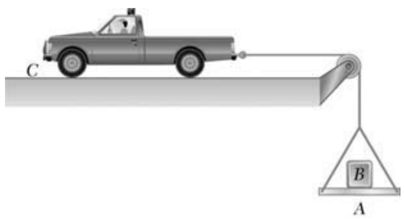
CHAPTER 12



PROBLEM 12.CQ1

A 1000 lb boulder B is resting on a 200 lb platform A when truck C accelerates to the left with a constant acceleration. Which of the following statements are true (more than one may be true)?

(a) The tension in the cord connected to the truck is 200 lb

(b) The tension in the cord connected to the truck is 1200 lb

(c) The tension in the cord connected to the truck is greater than 1200 lb

(d) The normal force between A and B is 1000 lb

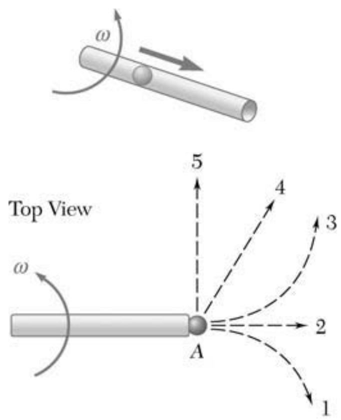
(e) The normal force between A and B is 1200 lb

(f) None of the above

SOLUTION

Answer: (c) The tension will be greater than 1200 lb and the normal force will be greater than 1000 lb.

PROBLEM 12.CQ2



Marble A is placed in a hollow tube, and the tube is swung in a horizontal plane causing the marble to be thrown out. As viewed from the top, which of the following choices best describes the path of the marble after leaving the tube?

(*a*) 1

(*b*) 2

(*c*) 3

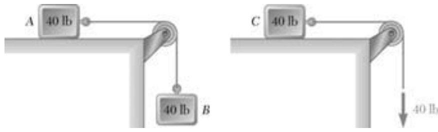
(*d*) 4

(*e*) 5

SOLUTION

Answer: (*d*) The particle will have velocity components along the tube and perpendicular to the tube. After it leaves, it will travel in a straight line.

PROBLEM 12.CQ3



The two systems shown start from rest. On the left, two 40 lb weights are connected by an inextensible cord, and on the right, a constant 40 lb force pulls on the cord. Neglecting all frictional forces, which of the following statements is true?

(*a*) Blocks *A* and *C* will have the same acceleration

(*b*) Block *C* will have a larger acceleration than block *A*

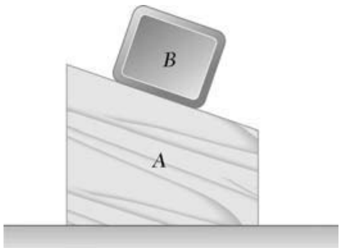
(*c*) Block *A* will have a larger acceleration than block *C*

(*d*) Block *A* will not move

(*e*) None of the above

SOLUTION

Answer: (*b*) If you draw a FBD of *B*, you will see that since it is accelerating downward, the tension in the cable will be less than 40 lb, so the acceleration of *A* will be less than the acceleration of *C*. Also, the system on the left has more inertia, so it is harder to accelerate than the system on the right.



PROBLEM 12.CQ4

The system shown is released from rest in the position shown. Neglecting friction, the normal force between block *A* and the ground is

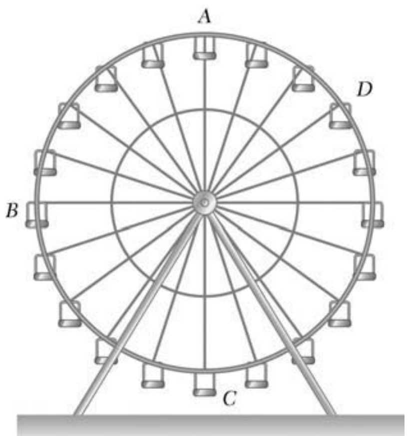
(*a*) less than the weight of *A* plus the weight of *B*

(*b*) equal to the weight of *A* plus the weight of *B*

(*c*) greater than the weight of *A* plus the weight of *B*

SOLUTION

Answer: (*a*) Since *B* has an acceleration component downward the normal force between *A* and the ground will be less than the sum of the weights.



PROBLEM 12.CQ5

People sit on a Ferris wheel at Points *A*, *B*, *C* and *D*. The Ferris wheel travels at a constant angular velocity. At the instant shown, which person experiences the largest force from his or her chair (back and seat)? Assume you can neglect the size of the chairs, that is, the people are located the same distance from the axis of rotation.

