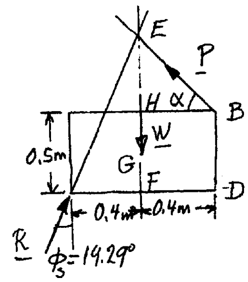
SOLUTION

(*a*) Free-body diagram

If the crate is about to tip about *C*, contact between crate and ground is only at *C* and the reaction **R** is applied at *C*. As the crate is about to slide, **R** must form with the vertical an angle

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}$$

$$0.35 = 19.29°$$



Since the crate is a 3-force body. **P** must pass through *E* where **R** and **W** intersect.

$$EF = \frac{CF}{\tan\theta_s} = \frac{0.4\text{ m}}{0.35} = 1.1429\text{ m}$$

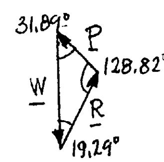
$$EH = EF - HF = 1.1429 - 0.5 = 0.6429\text{ m}$$

$$\tan\alpha = \frac{EH}{HB} = \frac{0.6429\text{ m}}{0.4\text{ m}}$$

$$\alpha = 58.11°$$

$\alpha = 58.1°$ ◀

(*b*) Force Triangle



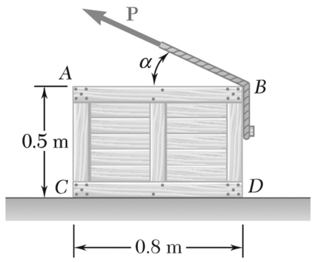
$$\frac{P}{\sin 19.29°} = \frac{W}{\sin 128.82°} \qquad P = 0.424\,W$$

$$P = 0.424(40\text{ kg})(9.81\text{ m/s}^2),$$

$P = 166.4$ N ◀

Note: After the crate starts moving, μ_s should be replaced by the lower value μ_k. This will yield a larger value of α.

PROBLEM 8.16

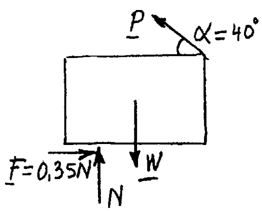


A 40-kg packing crate is pulled by a rope as shown. The coefficient of static friction between the crate and the floor is 0.35. If $\alpha = 40°$, determine (*a*) the magnitude of the force **P** required to move the crate, (*b*) whether the crate will slide or tip.

SOLUTION

Force P for which sliding is impending

(We assume that crate does not tip)



$$W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad N - W + P\sin 40° = 0$$

$$N = W - P\sin 40° \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 0.35\,N - P\cos 40° = 0$$

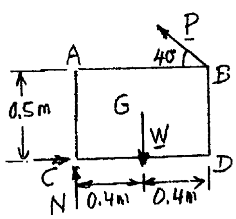
Substitute for N from Eq. (1):

$$0.35(W - P\sin 40°) - P\cos 40° = 0$$

$$P = \frac{0.35W}{0.35\sin 40° + \cos 40°} \qquad P = 0.3532W \ \triangleleft$$

Force P for which crate rotates about C

(We assume that crate does not slide)



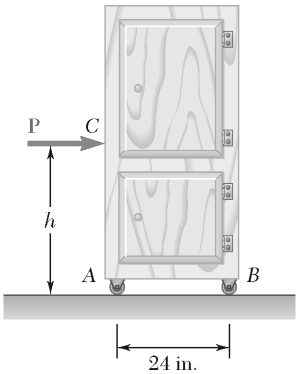
$$+\circlearrowleft \Sigma M_C = 0: \quad (P\sin 40°)(0.8 \text{ m}) + (P\cos 40°)(0.5 \text{ m})$$

$$-W(0.4 \text{ m}) = 0$$

$$P = \frac{0.4W}{0.8\sin 40° + 0.5\cos 40°} = 0.4458W \ \blacktriangleleft$$

Crate will first slide $\triangleleft$

$$P = 0.3532(392.4 \text{ N}) \qquad P = 138.6 \text{ N} \ \blacktriangleleft$$

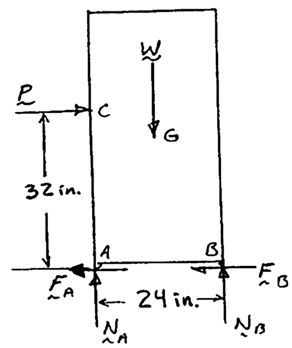


PROBLEM 8.17

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. If $h = 32$ in., determine the magnitude of the force **P** required to move the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION

FBD cabinet: *Note:* for tipping,



$$N_A = F_A = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P_{\text{tip}} = 0$$

$$P_{\text{tip}} = 2.66667$$

(*a*) All casters locked. Impending slip:

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W \qquad W = 120 \text{ lb}$$

So $\quad F_A + F_B = \mu_s W \qquad \mu_s = 0.3$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$
$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(120 \text{ lb}) \qquad \text{or} \quad \mathbf{P} = 36.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$(P = 0.3W < P_{\text{tip}} \quad \text{OK})$$

(*b*) Casters at *A* free, so $\quad F_A = 0$

Impending slip: $\quad F_B = \mu_s N_B$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

PROBLEM 8.17 (Continued)

$$\circlearrowleft \Sigma M_A = 0: \quad (32 \text{ in.})P + (12 \text{ in.})W - (24 \text{ in.})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \quad P = 0.25W$$

$$(P = 0.25W < P_{\text{tip}} \quad \text{OK})$$

$$P = 0.25(120 \text{ lb}) \qquad \text{or} \quad \mathbf{P} = 30.0 \text{ lb} \longrightarrow ◀$$

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A = 0 \quad P = F_A = \mu_s N_A$$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

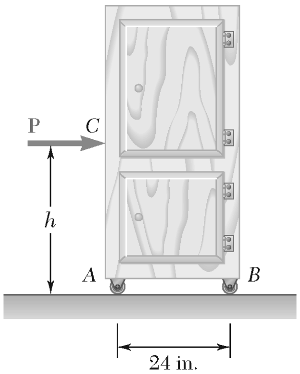
$$\circlearrowleft \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P - (24 \text{ in.})N_A = 0$$

$$3W - 8P - 6\frac{P}{0.3} = 0$$

$$P = 0.107143W = 12.8572$$

$$(P < P_{\text{tip}} \quad \text{OK})$$

$$\mathbf{P} = 12.86 \text{ lb} \longrightarrow ◀$$

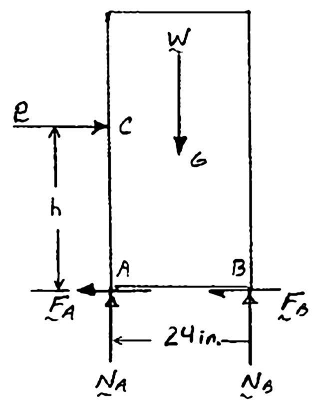


PROBLEM 8.18

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at both A and B are locked, determine (a) the force **P** required to move the cabinet to the right, (b) the largest allowable value of h if the cabinet is not to tip over.

SOLUTION

FBD cabinet:



(a) $\uparrow \Sigma F_y = 0$: $N_A + N_B - W = 0$

$$N_A + N_B = W$$

Impending slip:

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

So

$$F_A + F_B = \mu_s W$$

$\longrightarrow \Sigma F_x = 0$: $P - F_A - F_B = 0$

$$P = F_A + F_B = \mu_s W$$

$W = 120$ lb

$\mu_s = 0.3$

$$P = 0.3(120 \text{ lb}) = 141.26 \text{ N}$$

$\mathbf{P} = 36.0 \text{ lb} \longrightarrow$ ◀

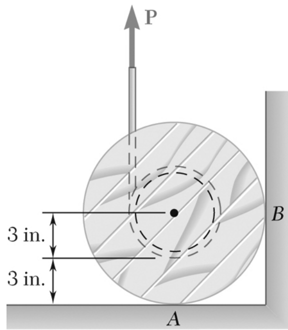
(b) For tipping, $N_A = F_A = 0$

$\circlearrowleft \Sigma M_B = 0$: $hP - (12 \text{ in.})W = 0$

$$h_{\max} = (12 \text{ in.})\frac{W}{P} = (12 \text{ in.})\frac{1}{\mu_s} = \frac{12 \text{ in.}}{0.3}$$

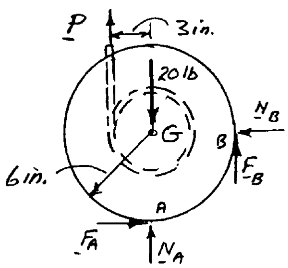
$h_{\max} = 40.0$ in. ◀

PROBLEM 8.19



Wire is being drawn at a constant rate from a spool by applying a vertical force **P** to the wire as shown. The spool and the wire wrapped on the spool have a combined weight of 20 lb. Knowing that the coefficients of friction at both A and B are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine the required magnitude of the force **P**.

SOLUTION



Since spool is rotating

$$F_A = \mu_k N_A \qquad F_B = \mu_k N_B$$

$$+\circlearrowleft \Sigma M_G = 0: \quad P(3\text{ in.}) - F_A(6\text{ in.}) - F_B(6\text{ in.}) = 0$$

$$3P - 6\mu_k(N_A + N_B) = 0 \tag{1}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_A - N_B = 0$$

$$N_B = \mu_k N_A \tag{2}$$

$$+\uparrow \Sigma F_y = 0: \quad P + N_A + F_B - 20\text{ lb} = 0$$

$$P + N_A + \mu_k N_B - 20 = 0$$

Substitute for N_B from (2):

$$P + N_A + \mu_k N_A - 20 = 0$$

$$N_A = \frac{20 - P}{1 + \mu_{k^2}} \tag{3}$$

Substitute from (2) into (1):

$$3P - 6\mu_k(N_A + \mu_k N_A) = 0$$

$$N_A = \frac{1}{2}\frac{P}{\mu_k(1 + \mu_k)} \tag{4}$$

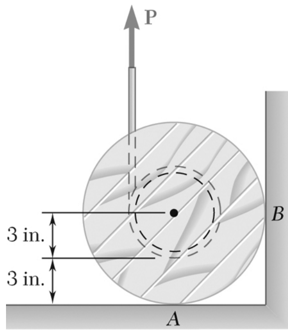
(3)=(4):

$$\frac{20 - P}{1 + \mu_k^2} = \frac{P}{2(\mu_k + \mu_k^2)}$$

Substitute $\mu_k = 0.30$:

$$\frac{20 - P}{1 + (0.3)^2} \quad \frac{P}{2(0.3)(1.03)}$$

$$20 - P = 1.3974P; \quad 2.3974P = 20; \qquad P = 8.34\text{ lb} \blacktriangleleft$$

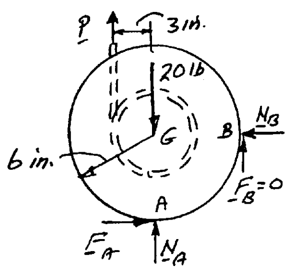


PROBLEM 8.20

Solve Problem 8.19 assuming that the coefficients of friction at B are zero.

PROBLEM 8.19 Wire is being drawn at a constant rate from a spool by applying a vertical force **P** to the wire as shown. The spool and the wire wrapped on the spool have a combined weight of 20 lb. Knowing that the coefficients of friction at both A and B are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine the required magnitude of the force **P**.

SOLUTION



Since spool is rotating

$$F_A = \mu_k N_A$$

$$+\circlearrowleft \Sigma M_G = 0: \quad P(3 \text{ in.}) - F_A(6 \text{ in.}) = 0$$

$$P = 2F_A = 2\mu_k N_A \qquad (1)$$

$$+\uparrow \Sigma F_y = 20: \quad P - 20 \text{ lb} + N_A = 0$$

$$N_A = 20 - P \qquad (2)$$

Substitute for N_A from (2) into (1) $\quad P = 2\mu_k(20 - P)$

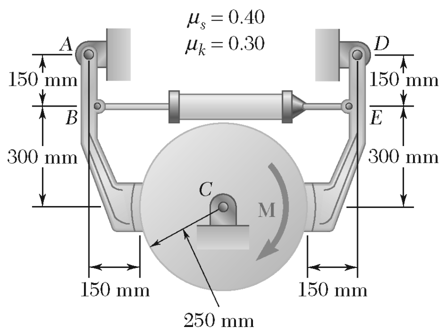
Substitute $\mu_k = 0.30$: $\quad P = 2(0.3)(20 - P)$

$$1.667P = 20 - P$$

$$2.667P = 20$$

$$P = 7.50 \text{ lb}$$

$P = 7.50$ lb ◀

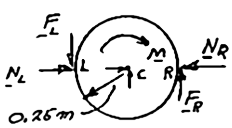


PROBLEM 8.21

The hydraulic cylinder shown exerts a force of 3 kN directed to the right on Point B and to the left on Point E. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

Free body: Drum



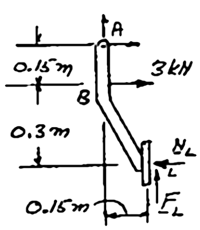
$$+\circlearrowright \Sigma M_C = 0: \quad M - (0.25\text{ m})(F_L + F_R) = 0$$

$$M = (0.25\text{ m})(F_L + F_R) \qquad (1)$$

Since drum is rotating

$$F_L = \mu_k N_L = 0.3 N_L$$
$$F_R = \mu_k N_R = 0.3 N_R$$

Free body: Left arm ABL



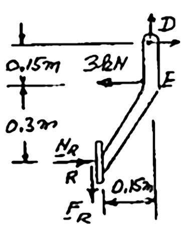
$$+\circlearrowright \Sigma M_A = 0: \quad (3\text{ kN})(0.15\text{ m}) + F_L(0.15\text{ m}) - N_L(0.45\text{ m}) = 0$$

$$0.45\text{ kN}\cdot\text{m} + (0.3N_L)(0.15\text{ m}) - N_L(0.45\text{ m}) = 0$$

$$0.405N_L = 0.45$$

$$N_L = 1.111\text{ kN}$$
$$F_L = 0.3N_L = 0.3(1.111\text{ kN})$$
$$= 0.3333\text{ kN} \qquad (2)$$

Free body: Right arm DER



$$+\circlearrowright \Sigma M_D = 0: \quad (3\text{ kN})(0.15\text{ m}) - F_R(0.15\text{ m}) - N_R(0.45\text{ m}) = 0$$

$$0.45\text{ kN}\cdot\text{m} - (0.3N_R)(0.15\text{ m}) - N_R(0.45\text{ m}) = 0$$

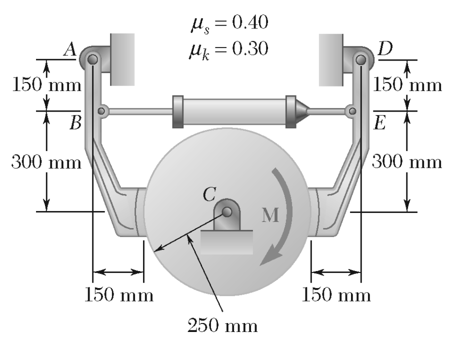
$$0.495N_R = 0.45$$

$$N_R = 0.9091\text{ kN}$$
$$F_R = \mu_k N_R = 0.3(0.9091\text{ kN})$$
$$= 0.2727\text{ kN} \qquad (3)$$

Substitute for F_L and F_R into (1): $\quad M = (0.25\text{ m})(0.333\text{ kN} + 0.2727\text{ kN})$

$$M = 0.1515\text{ kN}\cdot\text{m}$$

$$M = 151.5\text{ N}\cdot\text{m} \blacktriangleleft$$

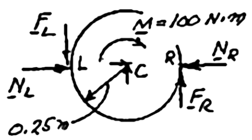


PROBLEM 8.22

A couple **M** of magnitude 100 N · m is applied to the drum as shown. Determine the smallest force that must be exerted by the hydraulic cylinder on joints B and E if the drum is not to rotate.

SOLUTION

Free body: Drum



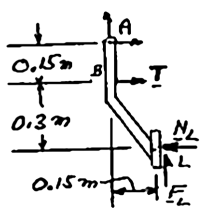
$$+\circlearrowright \Sigma M_C = 0: \quad 100 \text{ N}\cdot\text{m} - (0.25 \text{ m})(F_L + F_R) = 0$$

$$F_L + F_R = 400 \text{ N} \qquad (1)$$

Since motion impends

$$F_L = \mu_s N_L = 0.4 N_L$$
$$F_R = \mu_s N_R = 0.4 N_R$$

Free body: Left arm *ABL*



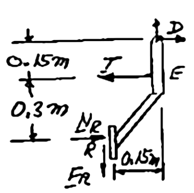
$$+\circlearrowright \Sigma M_A = 0: \quad T(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.15T + (0.4N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.39N_L = 0.15T; \quad N_L = 0.38462T$$

$$F_L = 0.4N_L = 0.4(0.38462T)$$
$$F_L = 0.15385T \qquad (2)$$

Free body: Right arm *DER*



$$+\circlearrowright \Sigma M_D = 0: \quad T(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.15T - (0.4N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

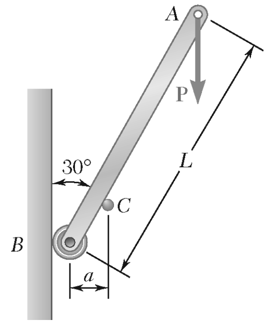
$$0.51N_R = 0.15T; \quad N_R = 0.29412T$$

$$F_R = 0.4N_R = 0.4(0.29412T)$$
$$F_R = 0.11765T \qquad (3)$$

Substitute for F_L and F_R into Eq. (1):

$$0.15385T + 0.11765T = 400$$
$$T = 1473.3 \text{ N}$$

$$T = 1.473 \text{ kN} \blacktriangleleft$$



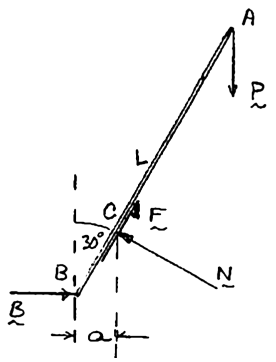
PROBLEM 8.23

A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:

Free-body diagram: For motion of B impending upward:



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu_s N_C\cos\theta - P = 0$$

$$N_C(\sin\theta - \mu\cos\theta) = P$$

Substitute for N_C from Eq. (1), and solve for a/L.

$$\frac{a}{L} = \sin^2\theta(\sin\theta - \mu_s\cos\theta) \qquad (2)$$

For $\theta = 30°$ and $\mu_s = 0.15$:

$$\frac{a}{L} = \sin^2 30°(\sin 30° - 0.15\cos 30°)$$

$$\frac{a}{L} = 0.092524 \qquad \frac{L}{a} = 10.808$$

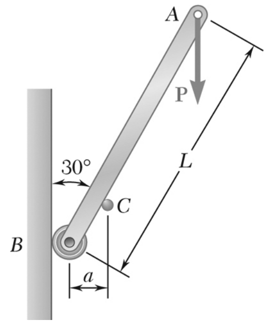
For motion of B impending downward, reverse sense of friction force F_C. To do this we make $\mu_s = -0.15$ in. Eq. (2).

Eq. (2):

$$\frac{a}{L} = \sin^2 30°(\sin 30° - (-0.15)\cos 30°)$$

$$\frac{a}{L} = 0.15748 \qquad \frac{L}{a} = 6.350$$

Range of values of L/a for equilibrium: $6.35 \le \dfrac{L}{a} \le 10.81$ ◀



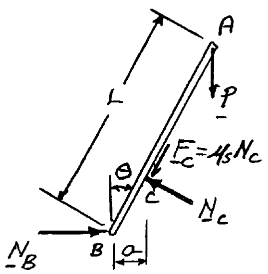
PROBLEM 8.24

Solve Problem 8.23 assuming that the coefficient of static friction between the peg and the rod is 0.60.

PROBLEM 8.23 A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

<u>Free-body diagram</u>: <u>For motion of B impending upward</u>



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu_s N_C\cos\theta - P = 0$$

$$N_C(\sin\theta - \mu_s\cos\theta) = P$$

Substitute for N_C from (1), and solve for $\frac{a}{L}$

$$\frac{a}{L} = \sin^2\theta(\sin\theta - \mu_s\cos\theta)$$

For $\theta = 30°$ and $\mu_s = 0.60$:

$$\frac{a}{L} = \sin^2 30°(\sin 30° - 0.60\cos 30°) \qquad (2)$$

$$\frac{a}{L} = -0.0049 < 0$$

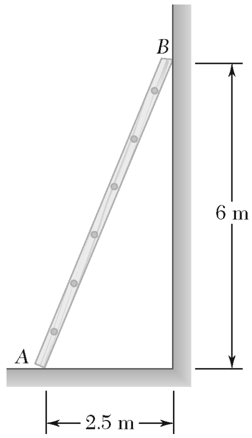
Thus, slipping of B upward does not occur for motion of B impending downward, reverse sense of friction force F_C. To do this we make $\mu_C = -0.60$ in Eq. (2).

$$\frac{a}{L} = \sin^2 30°(\sin 30° - (-0.60)\cos 30°)$$

$$\frac{a}{L} = 0.2459 \qquad \frac{L}{a} = 3.923$$

Range of L/a for equilibrium: $L/a \geq 3.92$ ◀

PROBLEM 8.25



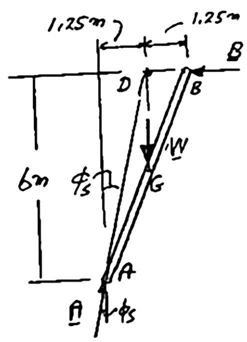
A 6.5-m ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is zero at B, determine the smallest value of μ_s at A for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Three-force body.

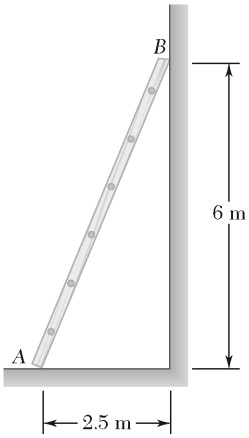
Line of action of **A** must pass through D, where **W** and **B** intersect.



At A: $$\mu_s = \tan\phi_s = \frac{1.25 \text{ m}}{6 \text{ m}} = 0.2083$$

$\mu_s = 0.208$ ◀

PROBLEM 8.26



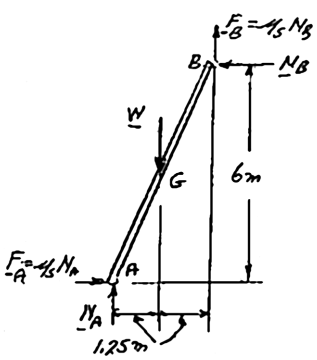
A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at *A* and *B*, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

<u>Free body: Ladder</u>

Motion impending:

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$



$$+\circlearrowleft \Sigma M_A = 0: \quad W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0$$

$$N_B = \frac{1.25W}{6 + 2.5\mu_s} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$

$$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \mu_s N_A - N_B = 0$$

Substitute for N_A and N_B from Eqs. (1) and (2):

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$

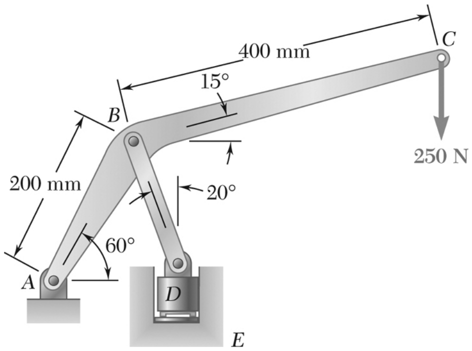
$$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$$

$$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$$

$$\mu_s = 0.2$$

and $$\mu_s = -5 \quad \text{(Discard)}$$

$$\mu_s = 0.200 \blacktriangleleft$$

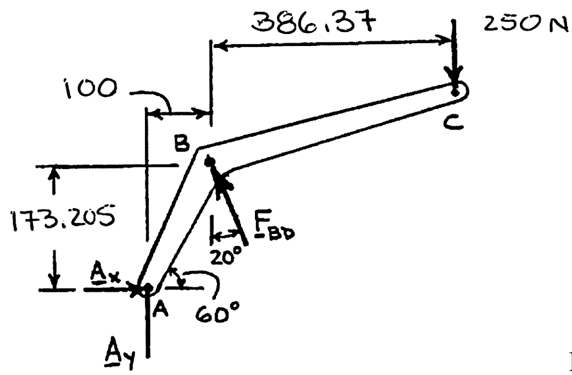


PROBLEM 8.27

The press shown is used to emboss a small seal at *E*. Knowing that the coefficient of static friction between the vertical guide and the embossing die *D* is 0.30, determine the force exerted by the die on the seal.

SOLUTION

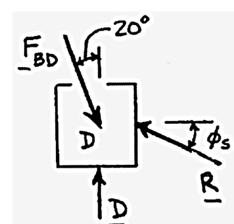
Free body: Member *ABC*



Dimensions in mm

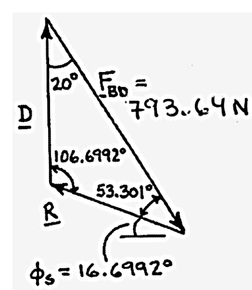
$$+\circlearrowleft \Sigma M_A = 0: \quad F_{BD}\cos 20°(100) + F_{BD}\sin 20°(173.205) - (250\text{ N})(100 + 386.37) = 0$$

$$F_{BD} = 793.64\text{ N}$$



Free body: Die *D*

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.3 = 16.6992°$$

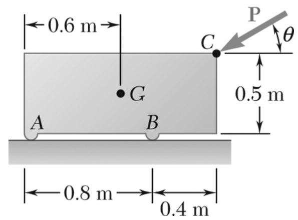


Force triangle:

$$\frac{D}{\sin 53.301°} = \frac{793.64\text{ N}}{\sin 106.6992°}$$

$$D = 664.35\text{ N}$$

On seal: $\mathbf{D} = 664\text{ N}\downarrow$ ◀



PROBLEM 8.28

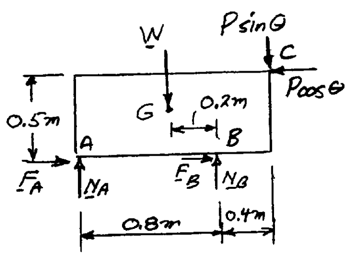
The machine base shown has a mass of 75 kg and is fitted with skids at A and B. The coefficient of static friction between the skids and the floor is 0.30. If a force **P** of magnitude 500 N is applied at corner C, determine the range of values of θ for which the base will not move.

SOLUTION

Free-body: Machine base

$$m = (75\text{ kg})(9.81\text{ m/s}^2) = 735.75\text{ N}$$

Assume sliding impends



$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W - P\sin\theta = 0$$

$$(N_A + N_B) = W + P\sin\theta \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_A + F_B - P\cos\theta = 0$$

$$\mu_s(N_A + N_B) = P\cos\theta = 0 \qquad (2)$$

$\dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}$:

$$\mu_s = \frac{P\cos\theta}{W + P\sin\theta}$$

$$\mu_s W + \mu_s P\sin\theta = P\cos\theta$$

$$0.30(735.75\text{ N}) + 0.30(500\text{ N})\sin\theta = 500\cos\theta$$

$$500\cos\theta - 150\sin\theta = 220.73$$

Solve for θ: $\qquad \theta = 48.28°$

Assume tipping about B impends: $\quad \therefore N_A = 0$

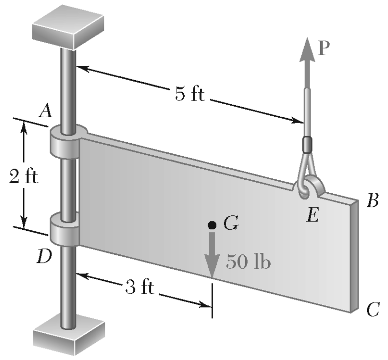
$$+\circlearrowleft \Sigma M_B = 0: \quad P\sin\theta(0.4\text{ m}) - P\cos\theta(0.5\text{ m}) - W(0.2\text{ m}) = 0$$

$$500\sin\theta(0.4) - 500\cos\theta(0.5) - 735.75(0.2\text{ m}) = 0$$

$$200\sin\theta - 250\cos\theta = 147.15$$

Solve for θ: $\qquad \theta = 78.70°$

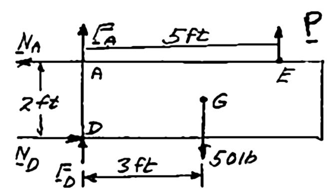
Range for no motion: $\qquad 48.3° \le \theta \le 78.7°$ ◀



PROBLEM 8.29

The 50-lb plate *ABCD* is attached at *A* and *D* to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 20$ lb.

SOLUTION



(*a*) $\underline{P = 0}$ $\quad +\circlearrowleft \Sigma M_D = 0{:}\quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) = 0$

$$N_A = 75\text{ lb}$$

$$\Sigma F_x = 0{:}\quad N_D = N_A = 75\text{ lb}$$

$$+\uparrow \Sigma F_y = 0{:}\quad F_A + F_D - 50\text{ lb} = 0$$

$$F_A + F_D = 50\text{ lb}$$

But: $\quad (F_A)_m = \mu_s N_A = 0.40(75\text{ lb}) = 30\text{ lb}$

$\quad (F_D)_m = \mu_s N_D = 0.40(75\text{ lb}) = 30\text{ lb}$

Thus: $\quad (F_A)_m + (F_D)_m = 60\text{ lb}$

and $\quad (F_A)_m + (F_D)_m > F_A + F_D$ $\quad$ Plate is in equilibrium ◀

(*b*) $\underline{P = 20\text{ lb}}$ $\quad +\circlearrowleft \Sigma M_D = 0{:}\quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) + (20\text{ lb})(5\text{ ft}) = 0$

$$N_A = 25\text{ lb}$$

$$\Sigma F_x = 0{:}\quad N_D = N_A = 25\text{ lb}$$

$$+\uparrow \Sigma F_y = 0{:}\quad F_A + F_D - 50\text{ lb} + 20\text{ lb} = 0$$

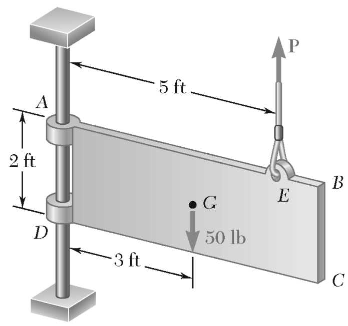
$$F_A + F_D = 30\text{ lb}$$

But: $\quad (F_A)_m = \mu_s N_A = 0.4(25\text{ lb}) = 10\text{ lb}$

$\quad (F_D)_m = \mu_s N_D = 0.4(25\text{ lb}) = 10\text{ lb}$

Thus: $\quad (F_A)_m + (F_D)_m = 20\text{ lb}$

and $\quad F_A + F_D > (F_A)_m + (F_D)_m$ $\quad$ Plate moves downward ◀

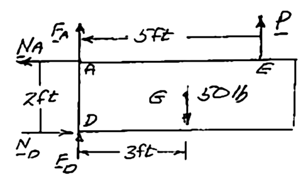


PROBLEM 8.30

In Problem 8.29, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

PROBLEM 8.29 The 50-lb plate $ABCD$ is attached at A and D to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at E is (*a*) $P = 0$, (*b*) $P = 20$ lb.

SOLUTION



We shall consider the following two cases:

(1) $0 < P < 30$ lb

$$+\circlearrowleft \Sigma M_D = 0: \quad N_A(2 \text{ ft}) - (50 \text{ lb})(3 \text{ ft}) + P(5 \text{ ft}) = 0$$

$$N_A = 75 \text{ lb} - 2.5P$$

(*Note:* $N_A \geq 0$ and directed $\longleftarrow$ for $P \leq 30$ lb as assumed here)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$

But:

$$(F_A)_m = (F_0)_m = \mu_s N_A$$
$$= 0.40(75 - 2.5P)$$
$$= 30 - P$$

Plate moves $\downarrow$ if:

$$F_A + F_D > (F_A)_m + (F_D)_m$$

or

$$50 - P > (30 - P) + (30 - P)$$

$P > 10$ lb ◁

(2) $30 \text{ lb} < P < 50$ lb

$$+\circlearrowleft \Sigma M_D = 0: \quad -N_A(2 \text{ ft}) - (50 \text{ lb})(3 \text{ ft}) + P(5 \text{ ft}) = 0$$

$$N_A = 2.5P - 75$$

(*Note:* $N_A >$ and directed $\longrightarrow$ for $P > 30$ lb as assumed)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$

PROBLEM 8.30 (Continued)

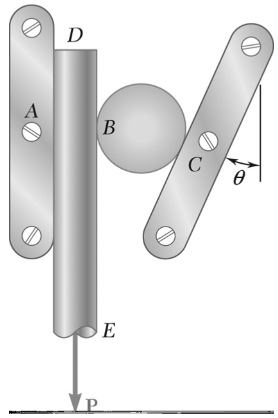
But: $(F_A)_m = (F_D)_m = \mu_s N_A$
$= 0.40(2.5P - 75)$
$= P - 30 \text{ lb}$

<u>Plate moves $\downarrow$ if</u>: $F_A + F_D > (F_A)_m + (F_D)_m$

$50 - P > (P - 30) + (P - 30)$ $\qquad P < \dfrac{110}{3} = 36.7 \text{ lb}$ ◁

Thus, plate moves downward for: $10.00 \text{ lb} < P < 36.7 \text{ lb}$ ◀

(*Note:* For $P > 50$ lb, plate is in equilibrium)



PROBLEM 8.31

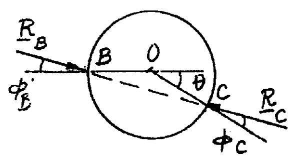
A rod DE and a small cylinder are placed between two guides as shown. The rod is not to slip downward, however large the force $\mathbf{P}$ may be; i.e., the arrangement is said to be self-locking. Neglecting the weight of the cylinder, determine the minimum allowable coefficients of static friction at A, B, and C.

SOLUTION

<u>Free body: Cylinder</u>

Since cylinder is a two-force body, $\mathbf{R}_B$ and $\mathbf{R}_C$ have the same line of action. Thus $\phi_B = \phi_C$:

From triangle OBC:



$$\phi_B + \phi_C = \theta$$

Thus:

$$\phi_B = \phi_C = \frac{\theta}{2}$$

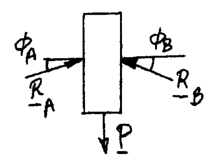
For no sliding, we must have $\tan\phi_B \le (\mu_B)_s$, $\tan\phi_C \le (\mu_C)_s$

Therefore:

$$(\mu_B)_s \ge \tan\frac{\theta}{2}, \qquad (\mu_c)_s \ge \tan\frac{\theta}{2} \blacktriangleleft$$

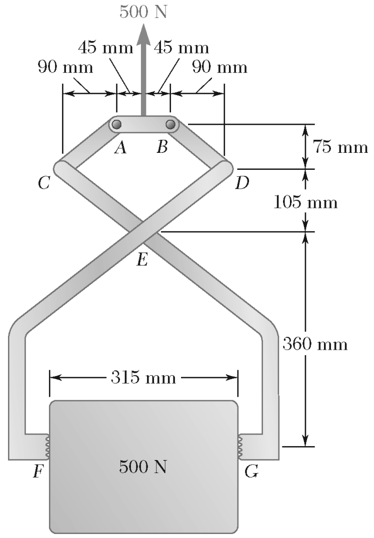
We also note that R_B and R_C are indeterminate.

<u>Free body: Rod</u>



Since R_B is indeterminate, it may be as large as necessary to satisfy equation $\Sigma F_y = 0$, no matter how large $\mathbf{P}$ is or how small ϕ_A is. Therefore

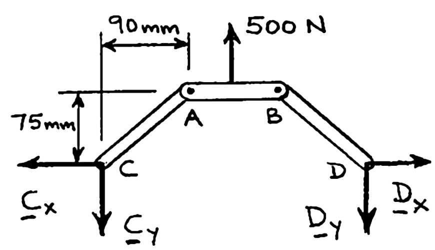
$(\mu_A)_s$ may have any value $\blacktriangleleft$



PROBLEM 8.32

A 500-N concrete block is to be lifted by the pair of tongs shown. Determine the smallest allowable value of the coefficient of static friction between the block and the tongs at *F* and *G*.

SOLUTION



<u>Free body: Members *CA*, *AB*, *BD*</u>

<u>By symmetry</u>: $C_y = D_y = \frac{1}{2}(500) = 250 \text{ N}$

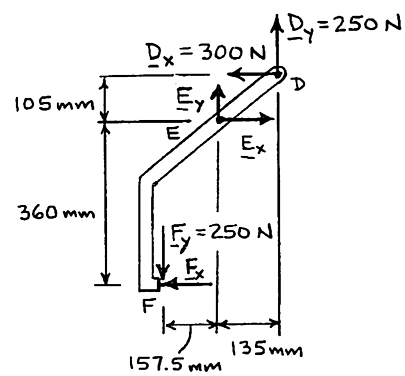
Since *CA* is a two-force member,

$$\frac{C_x}{90 \text{ mm}} = \frac{C_y}{75 \text{ mm}} = \frac{250 \text{ N}}{75 \text{ mm}}$$

$$C_x = 300 \text{ N}$$

$$\Sigma F_x = 0: \quad D_x = C_x$$

$$D_x = 300 \text{ N}$$



<u>Free body: Tong *DEF*</u>

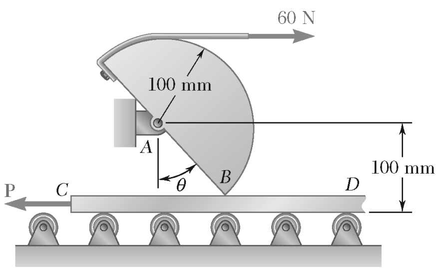
$$+\circlearrowleft \Sigma M_E = 0: \quad (300 \text{ N})(105 \text{ mm}) + (250 \text{ N})(135 \text{ mm}) + (250 \text{ N})(157.5 \text{ mm}) - F_x(360 \text{ mm}) = 0$$

$$F_x = +290.625 \text{ N}$$

Minimum value of μ_s: $\mu_s = \frac{F_y}{F_x} = \frac{250 \text{ N}}{290.625 \text{ N}}$

$$\mu_s = 0.860 \blacktriangleleft$$

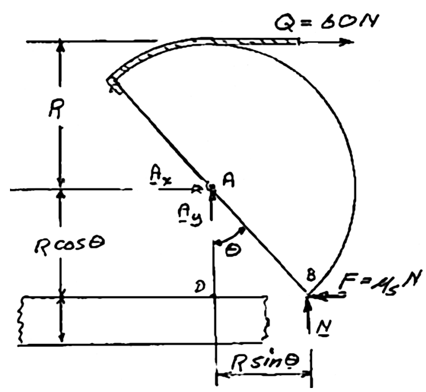
PROBLEM 8.33



The 100-mm-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** required to maintain the motion of the plate, knowing that the plate is 20 mm thick, (*b*) the largest thickness of the plate for which the mechanism is self-locking (i.e., for which the plate cannot be moved however large the force **P** may be).

SOLUTION

Free body: Cam



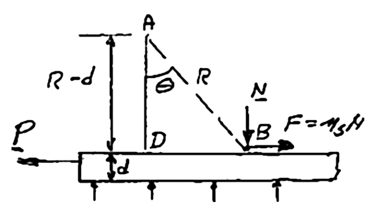
Impending motion: $F = \mu_s N$

$$+\circlearrowright \Sigma M_A = 0: \quad QR - NR\sin\theta + (\mu_s N)R\cos\theta = 0$$

$$N = \frac{Q}{\sin\theta - \mu_s \cos\theta} \qquad (1)$$

Free body: Plate $\quad \Sigma F_x = 0 \quad P = \mu_s N \qquad (2)$

Geometry in ΔABD with $R = 100$ mm and $d = 20$ mm



$$\cos\theta = \frac{R-d}{R} = \frac{80 \text{ mm}}{100 \text{ mm}} = 0.8$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = 0.6$$

PROBLEM 8.33 (Continued)

(*a*) Eq. (1) using $Q = 60 \text{ N}$ and $\mu_s = 0.45$

$$N = \frac{60 \text{ N}}{0.6 - (0.45)(0.8)}$$

$$= \frac{60}{0.24} = 250 \text{ N}$$

Eq. (2) $P = \mu_s N = (0.45)(250 \text{ N})$

$P = 112.5 \text{ N}$ ◀

(*b*) <u>For $P = \infty$</u>, $N = \infty$. Denominator is zero in Eq. (1).

$$\sin\theta - \mu_s \cos\theta = 0$$

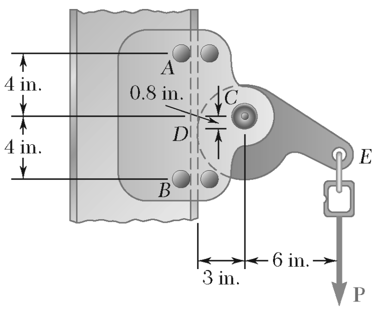
$$\tan\theta = \mu_s = 0.45$$

$$\theta = 24.23°$$

$$\cos\theta = \frac{R - d}{R}$$

$$\cos 24.23 = \frac{100 - d}{100}$$

$d = 8.81 \text{ mm}$ ◀

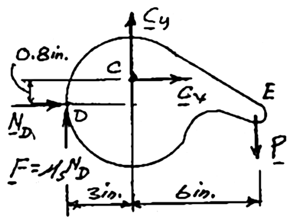


PROBLEM 8.34

A safety device used by workers climbing ladders fixed to high structures consists of a rail attached to the ladder and a sleeve that can slide on the flange of the rail. A chain connects the worker's belt to the end of an eccentric cam that can be rotated about an axle attached to the sleeve at C. Determine the smallest allowable common value of the coefficient of static friction between the flange of the rail, the pins at A and B, and the eccentric cam if the sleeve is not to slide down when the chain is pulled vertically downward.

SOLUTION

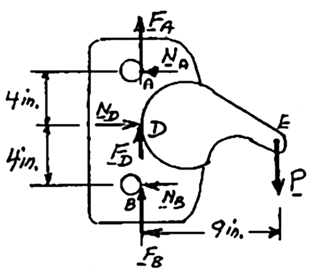
Free body: Cam



$$+\circlearrowleft \Sigma M_C = 0: \quad N_D(0.8 \text{ in.}) - \mu_s N_D(3\text{in.}) - P(6 \text{ in.}) = 0$$

$$N_D = \frac{6P}{0.8 - 3\mu_s} \qquad (1)$$

Free body: Sleeve and cam



$$\xrightarrow{+} \Sigma F_x = 0: \quad N_D - N_A - N_B = 0$$

$$N_A + N_B = N_D \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_B + F_D - P = 0$$

or

$$\mu_s(N_A + N_B + N_D) = P \qquad (3)$$

Substitute from Eq. (2) into Eq. (3): $\quad \mu_s(2N_D) = P \qquad N_D = \dfrac{P}{2\mu_s} \qquad (4)$

Equate expressions for N_D from Eq. (1) and Eq. (4):

$$\frac{P}{2\mu_s} = \frac{6P}{0.8 - 3\mu_s}; \quad 0.8 - 3\mu_s = 12\mu_s$$

$$\mu_s = \frac{0.8}{15} \qquad \mu_s = 0.0533 \blacktriangleleft$$

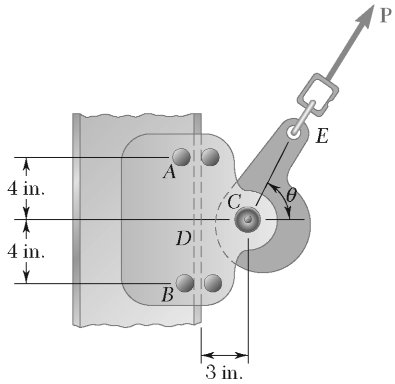
(*Note:* To verify that contact at pins A and B takes places as assumed, we shall check that $N_A > 0$ and $N_B = 0$.)

From Eq. (4):
$$N_D = \frac{P}{2\mu_s} = \frac{P}{2(0.0533)} = 9.375P$$

From free body of cam and sleeve:

$$+\circlearrowleft \Sigma M_B = 0: \quad N_A(8 \text{ in.}) - N_D(4 \text{ in.}) - P(9 \text{ in.}) = 0$$
$$8N_A = (9.375P)(4) + 9P$$
$$N_A = 5.8125P > 0 \quad \text{OK}$$

From Eq. (2):
$$N_A + N_B = N_D$$
$$5.8125P + N_B = 9.375P$$
$$N_B = 3.5625P > 0 \quad \text{OK}$$



PROBLEM 8.35

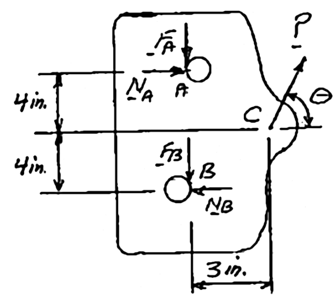
To be of practical use, the safety sleeve described in Problem 8.34 must be free to slide along the rail when pulled upward. Determine the largest allowable value of the coefficient of static friction between the flange of the rail and the pins at A and B if the sleeve is to be free to slide when pulled as shown in the figure, assuming (*a*) $\theta = 60°$, (*b*) $\theta = 50°$, (*c*) $\theta = 40°$.

SOLUTION

Note the cam is a two-force member.

<u>Free body: Sleeve</u>

We assume contact between rail and pins as shown.



$$+\circlearrowleft \Sigma M_C = 0: \quad F_A(3 \text{ in.}) + F_B(3 \text{ in.}) - N_A(4 \text{ in.}) - N_B(4 \text{ in.}) = 0$$

But $\quad F_A = \mu_s N_A$

$\quad F_B = \mu_s N_B$

We find $\quad 3\mu_s(N_A + N_B) - 4(N_A + N_B) = 0$

$$\mu_s = \frac{4}{3} = 1.33333$$

We now verify that our assumption was correct.

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B + P\cos\theta = 0$$

$$N_B - N_A = P\cos\theta \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - F_B + P\sin\theta = 0$$

$$\mu_s N_A + \mu_s N_B = P\sin\theta$$

$$N_A + N_B = \frac{P\sin\theta}{\mu_s} \tag{2}$$

Add Eqs. (1) and (2): $\quad 2N_B = P\left(\cos\theta + \dfrac{\sin\theta}{\mu_s}\right) > 0 \quad \text{OK}$

PROBLEM 8.35 (Continued)

Subtract Eq. (1) from Eq. (2): $$2N_A = P\left(\frac{\sin\theta}{\mu_s} - \cos\theta\right)$$

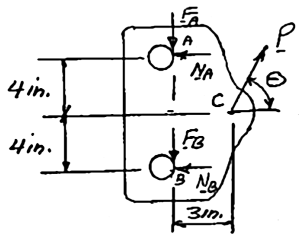
$N_A > 0$ only if $$\frac{\sin\theta}{\mu_s} - \cos\theta > 0$$

$$\tan\theta > \mu_s = 1.33333$$
$$\theta = 53.130°$$

(*a*) For case (*a*): Condition is satisfied, contact takes place as shown. Answer is correct.

$\mu_s = 1.333$ ◀

But for (*b*) and (*c*): $\theta < 53.130°$ and our assumption is wrong, N_A is directed to left.



$$\xrightarrow{+} \Sigma F_x = 0: \quad -N_A - N_B + P\cos\theta = 0$$

$$N_A + N_B = P\cos\theta \tag{3}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - F_A + P\sin\theta = 0$$

$$\mu_s(N_A + N_B) = P\sin\theta \tag{4}$$

Divide Eq. (4) by Eq. (3):

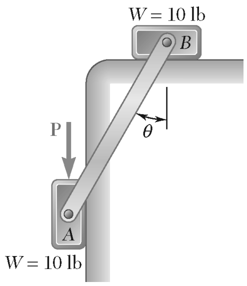
$$\mu_s = \tan\theta \tag{5}$$

(*b*) We make $\theta = 50°$ in Eq. (5):

$$\mu_s = \tan 50°$$ $\mu_s = 1.192$ ◀

(*c*) We make $\theta = 40°$ in Eq. (5):

$$\mu_s = \tan 40°$$ $\mu_s = 0.839$ ◀



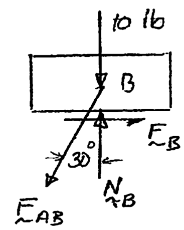
PROBLEM 8.36

Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$. with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD block B:

(a) Since $P = 2.69$ lb to initiate motion, equilibrium exists with $P = 0$ ◀



(b) For P_{max}, motion impends at both surfaces:

Block B: $+\uparrow \Sigma F_y = 0$: $N_B - 10\,\text{lb} - F_{AB}\cos 30° = 0$

$$N_B = 10\text{ lb} + \frac{\sqrt{3}}{2}F_{AB} \quad (1)$$

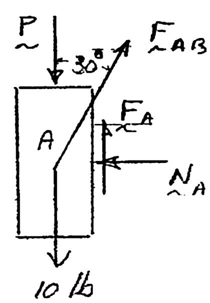
Impending motion: $F_B = \mu_s N_B = 0.3N_B$

$\xrightarrow{+} \Sigma F_x = 0$: $F_B - F_{AB}\sin 30° = 0$

$$F_{AB} = 2F_B = 0.6N_B \quad (2)$$

Solving Eqs. (1) and (2): $N_B = 10\text{ lb} + \dfrac{\sqrt{3}}{2}(0.6N_B) = 20.8166\text{ lb}$

FBD block A:



Then $F_{AB} = 0.6N_B = 12.4900\text{ lb}$

Block A: $\xrightarrow{+} \Sigma F_x = 0$: $F_{AB}\sin 30° - N_A = 0$

$$N_A = \frac{1}{2}F_{AB} = \frac{1}{2}(12.4900\text{ lb}) = 6.2450\text{ lb}$$

Impending motion: $F_A = \mu_s N_A = 0.3(6.2450\text{ lb}) = 1.8735\text{ lb}$

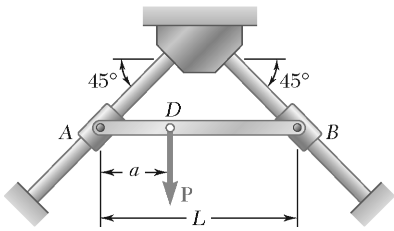
$+\uparrow \Sigma F_y = 0$: $F_A + F_{AB}\cos 30° - P - 10\text{ lb} = 0$

$$P = F_A + \frac{\sqrt{3}}{2}F_{AB} - 10\text{ lb}$$

$$= 1.8735\text{ lb} + \frac{\sqrt{3}}{2}(12.4900\text{ lb}) - 10\text{ lb}$$

$$= 2.69\text{ lb}$$

$P = 2.69$ lb ◀

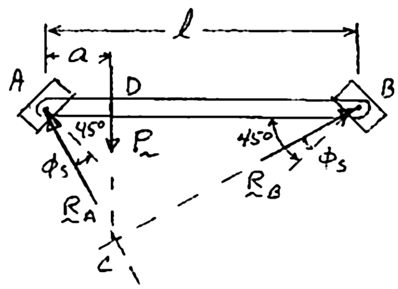


PROBLEM 8.37

Bar AB is attached to collars that can slide on the inclined rods shown. A force **P** is applied at Point D located at a distance a from end A. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio a/L for which equilibrium is maintained.

SOLUTION

FBD bar and collars:



Impending motion: $\phi_s = \tan^{-1}\mu_s$

$$= \tan^{-1} 0.3$$

$$= 16.6992°$$

Neglect weights: 3-force FBD and $\measuredangle ACB = 90°$

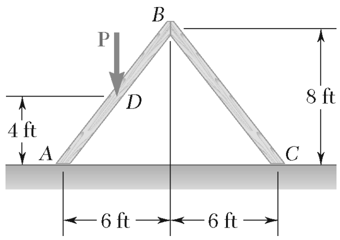
so
$$AC = \frac{a}{\cos(45° + \phi_s)}$$

$$= l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

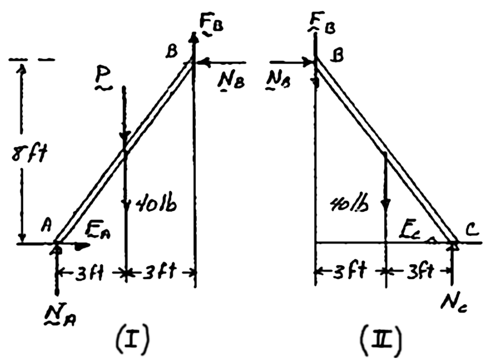
PROBLEM 8.38



Two identical uniform boards, each of weight 40 lb, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (*a*) the largest magnitude of the force **P** for which equilibrium will be maintained, (*b*) the surface at which motion will impend.

SOLUTION

Board FBDs:



Assume impending motion at C, so

$$F_C = \mu_s N_C = 0.4N_C$$

FBD II: $\circlearrowleft \Sigma M_B = 0$: $(6\text{ ft})N_C - (8\text{ ft})F_C - (3\text{ ft})(40\text{ lb}) = 0$

$$[6\text{ ft} - 0.4(8\text{ ft})]N_C = (3\text{ ft})(40\text{ lb})$$

or $$N_C = 42.857\text{ lb}$$

and $$F_C = 0.4N_C = 17.143\text{ lb}$$

$\xrightarrow{} \Sigma F_x = 0$: $N_B - F_C = 0$

$$N_B = F_C = 17.143\text{ lb}$$

$\uparrow \Sigma M_y = 0$: $-F_B - 40\text{ lb} + N_C = 0$

$$F_B = N_C - 40\text{ lb} = 2.857\text{ lb}$$

Check for motion at B: $$\frac{F_B}{N_B} = \frac{2.857\text{ lb}}{17.143\text{ lb}} = 0.167 < \mu_s,\ \text{OK, no motion.}$$

FBD I: $\circlearrowleft \Sigma M_A = 0$: $(8\text{ ft})N_B + (6\text{ ft})F_B - (3\text{ ft})(P + 40\text{ lb}) = 0$

$$P = \frac{(8\text{ ft})(17.143\text{ lb}) + (6\text{ ft})(2.857\text{ lb})}{3\text{ ft}} - 40\text{ lb}$$

$$= 11.429\text{ lb}$$

PROBLEM 8.38 (Continued)

Check for slip at A (unlikely because of P):

$$\xrightarrow{} \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad F_A = N_B = 17.143 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad N_A - P - 40 \text{ lb} + F_B = 0$$

or

$$N_A = 11.429 \text{ lb} + 40 \text{ lb} - 2.857 \text{ lb}$$
$$= 48.572 \text{ lb}$$

Then

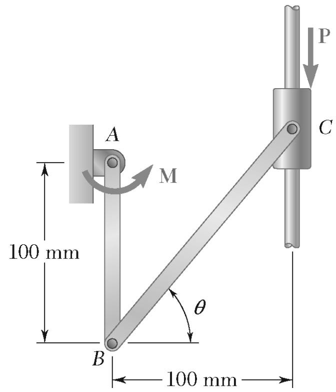
$$\frac{F_A}{N_A} = \frac{17.143 \text{ lb}}{48.572 \text{ lb}} = 0.353 < \mu_s$$

OK, no slip $\Rightarrow$ assumption is correct.

Therefore

(*a*) $P_{\max} = 11.43 \text{ lb}$ ◀

(*b*) Motion impends at C ◀

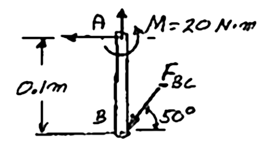


PROBLEM 8.39

Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of P for which equilibrium is maintained when $\theta = 50°$ and $M = 20$ N·m.

SOLUTION

Free body member *AB*:

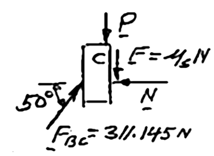


BC is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad 20 \text{ N}\cdot\text{m} - F_{BC} \cos 50°(0.1 \text{ m}) = 0$$

$$F_{BC} = 311.145 \text{ N}$$

<u>Motion of *C* impending upward</u>:



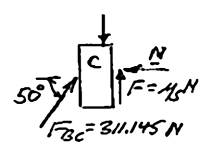
$$\xrightarrow{+} \Sigma F_x = 0: \quad (311.145 \text{ N})\cos 50° - N = 0$$

$$N = 200 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad (311.145 \text{ N})\sin 50° - P - (0.35)(200 \text{ N}) = 0$$

$P = 168.351$ N ◁

<u>Motion of *C* impending downward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad (311.145 \text{ N})\cos 50° - N = 0$$

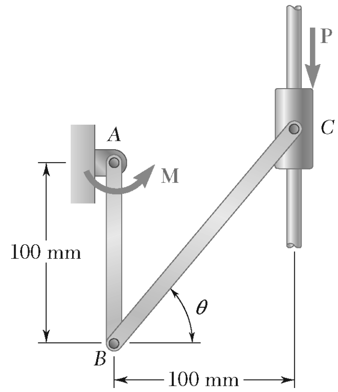
$$N = 200 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad (311.145 \text{ N})\sin 50° - P + (0.35)(200 \text{ N}) = 0$$

$P = 308.35$ N ◁

<u>Range of *P*</u>: $168.4 \text{ N} \le P \le 308 \text{ N}$ ◀

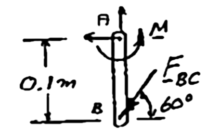
PROBLEM 8.40



Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of M for which equilibrium is maintained when $\theta = 60°$ and $P = 200$ N.

SOLUTION

Free body member *AB*:

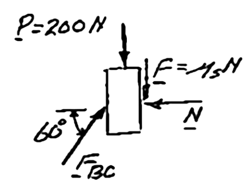


BC is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad M - F_{BC}\cos 60°(0.1\text{ m}) = 0$$

$$M = 0.05F_{BC} \qquad (1)$$

Motion of C impending upward:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC}\cos 60° - N = 0$$

$$N = 0.5F_{BC}$$

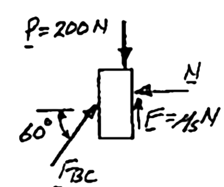
$$+\uparrow \Sigma F_y = 0: \quad F_{BC}\sin 60° - 200\text{ N} - (0.40)(0.5F_{BC}) = 0$$

$$F_{BC} = 300.29\text{ N}$$

Eq. (1): $\quad M = 0.05(300.29)$

$$M = 15.014\text{ N}\cdot\text{m} \quad \triangleleft$$

Motion of C impending downward:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC}\cos 60° - N = 0$$

$$N = 0.5F_{BC}$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BC}\sin 60° - 200\text{ N} + (0.40)(0.5F_{BC}) = 0$$

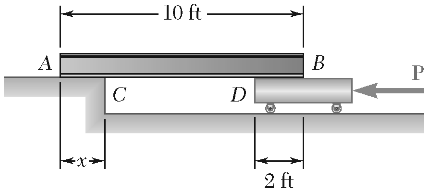
$$F_{BC} = 187.613\text{ N}$$

Eq. (1): $\quad M = 0.05(187.613)$

$$M = 9.381\text{ N}\cdot\text{m} \quad \triangleleft$$

Range of M: $\quad 9.38\text{ N}\cdot\text{m} \le M \le 15.01\text{ N}\cdot\text{m} \quad \blacktriangleleft$

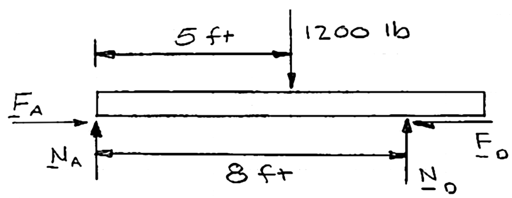
PROBLEM 8.41



A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

SOLUTION

FBD beam:



$$+\circlearrowleft \Sigma M_A = 0: \quad N_D(8\,\text{ft}) - (1200\text{ lb})(5\,\text{ft}) = 0$$

$$\mathbf{N}_D = 750\text{ lb}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad N_A - 1200 + 750 = 0$$

$$\mathbf{N}_A = 450\text{ lb}\uparrow$$

$$(F_A)_m = \mu_s N_A = 0.3(450) = 135.0\text{ lb}$$

$$(F_D)_m = \mu_s N_D = 0.3(750) = 225\text{ lb}$$

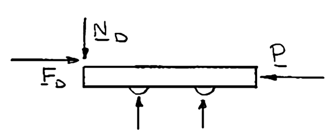
Since $(F_A)_m < (F_D)_m$, sliding first impends at A with

$$F_A = (F_A)_m = 135\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_A - F_D = 0$$

$$F_D = F_A = 135.0\text{ lb}$$

FBD dolly:



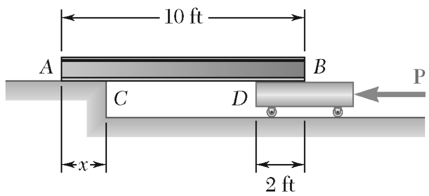
From *FBD* of dolly:

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_D - P = 0$$

$$P = F_D = 135.0\text{ lb}$$

$P = 135.0$ lb ◀

PROBLEM 8.42

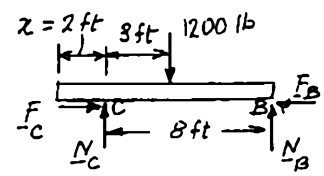


(*a*) Show that the beam of Problem 8.41 *cannot* be moved if the top surface of the dolly is slightly *lower* than the platform. (*b*) Show that the beam *can* be moved if two 175-lb workers stand on the beam at *B* and determine how far to the left the beam can be moved.

PROBLEM 8.41 A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

SOLUTION

(*a*) Beam alone



$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(8\text{ ft}) - (1200\text{ lb})(3\text{ ft}) = 0$$

$$\mathbf{N}_B = 450\text{ lb}\uparrow$$

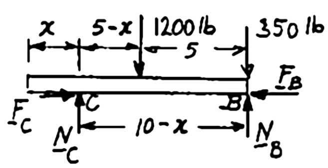
$$+\uparrow F_y = 0: \quad N_C + 450 - 1200 = 0$$

$$\mathbf{N}_C = 750\text{ lb}\uparrow$$

$$(F_C)_m = \mu_s N_C = 0.3(750) = 225\text{ lb}$$
$$(F_B)_m = \mu_s N_B = 0.3(450) = 135\text{ lb}$$

Since $(F_B)_m < (F_C)_m$, sliding first impends at *B*, and Beam cannot be moved ◀

(*b*) Beam with workers standing at *B*



$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(10-x) - (1200)(5-x) - 350(10-x) = 0$$

$$N_B = \frac{9500 - 1550x}{10-x}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (1200)(5) - N_C(10-x) = 0$$

$$N_C = \frac{6000}{10-x}$$

PROBLEM 8.42 (Continued)

Check that beam starts moving for $x = 2$ ft:

For $x = 2$ ft: $$N_B = \frac{9500 - 1550(2)}{10 - 2} = 800 \text{ lb}$$

$$N_C = \frac{6000}{10 - 2} = 750 \text{ lb}$$

$$(F_C)_m = \mu_s N_C = 0.3(750) = 225 \text{ lb}$$

$$(F_B)_m = \mu_s N_B = 0.3(800) = 240 \text{ lb}$$

Since $(F_C)_m < (F_B)_m$, sliding first impends at C, Beam moves ◀

How far does beam move?

Beam will stop moving when

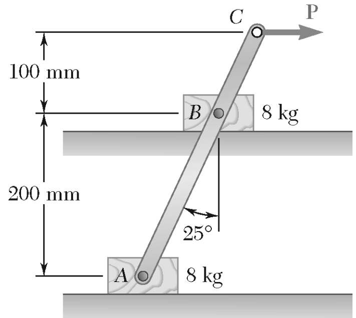
$$F_C = (F_B)_m$$

But $$F_C = \mu_k N_C = 0.25 \frac{6000}{10 - x} = \frac{1500}{10 - x}$$

and $$(F_B)_m = \mu_s N_B = 0.30 \frac{9500 - 1550x}{10 - x} = \frac{2850 - 465x}{10 - x}$$

Setting $F_C = (F_B)_m$: $1500 = 2850 - 465x$ $x = 2.90 \text{ ft}$ ◀

(*Note:* We have assumed that, once started, motion is continuous and uniform (no acceleration).)

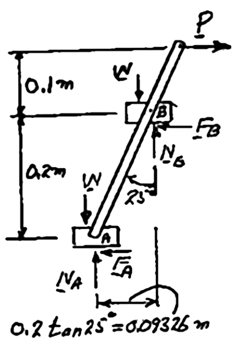


PROBLEM 8.43

Two 8-kg blocks A and B resting on shelves are connected by a rod of negligible mass. Knowing that the magnitude of a horizontal force **P** applied at C is slowly increased from zero, determine the value of P for which motion occurs, and what that motion is, when the coefficient of static friction between all surfaces is (*a*) $\mu_s = 0.40$, (*b*) $\mu_s = 0.50$.

SOLUTION

(*a*) $\underline{\mu_s = 0.40}$: Assume blocks slide to right.



$$W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$$
$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

$+\uparrow \Sigma F_y = 0$: $N_A + N_B - 2W = 0$

$$N_A + N_B = 2W$$

$\xrightarrow{+} \Sigma F_x = 0$: $P - F_A - F_B = 0$

$$P = F_A + F_B = \mu_s(N_A + N_B) = \mu_s(2W) \qquad (1)$$
$$P = 0.40(2)(78.48\text{ N}) = 62.78\text{ N}$$

$+\circlearrowright \Sigma M_B = 0$: $P(0.1\text{ m}) - (N_A - W)(0.09326\text{ m}) + F_A(0.2\text{ m}) = 0$

$$(62.78)(0.1) - (N_A - 78.48)(0.09326) + (0.4)(N_B)(0.2) = 0$$
$$0.17326 N_A = 1.041$$
$$N_A = 6.01\text{ N} > 0 \quad \text{OK}$$

<u>System slides</u>: $P = 62.8\text{ N}$ ◀

(*b*) $\underline{\mu_s = 0.50}$: See part *a*.

Eq. (1): $P = 0.5(2)(78.48\text{ N}) = 78.48\text{ N}$

$+\circlearrowright \Sigma M_B = 0$: $P(0.1\text{ m}) + (N_A - W)(0.09326\text{ m}) + F_A(0.2\text{ m}) = 0$

$$(78.48)(0.1) + (N_A - 78.48)(0.09326) + (0.5)N_A(0.2) = 0$$
$$0.19326 N_A = -0.529$$
$$N_A = -2.73\text{ N} < 0 \text{ uplift, rotation about } B$$

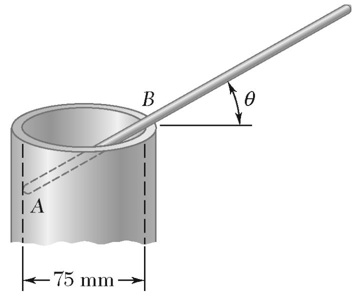
PROBLEM 8.43 (Continued)

For $N_A = 0$: $\circlearrowright \Sigma M_B = 0$: $P(0.1\text{ m}) - W(0.09326\text{ m}) = 0$

$$P = (78.48\text{ N})(0.09326\text{ m})/(0.1) = 73.19$$

System rotates about B: $P = 73.2\text{ N}$ ◀

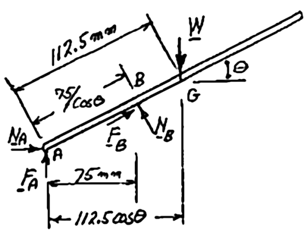
PROBLEM 8.44



A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends down at *A* and to left at *B*.