(*a*) *A*

(*b*) *B*

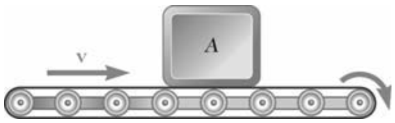
(*c*) *C*

(*d*) *D*

(*e*) The force is the same for all the passengers.

SOLUTION

Answer: (*c*) Draw a FBD and KD at each location and it will be clear that the maximum force will be experiences by the person at Point *C*.

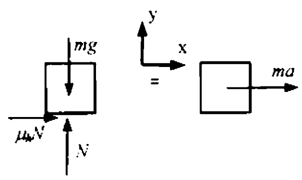


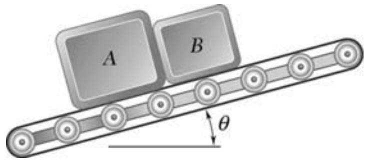
PROBLEM 12.F1

Crate A is gently placed with zero initial velocity onto a moving conveyor belt. The coefficient of kinetic friction between the crate and the belt is μ_k. Draw the FBD and KD for A immediately after it contacts the belt.

SOLUTION

Answer:





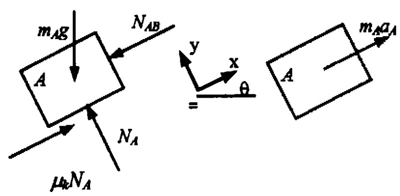
PROBLEM 12.F2

Two blocks weighing W_A and W_B are at rest on a conveyor that is initially at rest. The belt is suddenly started in an upward direction so that slipping occurs between the belt and the boxes. Assuming the coefficient of friction between the boxes and the belt is μ_k, draw the FBDs and KDs for blocks A and B. How would you determine if A and B remain in contact?

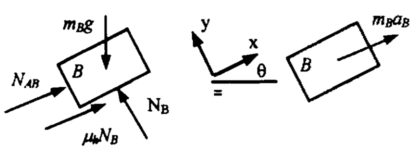
SOLUTION

Answer:

Block A

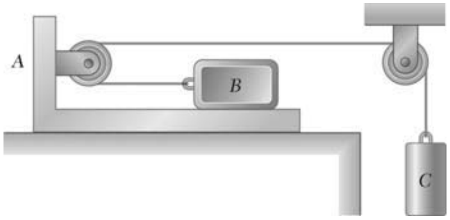


Block B



To see if they remain in contact assume $a_A = a_B$ and then check to see if N_{AB} is greater than zero.

PROBLEM 12.F3

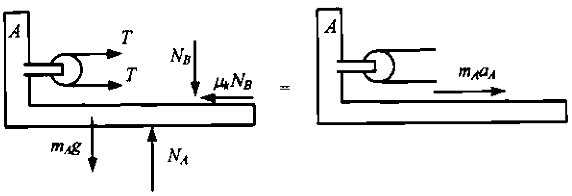


Objects A, B, and C have masses m_A, m_B, and m_C respectively. The coefficient of kinetic friction between A and B is μ_k, and the friction between A and the ground is negligible and the pulleys are massless and frictionless. Assuming B slides on A draw the FBD and KD for each of the three masses A, B and C.

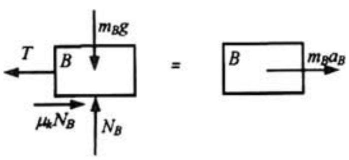
SOLUTION

Answer:

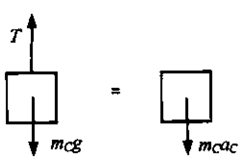
Block A



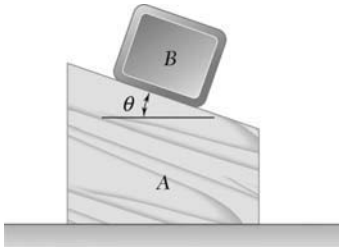
Block B



Block C



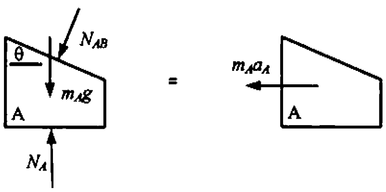
PROBLEM 12.F4



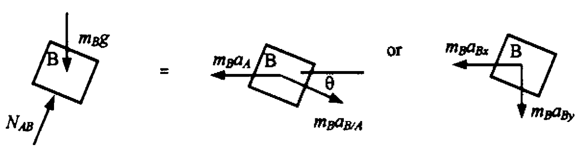
Blocks A and B have masses m_A and m_B respectively. Neglecting friction between all surfaces, draw the FBD and KD for each mass.