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta + F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta + \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta - \mu_s \cos\theta) \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + N_B \cos\theta + F_B \sin\theta - W = 0$$

$$\mu_s N_A + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0 \qquad (2)$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$\mu_s N_B(\sin\theta - \mu_s \cos\theta) + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta + 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta + 2(0.2)\sin\theta} \qquad (3)$$

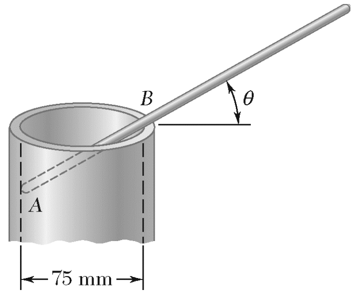
$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel *W*, and simplify to find

$$9.6\cos^3\theta + 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 + \tan\theta) = 1.6667$$

Solve by trial & error: $\theta = 35.8°$ ◀



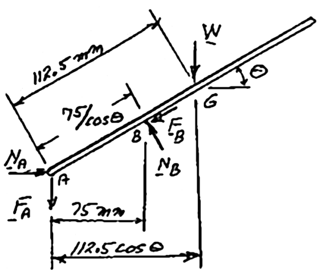
PROBLEM 8.45

In Problem 8.44, determine the smallest value of θ for which the rod will not fall out the pipe.

PROBLEM 8.44 A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends up at A and right at B.



$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta - F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta - \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_s \cos\theta) \tag{1}$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_A + N_B \cos\theta - F_B \sin\theta - W = 0$$

$$-\mu_s N_A + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0 \tag{2}$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$-\mu_s N_B(\sin\theta + \mu_s \cos\theta) + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta - 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta - 2(0.2)\sin\theta} \tag{3}$$

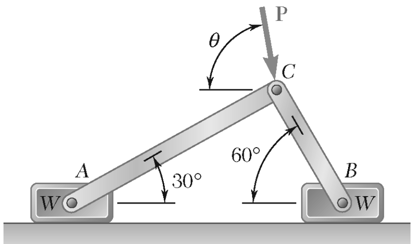
$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel W, and simplify to find

$$9.6\cos^3\theta - 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 - \tan\theta) = 1.6667$$

Solve by trial + error: $\theta = 20.5°$ ◀

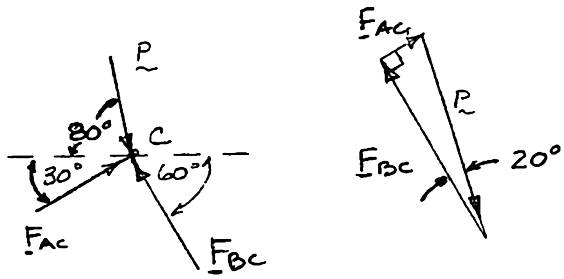


PROBLEM 8.46

Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:

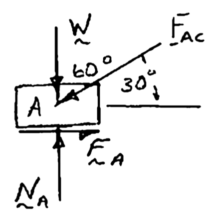


$$F_{AC} = P\sin 20° = 0.34202P$$
$$F_{BC} = P\cos 20° = 0.93969P$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W - F_{AC}\sin 30° = 0$$

or
$$N_A = W + 0.34202P\sin 30° = W + 0.171010P$$

FBD block A:
$$\rightarrow \Sigma F_x = 0: \quad F_A - F_{AC}\cos 30° = 0$$



or
$$F_A = 0.34202P\cos 30° = 0.29620P$$

For impending motion at A:
$$F_A = \mu_s N_A$$

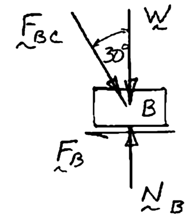
Then
$$N_A = \frac{F_A}{\mu_s}: \quad W + 0.171010P = \frac{0.29620}{0.3}P$$

or
$$P = 1.22500W$$

$$\uparrow \Sigma F_y = 0: \quad N_B - W - F_{BC}\cos 30° = 0$$
$$N_B = W + 0.93969P\cos 30° = W + 0.81380P$$

$$\rightarrow \Sigma F_x = 0: \quad F_{BC}\sin 30° - F_B = 0$$
$$F_B = 0.93969P\sin 30° = 0.46985P$$

FBD block *B*:

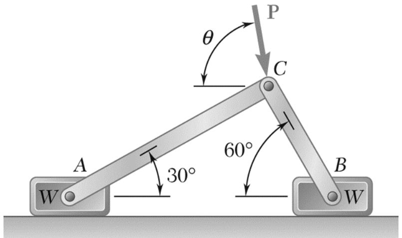


For impending motion at *B*: $F_B = \mu_s N_B$

Then $N_B = \dfrac{F_B}{\mu_s}$: $W + 0.81380P = \dfrac{0.46985P}{0.3}$

or $P = 1.32914W$

Thus, maximum *P* for equilibrium $P_{max} = 1.225W$ ◀



PROBLEM 8.47

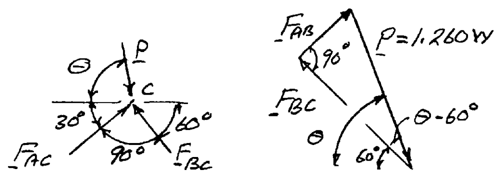
Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $P = 1.260W$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the range of values of θ, between 0 and 180°, for which equilibrium is maintained.

SOLUTION

AC and BC are two-force members

Free body: Joint C

Force triangle:



From Force triangle:

$$F_{AB} = P\sin(\theta - 60°) = 1.26W\sin(\theta - 60°) \quad (1)$$

$$F_{BC} = P\cos(\theta - 60°) = 1.26W\cos(\theta - 60°) \quad (2)$$

We shall, in turn, seek θ corresponding to impending motion of each block

For motion of A impending to left

from solution of Prob. 8.46: $F_{AC} = 0.419W$

EQ (1):

$$F_{AC} = 0.419W = 1.26W\sin(\theta - 60°)$$

$$\sin(\theta - 60°) = 0.33254$$

$$\theta - 60° = 19.423°$$

$$\theta = 79.42° \quad \triangleleft$$

For motion of B impending to right.

from solution of Prob. 8.46: $F_{BC} = 1.249W$

Eq. (2):

$$F_{BC} = 1.249W = 1.26W\cos(\theta - 60°)$$

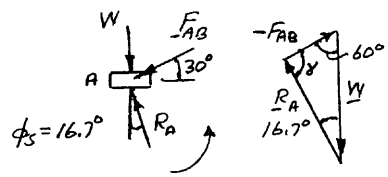
$$\cos(\theta - 60°) = 0.99127$$

$$\theta - 60° = \pm 7.58°$$

$$\theta - 60° = +7.58° \qquad \theta = 67.6° \quad \triangleleft$$

$$\theta - 60° = -7.58° \qquad \theta = 52.4° \quad \triangleleft$$

PROBLEM 8.47 (Continued)



<u>For motion of *A* impending to right</u>

$$\gamma = 180° - 60° - 16.7° = 103.3°$$

<u>Law of sines</u>:

$$\frac{-F_{AB}}{\sin 16.7°} = \frac{W}{\sin 103.3°}$$

$$F_{AB} = -0.29528W$$

Note: Direction of $+F_{AB}$ is kept same as in free body of Joint *C*.

Eq. (1):

$$F_{AB} = -0.29528W = 1.26W\sin(\theta - 60°)$$

$$\sin(\theta - 60°) = -0.23435$$

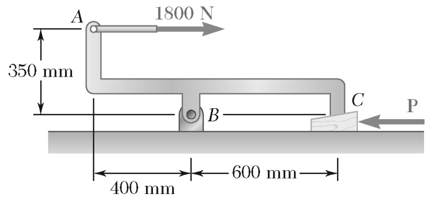
$$(\theta - 60°) = -13.553°$$

$\theta = 46.4°$ ◁

<u>Summary:</u>

A moves to right	No motion	B moves to right	No motion	A moves to left
46.4°	52.4°	67.6°	79.4°	θ

<u>No motion for</u>: $46.4° \le \theta \le 52.4°$ and $67.6° \le \theta \le 79.4°$ ◀



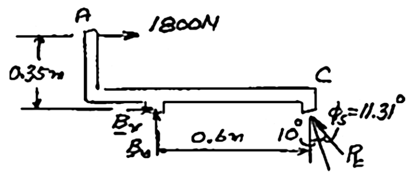
PROBLEM 8.48

The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (*a*) the force **P** required to move the wedge, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

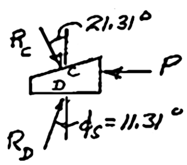
Free body: Part *ABC*



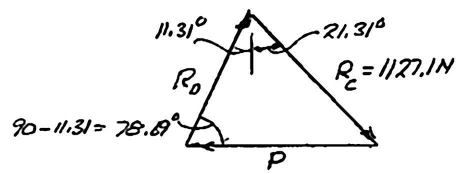
$$+\circlearrowleft \Sigma M_B = 0 \quad (1800 \text{ N})(0.35 \text{ m}) - R\cos 21.31°(0.6 \text{ m}) = 0$$

$$R_C = 1127.1 \text{ N}$$

Free body: Wedge



Force triangle:



(*a*) Law of sines:

$$\frac{P}{\sin(11.31° + 21.31°)} = \frac{1127.1 \text{ N}}{\sin 78.69°}$$

$$P = 619.6 \text{ N} \qquad \mathbf{P} = 620 \text{ N} \longleftarrow ◀$$

(*b*) Return to part *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 1800 \text{ N} - R_C \sin 21.31° = 0$$

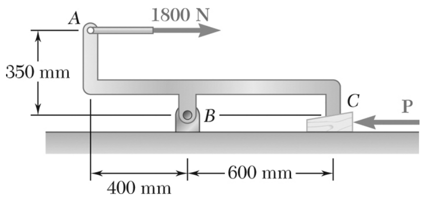
$$B_x + 1800 \text{ N} - (1127.1 \text{ N})\sin 21.31°$$

$$B_x = -1390.4 \text{ N} \qquad \mathbf{B}_x = 1390 \text{ N} \longleftarrow ◀$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + R_C \cos 21.31° = 0$$

$$B_y + (1127.1 \text{ N})\cos 21.31° = 0$$

$$B_y = -1050 \text{ N} \qquad \mathbf{B}_y = 1050 \text{ N} \downarrow ◀$$



PROBLEM 8.49

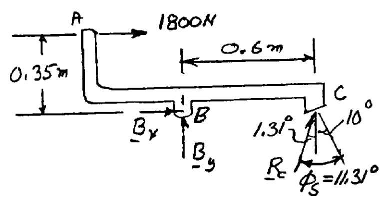
Solve Problem 8.48 assuming that the force **P** is directed to the right.

PROBLEM 8.48 The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (*a*) the force **P** required to move the wedge, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

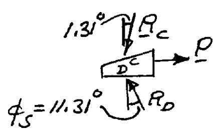
Free body: Part *ABC*



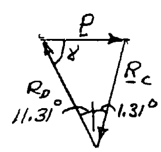
$$+\circlearrowright \Sigma M_B = 0: \quad (1800 \text{ N})(0.35 \text{ m}) - R_C \cos 1.31°(0.6 \text{ m}) = 0$$

$$R_C = 1050.3 \text{ N}$$

Free body: Wedge



Force triangle:



$$\gamma = 90° - 11.31° = 78.69°$$

(*a*) Law of sines:

$$\frac{P}{\sin(11.31° + 1.31°)} = \frac{1050.3 \text{ N}}{\sin 78.69°}$$

$$P = 234 \text{ N} \qquad \mathbf{P} = 234 \text{ N} \longrightarrow \blacktriangleleft$$

(*b*) Return to Part *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 1800 \text{ N} + R_C \sin 1.31° = 0$$

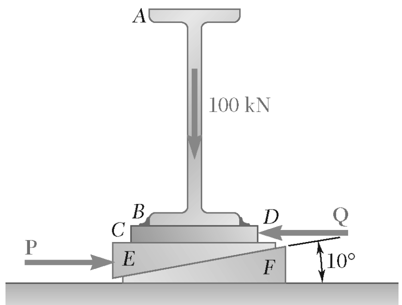
$$B_x + 1800 \text{ N} + (1050.3 \text{ N}) \sin 1.31° = 0$$

$$B_x = -1824 \text{ N} \qquad \mathbf{B}_x = 1824 \text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + R_C \cos 1.31° = 0$$

$$B_y + (1050.3 \text{ N}) \cos 1.31° = 0$$

$$B_y = -1050 \text{ N} \qquad \mathbf{B}_y = 1050 \text{ N} \downarrow \blacktriangleleft$$

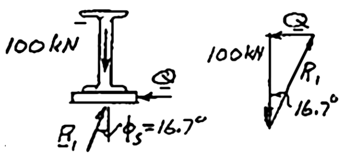


PROBLEM 8.50

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

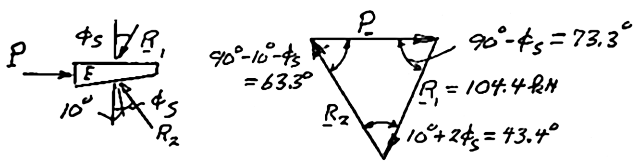
SOLUTION

Free body: Beam and plate *CD*



$$R_1 = \frac{(100\text{ kN})}{\cos 16.7°}$$

$$R_1 = 104.4\text{ kN}$$



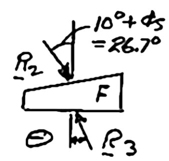
(*a*) $$\frac{P}{\sin 43.4°} = \frac{104.4\text{ kN}}{\sin 63.3°}$$ $\mathbf{P} = 80.3\text{ kN} \longrightarrow$ ◀

(*b*) $$\phi_s = \tan^{-1} 0.3 = 16.7°$$

$$Q = (100\text{ kN})\tan 16.7°$$ $\mathbf{Q} = 30\text{ kN} \longleftarrow$ ◀

Free body: Wedge *F*

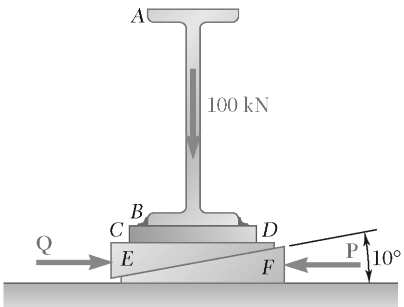
(To check that it does not move.)



Since wedge *F* is a two-force body, R_2 and R_3 are colinear

Thus $$\theta = 26.7°$$

But $$\phi_{\text{concrete}} = \tan^{-1} 0.6 = 31.0° > \theta \quad \text{OK}$$

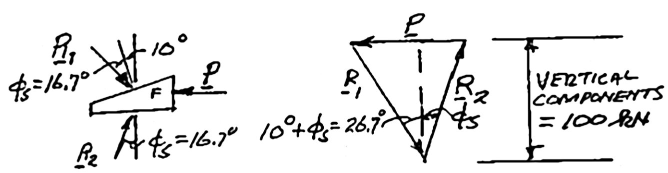


PROBLEM 8.51

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

<u>Free body: Wedge *F*</u> $\phi_s = \tan^{-1} 0.30 = 16.7°$

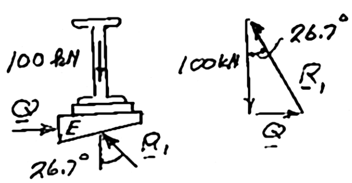


(*a*)
$$P = (100 \text{ kN}) \tan 26.7° + (100 \text{ kN}) \tan \phi_s$$
$$P = 50.29 \text{ kN} + 30 \text{ kN}$$
$$P = 80.29 \text{ kN}$$

P = 80.3 kN ← ◀

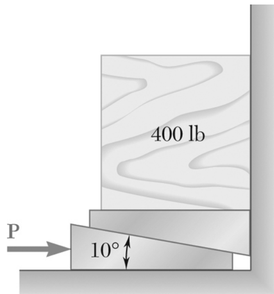
$$R_1 = \frac{(100 \text{ kN})}{\cos 26.7°} = 111.94 \text{ kN}$$

<u>Free body: Beam, plate, and wedge *E*</u>



(*b*)
$$Q = W \tan 26.7° = (100 \text{ kN}) \tan 26.7°$$
$$Q = 50.29 \text{ kN}$$

Q = 50.3 kN → ◀



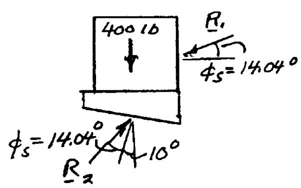
PROBLEM 8.52

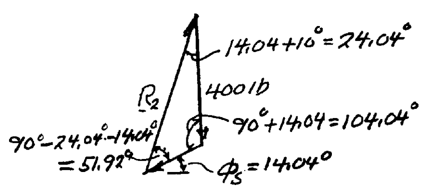
Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that the coefficient of static friction is 0.25 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

<u>Free body: Block and top wedge</u> $\qquad \phi_s = \tan^{-1} 0.25 = 14.04°$

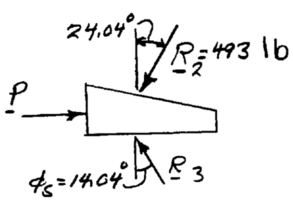
<u>Force triangle</u>

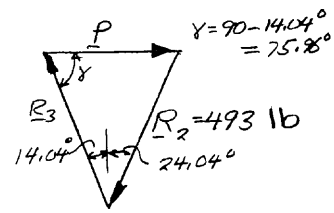




<u>Law of sines:</u>
$$\frac{R_2}{\sin 104.04°} = \frac{400 \text{ lb}}{\sin 51.92°}$$
$$R_2 = 493 \text{ lb}$$

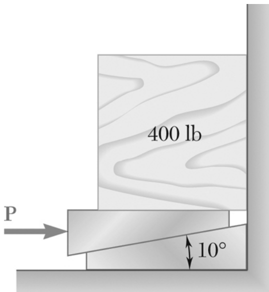
<u>Free body: Lower wedge</u> $\qquad$ <u>Force triangle</u>





<u>Law of sine</u>:
$$\frac{P}{\sin(14.04° + 24.04°)} = \frac{493 \text{ lb}}{\sin 75.96°}$$
$$P = 313.4 \text{ N}$$

P = 313 lb ⟶ ◀



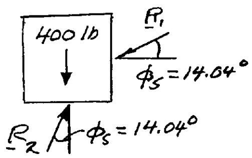
PROBLEM 8.53

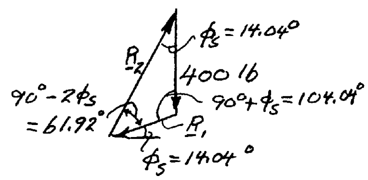
Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that the coefficient of static friction is 0.25 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

Free body: Block $\qquad \phi_s = \tan 0.25 = 14.04°$

Force triangle



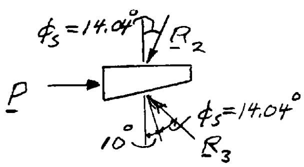


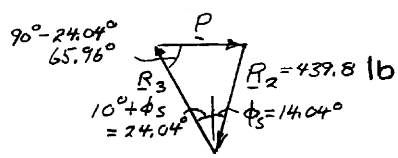
Law of sines:

$$\frac{R_2}{\sin 104.04°} = \frac{400 \text{ lb}}{\sin 61.92°}$$

$$R_2 = 439.8 \text{ lb}$$

Free body: Wedge $\qquad$ Force triangle



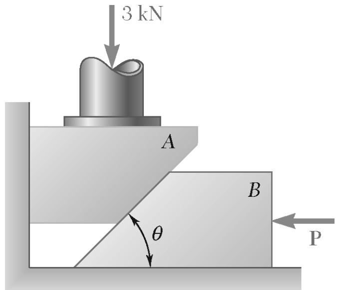


Law of sines:

$$\frac{P}{\sin(24.04° + 14.04°)} = \frac{439.8 \text{ lb}}{\sin 65.96°}$$

$$P = 297.0 \text{ lb}$$

$P = 297 \text{ lb} \longrightarrow$ ◀



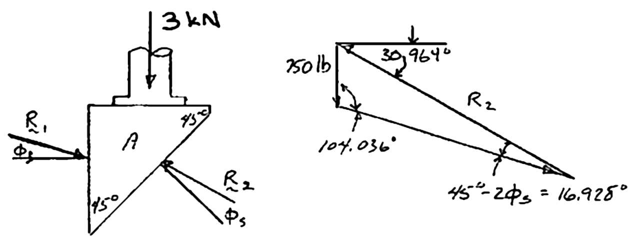
PROBLEM 8.54

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force $\mathbf{P}$ required to raise block A.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

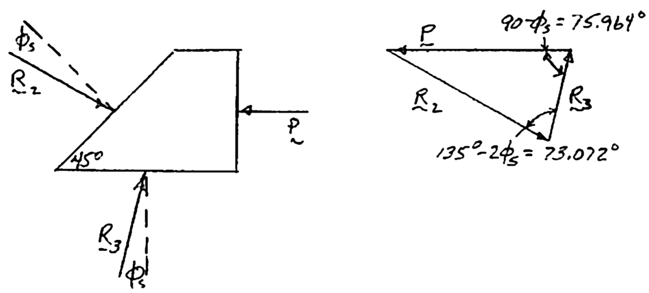
FBD block A:



$$\frac{R_2}{\sin 104.036°} = \frac{3\text{ kN}}{\sin 16.928°}$$

$$R_2 = 10.0000\text{ kN}$$

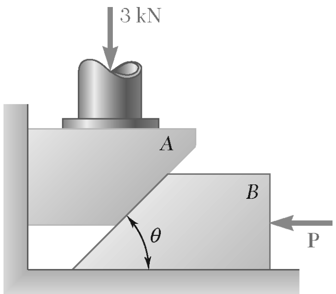
FBD wedge B:



$$\frac{P}{\sin 73.072°} = \frac{10.0000\text{ kN}}{\sin 75.964°}$$

$$P = 9.8611\text{ kN}$$

$\mathbf{P} = 9.86\text{ kN} \longleftarrow$ ◀



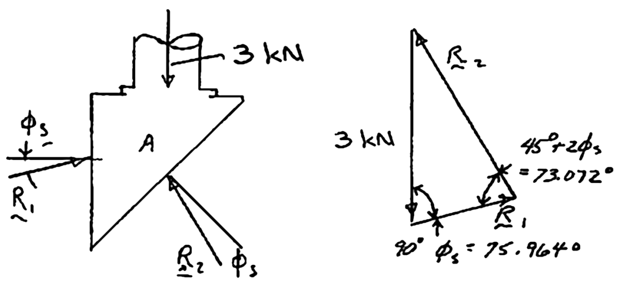
PROBLEM 8.55

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

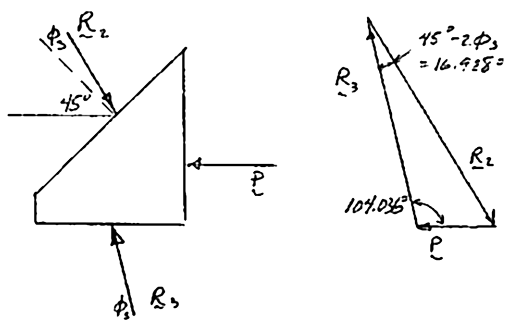
FBD block A:



$$\frac{R_2}{\sin(75.964°)} = \frac{3\text{ kN}}{\sin(73.072°)}$$

$$R_2 = 3.0420\text{ kN}$$

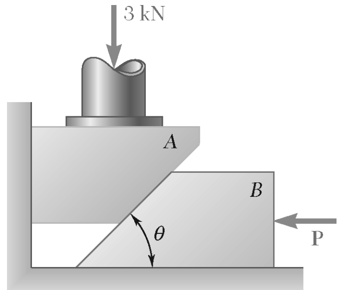
FBD wedge B:



$$\frac{P}{\sin 16.928°} = \frac{3.0420\text{ kN}}{\sin 104.036°}$$

$$P = 0.91300\text{ kN}$$

P = 913 N ← ◀



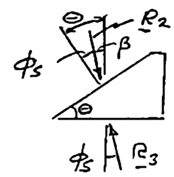
PROBLEM 8.56

Block A supports a pipe column and rests as shown on wedge B. The coefficient of static friction at all surfaces of contact is 0.25. If $\mathbf{P} = 0$, determine (a) the angle θ for which sliding is impending, (b) the corresponding force exerted on the block by the vertical wall.

SOLUTION

Free body: Wedge B $\qquad \phi_s = \tan^{-1} 0.25 = 14.04°$

(a) Since wedge is a two-force body, $\mathbf{R}_2$ and $\mathbf{R}_3$ must be equal and opposite. Therefore, they form equal angles with vertical



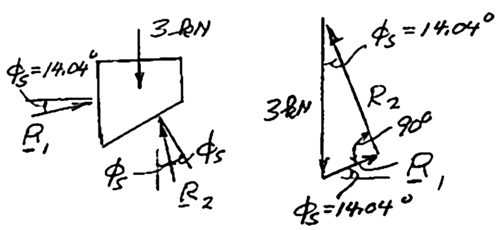
$$\beta = \phi_s$$

and

$$\theta - \phi_s = \phi_s$$
$$\theta = 2\phi_s = 2(14.04°)$$

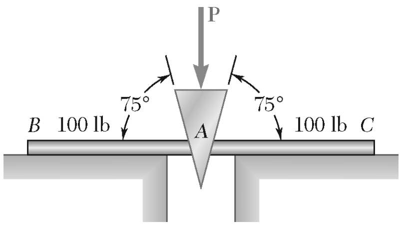
$\theta = 28.1°$ ◀

Free body: Block A



$$R_1 = (3\text{ kN})\sin 14.04° = 0.7278\text{ kN}$$

(b) Force exerted by wall: $\qquad \mathbf{R}_1 = 728\text{ N} \measuredangle 14.04°$ ◀



PROBLEM 8.57

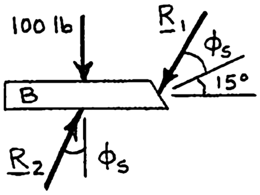
A wedge A of negligible weight is to be driven between two 100-lb plates B and C. The coefficient of static friction between all surfaces of contact is 0.35. Determine the magnitude of the force **P** required to start moving the wedge (a) if the plates are equally free to move, (b) if plate C is securely bolted to the surface.

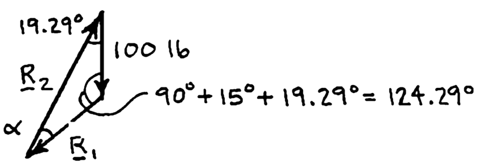
SOLUTION

(a) <u>With plates equally free to move</u>

<u>Free body: Plate B</u> $\qquad \phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.35 = 19.2900°$

Force triangle:





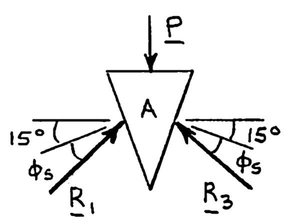
$$\alpha = 180° - 124.29° - 19.29° = 36.42°$$

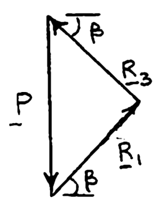
Law of sines: $$\frac{R_1}{\sin 19.29°} = \frac{100\text{ lb}}{\sin 36.42°}$$

$$R_1 = 55.643\text{ lb}$$

<u>Free body: Wedge A</u>

Force triangle:





By symmetry, $\qquad R_3 = R_1 = 55.643\text{ lb}$

$$\beta = 19.29° + 15° = 34.29°$$

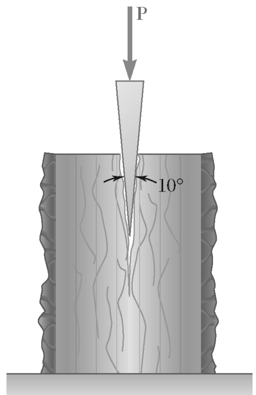
Then $\qquad P = 2R_1 \sin\beta$

or $\qquad P = 2(55.643)\sin 34.29° \qquad P = 62.7\text{ lb}$ ◀

(b) <u>With plate C bolted</u>

The free body diagrams of plate B and wedge A (the only members to move) are same as above. Answer is thus the same.

$P = 62.7\text{ lb}$ ◀

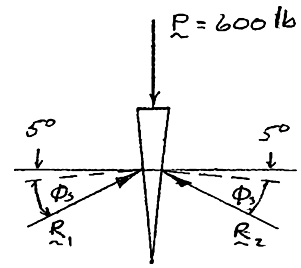


PROBLEM 8.58

A 10° wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force **P** of magnitude 600 lb was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

SOLUTION

FBD wedge (impending motion ↓):



$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1} 0.35$$
$$= 19.29°$$

By symmetry: $R_1 = R_2$

$$\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(5° + \phi_s) - 600 \text{ lb} = 0$$

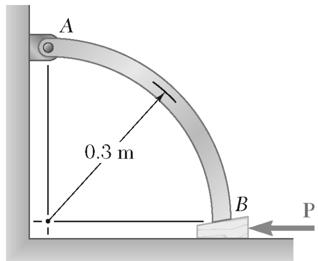
or
$$R_1 = R_2 = \frac{300 \text{ lb}}{\sin(5° + 19.29°)} = 729.30 \text{ lb}$$

When P is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components

$$R_{1x} = R_{2x}$$
$$= R_1 \cos(5° + \phi_s)$$
$$= (729.30 \text{ lb})\cos(5° + 19.29°)$$

$R_{1x} = R_{2x} = 665 \text{ lb}$ ◀

(Note that $\phi_s > 5°$, so wedge is self-locking.)

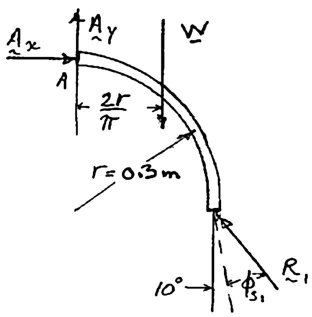


PROBLEM 8.59

A 10° wedge is to be forced under end *B* of the 5-kg rod *AB*. Knowing that the coefficient of static friction is 0.40 between the wedge and the rod and 0.20 between the wedge and the floor, determine the smallest force **P** required to raise end *B* of the rod.

SOLUTION

FBD *AB*:



$$W = mg$$

$$W = (5\text{ kg})(9.81\text{ m/s}^2)$$

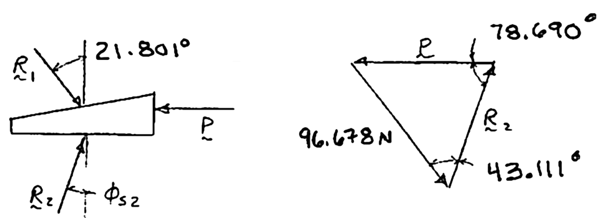
$$W = 49.050\text{ N}$$

$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.40 = 21.801°$$

$$\circlearrowleft \Sigma M_A = 0: \quad rR_1 \cos(10° + 21.801°) - rR_1 \sin(10° + 21.801°) - \frac{2r}{\pi}(49.050\text{ N}) = 0$$

$$R_1 = 96.678\text{ N}$$

FBD wedge:

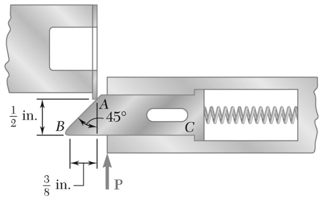


$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.20 = 11.3099$$

$$\frac{P}{\sin(43.111°)} = \frac{96.678\text{ N}}{\sin 78.690°}$$

P = 67.4 N ← ◀

PROBLEM 8.60



The spring of the door latch has a constant of 1.8 lb/in. and in the position shown exerts a 0.6-lb force on the bolt. The coefficient of static friction between the bolt and the strike plate is 0.40; all other surfaces are well lubricated and may be assumed frictionless. Determine the magnitude of the force **P** required to start closing the door.

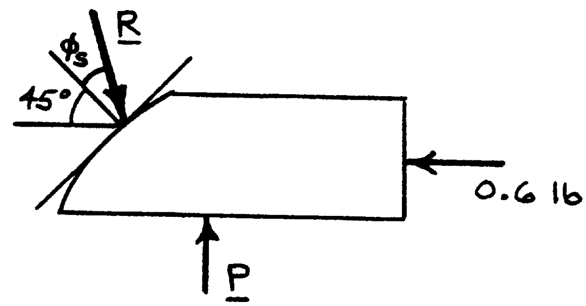
SOLUTION

Free body: Bolt

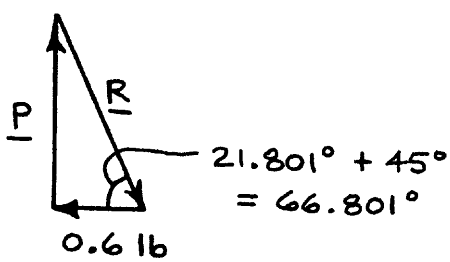
$$\mu_s = 0.40$$

$$\phi_s = \tan^{-1} 0.40$$

$$= 21.801°$$



Force triangle:



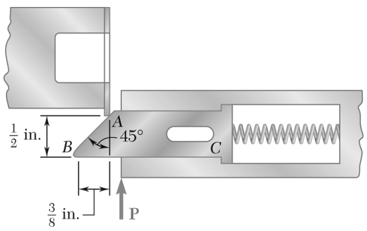
From force triangle,

$$P = (0.6 \text{ lb}) \tan 66.801°$$

$$P = 1.39997 \text{ lb}$$

$P = 1.400$ lb ◀

PROBLEM 8.61



In Problem 8.60, determine the angle that the face of the bolt should form with the line *BC* if the force **P** required to close the door is to be the same for both the position shown and the position when *B* is almost at the strike plate.

PROBLEM 8.60 The spring of the door latch has a constant of 1.8 lb/in. and in the position shown exerts a 0.6-lb force on the bolt. The coefficient of static friction between the bolt and the strike plate is 0.40; all other surfaces are well lubricated and may be assumed frictionless. Determine the magnitude of the force **P** required to start closing the door.

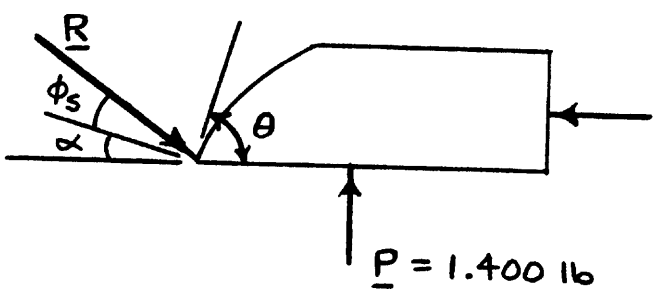
SOLUTION

<u>For position shown in figure</u>:

From Prob. 8.60: $P = 1.400$ lb

<u>For position when *B* reaches strike plate</u>:

<u>Free body: Bolt</u>



$$\mu_s = 0.40$$

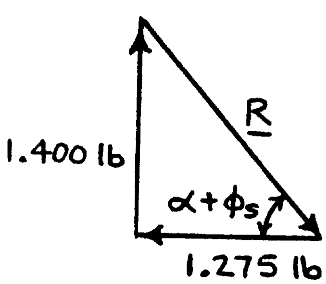
$$\phi_s = \tan^{-1} 0.40 = 21.801°$$

$$F = 0.6 \text{ lb} + kx$$

$$= 0.6 \text{ lb} + (1.8 \text{ lb/in.})\left(\frac{3}{8} \text{ in.}\right)$$

$$= 1.275 \text{ lb}$$

<u>Force triangle</u>:



From force triangle,

$$\tan(\alpha - \phi_s) = \frac{1.400 \text{ lb}}{1.275 \text{ lb}} = 1.09804$$

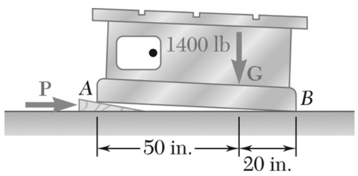
$$\alpha + \phi_s = 47.675°$$

$$\alpha = 47.675° - 21.801° = 25.874°$$

$$\theta = 180° - 90° - \alpha = 90° - \alpha = 90° - 25.874°$$

$\theta = 64.1°$ ◀

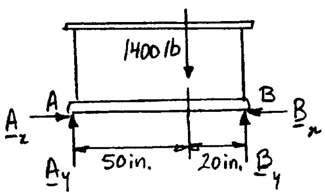
PROBLEM 8.62



A 5° wedge is to be forced under a 1400-lb machine base at A. Knowing that the coefficient of static friction at all surfaces is 0.20, (a) determine the force **P** required to move the wedge, (b) indicate whether the machine base will move.

SOLUTION

Free body: Machine base



$$+\circlearrowleft \Sigma M_B = 0: \quad (1400\text{ lb})(20\text{ in}) - A_y(70\text{ in}) = 0$$

$$A_y = 400\text{ lb}$$

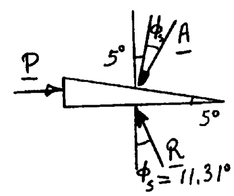
$$\xrightarrow{+} \Sigma F_y = 0: \quad A_y + B_y - 1400\text{ lb} = 0$$

$$B_y = 1400 - 400 = 1000\text{ lb}$$

Free body: Wedge

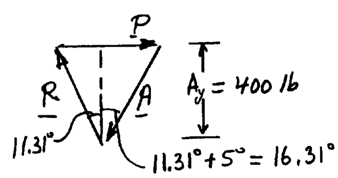
(Assume machine will not move)

$$\mu_s = 0.20, \quad \phi_s = \tan 0.20 = 11.31°$$



We know that $\qquad A_y = 400\text{ lb}$

Force triangle:



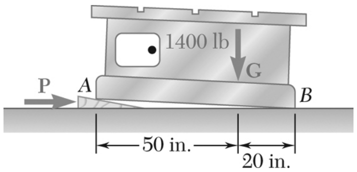
(a) $\qquad P = (400\text{ lb})(\tan 11.31° + \tan 16.31°) = 197.0\text{ lb}$ $\qquad$ $\mathbf{P} = 197.0\text{ lb} \longrightarrow$ ◀

(b) Total maximum friction force at A and B:

$$F_m = \mu_c W = 0.20(1400\text{ lb}) = 280\text{ lb}$$

Since $\qquad P < F_m$: $\qquad$ Machine will not move ◀

PROBLEM 8.63



Solve Problem 8.62 assuming that the wedge is to be forced under the machine base at B instead of A.

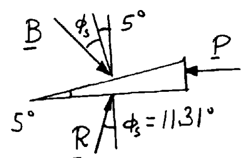
PROBLEM 8.62 A 5° wedge is to be forced under a 1400-lb machine base at A. Knowing that the coefficient of static friction at all surfaces is 0.20, (*a*) determine the force **P** required to move the wedge, (*b*) indicate whether the machine base will move.

SOLUTION

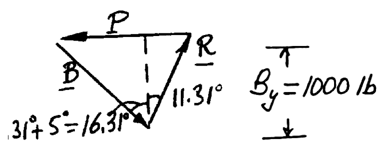
See solution to Prob. 8.62 for F.B.D. of machine base and determination of $B_y = 1000$ lb.

<u>Free body: Wedge</u> (Assume machine will not move)

$$\mu_s = 0.20 \qquad \phi_s = \tan^{-1} 0.20 = 11.31°$$



<u>Force triangle</u>:



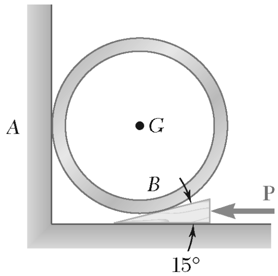
$$P = (1000 \text{ lb})(\tan 11.31° + \tan 16.31°) = 493 \text{ lb}$$

Total maximum friction force at A and B:

$$F_m = \mu_s W = 0.20(1400 \text{ lb}) = 280 \text{ lb} < 493 \text{ lb}$$

(*b*) Since $P > F_m$, <u>machine will move with wedge</u> ◀

(*a*) We then have $P = F_m$, **P** = 280 lb ⟵ ◀



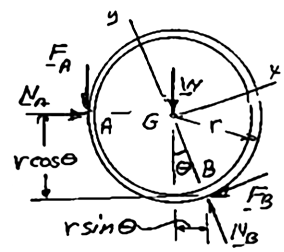
PROBLEM 8.64

A 15° wedge is forced under a 50-kg pipe as shown. The coefficient of static friction at all surfaces is 0.20. (*a*) Show that slipping will occur between the pipe and the vertical wall. (*b*) Determine the force **P** required to move the wedge.

SOLUTION

Free body: Pipe

$$+\circlearrowleft \Sigma M_B = 0: \quad Wr\sin\theta + F_A r(1+\sin\theta) - N_A r\cos\theta = 0$$



Assume slipping at *A*:

$$F_A = \mu_s N_A$$

$$N_A\cos\theta - \mu_s N_A(1+\sin\theta) = W\sin\theta$$

$$N_A = \frac{W\sin\theta}{\cos\theta - \mu_s(1+\sin\theta)}$$

$$N_A = \frac{W\sin 15^\circ}{\cos 15^\circ - (0.20)(1+\sin 15^\circ)}$$

$$= 0.36241W$$

$$+\nearrow \Sigma F_x = 0: \quad -F_B - W\sin\theta - F_A\sin\theta + N_A\cos\theta = 0$$

$$F_B = N_A\cos\theta - \mu_s N_A\sin\theta - W\sin\theta$$

$$F_B = 0.36241W\cos 15^\circ - 0.20(0.36241W)\sin 15^\circ - W\sin 15^\circ$$

$$F_B = 0.072482W$$

$$+\nwarrow \Sigma F_y = 0: \quad N_B - W\cos\theta - F_A\cos\theta - N_A\sin\theta = 0$$

$$N_B = N_A\sin\theta + \mu_s N_A\cos\theta + W\cos\theta$$

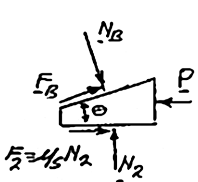
$$N_B = (0.36241W)\sin 15^\circ + 0.20(0.36241W)\cos 15^\circ + W\cos 15^\circ$$

$$N_B = 1.12974W$$

Maximum available: $F_B = \mu_s N_B = 0.22595W$

(*a*) We note that $F_B < F_{\max}$ No slip at *B* ◀

(*b*) Free body: Wedge $\quad +\uparrow \Sigma F_y = 0: \quad N_2 - N_B\cos\theta + F_B\sin\theta = 0$



$$N_2 = N_B\cos\theta - F_B\sin\theta$$

$$N_2 = (1.12974W)\cos 15^\circ - (0.07248W)\sin 15^\circ$$

$$N_2 = 1.07249W$$

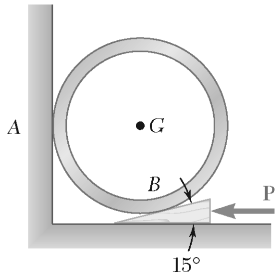
PROBLEM 8.64 (Continued)

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_B \cos\theta + N_B \sin\theta + \mu_s N_2 - P = 0$$

$$P = F_B \cos\theta + N_B \sin\theta + \mu_s N_2$$
$$P = (0.07248W)\cos 15° + (1.12974W)\sin 15° + 0.2(1.07249W)$$
$$P = 0.5769W$$

$$W = mg: \quad P = 0.5769(50 \text{ kg})(9.81 \text{ m/s}^2) \qquad \mathbf{P} = 283 \text{ N} \longleftarrow \blacktriangleleft$$

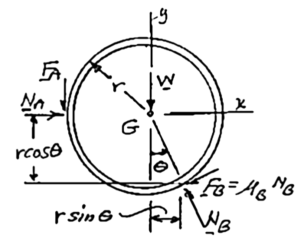
PROBLEM 8.65



A 15° wedge is forced under a 50-kg pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping will occur at A.

SOLUTION

<u>Free body: Pipe</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad N_B r\cos\theta - \mu_B N_B r - (\mu_B N_B \sin\theta)r - Wr = 0$$

$$N_B = \frac{W}{\cos\theta - \mu_B(1+\sin\theta)}$$

$$N_B = \frac{W}{\cos 15° - 0.2(1+\sin 15°)}$$

$$N_B = 1.4002W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta - \mu_B N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_B\cos\theta)$$

$$= (1.4002W)(\sin 15° + 0.2\times\cos 15°)$$

$$N_A = 0.63293W$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - W + N_B\cos\theta - \mu_B N_B \sin\theta = 0$$

$$F_A = N_B(\cos\theta - \mu_B\sin\theta) - W$$

$$F_A = (1.4002W)(\cos 15° - 0.2\times\sin\theta) - W$$

$$F_A = 0.28001W$$

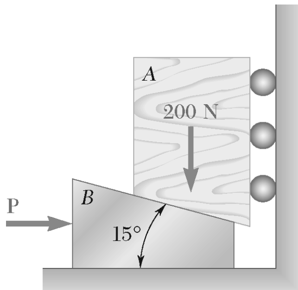
For slipping at A:

$$F_A = \mu_A N_A$$

$$\mu_A = \frac{F_A}{N_A} = \frac{0.28001W}{0.63293W}$$

$\mu_A = 0.442$ ◀

PROBLEM 8.66*

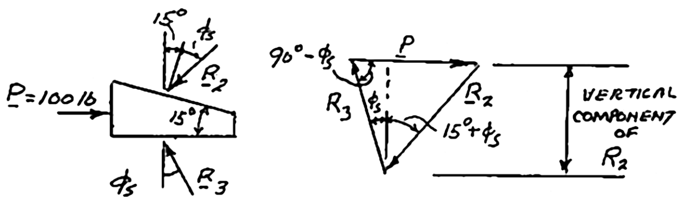


A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For P = 100 N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

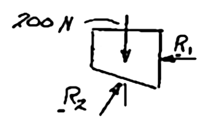
Force triangle:



Law of sines:
$$\frac{R_2}{\sin(90°-\phi_s)} = \frac{P}{\sin(15°+2\phi_s)}$$

$$R_2 = P\frac{\sin(90°-\phi_s)}{\sin(15°+2\phi_s)} \qquad (1)$$

Free body: Block



$$\Sigma F_y = 0$$

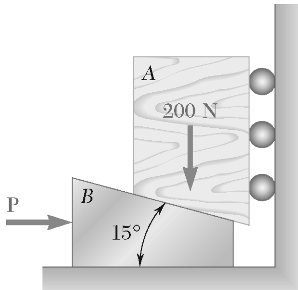
Vertical component of R_2 is 200 N

Return to force triangle of wedge. Note $P = 100$ N

$$100\,\text{N} = (200\,\text{N})\tan\phi + (200\,\text{N})\tan(15° + \phi_s)$$
$$0.5 = \tan\phi + \tan(15° + \phi_s)$$

Solve by trial and error $\qquad \phi_s = 6.292$

$$\mu_s = \tan\phi_s = \tan 6.292° \qquad \mu_s = 0.1103 \blacktriangleleft$$



PROBLEM 8.67*

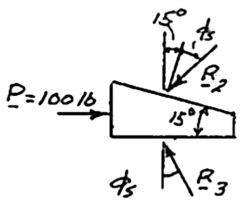
Solve Problem 8.66 assuming that the rollers are removed and that μ_s is the coefficient of friction at all surfaces of contact.

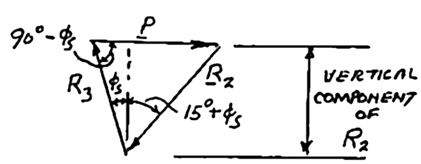
PROBLEM 8.66* A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For P = 100 N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

Force triangle:





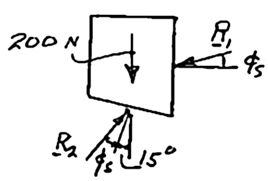
Law of sines:

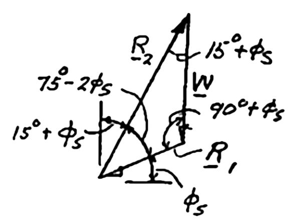
$$\frac{R_2}{\sin(90^\circ - \phi_s)} = \frac{P}{\sin(15^\circ + 2\phi_s)}$$

$$R_2 = P\frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} \tag{1}$$

Free body: Block (Rollers removed)

Force triangle:





Law of sines:

$$\frac{R_2}{\sin(90^\circ + \phi_s)} = \frac{W}{\sin(75^\circ - 2\phi_s)}$$

$$R_2 = W\frac{\sin(90^\circ + \phi_s)}{\sin(75^\circ - 2\theta_s)} \tag{2}$$

Equate R_2 from Eq. (1) and Eq. (2):

$$P\frac{\sin(90°-\phi_s)}{\sin(15°+2\phi_s)} = W\frac{\sin(90°+\phi_s)}{\sin(75°-2\phi_s)}$$

$$P = 100\text{ lb}$$

$$W = 200\text{ N}$$

$$0.5 = \frac{\sin(90°+\phi_s)\sin(15°+2\phi_s)}{\sin(75°-2\phi_s)\sin(90°-\phi_s)}$$

Solve by trial and error: $\phi_s = 5.784°$

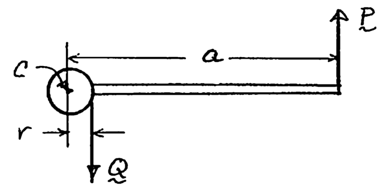
$$\mu_s = \tan\phi_s = \tan 5.784° \qquad \mu_s = 0.1013 ◀$$

PROBLEM 8.68

Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Section 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

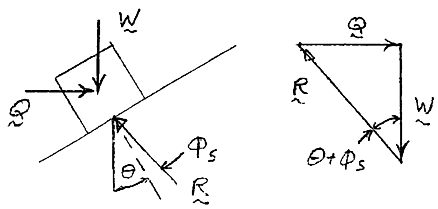
FBD jack handle:



See Section 8.6. $\quad \circlearrowleft \Sigma M_C = 0: \quad aP - rQ = 0 \quad \text{or} \quad P = \frac{r}{a}Q$

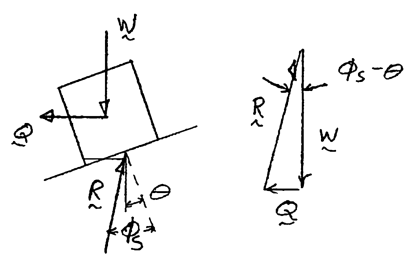
FBD block on incline:

(*a*) Raising load



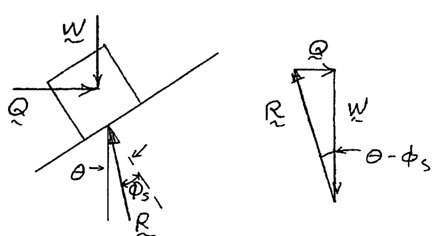
$$Q = W\tan(\theta + \phi_s) \qquad\qquad P = \frac{r}{a}W\tan(\theta + \phi_s) \blacktriangleleft$$

(*b*) Lowering load if screw is self-locking (i.e., if $\phi_s > \theta$)

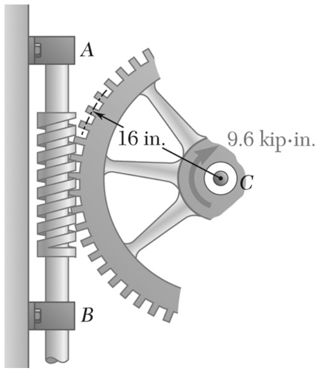


$$Q = W\tan(\phi_s - \theta) \qquad\qquad P = \frac{r}{a}W\tan(\phi_s - \theta) \blacktriangleleft$$

(*c*) Holding load is screw is not self-locking (i.e., if $\phi_s < \theta$)



$$Q = W\tan(\theta - \phi_s) \qquad\qquad P = \frac{r}{a}W\tan(\theta - \phi_s) \blacktriangleleft$$



PROBLEM 8.69

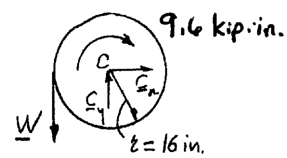
The square-threaded worm gear shown has a mean radius of 2 in. and a lead of 0.5 in. The large gear is subjected to a constant clockwise couple of 9.6 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

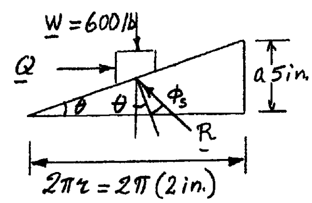
Free body: Large gear

$$+\circlearrowleft \Sigma M_C = 0: \quad W(16 \text{ in.}) - 9.6 \text{ kip} \cdot \text{in.} = 0$$

$$W = 0.6 \text{ kip} = 600 \text{ lb}$$



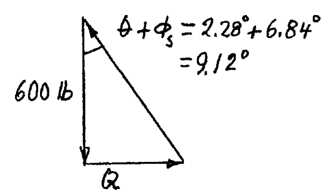
Block-and-Incline analysis of worn gear



$$\tan\theta = \frac{0.5 \text{ in.}}{2\pi(2 \text{ in.})} = 0.039789$$

$$\theta = 2.28°$$

$$\mu_s = 0.12, \ \phi_s = \tan^{-1} 0.12 = 6.84°$$



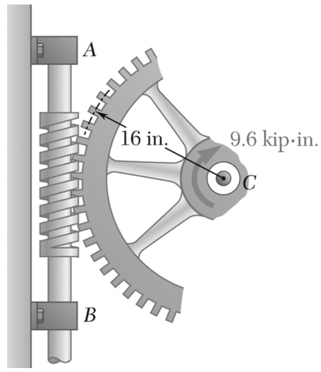
$$Q = (600 \text{ lb}) \tan 9.12°$$

$$= 96.32 \text{ lb}$$

$$\text{Torque} = Q_r = (96.32 \text{ lb})(2 \text{ in.})$$

$$= 192.6 \text{ lb} \cdot \text{in.}$$

$$\text{Torque} = 16.05 \text{ lb} \cdot \text{ft} \blacktriangleleft$$



PROBLEM 8.70

In Problem 8.69, determine the couple that must be applied to shaft *AB* in order to rotate the large gear clockwise.

PROBLEM 8.69 The square-threaded worm gear shown has a mean radius of 2 in. and a lead of 0.5 in. The large gear is subjected to a constant clockwise couple of 9.6 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

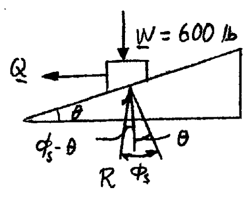
SOLUTION

Free body: Large gear

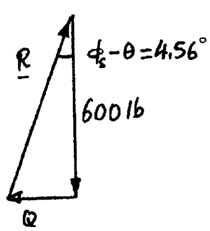
See solution to Prob. 8.69. We find $W = 600$ lb

Block-and-incline analysis of worm gear

From Prob. 8.69 we have $\theta = 2.28°$ and $\phi_s = 6.84°$. Since $\theta < \phi_s$, gear is self-looking and a torque must be applied to it to rotate large gear clockwise.



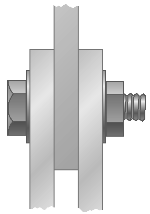
$$\phi_s - \theta = 6.84° - 2.28° = 4.56°$$



$$Q = (600 \text{ lb})\tan 4.56°$$
$$= 47.85 \text{ lb}$$

$$\text{Torque} = Q_r = (47.85 \text{ lb})(2 \text{ in.})$$
$$= 95.70 \text{ lb} \cdot \text{in.}$$

$$\text{Torque} = 7.98 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

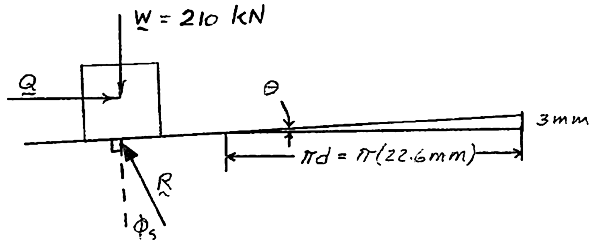


PROBLEM 8.71

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

FBD block on incline:



$$\theta = \tan^{-1}\frac{3\text{ mm}}{(22.6\text{ mm})\pi}$$

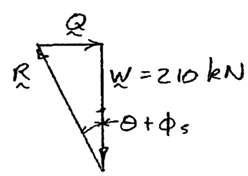
$$= 2.4195°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.40$$

$$\phi_s = 21.801°$$

$$Q = (210\text{ kN})\tan(21.801° + 2.4195°)$$

$$Q = 94.468\text{ kN}$$

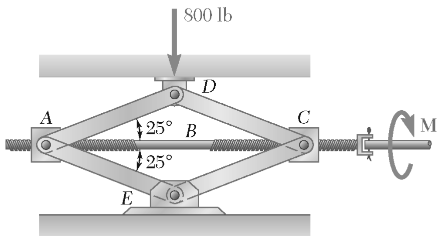


$$\text{Torque} = \frac{d}{2}Q$$

$$= \frac{22.6\text{ mm}}{2}(94.468\text{ kN})$$

$$= 1067.49\text{ N}\cdot\text{m}$$

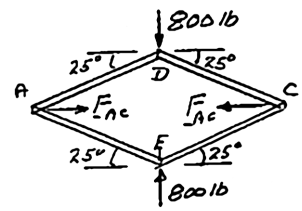
$\text{Torque} = 1068\text{ N}\cdot\text{m}$ ◀



PROBLEM 8.72

The position of the automobile jack shown is controlled by a screw ABC that is single-threaded at each end (right-handed thread at A, left-handed thread at C). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

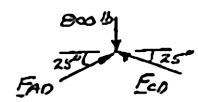
SOLUTION



Free body: Parts A, D, C, E

Two-force members

Joint D:

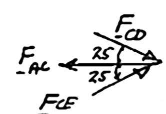


Symmetry: $F_{AD} = F_{CD}$

$$+\uparrow \Sigma F_y = 0: \quad 2F_{CD}\sin 25° - 800 \text{ lb} = 0$$

$$F_{CD} = 946.5 \text{ lb}$$

Joint C:



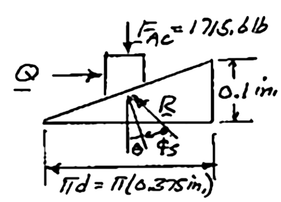
Symmetry: $F_{CE} = F_{CD}$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2F_{CD}\cos 25° - F_{AC} = 0$$

$$F_{AC} = 2(946.5 \text{ lb})\cos 25°$$

$$F_{AC} = 1715.6 \text{ lb}$$

Block-and-incline analysis of *one* screw:

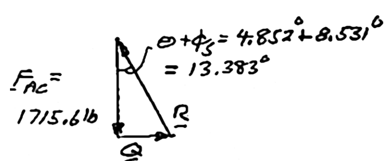


$$\tan\theta = \frac{0.1 \text{ in.}}{\pi(0.375 \text{ in.})}$$

$$\theta = 4.852°$$

$$\phi_s = \tan^{-1} 0.15$$

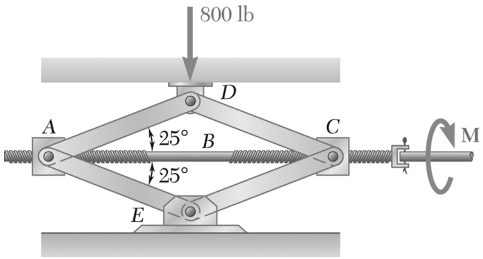
$$= 8.531°$$



$$Q = (1715.6 \text{ lb})\tan 13.383°$$

$$Q = 408.2 \text{ lb}$$

But, we have *two* screws: $\quad \text{Torque} = 2Qr = 2(408.2 \text{ lb})\left(\dfrac{0.375 \text{ in.}}{2}\right)$ $\quad \text{Torque} = 153.1 \text{ lb}\cdot\text{in.}$ ◀



PROBLEM 8.73

For the jack of Problem 8.72, determine the magnitude of the couple **M** that must be applied to lower the automobile.

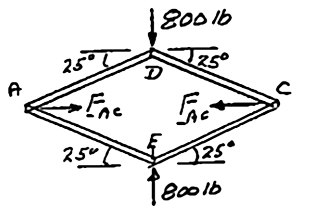
PROBLEM 8.72 The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION

<u>Free body: Parts *A*, *D*, *C*, *E*</u>

Two-force members

<u>Joint *D*:</u>

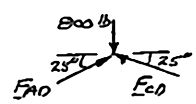


Symmetry: $F_{AD} = F_{CD}$

$$+\uparrow \Sigma F_y = 0: \quad 2F_{CD} \sin 25° - 800 \text{ lb} = 0$$

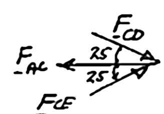
$$F_{CD} = 946.5 \text{ lb}$$

<u>Joint *C*:</u>



Symmetry: $F_{CE} = F_{CD}$

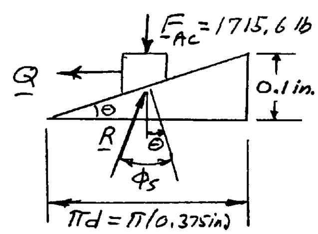
$$\xrightarrow{+} \Sigma F_x = 0: \quad 2F_{CD} \cos 25° - F_{AC} = 0$$



$$F_{AC} = 2(946.5 \text{ lb})\cos 25°;$$

$$F_{AC} = 1715.6 \text{ lb}$$

<u>Block-and-incline analysis of *one* screw:</u>



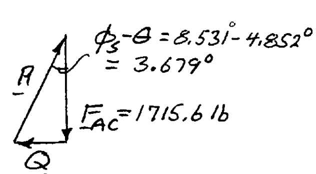
$$\tan\theta = \frac{0.1 \text{ in.}}{\pi(0.375 \text{ in.})}$$

$$\theta = 4.852°$$

$$\phi_s = \tan^{-1} 0.15$$

$$= 8.531°$$

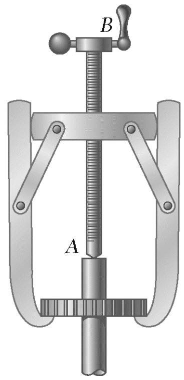
Since $\phi_s > \theta$, the screw is self-locking



$$Q = (1715.6 \text{ lb})\tan 3.679°$$

$$Q = 110.3 \text{ lb}$$

For two screws: $\text{Torque} = 2(110.3 \text{ lb})\frac{1}{2}(0.375 \text{ in.})$ $\qquad$ $\text{Torque} = 41.4 \text{ lb} \cdot \text{in.}$ ◀

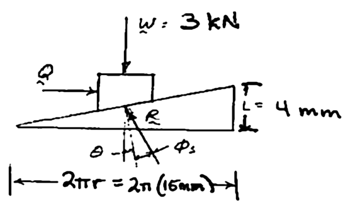


PROBLEM 8.74

In the gear-pulling assembly shown the square-threaded screw AB has a mean radius of 15 mm and a lead of 4 mm. Knowing that the coefficient of static friction is 0.10, determine the couple that must be applied to the screw in order to produce a force of 3 kN on the gear. Neglect friction at end A of the screw.

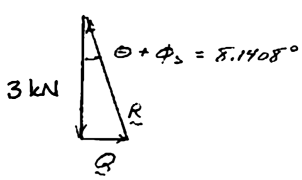
SOLUTION

Block/Incline:



$$\theta = \tan^{-1}\frac{4\text{ mm}}{30\pi\text{ mm}}$$
$$= 2.4302°$$

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.10)$$
$$= 5.7106°$$



$$Q = (3000\text{ N})\tan(8.1408°)$$
$$= 429.14\text{ N}$$

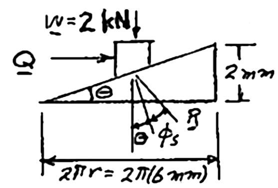
$$\text{Couple} = rQ$$
$$= (0.015\text{ m})(429.14\text{ N})$$
$$= 6.4371\text{ N}\cdot\text{m}$$

$M = 6.44\text{ N}\cdot\text{m}$ ◀

PROBLEM 8.75

The ends of two fixed rods A and B are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod A has a right-handed thread and rod B has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

SOLUTION



To draw rods together:

<u>Screw at A</u>

$$\tan\theta = \frac{2 \text{ mm}}{2\pi(6 \text{ mm})}$$

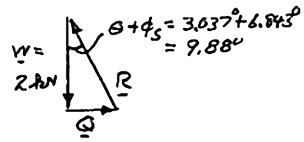
$$\theta = 3.037°$$

$$\phi_s = \tan^{-1} 0.12$$

$$= 6.843°$$

$$Q = (2 \text{ kN}) \tan 9.88°$$

$$= 348.3 \text{ N}$$



$$\text{Torque at } A = Qr$$

$$= (348.3 \text{ N})(0.006 \text{ m})$$

$$= 2.0898 \text{ N} \cdot \text{m}$$

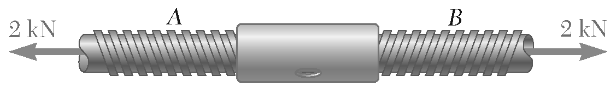
Same torque required at B

Total torque $= 4.18 \text{ N} \cdot \text{m}$ ◀

PROBLEM 8.76

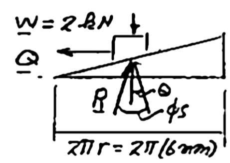
Assuming that in Problem 8.75 a right-handed thread is used on *both* rods A and B, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

PROBLEM 8.75 The ends of two fixed rods A and B are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod A has a right-handed thread and rod B has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.



SOLUTION

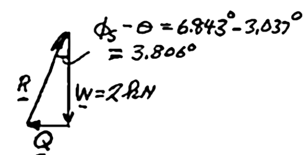
From the solution to Problem 8.70,



$$\text{Torque at } A = 2.09\ \text{N}\cdot\text{m}$$

Screw at B: Loosening

$$\theta = 3.037°$$
$$\phi_s = 6.843°$$
$$Q = (2\ \text{kN})\tan 3.806°$$
$$= 133.1\ \text{N}$$

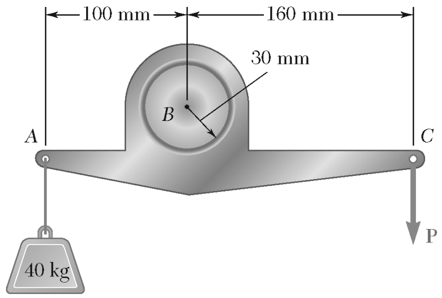


$$\text{Torque at } B = Qr$$
$$= (133.1\ \text{N})(0.006\ \text{m})$$
$$= 0.79860\ \text{N}\cdot\text{m}$$

$$\text{Total torque} = 2.0848\ \text{N}\cdot\text{m} + 0.79860\ \text{N}\cdot\text{m}$$

Total torque $= 2.89\ \text{N}\cdot\text{m}$ ◀

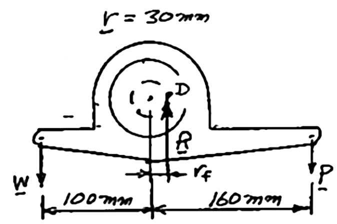
PROBLEM 8.77



A lever of negligible weight is loosely fitted onto a 30-mm-radius fixed shaft as shown. Knowing that a force **P** of magnitude 275 N will just start the lever rotating clockwise, determine (*a*) the coefficient of static friction between the shaft and the lever, (*b*) the smallest force **P** for which the lever does not start rotating counterclockwise.