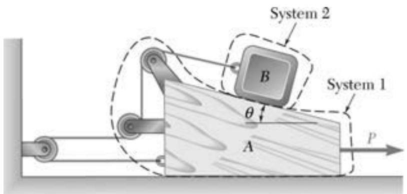
SOLUTION

Block A



Block B



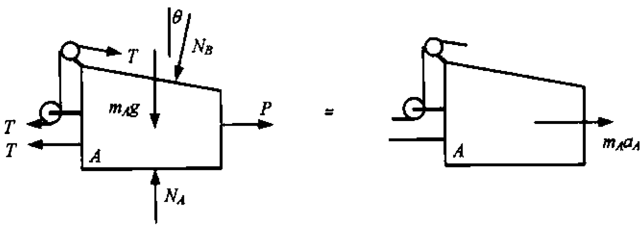


PROBLEM 12.F5

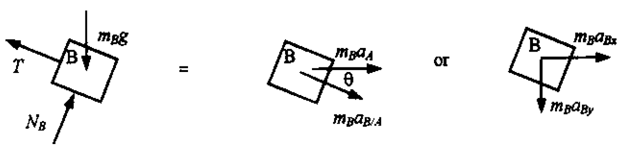
Blocks A and B have masses m_A and m_B respectively. Neglecting friction between all surfaces, draw the FBD and KD for the two systems shown.

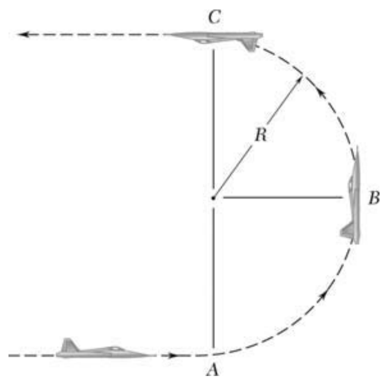
SOLUTION

System 1



System 2

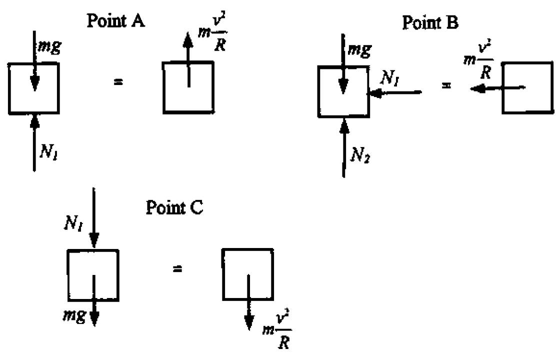




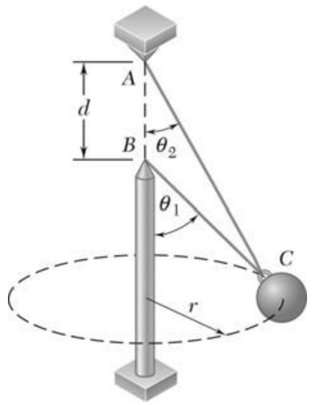
PROBLEM 12.F6

A pilot of mass m flies a jet in a half vertical loop of radius R so that the speed of the jet, v, remains constant. Draw a FBD and KD of the pilot at Points A, B and C.

SOLUTION

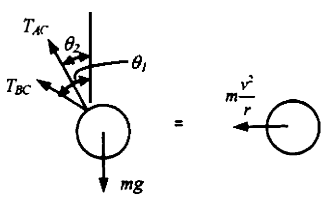


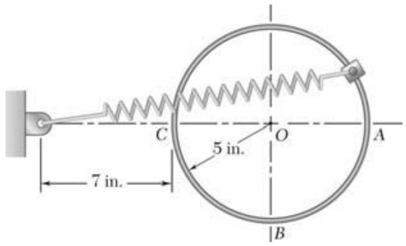
PROBLEM 12.F7



Wires AC and BC are attached to a sphere which revolves at a constant speed v in the horizontal circle of radius r as shown. Draw a FBD and KD of C.

SOLUTION

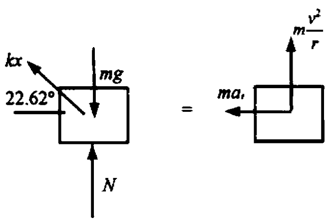




PROBLEM 12.F8

A collar of mass m is attached to a spring and slides without friction along a circular rod in a vertical plane. The spring has an undeformed length of 5 in. and a constant k. Knowing that the collar has a speed v at Point B, draw the FBD and KD of the collar at this point.

SOLUTION



where $x = 7/12$ ft and $r = 5/12$ ft.

PROBLEM 12.1

Astronauts who landed on the moon during the Apollo 15, 16 and 17 missions brought back a large collection of rocks to the earth. Knowing the rocks weighed 139 lb when they were on the moon, determine (*a*) the weight of the rocks on the earth, (*b*) the mass of the rocks in slugs. The acceleration due to gravity on the moon is 5.30 ft/s^2.