SOLUTION

(*a*) Impending motion $\circlearrowright$



$$W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(160 - r_f) - W(100 + r_f) = 0$$

$$r_f = \frac{160P - 100W}{P + W}$$

$$r_f = \frac{(160 \text{ mm})(275 \text{ N}) - (100 \text{ mm})(392.4 \text{ N})}{275 \text{ N} + 392.4 \text{ N}}$$

$$r_f = 7.132 \text{ mm}$$

$$r_f = r \sin \phi_s = r\mu_s$$

$$\mu_s = \frac{r_f}{r} = \frac{7.132 \text{ mm}}{30 \text{ mm}} = 0.2377$$

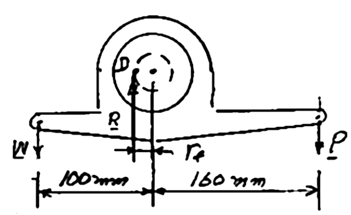
$\mu_s = 0.238$ ◀

(*b*) Impending motion $\circlearrowleft$

$$r_f = r \sin \phi_s = r\mu_s$$

$$= (30 \text{ mm})(0.2377)$$

$$r_f = 7.132 \text{ mm}$$



$$+\circlearrowright \Sigma M_D = 0: \quad P(160 + r_f) - W(100 - r_f) = 0$$

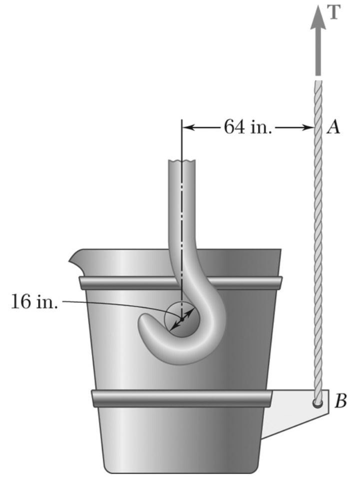
$$P = W \frac{100 - r_f}{160 + r_f}$$

$$P = (392.4 \text{ N}) \frac{100 \text{ mm} - 7.132 \text{ mm}}{160 \text{ mm} + 7.132 \text{ mm}}$$

$$P = 218.04 \text{ N}$$

$\mathbf{P} = 218 \text{ N} \downarrow$ ◀

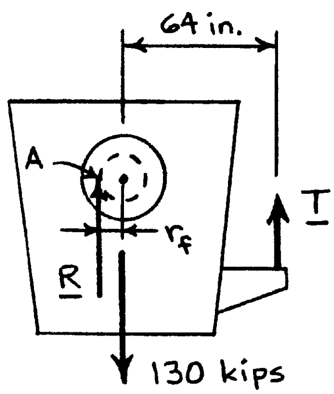
PROBLEM 8.78



A hot-metal ladle and its contents weigh 130 kips. Knowing that the coefficient of static friction between the hooks and the pinion is 0.30, determine the tension in cable *AB* required to start tipping the ladle.

SOLUTION

Free body: Ladle



$$\sin\phi_s \approx \tan\phi_s = \mu_s = 0.30$$

$$r_{\text{bearing}} = 8 \text{ in.}$$

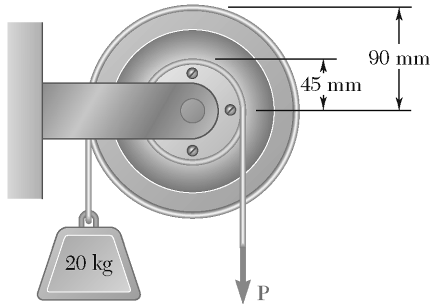
$$r_f = r_{\text{bearing}} \sin\phi_s = (8 \text{ in.})\,(0.30) = 2.4 \text{ in.}$$

R is tangent to friction circle at *A*.

$$+\circlearrowright \Sigma M_A = 0: \quad T(64 \text{ in.} + r_f) - (130 \text{ kips})r_f = 0$$

$$T(64 + 2.4) - (130)(2.4) = 0$$

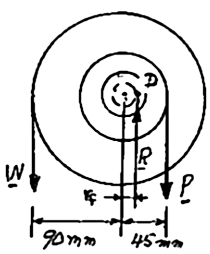
$$T = 4.70 \text{ kips} \blacktriangleleft$$



PROBLEM 8.79

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION



$$+\circlearrowright \Sigma M_D = 0: \quad P(45 - r_f) - W(90 + r_f) = 0$$

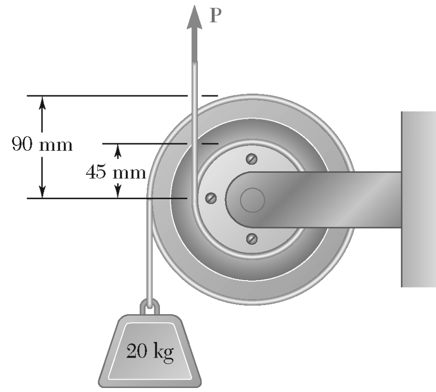
$$P = W\frac{90 + r_f}{45 - r_f}$$

$$= (196.2 \text{ N})\frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 449.82 \text{ N}$$

$P = 450$ N ◀

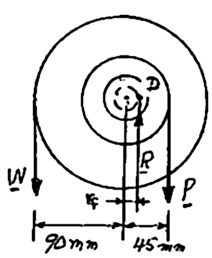
PROBLEM 8.80



The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

Find **P** required to start raising load



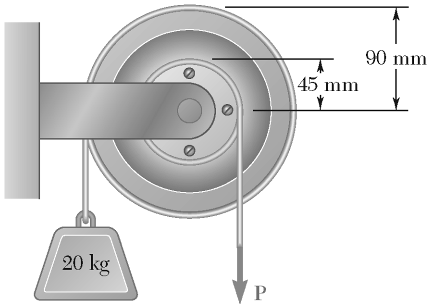
$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 - r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 - r_f}$$

$$= (196.2 \text{ N})\frac{90 \text{ mm} - 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 411.54 \text{ N}$$

$P = 412$ N ◀

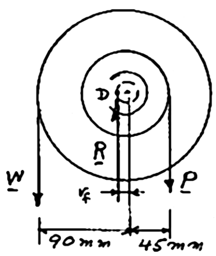


PROBLEM 8.81

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



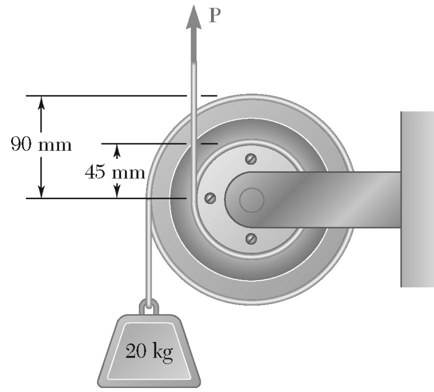
$$+\circlearrowright \Sigma M_D = 0: \quad P(45 + r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 + r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} - 4\text{ mm}}{45\text{ mm} + 4\text{ mm}}$$

$$P = 344.35\text{ N}$$

$P = 344$ N ◀

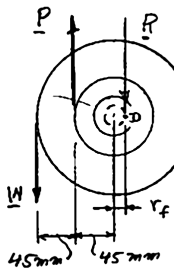


PROBLEM 8.82

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



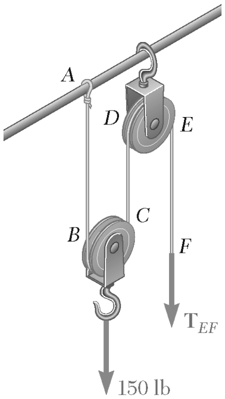
$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 + r_f) - W(90 + r_f) = 0$$

$$P = W\frac{90 + r_f}{45 + r_f}$$

$$= (196.2 \text{ N})\frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} + 4 \text{ mm}}$$

$$P = 376.38 \text{ N}$$

$P = 376$ N ◀



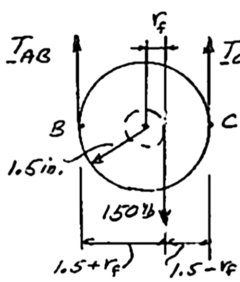
PROBLEM 8.83

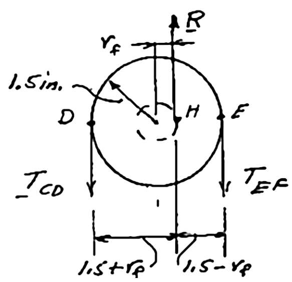
The block and tackle shown are used to raise a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

SOLUTION

For each pulley: Axle diameter $= 0.5$ in.

$$r_f = r\sin\phi_s \approx \mu_s r = 0.20\left(\frac{0.5\text{ in.}}{2}\right) = 0.05\text{ in.}$$





Pulley *BC*: $+\circlearrowleft \Sigma M_B = 0{:}\quad T_{CD}(3\text{ in.}) - (150\text{ lb})(1.5\text{ in.} + r_f) = 0$

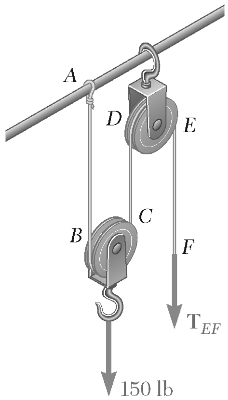
$$T_{CD} = \frac{1}{3}(150\text{ lb})(1.5\text{ in.} + 0.05\text{ in.}) \qquad T_{CD} = 77.5\text{ lb} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0{:}\quad T_{AB} + 77.5\text{ lb} - 150\text{ lb} = 0 \qquad T_{AB} = 72.5\text{ lb} \blacktriangleleft$$

Pulley *DE*: $+\circlearrowleft \Sigma M_B = 0{:}\quad T_{CD}(1.5 + r_f) - T_{EF}(1.5 - r_f) = 0$

$$T_{EF} = T_{CD}\frac{1.5 + r_f}{1.5 - r_f}$$

$$= (77.5\text{ lb})\frac{1.5\text{ in.} + 0.05\text{ in.}}{1.5\text{ in.} - 0.05\text{ in.}} \qquad T_{EF} = 82.8\text{ lb} \blacktriangleleft$$



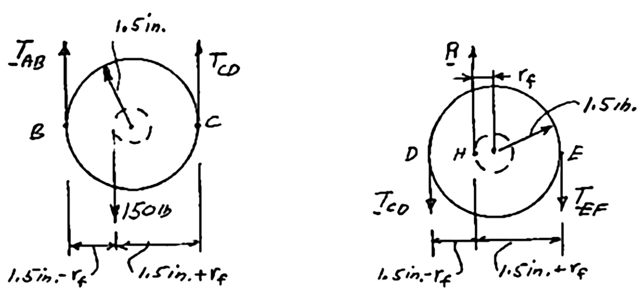
PROBLEM 8.84

The block and tackle shown are used to lower a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

For each pulley:

$$r_f = r\mu_s = \left(\frac{0.5 \text{ in.}}{2}\right)0.2 = 0.05 \text{ in.}$$



Pulley *BC*:

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(3 \text{ in.}) - (150 \text{ lb})(1.5 \text{ in.} - r_f) = 0$$

$$T_{CD} = \frac{(150 \text{ lb})(1.5 \text{ in.} - 0.05 \text{ in.})}{3 \text{ in.}}$$

$T_{CD} = 72.5$ lb ◀

$$+\uparrow \Sigma F_y = 0: \quad T_{AB} + 72.5 \text{ lb} - 150 \text{ lb} = 0$$

$T_{AB} = 77.5$ lb ◀

Pulley *DE*:

$$T_{CD}(1.5 \text{ in.} - r_f) - T_{EF}(1.5 \text{ in.} + r_f) = 0$$

$$T_{EF} = T_{CD}\frac{1.5 \text{ in.} - r_f}{1.5 \text{ in.} + r_f}$$

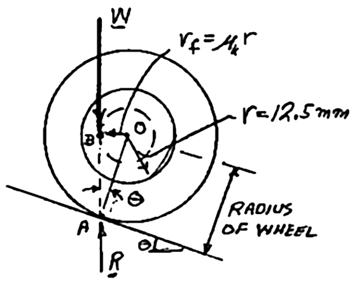
$$= (72.5 \text{ lb})\frac{1.5 \text{ in.} - 0.05 \text{ in.}}{1.5 \text{ in.} + 0.05 \text{ in.}}$$

$T_{EF} = 67.8$ lb ◀

PROBLEM 8.85

A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION



$$\tan\theta = \frac{2}{100} = 0.02$$

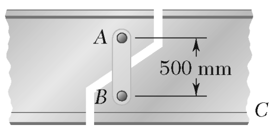
Since a scooter rolls at constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.

$$r_f = \mu_k r = (0.10)(12.5 \text{ mm})$$
$$= 1.25 \text{ mm}$$

$$OA = \frac{OB}{\tan\theta} = \frac{r_f}{\tan\theta} = \frac{1.25 \text{ mm}}{0.02}$$
$$= 62.5 \text{ mm}$$

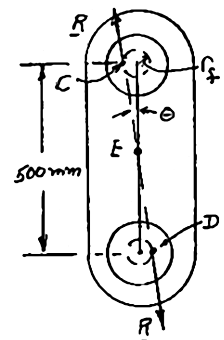
$$\text{Diameter of wheel} = 2(OA) = 125.0 \text{ mm} \blacktriangleleft$$

PROBLEM 8.86



The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 60-mm-diameter pins A and B the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by BC on the link is 200 kN, determine (a) the horizontal force that should be exerted on beam BC to just move the link, (b) the angle that the resulting force exerted by beam BC on the link will form with the vertical.

SOLUTION



Bearing:

$$r = 30\text{ mm}$$

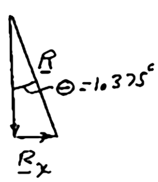
$$r_f = \mu_s r$$

$$= 0.20(30\text{ mm})$$

$$= 6\text{ mm}$$

Resultant forces **R** must be tangent to friction circles at Points C and D.

(a)



$$R_y = \text{Vertical component} = 200\text{ kN}$$

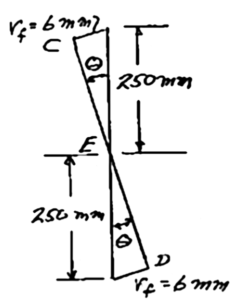
$$R_x = R_y \tan\theta$$

$$= (200\text{ kN})\tan 1.375°$$

$$= 4.80\text{ kN}$$

Horizontal force = 4.80 kN ◀

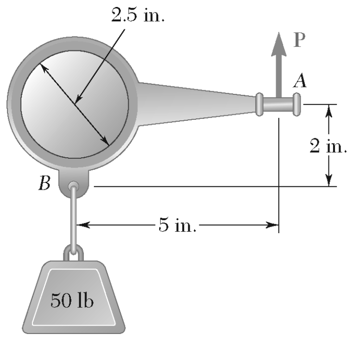
(b)



$$\sin\theta = \frac{6\text{ mm}}{250\text{ mm}}$$

$$\sin\theta = 0.024$$

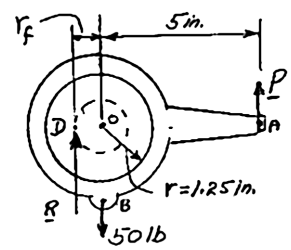
$\theta = 1.375°$ ◀



PROBLEM 8.87

A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION



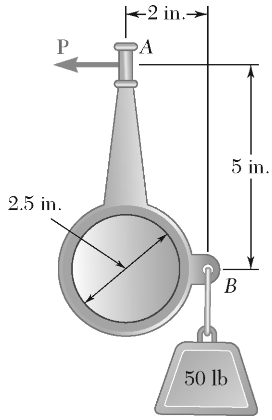
$$r_f = \mu_s r$$
$$= 0.15(1.25 \text{ in.})$$
$$= 0.1875 \text{ in.}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad P(5 \text{ in.} + r_f) - (50 \text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5.1875}$$
$$= 1.807 \text{ lb}$$

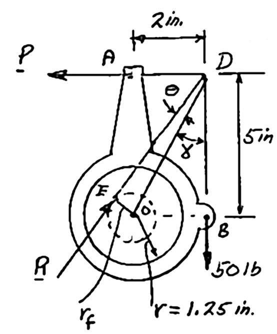
$\mathbf{P} = 1.807 \text{ lb} \uparrow$ ◀

PROBLEM 8.88



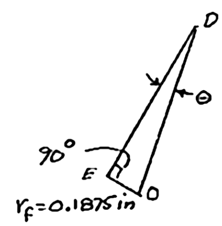
A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION



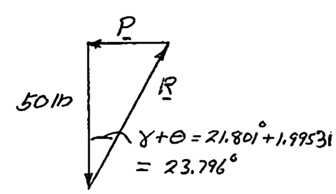
$$r_f = \mu_s r$$
$$= 0.15(1.25 \text{ in.})$$
$$r_f = 0.1875 \text{ in.}$$
$$\tan\gamma = \frac{2 \text{ in.}}{5 \text{ in.}}$$
$$\gamma = 21.801°$$

In ΔEOD:



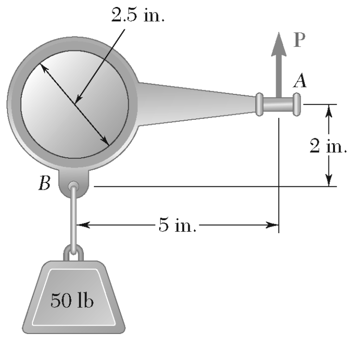
$$OD = \sqrt{(2 \text{ in.})^2 + (5 \text{ in.})^2}$$
$$= 5.3852 \text{ in.}$$
$$\sin\theta = \frac{OE}{OD} = \frac{r_f}{OD}$$
$$= \frac{0.1875 \text{ in.}}{5.3852 \text{ in.}}$$
$$\theta = 1.99531°$$

Force triangle:



$$P = (50 \text{ lb})\tan(\gamma + \theta)$$
$$= (50 \text{ lb})\tan 23.796°$$
$$= 22.049 \text{ lb}$$

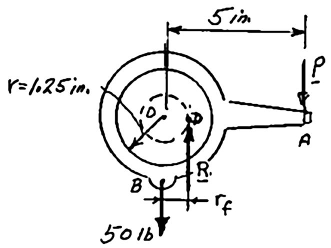
$\mathbf{P} = 22.0 \text{ lb} \longleftarrow$ ◀



PROBLEM 8.89

A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION

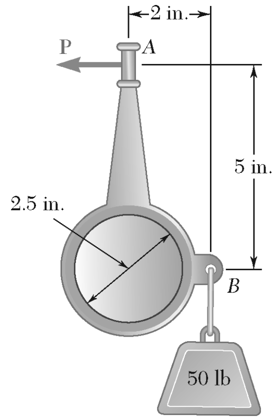


$$r_f = \mu_s r$$
$$= 0.15(1.25 \text{ in.})$$
$$r_f = 0.1875 \text{ in.}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(5 \text{ in.} - r_f) - (50 \text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5 - 0.1875}$$
$$= 1.948 \text{ lb}$$

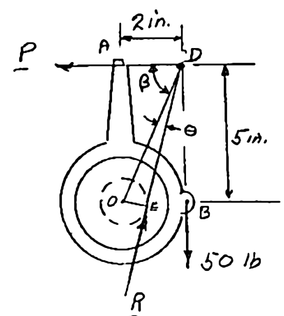
$\mathbf{P} = 1.948 \text{ lb} \downarrow$ ◀



PROBLEM 8.90

A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION

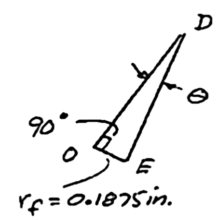


$$
\begin{aligned}
r_f &= \mu_s r \\
&= 0.15(1.25 \text{ in.}) \\
&= 0.1875 \text{ in.}
\end{aligned}
$$

$$\tan\beta = \frac{5 \text{ in.}}{2 \text{ in.}}$$

$$\beta = 68.198°$$

In ΔEOD:



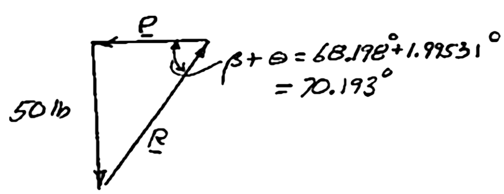
$$OD = \sqrt{(2 \text{ in.})^2 + (5 \text{ in.})^2}$$

$$OD = 5.3852 \text{ in.}$$

$$\sin\theta = \frac{OE}{OD} = \frac{0.1875 \text{ in.}}{5.3852 \text{ in.}}$$

$$\theta = 1.99531°$$

Force triangle:



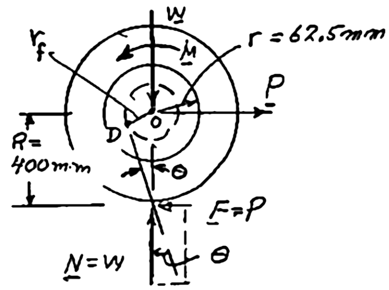
$$P = \frac{50}{\tan(\beta + \theta)} = \frac{50 \text{ lb}}{\tan 70.193°}$$

$\mathbf{P} = 18.01 \text{ lb} \longleftarrow$ ◀

PROBLEM 8.91

A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION



$$r_f = \mu r; \quad R = 400 \text{ mm}$$

$$\sin\theta = \tan\theta = \frac{r_f}{R} = \frac{\mu r}{R}$$

$$P = W\tan\theta = W\frac{\mu r}{R}$$

$$P = W\mu\frac{62.5 \text{ mm}}{400 \text{ mm}}$$

$$= 0.15625W\mu$$

<u>For one wheel</u>: $W = \frac{1}{8}(30 \text{ mg})(9.81 \text{ m/s}^2)$

$$= \frac{1}{8}(294.3 \text{ kN})$$

<u>For eight wheels of railroad car</u>: $\Sigma P = 8(0.15625)\frac{1}{8}(294.3 \text{ kN})\mu$

$$= (45.984\mu) \text{ kN}$$

(*a*) To start motion: $\mu_s = 0.020$

$$\Sigma P = (45.984)(0.020)$$

$$= 0.9197 \text{ kN}$$

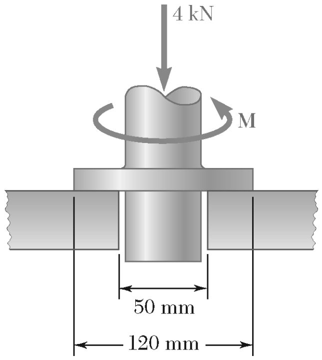
$\Sigma P = 920 \text{ N}$ ◀

(*b*) To maintain motion: $\mu_k = 0.015$

$$\Sigma P = (45.984)(0.015)$$

$$= 0.6897 \text{ kN}$$

$\Sigma P = 690 \text{ N}$ ◀



PROBLEM 8.92

Knowing that a couple of magnitude $30\ \text{N}\cdot\text{m}$ is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

SOLUTION

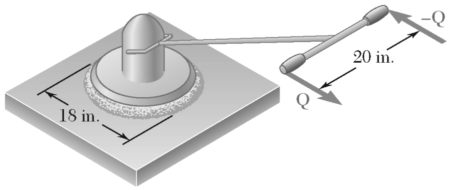
For annular contact regions, use Equation 8.8 with impending slipping:

$$M = \frac{2}{3}\mu_s N \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

So,

$$30\ \text{N}\cdot\text{m} = \frac{2}{3}\mu_s (4000\ \text{N}) \frac{(0.06\ \text{m})^3 - (0.025\ \text{m})^3}{(0.06\ \text{m})^2 - (0.025\ \text{m})^2}$$

$\mu_s = 0.1670$ ◀



PROBLEM 8.93

A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

See Figure 8.12 and Eq. (8.9).

Using:

$$R = 9 \text{ in.}$$
$$P = 50 \text{ lb}$$

and

$$\mu_k = 0.25$$
$$M = \frac{2}{3}\mu_k PR = \frac{2}{3}(0.25)(50 \text{ lb})(9 \text{ in.})$$
$$= 75 \text{ lb} \cdot \text{in.}$$

$\Sigma M_y = 0$ yields:
$$M = Q(20 \text{ in.})$$
$$75 \text{ lb} \cdot \text{in.} = Q(20 \text{ in.})$$

$$Q = 3.75 \text{ lb} \blacktriangleleft$$

PROBLEM 8.94*

The frictional resistance of a thrust bearing decreases as the shaft and bearing surfaces wear out. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by Eq. (8.9) for a new bearing.

SOLUTION

Using Figure 8.12, we assume

$$\Delta N = \frac{k}{r}\Delta A: \quad \Delta A = r\Delta\theta\,\Delta r$$

$$\Delta N = \frac{k}{r} r\Delta\theta\Delta r = k\Delta\theta\Delta r$$

We write $\qquad P = \Sigma\Delta N \quad \text{or} \quad P = \int dN$

$$P = \int_0^{2\pi}\int_0^R k\Delta\theta\Delta r = 2\pi Rk; \quad k = \frac{P}{2\pi R}$$

$$\Delta N = \frac{P\Delta\theta\Delta r}{2\pi R}$$

$$\Delta M = r\Delta F = r\mu_k \Delta N = r\mu_k \frac{P\Delta\theta\Delta r}{2\pi R}$$

$$M = \int_0^{2\pi}\int_0^R \frac{\mu_k P}{2\pi R} r dr d\theta = \frac{2\pi\mu_k P}{2\pi R}\cdot\frac{R^2}{2} = \frac{1}{2}\mu_k PR$$

From Eq. (8.9) for a new bearing,

$$M_{\text{new}} = \frac{2}{3}\mu_k PR$$

Thus, $\qquad \dfrac{M}{M_{\text{new}}} = \dfrac{\frac{1}{2}}{\frac{2}{3}} = \dfrac{3}{4} \qquad\qquad M = 0.75 M_{\text{new}}$ ◀

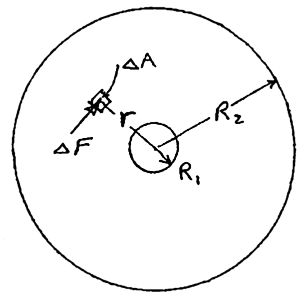
PROBLEM 8.95*

Assuming that bearings wear out as indicated in Problem 8.94, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is

$$M = \tfrac{1}{2}\mu_k P(R_1 + R_2)$$

where P = magnitude of the total axial force
R_1, R_2 = inner and outer radii of collar

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$.

As in the text, $\Delta F = \mu \Delta N \quad \Delta M = r\Delta F$

The total normal force P is

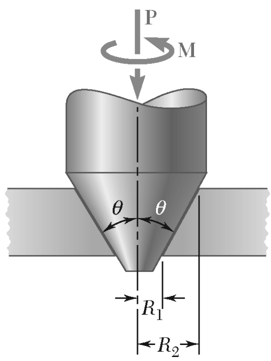
$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left(\int_{R_1}^{R_2} \frac{k}{r} r dr\right) d\theta$$

$$P = 2\pi \int_{R_1}^{R_2} k dr = 2\pi k(R_2 - R_1) \quad \text{or} \quad k = \frac{P}{2\pi(R_2 - R_1)}$$

Total couple: $$M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2} r\mu \frac{k}{r} r dr\right) d\theta$$

$$M_{\text{worn}} = 2\pi\mu k \int_{R_1}^{R_2} (r dr) = \pi\mu k\left(R_2^2 - R_1^2\right) = \frac{\pi\mu P\left(R_2^2 - R_1^2\right)}{2\pi(R_2 - R_1)}$$

$$M_{\text{worn}} = \frac{1}{2}\mu P(R_2 + R_1) \blacktriangleleft$$

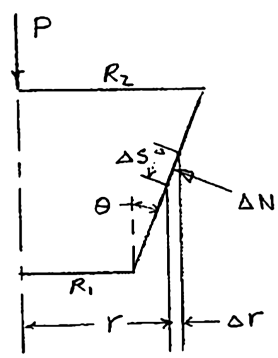


PROBLEM 8.96*

Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is

$$M = \frac{2}{3}\frac{\mu_k P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

SOLUTION



Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k$.

So $\quad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \dfrac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation.

$$\Delta F_y = \Delta N \sin\theta = kr\Delta r\Delta\phi$$

Total vertical force: $\quad P = \lim\limits_{\Delta A \to 0} \Sigma\Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \quad \text{or} \quad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

Friction force: $\quad \Delta F = \mu\Delta N = \mu k\Delta A$

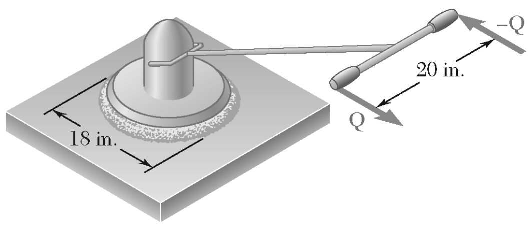
Moment: $\quad \Delta M = r\Delta F = r\mu kr\dfrac{\Delta r}{\sin\theta}\Delta\phi$

Total couple: $\quad M = \lim\limits_{\Delta A \to 0}\Sigma\Delta M = \displaystyle\int_0^{2\pi}\left(\int_{R_1}^{R_2}\frac{\mu k}{\sin\theta}r^2 dr\right)d\phi$

$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2 dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_3^2\right)}\left(R_2^3 - R_3^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$

PROBLEM 8.97

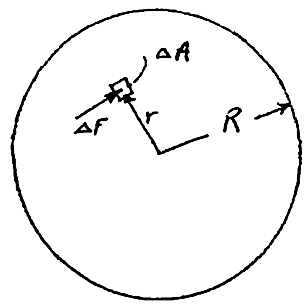


Solve Problem 8.93 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

PROBLEM 8.93 A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k\left(1-\dfrac{r}{R}\right)$.



$$\Delta F = \mu \Delta N = \mu k\left(1-\frac{r}{R}\right)\Delta A = \mu k\left(1-\frac{r}{R}\right) r\Delta r\Delta\theta$$

$$P = \lim_{\Delta A\to 0} \Sigma\Delta N = \int_0^{2\pi}\left[\int_0^R k\left(1-\frac{r}{R}\right) rdr\right] d\theta$$

$$P = 2\pi k\int_0^R \left(1-\frac{r}{R}\right) rdr = 2\pi k\left(\frac{R^2}{2}-\frac{R^3}{3R}\right)$$

$$P = \frac{1}{3}\pi kR^2 \quad \text{or} \quad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A\to 0} \Sigma r\Delta F = \int_0^{2\pi}\left[\int_0^R r\mu k\left(1-\frac{r}{R}\right) rdr\right] d\theta$$

$$= 2\pi\mu k\int_0^R \left(r^2 - \frac{r^3}{R}\right) dr$$

$$= 2\pi\mu k\left(\frac{R^3}{3}-\frac{R^4}{4R}\right) = \frac{1}{6}\pi\mu kR^3$$

$$= \frac{\pi\mu}{6}\frac{3P}{\pi R^2}R^3 = \frac{1}{2}\mu PR$$

where $\qquad \mu = \mu_k = 0.25 \quad R = 9\text{ in.}$

PROBLEM 8.97 (Continued)

$$P = W = 50 \text{ lb}$$

Then
$$M = \frac{1}{2}(0.25)(50 \text{ lb})(9 \text{ in.})$$
$$= 56.250 \text{ lb} \cdot \text{in.}$$

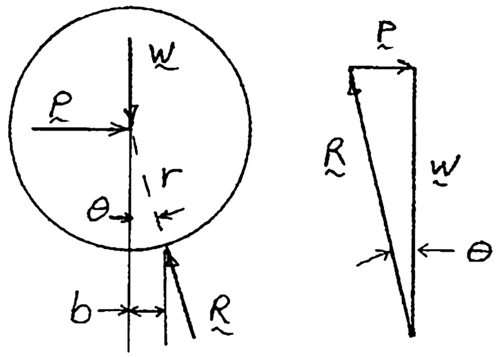
Finally
$$Q = \frac{M}{d} = \frac{56.250 \text{ lb} \cdot \text{in.}}{20 \text{ in.}}$$
$Q = 2.81 \text{ lb}$ ◀

PROBLEM 8.98

Determine the horizontal force required to move a 2500-lb automobile with 23-in.-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 0.05 in.

SOLUTION

FBD wheel:



$$r = 11.5 \text{ in.}$$
$$b = 0.05 \text{ in.}$$
$$\theta = \sin^{-1}\frac{b}{r}$$

$P = W\tan\theta = W\tan\left(\sin^{-1}\frac{b}{r}\right)$ for each wheel, so for total

$$P = 2500 \text{ lb} \tan\left(\sin^{-1}\frac{0.05}{11.5}\right)$$

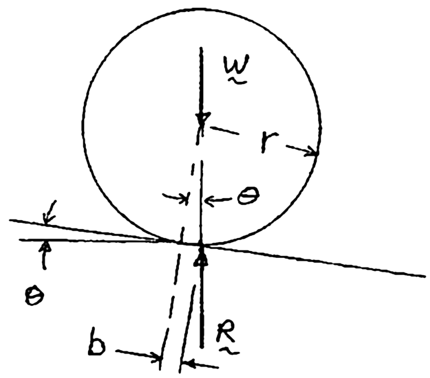
$P = 10.87$ lb ◀

PROBLEM 8.99

Knowing that a 6-in.-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

SOLUTION

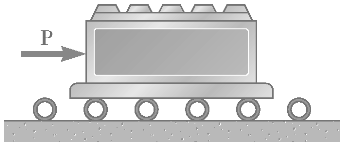
FBD disk:



$$\tan\theta = \text{slope} = 0.02$$

$$b = r\tan\theta = (3 \text{ in.})(0.02) \qquad b = 0.0600 \text{ in.} \blacktriangleleft$$

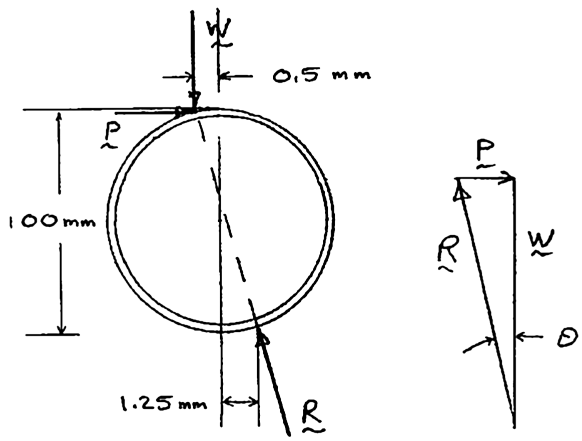
PROBLEM 8.100



A 900-kg machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 100 mm. Knowing that the coefficient of rolling resistance is 0.5 mm between the pipes and the base and 1.25 mm between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$W = mg$$
$$= (900 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 8829.0 \text{ N}$$
$$\theta = \sin^{-1} \frac{.5 \text{ mm} + 1.25 \text{ mm}}{100 \text{ mm}}$$
$$= 1.00273°$$

$P = W \tan\theta$ for each pipe, so also for total

$$P = (8829.0 \text{ N}) \tan(1.00273°) \qquad P = 154.4 \text{ N} \blacktriangleleft$$

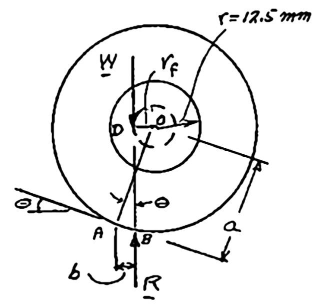
PROBLEM 8.101

Solve Problem 8.85 including the effect of a coefficient of rolling resistance of 1.75 mm.

PROBLEM 8.85 A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION

Since the scooter rolls at a constant speed, each wheel is in <u>equilibrium</u>. Thus, **W** and **R** must have a <u>common line of action</u> tangent to the friction circle.



$$a = \text{Radius of wheel}$$

$$\tan\theta = \frac{2}{100} = 0.02$$

Since b and r_f are small compared to a,

$$\tan\theta \approx \frac{r_f + b}{a} = \frac{\mu_k r + b}{a} = 0.02$$

Data: $\mu_k = 0.10, \quad b = 1.75\text{ mm}, \quad r = 12.5\text{ mm}$

$$\frac{(0.10)(12.5\text{ mm}) + 1.75\text{ mm}}{a} = 0.02$$

$a = 150$ mm Diameter $= 2a = 300$ mm ◀

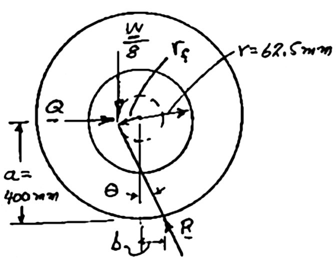
PROBLEM 8.102

Solve Problem 8.91 including the effect of a coefficient of rolling resistance of 0.5 mm.

PROBLEM 8.91 A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

For one wheel:



$$r_f = \mu r$$

$$\tan\theta \approx \sin\theta \approx \frac{r_f + b}{a}$$

$$\tan\theta = \frac{\mu r + b}{a}$$

$$Q = \frac{W}{8}\tan\theta = \frac{W}{8}\frac{\mu r + b}{a}$$

For eight wheels of car:

$$P = W\frac{\mu r + b}{a}$$

$$W = mg = (30 \text{ Mg})(9.81 \text{ m/s}^2) = 294.3 \text{ kN}$$

$$a = 400 \text{ mm}, \quad r = 62.5 \text{ mm}, \quad b = 0.5 \text{ mm}$$

(*a*) To start motion:

$$\mu = \mu_s = 0.02$$

$$P = (294.3 \text{ kN})\frac{(0.020)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

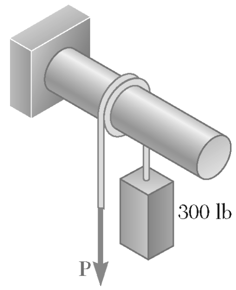
$P = 1.288$ kN ◀

(*b*) To maintain constant speed

$$\mu = \mu_k = 0.015$$

$$P = (294.3 \text{ kN})\frac{(0.015)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

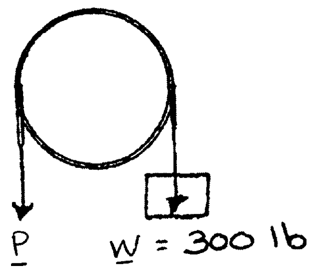
$P = 1.058$ kN ◀



PROBLEM 8.103

A 300-lb block is supported by a rope that is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



$$\beta = 1.5 \text{ turns} = 3\pi \text{ rad}$$

For impending motion of W up,

$$P = We^{\mu_s \beta} = (300 \text{ lb})e^{(0.15)3\pi}$$
$$= 1233.36 \text{ lb}$$

For impending motion of W down,

$$P = We^{-\mu_s \beta} = (300 \text{ lb})e^{-(0.15)3\pi}$$
$$= 72.971 \text{ lb}$$

For equilibrium, $73.0 \text{ lb} \leq P \leq 1233 \text{ lb}$ ◀

PROBLEM 8.104

A hawser is wrapped two full turns around a bollard. By exerting an 80-lb force on the free end of the hawser, a dockworker can resist a force of 5000 lb on the other end of the hawser. Determine (*a*) the coefficient of static friction between the hawser and the bollard, (*b*) the number of times the hawser should be wrapped around the bollard if a 20,000-lb force is to be resisted by the same 80-lb force.

SOLUTION

(*a*)

$$\beta = 2 \text{ turns} = 2(2\pi) = 4\pi$$

$$T_1 = 80 \text{ lb}, \qquad T_2 = 5000 \text{ lb}$$

$$\ln \frac{T_2}{T_1} = \mu_s \beta$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{4\pi} \ln \frac{5000 \text{ lb}}{80 \text{ lb}}$$

$$\mu_s = \frac{1}{4\pi} \ln 62.5 = \frac{4.1351}{4\pi}$$

$\mu_s = 0.329$ ◀

(*b*)

$$T_1 = 80 \text{ lb}, \quad T_2 = 20{,}000 \text{ lb}, \quad \mu_s = 0.329$$

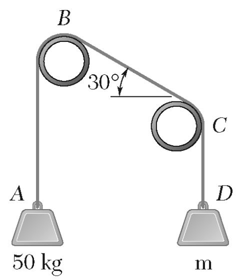
$$\ln \frac{T_2}{T_1} = \mu_s \beta$$

$$\beta = \frac{1}{\mu} \ln \frac{T_2}{T_1} = \frac{1}{0.329} \ln \frac{20{,}000 \text{ lb}}{80 \text{ lb}}$$

$$\beta = \frac{1}{0.329} \ln(250) = \frac{5.5215}{0.329} = 16.783$$

$$\text{Number of turns} = \frac{16.783}{2\pi}$$

Number of turns $= 2.67$ ◀



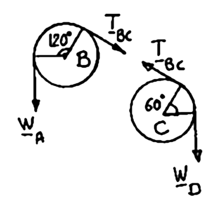
PROBLEM 8.105

A rope $ABCD$ is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the smallest value of the mass m for which equilibrium is possible, (*b*) the corresponding tension in portion BC of the rope.

SOLUTION

We apply Eq. (8.14) to pipe B and pipe C.

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad (8.14)$$



Pipe B: $\quad T_2 = W_A, \quad T_1 = T_{BC}$

$$\mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$$

$$\frac{W_A}{T_{BC}} = e^{0.25(2\pi/3)} = e^{\pi/6} \qquad (1)$$

Pipe C: $\quad T_2 = T_{BC}, \quad T_1 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$

$$\frac{T_{BC}}{W_D} = e^{0.025(\pi/3)} = e^{\pi/12} \qquad (2)$$

(*a*) Multiplying Eq. (1) by Eq. (2):

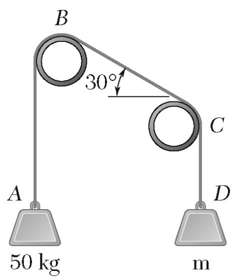
$$\frac{W_A}{W_D} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_D = \frac{W_A}{2.193} \quad m = \frac{W_D}{g} = \frac{\frac{W_A}{g}}{2.193} = \frac{m_A}{2.193} = \frac{50 \text{ kg}}{2.193}$$

$$m = 22.8 \text{ kg} \blacktriangleleft$$

(*b*) From Eq. (1):

$$T_{BC} = \frac{W_A}{e^{\pi/6}} = \frac{(50 \text{ kg})(9.81 \text{ m/s}^2)}{1.688} = 291 \text{ N} \blacktriangleleft$$



PROBLEM 8.106

A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the largest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

SOLUTION

See *FB* diagrams of Problem 8.105. We apply Eq. (8.14) to pipes *B* and *C*.

Pipe *B*: $T_1 = W_A, \quad T_2 = T_{BC}, \quad \mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{T_{BC}}{W_A} = e^{0.25(2\pi/3)} = e^{\pi/6} \tag{1}$$

Pipe *C*: $T_1 = T_{BC}, \quad T_2 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$

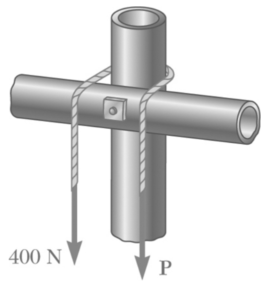
$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{W_D}{T_{BC}} = e^{0.25(\pi/3)} = e^{\pi/12} \tag{2}$$

(*a*) Multiply Eq. (1) by Eq. (2):

$$\frac{W_D}{W_A} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_0 = 2.193 W_A \quad m = 2.193 m_A = 2.193(50 \text{ kg})$$ $m = 109.7$ kg ◀

(*b*) From Eq. (1): $T_{BC} = W_A e^{\pi/6} = (50 \text{ kg})(9.81 \text{ m/s}^2)(1.688) = 828 \text{ N}$ ◀



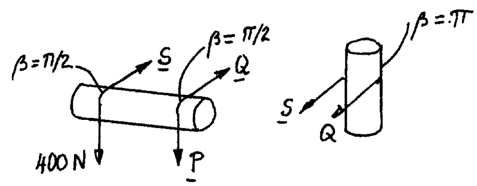
PROBLEM 8.107

Knowing that the coefficient of static friction is 0.25 between the rope and the horizontal pipe and 0.20 between the rope and the vertical pipe, determine the range of values of P for which equilibrium is maintained.

SOLUTION

Horizontal pipe $\mu_s = \mu_h$

Vertical pipe $\mu_s = \mu_v$



For motion of P to be impending downward:

$$\frac{P}{Q} = e^{\mu_h(\pi/2)}, \quad \frac{Q}{S} = e^{\mu_v \pi}, \quad \frac{S}{400\text{ N}} = e^{\mu_h(\pi/2)}$$

Multiply equation member by member:

$$\frac{P}{Q}\frac{Q}{S}\frac{S}{400} = e^{(\mu_h + 2\mu_v + \mu_h)(\pi/2)}$$

or:

$$\frac{P}{400\text{ N}} = e^{(\mu_h + \mu_v)\pi} \qquad (1)$$

For motion of **P** to be impending upward, we find in a similar way

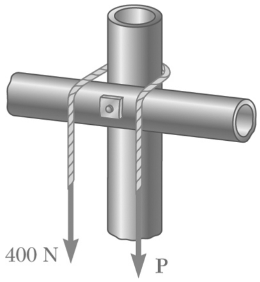
$$\frac{P}{400\text{ N}} = e^{-(\mu_h + \mu_v)\pi} \qquad (2)$$

Given data: $\mu_h = 0.25, \quad \mu_v = 0.20$

From (1): $P = (400\text{ N})e^{0.45\pi} = 1644\text{ N}$

From (2): $P = (400\text{ N})e^{-0.45\pi} = 97.3\text{ N}$

Range for equilibrium: $97.3\text{ N} \le P \le 1644\text{ N}$ ◀



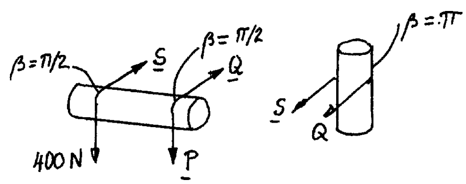
PROBLEM 8.108

Knowing that the coefficient of static friction is 0.30 between the rope and the horizontal pipe and that the smallest value of P for which equilibrium is maintained is 80 N, determine (*a*) the largest value of P for which equilibrium is maintained, (*b*) the coefficient of static friction between the rope and the vertical pipe.

SOLUTION

Horizontal pipe: $\mu_s = \mu_h$

Vertical pipe: $\mu_s = \mu_v$



For motion of P to be impending downward:

$$\frac{P}{Q} = e^{\mu_h(\pi/2)}, \quad \frac{Q}{S} = e^{\mu_v \pi}, \quad \frac{S}{400\text{ N}} = e^{\mu_h(\pi/2)}$$

Multiply equation member by member:

$$\frac{P}{Q}\frac{Q}{S}\frac{S}{400} = e^{(\mu_h + 2\mu_v + \mu_h)(\pi/2)}$$

or: $$\frac{P}{400\text{ N}} = e^{(\mu_h + \mu_v)\pi} \qquad (1)$$

For motion of **P** to be impending upward, we find in a similar way

$$\frac{P}{400\text{ N}} = e^{-(\mu_h + \mu_v)\pi} \qquad (2)$$

Setting $P = P_{\max}$ in Eq. (1), $P = 80\text{ N}$ in Eq. (2) and $\mu_h = 0.30$ in both, we get

$$\frac{P_{\max}}{400\text{ N}} = e^{(0.30+\mu_v)\pi} \ (1'); \quad \frac{80\text{ N}}{400\text{ N}} = e^{-(0.30+\mu_v)\pi} \ (2')$$

(a) Multiplying $(1')$ by $(2')$:

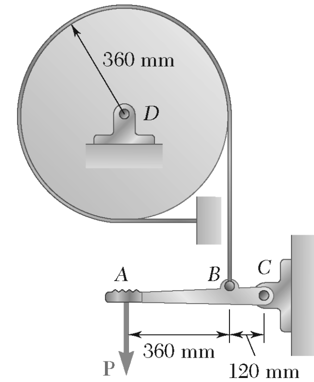
$$\frac{P_{\max}}{400} \cdot \frac{80}{400} = e^0 = 1$$

$P_{\max} = 2000\text{ N}$ ◀

(b) From $(2'): (0.30 + \mu_v)\pi = \ln \dfrac{400}{80} = \ln 5 = 1.60944$

$$0.30 + \mu_v = 0.512$$

$\mu_v = 0.212$ ◀

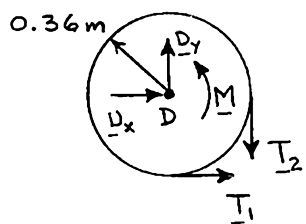


PROBLEM 8.109

A band brake is used to control the speed of a flywheel as shown. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$. Determine the magnitude of the couple being applied to the flywheel, knowing that $P = 45$ N and that the flywheel is rotating counterclockwise at a constant speed.

SOLUTION

Free body: Cylinder



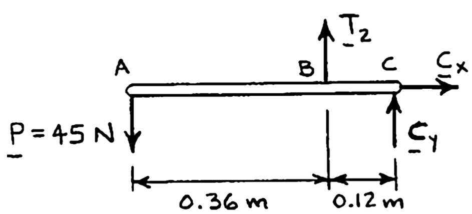
Since slipping of band relative to cylinder is clockwise, $\mathbf{T}_1$ and $\mathbf{T}_2$ are located as shown.

From free body: Lever ABC

$$+\circlearrowleft \Sigma M_C = 0: \quad (45 \text{ N})(0.48 \text{ m}) - T_2(0.12 \text{ m}) = 0$$

$$T_2 = 180 \text{ N}$$

Free body: Lever ABC



From free body: Cylinder

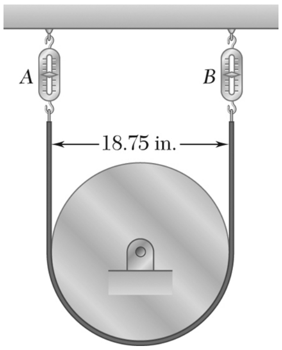
Using Eq. (8.14) with $\mu_k = 0.25$ and $\beta = 270° = \dfrac{3\pi}{2}$ rad:

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{(0.25)(3\pi/2)} = e^{3\pi/8}$$

$$T_1 = \frac{T_2}{e^{3\pi/8}} = \frac{180 \text{ N}}{3.2482} = 55.415 \text{ N}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (55.415 \text{ N})(0.36 \text{ m}) - (180 \text{ N})(0.36 \text{ m}) + M = 0$$

$$\mathbf{M} = 44.9 \text{ N} \cdot \text{m} \circlearrowleft \blacktriangleleft$$



PROBLEM 8.110

The setup shown is used to measure the output of a small turbine. When the flywheel is at rest, the reading of each spring scale is 14 lb. If a 105-lb · in. couple must be applied to the flywheel to keep it rotating clockwise at a constant speed, determine (*a*) the reading of each scale at that time, (*b*) the coefficient of kinetic friction. Assume that the length of the belt does not change.

SOLUTION

(*a*) Since the length of the belt is constant, the spring in scale B will increase in length by δ and the spring in scale A will decrease by the same amount. Thus, the sum of the readings in scales A and B remains constant:

$$T_A + T_B = 14 \text{ lb} + 14 \text{ lb} \qquad T_A + T_B = 28 \text{ lb} \quad (1)$$

On the other hand, the sum of the moments of T_A and T_B about axle must be equal to moment of couple:

$$(T_B - T_A)(9.375 \text{ in.}) = 105 \text{ lb} \cdot \text{in.}, \qquad T_B - T_A = 11.2 \text{ lb} \quad (2)$$

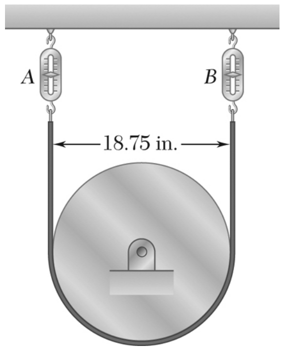
Solving (1) and (2) simultaneously

$$T_A = 8.40 \text{ lb}; \qquad T_B = 19.60 \text{ lb} \blacktriangleleft$$

(*b*) Apply Eq. (8.13) with $T_2 = T_B, \quad T_1 = T_A, \ \beta = 180° = \pi$ rad.

$$\ln\frac{T_2}{T_1} = \mu_s \beta: \qquad \mu_s \pi = \ln\frac{19.60}{8.40} = 0.84730 \qquad \mu_s = 0.270 \blacktriangleleft$$

PROBLEM 8.111



The setup shown is used to measure the output of a small turbine. The coefficient of kinetic friction is 0.20 and the reading of each spring scale is 16 lb when the flywheel is at rest. Determine (*a*) the reading of each scale when the flywheel is rotating clockwise at a constant speed, (*b*) the couple that must be applied to the flywheel. Assume that the length of the belt does not change.

SOLUTION

(*a*) Since the length of the belt is constant, the spring in scale *B* will increase in length by δ and the spring in scale *A* will decrease by the same amount. Thus, the sum of the readings in scales *A* and *B* remains constant:

$$T_A + T_B = 16 \text{ lb} + 16 \text{ lb} \qquad T_A + T_B = 32 \text{ lb} \quad (1)$$

We now apply Eq. (8.14) with

$$T_1 = T_A, \quad T_2 = T_B, \quad \mu_k = 0.20, \quad \beta = 180° = \pi \text{ rad.}$$

$$\frac{T_2}{T_1} = e^{\mu_k B}: \quad \frac{T_B}{T_A} = e^{0.20\pi} = 1.87446$$

$$T_B = 1.87446\ T_A \quad (2)$$

Substituting (2) into (1):

$$T_A + 1.87446 T_A = 32 \text{ lb}$$

$$T_A = 11.1325 \text{ lb} \qquad T_A = 11.13 \text{ lb} \blacktriangleleft$$

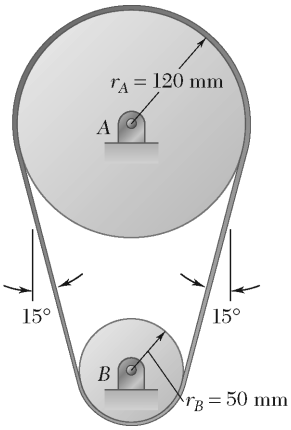
From (1): $T_B = 32 \text{ lb} - 11.1325 \text{ lb}$

$$T_B = 20.868 \text{ lb} \qquad T_B = 20.9 \text{ lb} \blacktriangleleft$$

(*b*) Couple applied to flywheel:

$$M = (T_B - T_A)r$$
$$= (20.868 \text{ lb} - 11.1325 \text{ lb})(9.375 \text{ in.})$$

$$M = 91.3 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

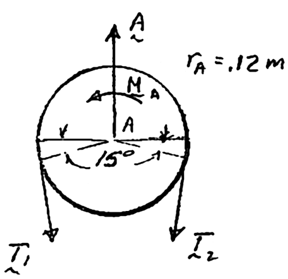


PROBLEM 8.112

A flat belt is used to transmit a couple from drum B to drum A. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum A.

SOLUTION

FBD's drums:



$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

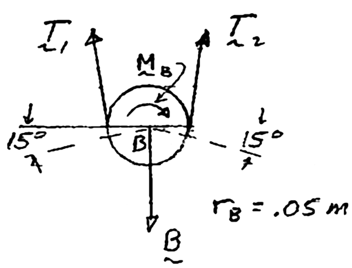
$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on B (friction coefficients being equal)

So

$$T_2 = T_{\max} = T_1 e^{\mu_s \beta_B}$$

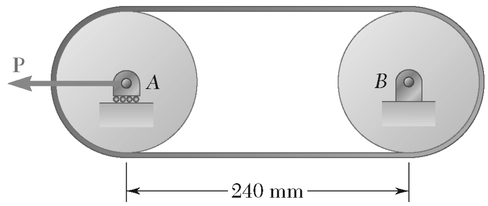
$$450 \text{ N} = T_1 e^{(0.4)5\pi/6} \quad \text{or} \quad T_1 = 157.914 \text{ N}$$



$$\circlearrowleft \Sigma M_A = 0\text{:} \quad M_A + (0.12 \text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12 \text{ m})(450 \text{ N} - 157.914 \text{ N}) = 35.05 \text{ N} \cdot \text{m}$$

$$M_A = 35.1 \text{ N} \cdot \text{m} \blacktriangleleft$$

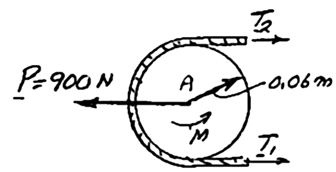


PROBLEM 8.113

A flat belt is used to transmit a couple from pulley A to pulley B. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Drum A:



$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{(0.35)\pi}$$

$$T_2 = 3.0028 T_1$$

$$\beta = 180° = \pi \text{ radians}$$

(*a*) Torque: $+\circlearrowleft \Sigma M_A = 0: \quad M - (675.15 \text{ N})(0.06 \text{ m}) + (224.84 \text{ N})(0.06 \text{ m})$

$$M = 27.0 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900 \text{ N} = 0$

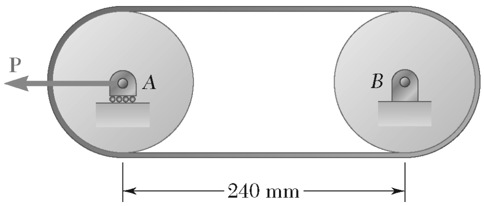
$$T_1 + 3.0028 T_1 - 900 \text{ N} = 0$$

$$4.00282 T_1 = 900$$

$$T_1 = 224.841 \text{ N}$$

$$T_2 = 3.0028(224.841 \text{ N}) = 675.15 \text{ N}$$

$$T_{\max} = 675 \text{ N} \blacktriangleleft$$



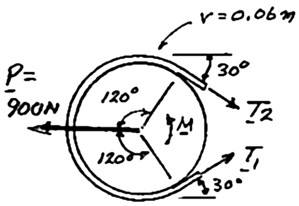
PROBLEM 8.114

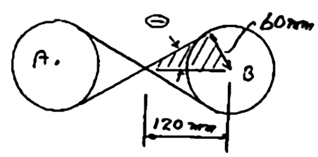
Solve Problem 8.113 assuming that the belt is looped around the pulleys in a figure eight.

PROBLEM 8.113 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude *P* = 900 N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

<u>Drum *A*</u>:





$$\beta = 240° = 240° \frac{\pi}{180°} = \frac{4}{3}\pi$$

$$\sin\theta = \frac{60}{120} = \frac{1}{2}$$

$$\theta = 30°$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35(4/3\pi)}$$

$$T_2 = 4.3322 T_1$$

(*a*) <u>Torque</u>: $+\circlearrowleft \Sigma M_B = 0: \quad M - (844.3\text{ N})(0.06\text{ m}) + (194.9\text{ N})(0.06\text{ m}) = 0$

$$M = 39.0\text{ N}\cdot\text{m} \blacktriangleleft$$

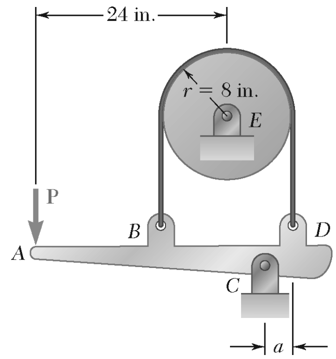
(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad (T_1 + T_2)\cos 30° - 900\text{ N}$

$$(T_1 + 4.3322 T_1)\cos 30° = 900$$

$$T_1 = 194.90\text{ N}$$

$$T_2 = 4.3322(194.90\text{ N}) = 844.3\text{ N}$$

$$T_{max} = 844\text{ N} \blacktriangleleft$$



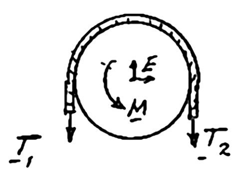
PROBLEM 8.115

The speed of the brake drum shown is controlled by a belt attached to the control bar AD. A force **P** of magnitude 25 lb is applied to the control bar at A. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that a = 4 in., and that the drum is rotating at a constant speed (a) counterclockwise, (b) clockwise.

SOLUTION

(a) Counterclockwise rotation

Free body: Drum

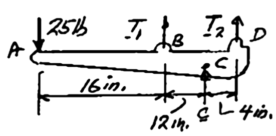


$$r = 8\text{ in.} \quad \beta = 180° = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933T_1$$

Free body: Control bar



$$+\circlearrowright \Sigma M_C = 0: \quad T_1(12\text{ in.}) - T_2(4\text{ in.}) - (25\text{ lb})(28\text{ in.}) = 0$$

$$T_1(12) - 2.1933T_1(4) - 700 = 0$$

$$T_1 = 216.93\text{ lb}$$

$$T_2 = 2.1933(216.93\text{ lb}) = 475.80\text{ lb}$$

Return to free body of drum

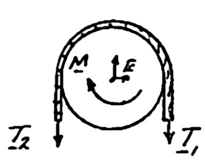
$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(8\text{ in.}) - T_2(8\text{ in.}) = 0$$

$$M + (216.96\text{ lb})(8\text{ in.}) - (475.80\text{ lb})(8\text{ in.}) = 0$$

$$M = 2070.9\text{ lb}\cdot\text{in.}$$

$M = 2070\text{ lb}\cdot\text{in.}$ ◀

(b) Clockwise rotation



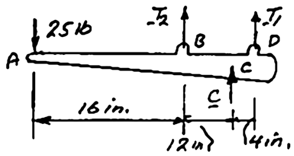
$$r = 8\text{ in.} \quad \beta = \pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933T_1$$

PROBLEM 8.115 (Continued)

Free body: Control rod



$$+\circlearrowleft \Sigma M_C = 0: \quad T_2(12 \text{ in.}) - T_1(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$

$$2.1933T_1(12) - T_1(4) - 700 = 0$$

$$T_1 = 31.363 \text{ lb}$$

$$T_2 = 2.1933(31.363 \text{ lb})$$

$$T_2 = 68.788 \text{ lb}$$

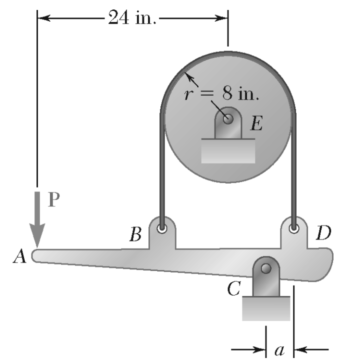
Return to free body of drum

$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (31.363 \text{ lb})(8 \text{ in.}) - (68.788 \text{ lb})(8 \text{ in.}) = 0$$

$$M = 299.4 \text{ lb} \cdot \text{in.}$$

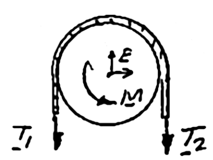
$M = 299 \text{ lb} \cdot \text{in.}$ ◀



PROBLEM 8.116

The speed of the brake drum shown is controlled by a belt attached to the control bar AD. Knowing that $a = 4$ in., determine the maximum value of the coefficient of static friction for which the brake is not self-locking when the drum rotates counterclockwise.

SOLUTION

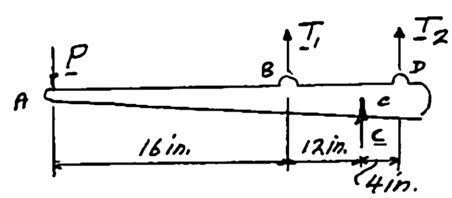


$$r = 8 \text{ in.}, \quad \beta = 180° = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{\mu_s \pi}$$

$$T_2 = e^{\mu_s \pi} T_1$$

Free body: Control rod



$$+\circlearrowleft \Sigma M_C = 0: \quad P(28 \text{ in.}) - T_1(12 \text{ in.}) + T_2(4 \text{ in.}) = 0$$

$$28P - 12T_1 + e^{\mu\pi} T_1(4) = 0$$

For self-locking brake: $P = 0$

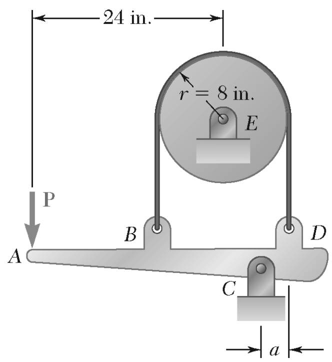
$$12T_1 = 4T_1 e^{\mu_s \pi}$$

$$e^{\mu_s \pi} = 3$$

$$\mu_s \pi = \ln 3 = 1.0986$$

$$\mu_s = \frac{1.0986}{\pi} = 0.3497$$

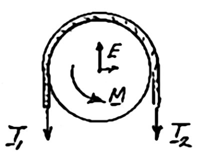
$\mu_s = 0.350$ ◀



PROBLEM 8.117

The speed of the brake drum shown is controlled by a belt attached to the control bar AD. Knowing that the coefficient of static friction is 0.30 and that the brake drum is rotating counterclockwise, determine the minimum value of a for which the brake is not self-locking.

SOLUTION

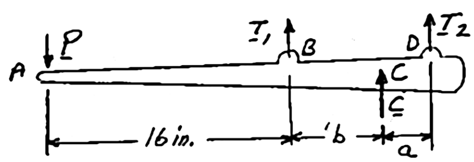


$r = 8$ in., $\quad \beta = \pi$ radians

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.30\pi} = 2.5663$$

$$T_2 = 2.5663 T_1$$

Free body: Control rod



$$b = 16 \text{ in.} - a$$

$$+\circlearrowleft \Sigma M_C = 0: \quad P(16 \text{ in.} + b) - T_1 b + T_2 a = 0$$

For brake to be self-locking, $\quad P = 0$

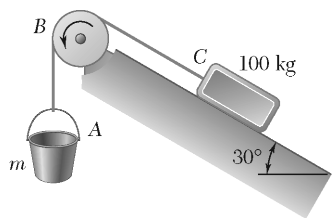
$$T_2 a = T_1 b; \quad 2.5663 T_1 a = T_1(16 - a)$$

$$2.5663a = 16 - a$$

$$3.5663a = 16$$

$$a = 4.49 \text{ in.} \blacktriangleleft$$

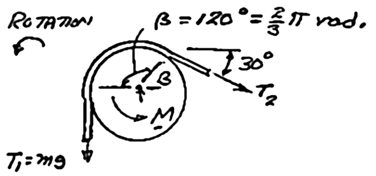
PROBLEM 8.118



Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will (a) remain at rest, (b) start moving up the incline, (c) continue moving up the incline at a constant speed.

SOLUTION

Free body: Drum



$$\frac{T_2}{mg} = e^{\mu \frac{2}{3}\pi}$$

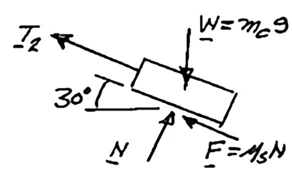
$$T_2 = mge^{2\mu\pi/3} \qquad (1)$$

(a) Smallest m for block C to remain at rest

Cable slips on drum.

Eq. (1) with $\mu_k = 0.25$; $T_2 = mge^{2(0.25)\pi/3} = 1.6881mg$

Block C: At rest, motion impending ↘



$+\nearrow \Sigma F = 0$: $N - m_C g \cos 30°$

$N = m_C g \cos 30°$

$F = \mu_s N = 0.35 m_C g \cos 30°$

$m_C = 100 \text{ kg}$

$+\nwarrow \Sigma F = 0$: $T_2 + F - m_C g \sin 30° = C$

$$1.6881mg + 0.35m_C g \cos 30° - m_C g \sin 30° = 0$$

$$1.6881m = 0.19689m_C$$

$m = 0.11663m_C = 0.11663(100 \text{ kg})$; $\qquad m = 11.66 \text{ kg}$ ◀

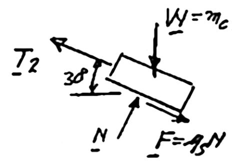
(b) Smallest m to start block moving up

No slipping at both drum and block: $\mu_s = 0.35$

Eq. (1): $T_2 = mge^{2(0.35)\pi/3} = 2.0814mg$

Block C:

Motion impending ↖



$$m_C = 100\,\text{kg}$$

$$+\nearrow \Sigma F = 0: \quad N - mg\cos 30°$$

$$N = m_C g\cos 30°$$
$$F = \mu_s N = 0.35 m_C g\cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g\sin 30° = 0$$

$$2.0814mg - 0.35m_C g\cos 30° - m_C g\sin 30° = 0$$
$$2.0814m = 0.80311m_C$$
$$m = 0.38585m_C = 0.38585(100\,\text{kg})$$

$m = 38.6\,\text{kg}$ ◀

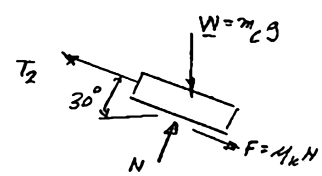
(c) Smallest m to keep block moving up drum: No slipping: $\mu_s = 0.35$

Eq. (1) with $\mu_s = 0.35$

$$T_2 = mg^{2\mu_s\pi/3} = mge^{2(0.35)\pi/3}$$
$$T_2 = 2.0814mg$$

Block C: Moving up plane, thus $\mu_k = 0.25$

Motion up ↖



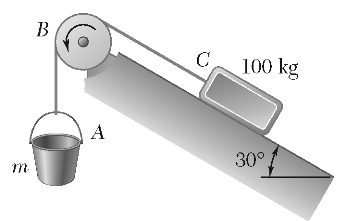
$$+\nearrow \Sigma F = 0: \quad N - m_C g\cos 30° = 0$$

$$N = m_C g\cos 30°$$
$$F = \mu_k N = 0.25 m_C g\cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g\sin 30° = 0$$

$$2.0814mg - 0.25m_C g\cos 30° - m_C g\sin 30° = 0$$
$$2.0814m = 0.71651m_C$$
$$m = 0.34424m_C = 0.34424(100\,\text{kg})$$

$m = 34.4\,\text{kg}$ ◀

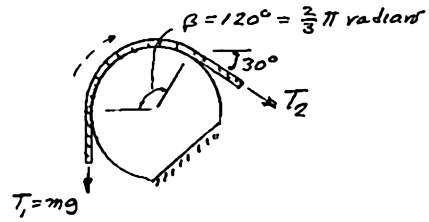


PROBLEM 8.119

Solve Problem 8.118 assuming that drum B is frozen and cannot rotate.

PROBLEM 8.118 Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will (*a*) remain at rest, (*b*) start moving up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION



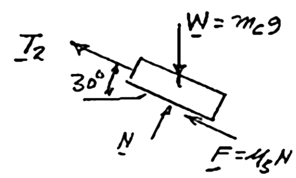
(*a*) <u>Block C remains at rest:</u> Motion impends ↘

<u>Drum:</u>

$$\frac{T_2}{mg} = e^{\mu_k \beta} = e^{0.35(2\pi/3)}$$

$$T_2 = 2.0814mg$$

<u>Block C</u>: Motion impends ↘



$$\nearrow \Sigma F = 0: \quad N - m_C g\cos 30° = 0$$

$$N = m_C g\cos 30°$$

$$F = \mu_s N = 0.35 m_C g\cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_C g\sin 30° = 0$$

$$2.0814mg + 0.35 m_C g\cos 30° - m_C g\sin 30° = 0$$

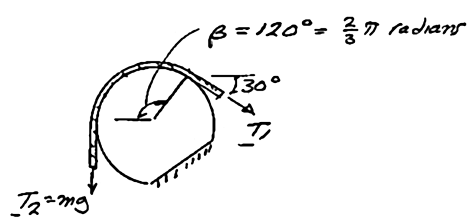
$$2.0814mg = 0.19689 m_C$$

$$m = 0.09459 m_C = 0.09459(100 \text{ kg})$$

$m = 9.46$ kg ◀

(*b*) <u>Block C</u>: Starts moving up $\quad \mu_s = 0.35$

<u>Drum</u>: Impending motion of cable ↺



$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

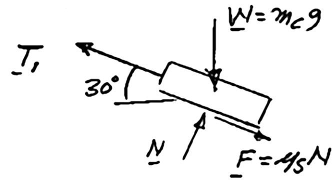
$$\frac{mg}{T_1} = e^{0.35(2/3\pi)}$$

$$T_1 = \frac{mg}{2.0814}$$

$$= 0.48045mg$$

PROBLEM 8.119 (Continued)

Block C: Motion impends $\nwarrow$



$$+\nearrow \Sigma F = 0: \quad N - m_C g \cos 30°$$

$$N = m_C g \cos 30°$$

$$F = \mu_s N = 0.35 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_1 - F - m_C g \sin 30° = 0$$

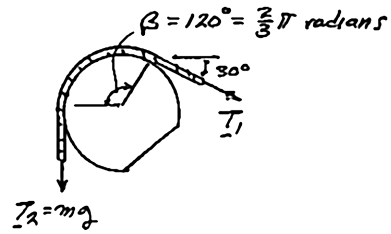
$$0.48045 mg - 0.35 m_C g \cos 30° - 0.5 m_C g = 0$$

$$0.48045 m = 0.80311 m_C$$

$$m = 1.67158 m_C = 1.67158(100\text{ kg}) \qquad m = 167.2\text{ kg} \blacktriangleleft$$

(c) Smallest m to keep block moving $\nwarrow$

Drum: Motion of cable $\circlearrowleft$



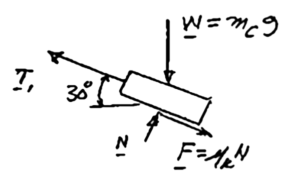
$$\mu_k = 0.25$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25(2/3\pi)}$$

$$\frac{mg}{T_1} = 1.6881$$

$$T_1 = \frac{mg}{1.6881} = 0.59238 mg$$

Block C: Block moves $\nwarrow$



$$+\nearrow \Sigma F = 0: \quad N - m_C g \cos 30° = 0$$

$$N = m_C g \cos 30°$$

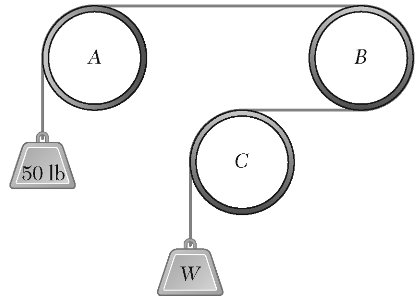
$$F = \mu_k N = 0.25 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_1 - F - m_C g \sin 30° = 0$$

$$0.59238 mg - 0.25 m_C g \cos 30° - 0.5 m_C g = 0$$

$$0.59238 m = 0.71651 m_C$$

$$m = 1.20954 m_C = 1.20954(100\text{ kg}) \qquad m = 121.0\text{ kg} \blacktriangleleft$$

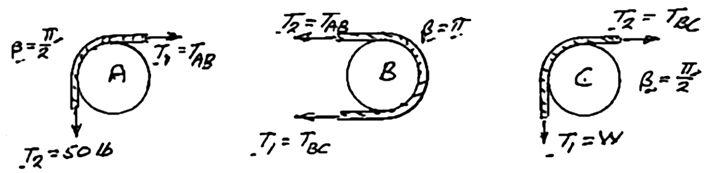


PROBLEM 8.120

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (a) the smallest weight W for which equilibrium is maintained, (b) the largest weight W that can be raised if pipe B is slowly rotated counterclockwise while pipes A and C remain fixed.

SOLUTION

(a) $\mu = \mu_s = 0.25$ at all pipes.



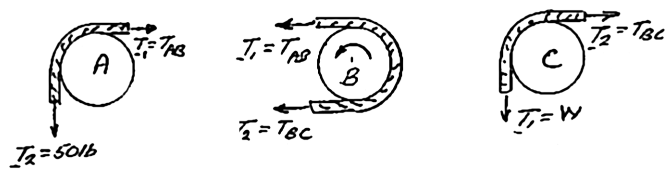
$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.25\pi/2} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.25\pi/2}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/8} \cdot e^{\pi/4} \cdot e^{\pi/8} = e^{\pi/8+\pi/4+\pi/8} = e^{\pi/2} = 4.8105$$

$$\frac{50 \text{ lb}}{W} = 4.8015; \quad W = 10.394 \text{ lb}$$

$W = 10.39$ lb ◀

(b) Pipe B rotated ↺ $\qquad \beta = \frac{\pi}{2};\ \mu = \mu_k \qquad \beta = \pi;\ \mu = \mu_s \qquad \beta = \frac{\pi}{2};\ \mu = \mu_k$



$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{BC}}{T_{AB}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

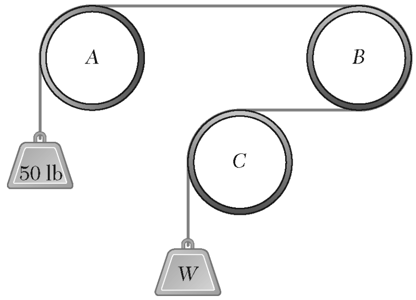
$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{-\pi/4} \cdot e^{\pi/10}$$

$$= e^{\pi/10-\pi/4+\pi/10} = e^{-\pi/20} = 0.85464$$

$$\frac{50 \text{ lb}}{W} = 0.85464$$

$$W = \frac{50 \text{ lb}}{0.85464} = 58.504 \text{ lb}$$

$W = 58.5$ lb ◀



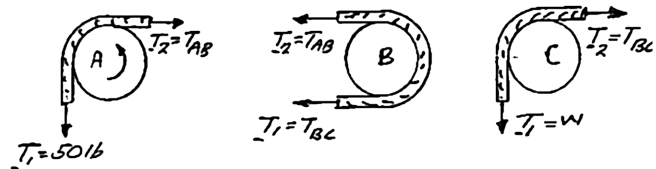
PROBLEM 8.121

A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe A is rotated counterclockwise, (*b*) if only pipe C is rotated clockwise.

SOLUTION

(*a*) Pipe *A* rotates

$$\beta = \frac{\pi}{2};\ \mu = \mu_s \qquad \beta = \pi,\ \mu = \mu_k \qquad \beta = \frac{\pi}{2};\ \mu = \mu_k$$



$$\frac{T_{AB}}{50 \text{ lb}} = e^{0.25\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

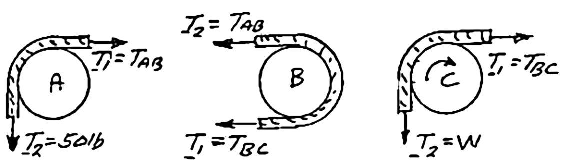
$$\frac{T_{AB}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AB}} \cdot \frac{W}{T_{BC}} = e^{\pi/8} \cdot e^{-\pi/5} \cdot e^{-\pi/10}$$

$$= e^{\pi(1/8-1/5-1/10)} = e^{-7\pi/40} = 0.57708$$

$$\frac{W}{50 \text{ lb}} = 0.57708; \quad W = 28.854 \text{ lb} \qquad W = 28.9 \text{ lb} \blacktriangleleft$$

(*b*) Pipe *C* rotates

$$\beta = \frac{\pi}{2};\ \mu = \mu_k \qquad \beta = \pi;\ \mu = \mu_k \qquad \beta = \frac{\pi}{2},\ \mu = \mu_s$$



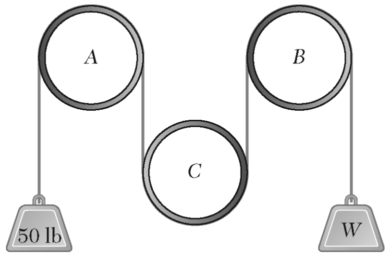
$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi/2}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{\pi/5} \cdot e^{-\pi/8} = e^{7\pi/40} = 0.57708$$

$$\frac{50 \text{ lb}}{W} = 0.57708$$

$$W = 28.854 \text{ lb} \qquad W = 28.9 \text{ lb} \blacktriangleleft$$

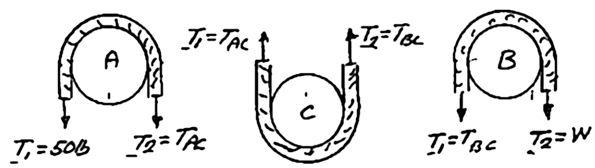
PROBLEM 8.122



A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight W for which equilibrium is maintained, (*b*) the largest weight W that can be raised if pipe B is slowly rotated counterclockwise while pipes A and C remain fixed.

SOLUTION

(*a*) <u>Smallest W for equilibrium</u> $\quad B = \pi, \quad \mu = \mu_s$



$$\frac{T_{AC}}{50 \text{ lb}} = e^{0.25\pi} \qquad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi}$$

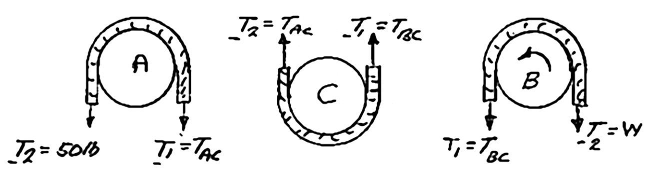
$$\frac{T_{AC}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{\pi/4} \cdot e^{\pi/4} = e^{3\pi/4} = 10.551$$

$$\frac{W}{50 \text{ lb}} = 10.551; \qquad W = 4.739 \text{ lb}$$

$W = 4.74$ lb ◀

(*b*) <u>Largest W which can be raised by pipe B rotated</u>

$$\beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_s$$



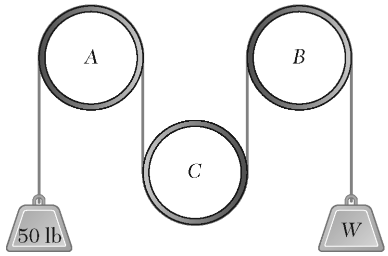
$$\frac{50 \text{ lb}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi}$$

$$\frac{50 \text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{\pi/5} \cdot e^{-\pi/4} = e^{\pi(1/5+1/5-1/4)}$$

$$= e^{3\pi/20} = 1.602$$

$$\frac{50 \text{ lb}}{W} = 1.602; \qquad W = \frac{50 \text{ lb}}{1.602} = 31.21 \text{ lb}$$

$W = 31.2$ lb ◀



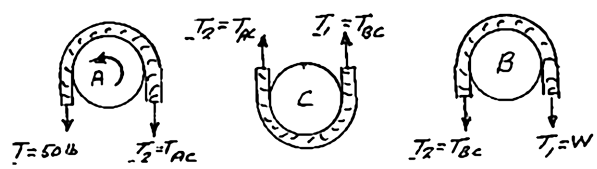
PROBLEM 8.123

A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe A is rotated counterclockwise, (*b*) if only pipe C is rotated clockwise.

SOLUTION

(*a*) Pipe *A* rotates

$\beta = \pi, \mu = \mu_s \qquad \beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_k$



$$\frac{T_{AC}}{50\text{ lb}} = e^{0.25\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{T_{AC}}{50\text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{-\pi/5} \cdot e^{-\pi/5}$$

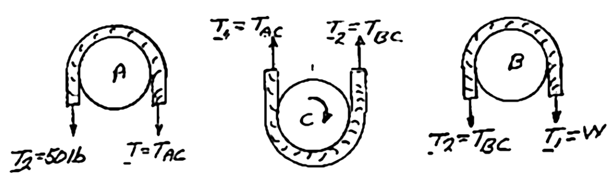
$$= e^{\pi(1/4-1/5-1/5)} = e^{-3\pi/20} = 0.62423$$

$$\frac{W}{50\text{ lb}} = 0.62423; \quad W = 31.21\text{ lb}$$

$W = 31.2\text{ lb}$ ◀

(*b*) Pipe *C* rotates

$\beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_s \qquad \beta = \pi, \mu = \mu_k$



$$\frac{50\text{ lb}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

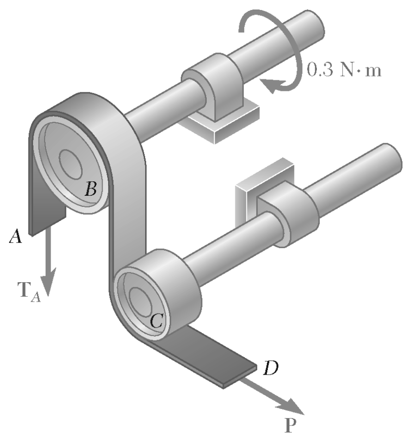
$$\frac{50\text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{-\pi/4} \cdot e^{\pi/5} = e^{\pi(1/5-1/4+1/5)} = e^{3\pi/20}$$

$$\frac{50\text{ lb}}{W} = e^{3\pi/20} = 1.602$$

$$W = \frac{50\text{ lb}}{1.602} = 31.21\text{ lb}$$

$W = 31.2\text{ lb}$ ◀

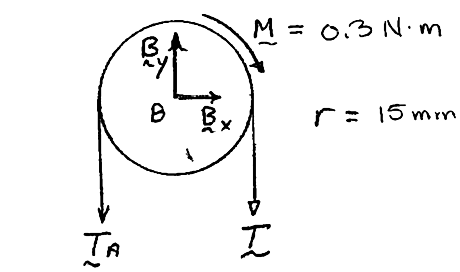
PROBLEM 8.124



A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

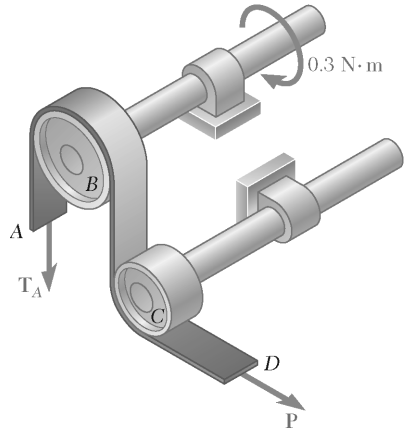
$$T_A - T = \frac{M}{r} = \frac{300 \text{ N} \cdot \text{mm}}{20 \text{ mm}} = 15.0000 \text{ N}$$

Impending slipping: $\quad T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

So $\quad T(e^{0.4\pi} - 1) = 15.0000 \text{ N}$

or $\quad T = 5.9676 \text{ N}$

If C is free to rotate, $P = T$ $\quad P = 5.97 \text{ N}$ ◀



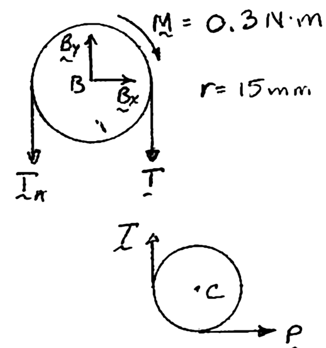
PROBLEM 8.125

Solve Problem 8.124 assuming that the idler drum C is frozen and cannot rotate.

PROBLEM 8.124 A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = 300\text{ N}\cdot\text{mm} = 15.0000\text{ N}$$

Impending slipping: $\quad T_A = Te^{\mu_s\beta} = Te^{0.4\pi}$

So $\quad (e^{0.4\pi} - 1)T = 15.000\text{ N}$

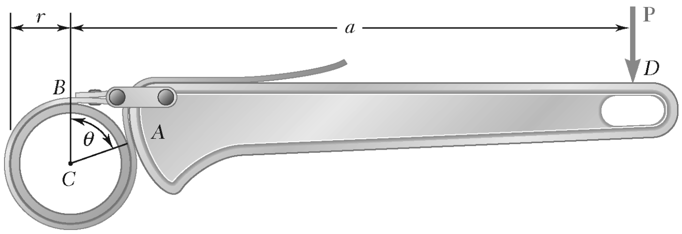
or $\quad T = 5.9676\text{ N}$

If C is fixed, the tape must slip $\circlearrowleft$

So $\quad P = Te^{\mu_k\beta_C} = (5.9676\text{ N})e^{0.3\pi/2} = 9.5600\text{ N} \qquad P = 9.56\text{ N}$ ◀

PROBLEM 8.126

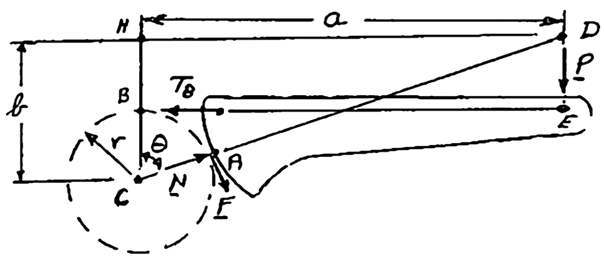
The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop "belt friction" between strap and pipe.

Free body: Wrench handle



Geometry In ΔCDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

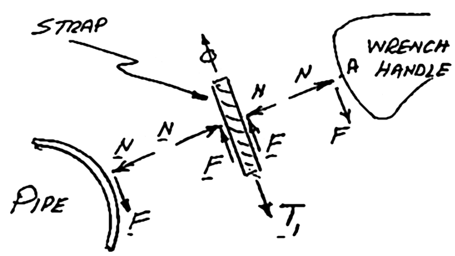
$$DE = \frac{a}{\tan\theta} - r$$

$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

On wrench handle $\quad +\circlearrowleft \Sigma M_D = 0\text{:} \quad T_B(DE) - F(AD) = 0$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \tag{1}$$

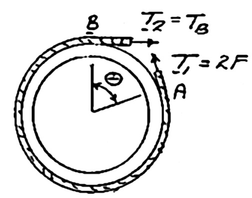
Free body: Strap at Point *A*



$$+\searrow \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \qquad (2)$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):
$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \qquad (3)$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N \sin\theta + F\cos\theta - T_B = 0$$

$$\frac{N}{F}\sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have
$$\frac{1}{\mu_s}\sin\theta = \frac{T_B}{F} - \cos\theta$$

or
$$\mu_s = \frac{\sin\theta}{\frac{T_B}{F} - \cos\theta} \qquad (4)$$

(*Note:* For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For $a = 200$ mm, $r = 30$ mm, $\theta = 65°$

Eq. (1):
$$\frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 65°} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 65°} - 30 \text{ mm}}$$

$$= \frac{190.676 \text{ mm}}{63.262 \text{ mm}} = 3.0141$$

$$\beta = 2\pi - \theta = 2\pi - 65°\frac{\pi}{180°} = 5.1487 \text{ radians}$$

PROBLEM 8.126 (Continued)

Eq. (3):
$$\mu_s = \frac{1}{5.1487 \text{ rad}} \ln \frac{3.0141}{2}$$
$$= \frac{0.41015}{5.1487}$$
$$= 0.0797 \quad \triangleleft$$

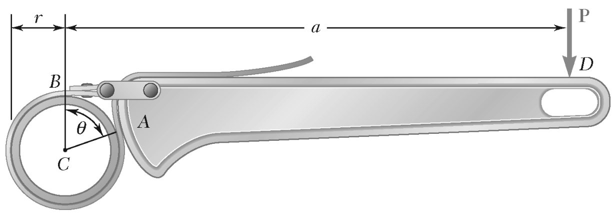
Eq. (4):
$$\mu_s = \frac{\sin 65°}{3.0141 - \cos 65°}$$
$$= \frac{0.90631}{2.1595}$$
$$= 0.3497 \quad \triangleleft$$

We choose the larger value: $\mu_s = 0.350$ ◀

PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75°$.

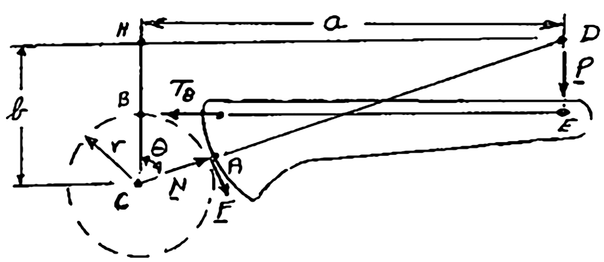
PROBLEM 8.126 The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop "belt friction" between strap and pipe.

Free body: Wrench handle



Geometry In ΔCDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

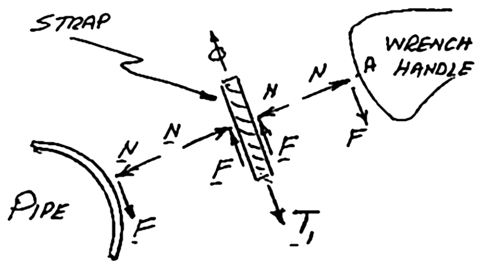
$$DE = \frac{a}{\tan\theta} - r$$

$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

On wrench handle $\quad +\circlearrowleft \Sigma M_D = 0: \quad T_B(DE) - F(AD) = 0$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \tag{1}$$

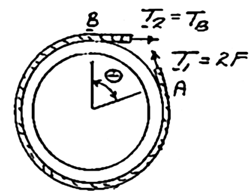
Free body: Strap at Point A



$$+\searrow \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \tag{2}$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):
$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \tag{3}$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N \sin\theta + F\cos\theta - T_B = 0$$

$$\frac{N}{F}\sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have
$$\frac{1}{\mu_s}\sin\theta = \frac{T_B}{F} - \cos\theta$$

or
$$\mu_s = \frac{\sin\theta}{\dfrac{T_B}{F} - \cos\theta} \tag{4}$$

(*Note:* For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For $\quad a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 75°$

Eq. (1):
$$\frac{T_B}{F} = \frac{\dfrac{200 \text{ mm}}{\sin 75°} - 30 \text{ mm}}{\dfrac{200 \text{ mm}}{\tan 75°} - 30 \text{ mm}}$$

$$= \frac{177.055 \text{ mm}}{23.590 \text{ mm}} = 7.5056$$

$$\beta = 2\pi - \theta = 2\pi - 75° \frac{\pi}{180°} = 4.9742$$

PROBLEM 8.127 (Continued)

Eq. (3): $$\mu_s = \frac{1}{4.9742 \text{ rad}} \ln \frac{7.5056}{2}$$

$$= \frac{1.3225}{4.9742}$$

$$= 0.2659 \quad \triangleleft$$

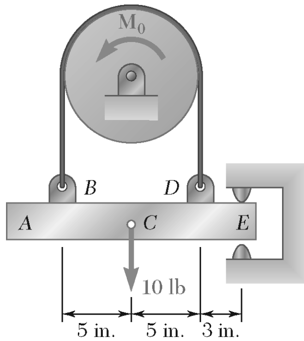
Eq. (4): $$\mu_s = \frac{\sin 75°}{7.5056 - \cos 75°}$$

$$= \frac{0.96953}{7.2468}$$

$$= 0.1333 \quad \triangleleft$$

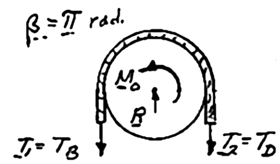
We choose the larger value: $\mu_s = 0.266$ ◀

PROBLEM 8.128



The 10-lb bar AE is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end E of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end E of the bar.

SOLUTION

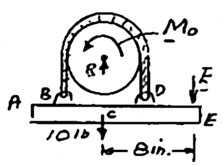


Drum: Slipping impends $\qquad \mu_s = 0.30$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663 T_B$$

(*a*) Free-body: Drum and bar



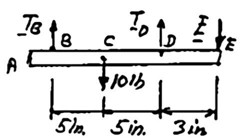
$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(8\text{ in.}) = 0$$

$$M_0 = (3.78649\text{ lb})(8\text{ in.})$$

$$= 30.27\text{ lb}\cdot\text{in.}$$

$$\mathbf{M}_0 = 30.3\text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

(*b*) Bar AE:



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D - E - 10\text{ lb} = 0$$

$$T_B + 2.5663 T_B - E - 10\text{ lb} = 0$$

$$3.5663 T_B - E - 10\text{ lb} = 0$$

$$E = 3.5663 T_B - 10\text{ lb} \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad E(3\text{ in.}) - (10\text{ lb})(5\text{ in.}) + T_B(10\text{ in.}) = 0$$

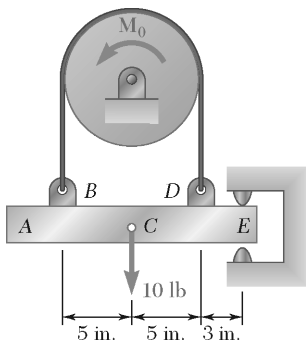
$$(3.5663 T_B - 10\text{ lb})(3\text{ in.}) - 50\text{ lb}\cdot\text{in.} + T_B(10\text{ in.}) = 0$$

$$20.699 T_B = 80 \quad T_B = 3.8649\text{ lb}$$

Eq. (1): $$E = 3.5663(3.8649\text{ lb}) - 10\text{ lb}$$

$$E = +3.78347\text{ lb}$$

$$\mathbf{E} = 3.78\text{ lb} \downarrow \blacktriangleleft$$



PROBLEM 8.129

Solve Problem 8.128 assuming that a clockwise couple $\mathbf{M}_0$ is applied to the drum.

PROBLEM 8.128 The 10-lb bar *AE* is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end *E* of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end *E* of the bar.

SOLUTION

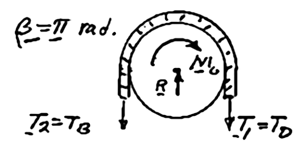
Drum: Slipping impends

$$\mu_s = 0.30$$

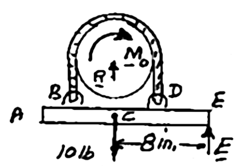
$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{T_B}{T_D} = e^{0.30\pi} = 2.5663$$

$$T_B = 2.5663 T_D$$



(*a*) Free body: Drum and bar

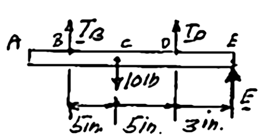


$$+\circlearrowright \Sigma M_C = 0: \quad M_0 - E(8 \text{ in.}) = 0$$

$$M_0 = (2.1538 \text{ lb})(8 \text{ in.})$$

$$\mathbf{M}_0 = 17.23 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

(*b*) Bar *AE*:



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D + E - 10 \text{ lb} = 0$$

$$= 2.5663 T_D + T_D + E - 10 \text{ lb}$$

$$E = -3.5663 T_D + 10 \text{ lb} \qquad (1)$$

$$+\circlearrowright \Sigma M_B = 0: \quad T_D(10 \text{ in.}) - (10 \text{ lb})(5 \text{ in.}) + E(13 \text{ in.}) = 0$$

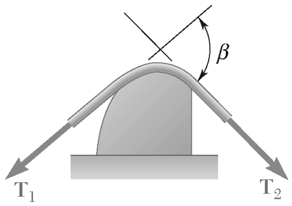
$$T_D(10 \text{ in.}) - 50 \text{ lb} \cdot \text{in.} + (-3.5663 T_D + 10 \text{ lb})(13 \text{ in.}) = 0$$

$$-36.362 T_D + 80 \text{ lb} \cdot \text{in.} = 0; \quad T_D = 2.200 \text{ lb}$$

Eq. (1):

$$E = -3.5633(2.200 \text{ lb}) + 10 \text{ lb}$$

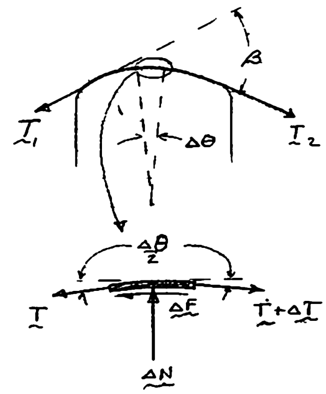
$$E = +2.1538 \text{ lb} \qquad \mathbf{E} = 2.15 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 8.130

Prove that Eqs. (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION



$$\uparrow \Sigma F_n = 0: \quad \Delta N - [T + (T + \Delta T)]\sin\frac{\Delta\theta}{2} = 0$$

or
$$\Delta N = (2T + \Delta T)\sin\frac{\Delta\theta}{2}$$

$$\longrightarrow \Sigma F_t = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

or
$$\Delta F = \Delta T\cos\frac{\Delta\theta}{2}$$

Impending slipping:
$$\Delta F = \mu_s \Delta N$$

So
$$\Delta T\cos\frac{\Delta\theta}{2} = \mu_s 2T\sin\frac{\Delta\theta}{2} + \mu_s \Delta T\frac{\sin\Delta\theta}{2}$$

In limit as
$$\Delta\theta \longrightarrow 0: \quad dT = \mu_s T d\theta \quad \text{or} \quad \frac{dT}{T} = \mu_s d\theta$$

So
$$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu_s d\theta$$

and
$$\ln\frac{T_2}{T_1} = \mu_s\beta \qquad \text{or} \quad T_2 = T_1 e^{\mu_s\beta} \blacktriangleleft$$

(*Note:* Nothing above depends on the shape of the surface, except it is assumed to be a smooth curve.)

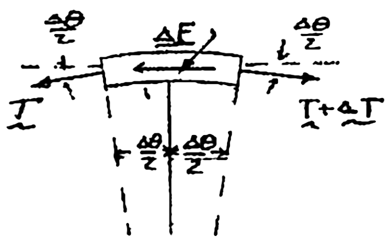
PROBLEM 8.131

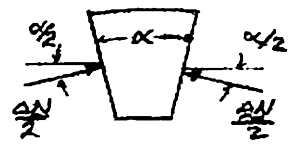
Complete the derivation of Eq. (8.15), which relates the tension in both parts of a V belt.

SOLUTION

Small belt section:

Side view: **End view:**





$$\uparrow \Sigma F_y = 0: \quad 2\frac{\Delta N}{2}\sin\frac{\alpha}{2} - [T + (T + \Delta T)]\sin\frac{\Delta\theta}{2} = 0$$

$$\longrightarrow \Sigma F_x = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

Impending slipping:

$$\Delta F = \mu_s \Delta N \Rightarrow \Delta T\cos\frac{\Delta\theta}{2} = \mu_s \frac{2T + \Delta T}{\sin\frac{\alpha}{2}}\sin\frac{\Delta\theta}{2}$$

In limit as

$$\Delta\theta \longrightarrow 0: \quad dT = \frac{\mu_s T d\theta}{\sin\frac{\alpha}{2}} \quad \text{or} \quad \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}} d\theta$$

so

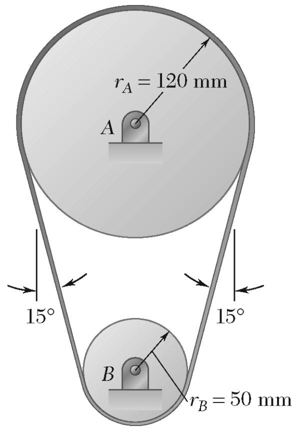
$$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}} \int_0^\beta d\theta$$

or

$$\ln\frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin\frac{\alpha}{2}}$$

or

$$T_2 = T_1 e^{\mu_s \beta / \sin\frac{\alpha}{2}} \blacktriangleleft$$



PROBLEM 8.132

Solve Problem 8.112 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.112 A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

SOLUTION

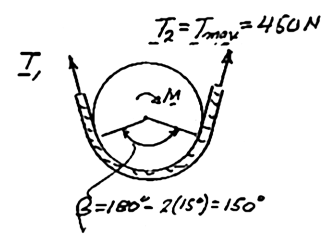
Since β is smaller for pulley *B*. The belt will slip first at *B*.

$$\beta = 150°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{5}{6}\pi \text{ rad}$$

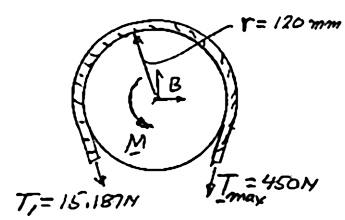
$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{450 \text{ N}}{T_1} = e^{(0.4)\frac{5}{6}\pi/\sin 18°} = e^{3.389}$$

$$\frac{450 \text{ N}}{T_1} = 29.63; \quad T_1 = 15.187 \text{ N}$$



Torque on pulley *A*:



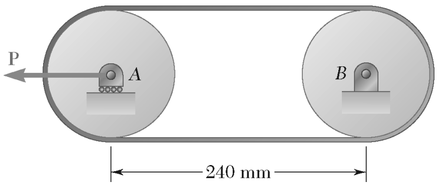
$$+\circlearrowleft \Sigma M_B = 0: \quad M - (T_{max} - T_1)(0.12 \text{ m}) = 0$$

$$M - (450 \text{ N} - 15.187 \text{ N})(0.12 \text{ m}) = 0$$

$$M = 52.18 \text{ N} \cdot \text{m}$$

$M = 52.2$ N · m ◀

PROBLEM 8.133

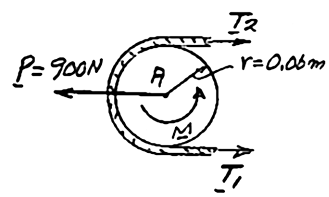


Solve Problem 8.113 assuming that the flat belt and pulleys are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.113 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Pulley *A*: $\beta = \pi$ rad



$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{T_2}{T_1} = e^{0.35\pi / \sin 18°}$$

$$\frac{T_2}{T_1} = e^{3.558} = 35.1$$

$$T_2 = 35.1 T_1$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900 \text{ N} = 0$$

$$T_1 + 35.1 T_1 - 900 \text{ N} = 0$$

$$T_1 = 24.93 \text{ N}$$

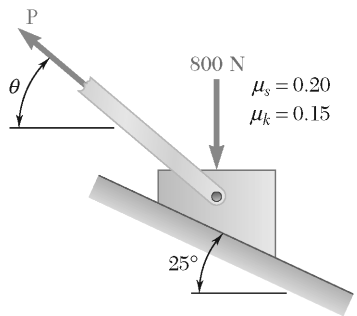
$$T_2 = 35.1(24.93 \text{ N}) = 875.03 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M - T_2(0.06 \text{ m}) + T_1(0.06 \text{ m}) = 0$$

$$M - (875.03 \text{ N})(0.06 \text{ m}) + (24.93 \text{ N})(0.06 \text{ m}) = 0$$

$$M = 51.0 \text{ N} \cdot \text{m} \blacktriangleleft$$

$$T_{max} = T_2 \qquad T_{max} = 875 \text{ N} \blacktriangleleft$$

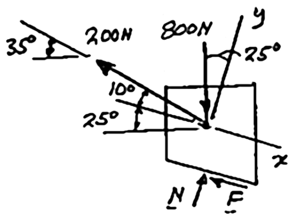


PROBLEM 8.134

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35°$ and $P = 200$ N.

SOLUTION

<u>Assume equilibrium</u>:



$+\nearrow \Sigma F_y = 0$: $N - (800 \text{ N})\cos 25° + (200 \text{ N})\sin 10° = 0$

$N = 690.3$ N $\qquad$ $\mathbf{N} = 690.3$ N $\uparrow$

$\searrow^{+} \Sigma F_x = 0$: $-F + (800 \text{ N})\sin 25° - (200 \text{ N})\cos 10° = 0$

$F = 141.13$ N $\qquad$ $\mathbf{F} = 141.13$ N $\nwarrow$

Maximum friction force:

$$F_m = \mu_s N$$
$$= (0.20)(690.3 \text{ N})$$
$$= 138.06 \text{ N}$$

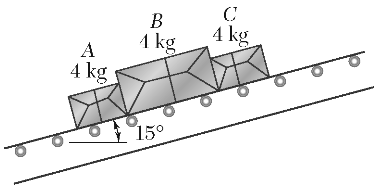
Since $F > F_m$, $\qquad$ <u>Block moves down</u> $\searrow$ ◀

Friction force:

$$F = \mu_k N$$
$$= (0.15)(690.3 \text{ N})$$
$$= 103.547 \text{ N}$$

$\mathbf{F} = 103.5$ N $\nwarrow$ ◀

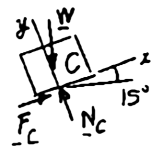
PROBLEM 8.135



Three 4-kg packages A, B, and C are placed on a conveyor belt that is at rest. Between the belt and both packages A and C the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package B and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

SOLUTION

Consider C by itself: Assume equilibrium



$$+\nwarrow \Sigma F_y = 0: \quad N_C - W\cos 15° = 0$$

$$N_C = W\cos 15° = 0.966W$$

$$+\nearrow \Sigma F_x = 0: \quad F_C - W\sin 15° = 0$$

$$F_C = W\sin 15° = 0.259W$$

But

$$\begin{aligned} F_m &= \mu_s N_C \\ &= 0.30(0.966W) \\ &= 0.290W \end{aligned}$$

Thus, $F_C < F_m$ Package C does not move ◀

$$\begin{aligned} F_C &= 0.259W \\ &= 0.259(4\text{ kg})(9.81\text{ m/s}^2) \\ &= 10.16\text{ N} \end{aligned}$$

$$\mathbf{F}_C = 10.16\text{ N} \nearrow \blacktriangleleft$$

Consider B by itself: Assume equilibrium. We find,

$$F_B = 0.259W$$
$$N_B = 0.966W$$

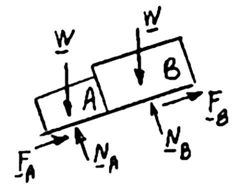
But

$$\begin{aligned} F_m &= \mu_s N_B \\ &= 0.10(0.966W) \\ &= 0.0966W \end{aligned}$$

Thus, $F_B > F_m$. Package B would move if alone ◀

PROBLEM 8.135 (Continued)

Consider A and B together: Assume equilibrium



$$F_A = F_B = 0.259W$$
$$N_A = N_B = 0.966W$$
$$F_A + F_B = 2(0.259W) = 0.518W$$
$$(F_A)_m + (F_B)_m = 0.3N_A + 0.1N_B = 0.386W$$

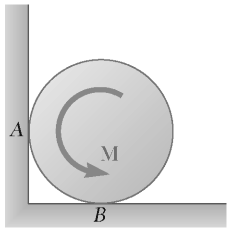
Thus, $$F_A + F_B > (F_A)_m + (F_B)_m$$ A and B move ◀

$$F_A = \mu_k N_A = 0.2(0.966)(4)(9.81)$$

$\mathbf{F}_A = 7.58\text{ N} \nearrow$ ◀

$$F_B = \mu_k N_B = 0.08(0.966)(4)(9.81)$$

$\mathbf{F}_B = 3.03\text{ N} \nearrow$ ◀

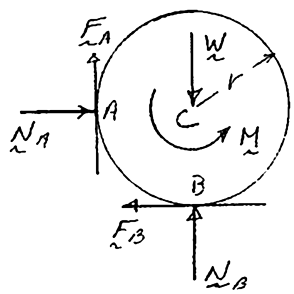


PROBLEM 8.136

The cylinder shown is of weight W and radius r. Express in terms W and r the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate, assuming the coefficient of static friction to be (*a*) zero at A and 0.30 at B, (*b*) 0.25 at A and 0.30 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A$$
$$F_B = \mu_B N_B$$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - F_B = 0$$
$$N_A = F_B = \mu_B N_B$$
$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$
$$N_B(1 + \mu_A \mu_B) = W$$

or
$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and
$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$
$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

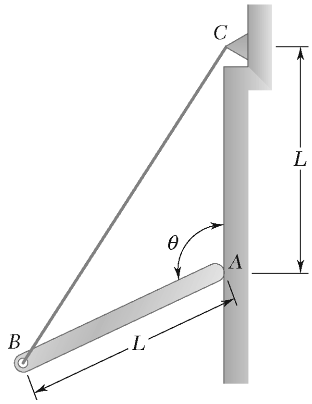
$$\Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$
$$M = Wr\mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

(*a*) For $\mu_A = 0$ and $\mu_B = 0.30$:

$M = 0.300Wr$ ◀

(*b*) For $\mu_A = 0.25$ and $\mu_B = 0.30$:

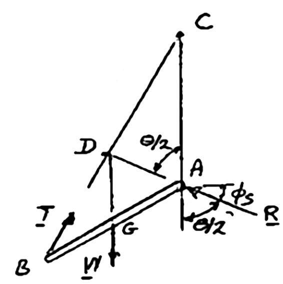
$M = 0.349Wr$ ◀



PROBLEM 8.137

End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (*a*) the largest value of θ for which motion is impending, (*b*) the corresponding value of the tension in the cord.

SOLUTION



Free-body diagram

Three-force body. Line of action of **R** must pass through D, where **T** and **R** intersect.

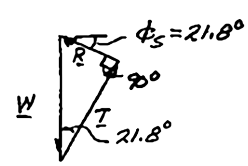
Motion impends:

$$\tan\phi_s = 0.4$$
$$\phi_s = 21.80°$$

(*a*) Since $BG = GA$, it follows that $BD = DC$ and AD bisects $\angle BAC$

$$\frac{\theta}{2} + \phi_s = 90°$$
$$\frac{\theta}{2} + 21.8° = 90°$$

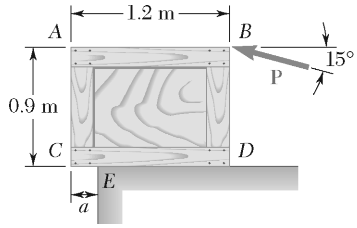
$\theta = 136.4°$ ◀



(*b*) Force triangle (right triangle):

$$T = W\cos 21.8°$$

$T = 0.928W$ ◀



PROBLEM 8.138

A worker slowly moves a 50-kg crate to the left along a loading dock by applying a force **P** at corner B as shown. Knowing that the crate starts to tip about the edge E of the loading dock when $a = 200$ mm, determine (a) the coefficient of kinetic friction between the crate and the loading dock, (b) the corresponding magnitude P of the force.

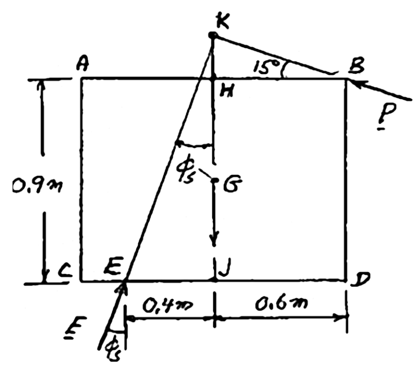
SOLUTION

<u>Free body: Crate</u> Three-force body.

Reaction E must pass through K where **P** and **W** intersect.

<u>Geometry</u>:

(a) $$HK = (0.6\text{ m})\tan 15° = 0.16077\text{ m}$$



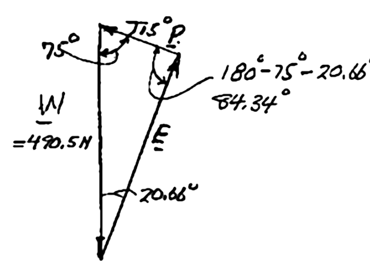
$$JK = 0.9\text{ m} + HK = 1.06077\text{ m}$$

$$\tan\phi_s = \frac{0.4\text{ m}}{1.06077\text{ m}} = 0.37708$$

$\mu_s = \tan\phi_s = 0.377$ ◀

$$\phi_s = 20.66°$$

<u>Force triangle</u>:



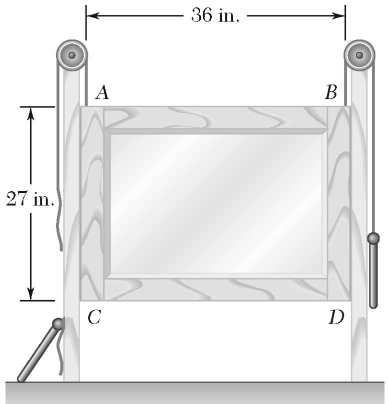
(b) $$W = (50\text{ kg})(9.81\text{ m/s}) = 490.5\text{ N}$$

<u>Law of sines</u>:

$$\frac{P}{\sin 20.66°} = \frac{490.5\text{ N}}{\sin 84.34°}$$

$$P = 173.91\text{ N}$$

$P = 173.9$ N ◀

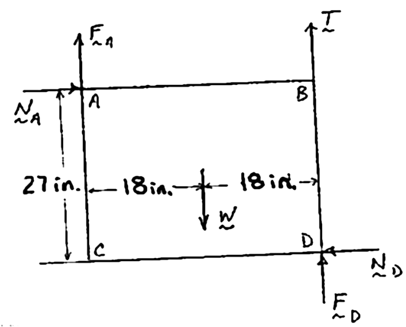


PROBLEM 8.139

A window sash weighing 10 lb is normally supported by two 5-lb sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller than the frame and will bind only at Points *A* and *D*.)

SOLUTION

FBD window: $T = 5\text{ lb}$



$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - N_D = 0$$

$$N_A = N_D$$

Impending motion:

$$F_A = \mu_s N_A$$

$$F_D = \mu_s N_D$$

$$\circlearrowleft \Sigma M_D = 0: \quad (18\text{ in.})W - (27\text{ in.})N_A - (36\text{ in.})F_A = 0 \qquad W = 10\text{ lb}$$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad F_A - W + T + F_D = 0$$

$$F_A + F_D = W - T = \frac{W}{2}$$

Now

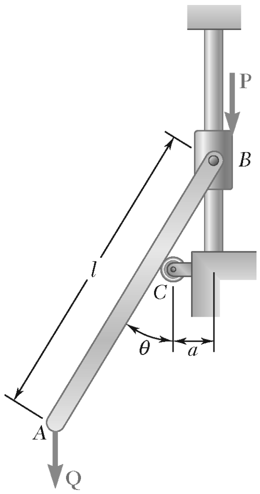
$$F_A + F_D = \mu_s(N_A + N_D)$$

$$= 2\mu_s N_A$$

Then

$$\frac{W}{2} = 2\mu_s \frac{2W}{3 + 4\mu_s}$$

or $\mu_s = 0.750$ ◀



PROBLEM 8.140

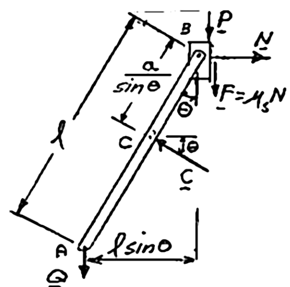
The slender rod AB of length $l = 600$ mm is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 80$ mm from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 100$ N and $\theta = 30°$.

SOLUTION

<u>For motion of collar at B impending upward</u>:

$$\mathbf{F} = \mu_s N \downarrow$$

$$+\circlearrowright \Sigma M_B = 0: \quad Ql\sin\theta - \frac{Ca}{\sin\theta} = 0$$



$$C = Q\left(\frac{l}{a}\right)\sin^2\theta$$

$$\Sigma F_x = 0: \quad N = C\cos\theta = Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta$$

$$+\downarrow \Sigma F_y = 0: \quad P + Q - C\sin\theta - \mu_s N = 0$$

$$P + Q - Q\left(\frac{l}{a}\right)\sin^3\theta - \mu_s Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta = 0$$

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta - \mu_s\cos\theta) - 1\right] \qquad (1)$$

Substitute data:

$$P = (100\text{ N})\left[\frac{600\text{ mm}}{80\text{ mm}}\sin^2 30°(\sin 30° - 0.25\cos 30°) - 1\right]$$

$$P = -46.84\text{ N } (P \text{ is directed } \uparrow)$$

$$P = -46.8\text{ N} \quad \triangleleft$$

PROBLEM 8.140 (Continued)

For motion of collar, impending downward:

$$\mathbf{F} = \mu_s N \uparrow$$

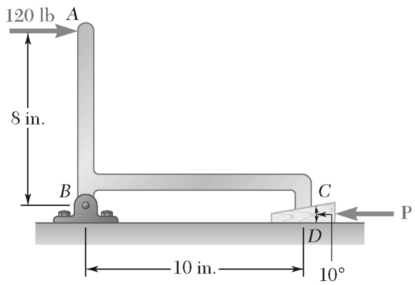
In Eq. (1) we substitute $-\mu_s$ for μ_s.

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta + \mu_s\cos\theta) - 1\right]$$

$$P = (100\text{ N})\left[\frac{600\text{ mm}}{80\text{ mm}}\sin^2 30°(\sin 30° + 0.25\cos\theta) - 1\right]$$

$$P = +34.34\text{ N} \quad \triangleleft$$

For equilibrium: $-46.8\text{ N} \le P \le 34.3\text{ N}$ ◀



PROBLEM 8.141

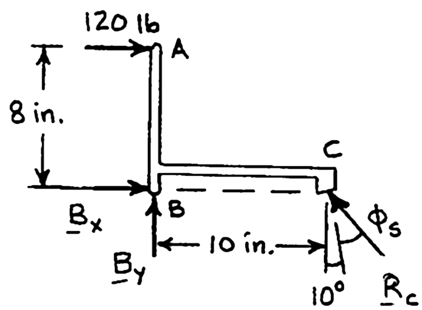
The machine part ABC is supported by a frictionless hinge at B and a 10° wedge at C. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (*a*) the force **P** required to move the wedge to the left, (*b*) the components of the corresponding reaction at B.

SOLUTION

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.20 = 11.3099°$$

<u>Free body: *ABC*</u>

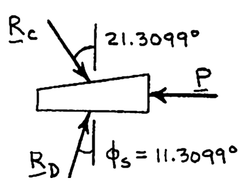


$$10° + 11.3099° = 21.3099°$$

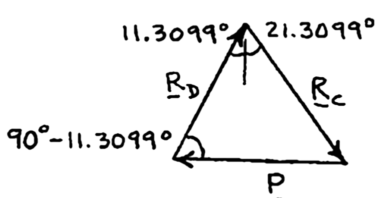
$$+\circlearrowleft \Sigma M_B = 0: \quad (R_C \cos 21.3099°)(10) - (120 \text{ lb})(8) = 0$$

$$R_C = 103.045 \text{ lb}$$

<u>Free body: Wedge</u>



Force triangle:



Law of sines:

$$\frac{P}{\sin 32.6198°} = \frac{(R_C = 103.045 \text{ lb})}{\sin 78.690°}$$

(a) $\mathbf{P} = 56.6 \text{ lb} \longleftarrow$ ◀

(b) Returning to free body of ABC:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 120 - (103.045)\sin 21.3099° = 0$$

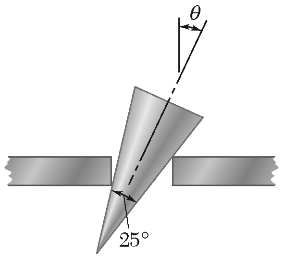
$$B_x = -82.552 \text{ lb}$$

$\mathbf{B}_x = 82.6 \text{ lb} \longleftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + (103.045)\cos 21.3099° = 0$$

$$B_y = -96.000 \text{ lb}$$

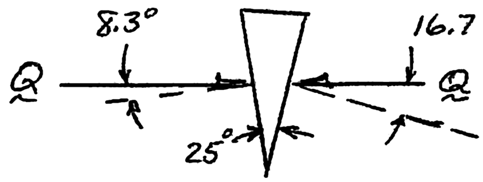
$\mathbf{B}_y = 96.0 \text{ lb} \downarrow$ ◀



PROBLEM 8.142

A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (*a*) if $\mu_s = 0.20$, (*b*) if $\mu_s = 0.30$.

SOLUTION



As the plates are moved, the angle θ will decrease.

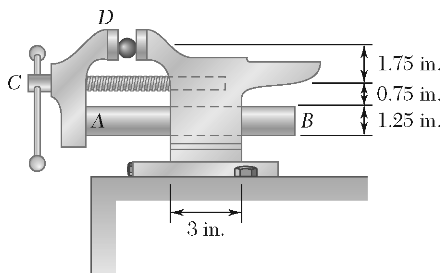
(*a*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.2 = 11.31°.$

As θ decrease, the minimum angle at the contact approaches

$12.5° > \phi_s = 11.31°,$ so the wedge will slide up and out from the slot. ◀

(*b*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.3 = 16.70°.$

As θ decreases, the angle at one contact reaches 16.7°. (At this time the angle at the other contact is $25° - 16.7° = 8.3° < \phi_s$). The wedge binds in the slot. ◀

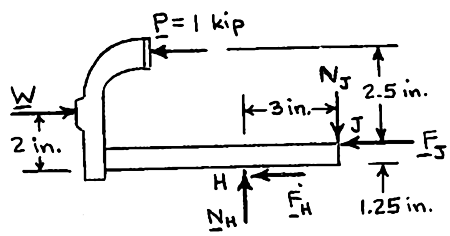


PROBLEM 8.143

In the machinist's vise shown, the movable jaw D is rigidly attached to the tongue AB that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 0.75 in. and a pitch of 0.25 in. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 1 kip.

SOLUTION

Free body: Jaw D and tongue AB



P is due to elastic forces in clamped object.

W is force exerted by screw.

$$+\uparrow \Sigma F_y = 0: \quad N_H - N_J = 0 \quad N_J = N_H = N$$

For final tightening,

$$F_H = F_J = \mu_s N = 0.25\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad W - P - 2(0.25\text{ N}) = 0$$

$$N = 2(W - P) \tag{1}$$

$$+\circlearrowleft \Sigma M_H = 0: \quad P(3.75) - W(2) - N(3) + (0.25\text{ N})(1.25) = 0$$

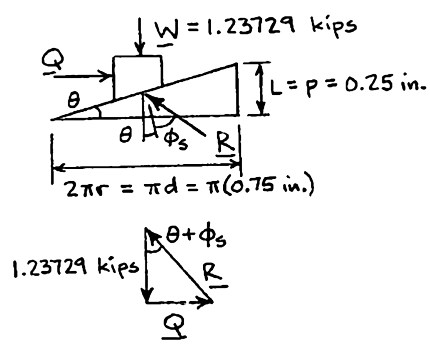
$$3.75P - 2W - 2.6875\text{ N} = 0 \tag{2}$$

Substitute Eq. (1) into Eq. (2): $\quad 3.75P - 2W - 2.6875[2(W - P)] = 0$

$$7.375W = 9.125P = 9.125(1\text{ kip})$$

$$W = 1.23729\text{ kips}$$

Block-and-incline analysis of screw:



$$\tan\phi_s = \mu_s = 0.25$$

$$\phi_s = 14.0362°$$

$$\tan\theta = \frac{0.25\text{ in.}}{\pi(0.75\text{ in.})}$$

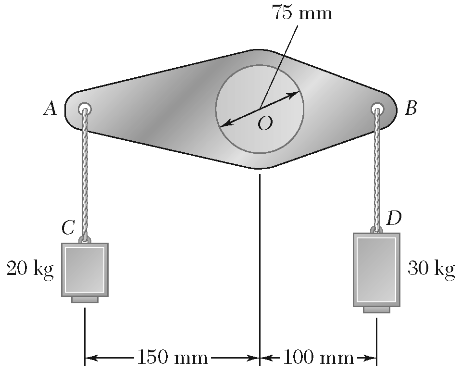
$$\theta = 6.0566°$$

$$\theta + \phi_s = 20.093°$$

$$Q = (1.23729\text{ kips})\tan 20.093°$$

$$= 0.45261\text{ kip}$$

$$T = Qr = (452.61\text{ lb})\left(\frac{0.75\text{ in.}}{2}\right) \qquad T = 169.7\text{ lb}\cdot\text{in.} \blacktriangleleft$$

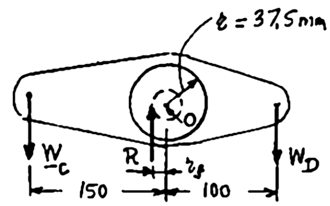


PROBLEM 8.144

A lever of negligible weight is loosely fitted onto a 75-mm-diameter fixed shaft. It is observed that the lever will just start rotating if a 3-kg mass is added at *C*. Determine the coefficient of static friction between the shaft and the lever.

SOLUTION

$$+\circlearrowleft \Sigma M_O = 0: \quad W_C(150) - W_D(100) - Rr_f = 0$$



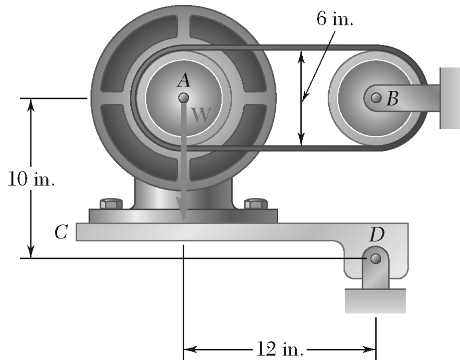
But
$$W_C = (23\text{ kg})(9.81\text{ m/s}^2)$$
$$W_D = (30\text{ kg})(9.81\text{ m/s}^2)$$
$$R = W_C + W_D = (53\text{ kg})(9.81)$$

Thus, after dividing by 9.81,

$$23(150) - 30(100) - 53\,r_f = 0$$
$$r_f = 8.49\text{ mm}$$

But
$$\mu_s \approx \frac{r_f}{r} = \frac{8.49\text{ mm}}{37.5\text{ mm}}$$

$$\mu_s \approx 0.226 \blacktriangleleft$$

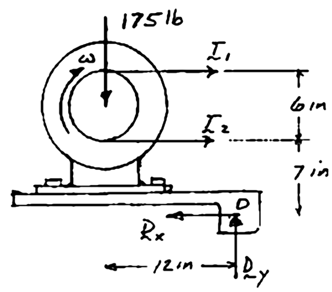


PROBLEM 8.145

In the pivoted motor mount shown the weight **W** of the 175-lb motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest couple that can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor and mount:



Impending belt slip: cw rotation

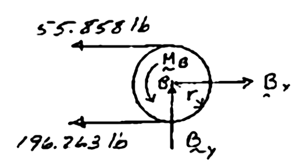
$$T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.40\pi} = 3.5136 T_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (12 \text{ in.})(175 \text{ lb}) - (7 \text{ in.})T_2 - (13 \text{ in.})T_1 = 0$$

$$2100 \text{ lb} = [(7 \text{ in.})(3.5136) + 13 \text{ in.}]T_1$$

$$T_1 = 55.858 \text{ lb}, \quad T_2 = 3.5136 T_1 = 196.263 \text{ lb}$$

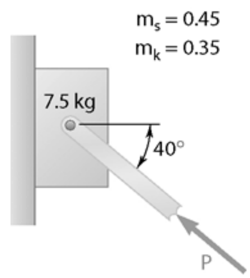
FBD drum at *B*:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (3 \text{ in.})(196.263 \text{ lb} - 55.858 \text{ lb}) = 0$$

$r = 3$ in.

$$M_B = 421 \text{ lb} \cdot \text{in.} \blacktriangleleft$$



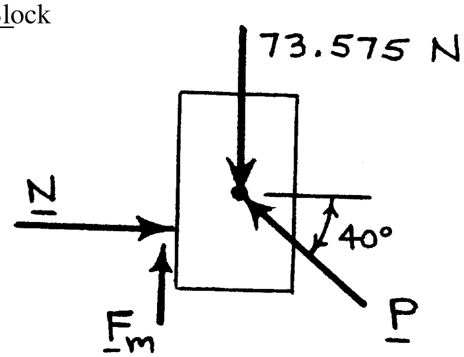
PROBLEM 8.F1

Draw the free-body diagram needed to determine the smallest force **P** for which equilibrium of the 7.5-kg block is maintained.

SOLUTION

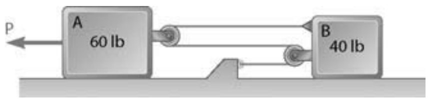
$$W = (7.5 \text{ kg})(9.81 \text{ m/s}^2) = 73.575 \text{ N}$$

Free body: Block



Since we seek smallest P, motion impends downward, with

$$F_m = \mu_s N = 0.45 \text{ N}$$

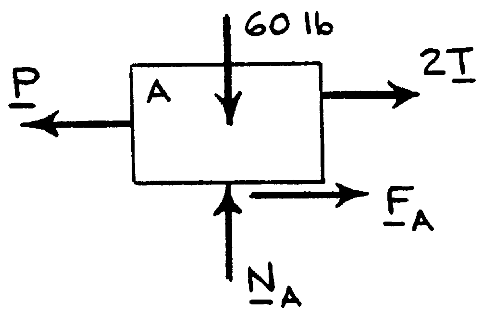


PROBLEM 8.F2

Two blocks A and B are connected by a cable as shown. Knowing that the coefficient of static friction at all surfaces of contact is 0.30 and neglecting the friction of the pulleys, draw the free-body diagrams needed to determine the smallest force **P** required to move the blocks.

SOLUTION

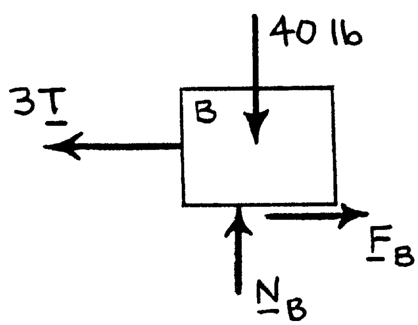
Free body: Block A



Motion impends to left, with

$$F_A = \mu_S N_A = 0.30 N_A$$

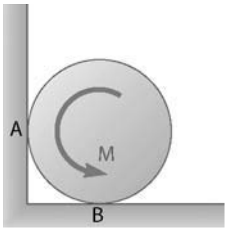
Free body: Block B



Motion impends to left, with

$$F_B = \mu_s N_B = 0.30\ N_B$$

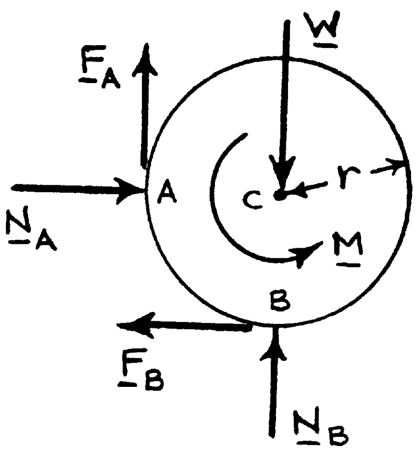
PROBLEM 8.F3



The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Draw the free-body diagram needed to determine the largest couple **M** that can be applied to the cylinder if it is not to rotate.

SOLUTION

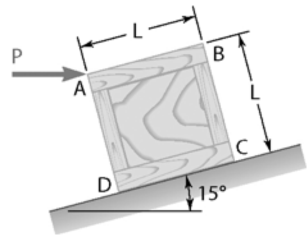
Free body: Cylinder



For maximum **M**, motion impends at both A and B, with

$$F_A = \mu_s N_A$$
$$F_B = \mu_S N_B$$

PROBLEM 8.F4

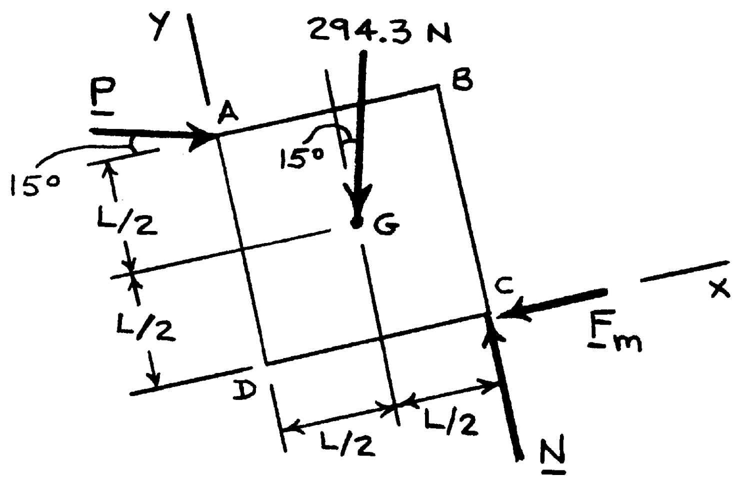


A uniform crate of mass 30 kg must be moved up along the 15° incline without tipping. Knowing that the force **P** is horizontal, draw the free-body diagram needed to determine the largest allowable coefficient of static friction between the crate and the incline, and the corresponding force **P**.

SOLUTION

$$W = (30\text{ kg})\,(9.81\text{ m/s}^2) = 294.3\text{ N}$$

Free body: Crate



For impending tip, **N** must act at C.

For impending slip, $F_m = \mu_{\max} N$