SOLUTION

Since the rocks weighed 139 lb on the moon, their mass is

$$m = \frac{W_{\text{moon}}}{g_{\text{moon}}} = \frac{139 \text{ lb}}{5.30 \text{ ft/s}^2} = 26.226 \text{ lb} \cdot \text{s}^2/\text{ft}$$

(*a*) On the earth, $W_{\text{earth}} = mg_{\text{earth}}$

$$w = (26.226 \text{ lb} \cdot \text{s}^2/\text{ft})(32.2 \text{ ft/s}^2) \qquad w = 844 \text{ lb} \blacktriangleleft$$

(*b*) Since $1 \text{ slug} = 1 \text{ lb} \cdot \text{s}^2/\text{ft}$, $\qquad m = 26.2 \text{ slugs} \blacktriangleleft$

PROBLEM 12.2

The value of g at any latitude ϕ may be obtained from the formula

$$g = 32.09(1 + 0.0053 \sin^2\phi) \text{ ft/s}^2$$

which takes into account the effect of the rotation of the earth, as well as the fact that the earth is not truly spherical. Determine to four significant figures (a) the weight in pounds, (b) the mass in pounds, (c) the mass in $\text{lb}\cdot\text{s}^2/\text{ft}$, at the latitudes of 0°, 45°, and 60°, of a silver bar, the mass of which has been officially designated as 5 lb.

SOLUTION

$$g = 32.09(1 + 0.0053 \sin^2\phi) \text{ ft/s}^2$$

$\phi = 0°$: $\quad g = 32.09 \text{ ft/s}^2$

$\phi = 45°$: $\quad g = 32.175 \text{ ft/s}^2$

$\phi = 90°$: $\quad g = 32.26 \text{ ft/s}^2$

(a) Weight: $\quad W = mg$

$\phi = 0°$: $\quad W = (0.1554 \text{ lb}\cdot\text{s}^2/\text{ft})(32.09 \text{ ft/s}^2) = 4.987 \text{ lb}$ ◀

$\phi = 45°$: $\quad W = (0.1554 \text{ lb}\cdot\text{s}^2/\text{ft})(32.175 \text{ ft/s}^2) = 5.000 \text{ lb}$ ◀

$\phi = 90°$: $\quad W = (0.1554 \text{ lb}\cdot\text{s}^2/\text{ft})(32.26 \text{ ft/s}^2) = 5.013 \text{ lb}$ ◀

(b) Mass: At all latitudes: $\quad m = 5.000 \text{ lb}$ ◀

(c) or $\quad m = \dfrac{5.00 \text{ lb}}{32.175 \text{ ft/s}^2} \qquad m = 0.1554 \text{ lb}\cdot\text{s}^2/\text{ft}$ ◀

PROBLEM 12.3

A 400-kg satellite has been placed in a circular orbit 1500 km above the surface of the earth. The acceleration of gravity at this elevation is 6.43 m/s^2. Determine the linear momentum of the satellite, knowing that its orbital speed is 25.6×10^3 km/h.

SOLUTION

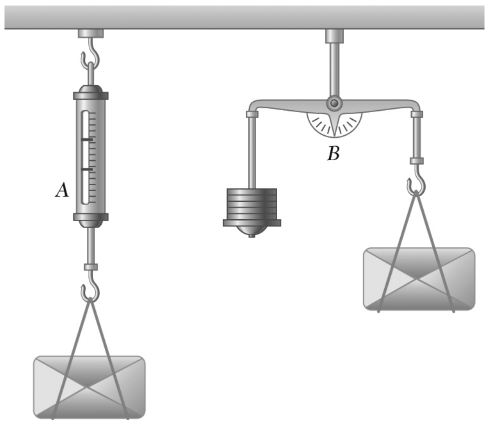
Mass of satellite is independent of gravity: $m = 400 \text{ kg}$

$$v = 25.6\times10^3 \text{ km/h}$$

$$= (25.6\times10^6 \text{ m/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 7.111\times10^3 \text{ m/s}$$

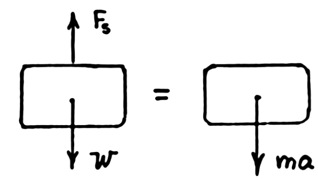
$$L = mv = (400 \text{ kg})(7.111\times10^3 \text{ m/s}) \qquad L = 2.84\times10^6 \text{ kg}\cdot\text{m/s} \blacktriangleleft$$

PROBLEM 12.4



A spring scale A and a lever scale B having equal lever arms are fastened to the roof of an elevator, and identical packages are attached to the scales as shown. Knowing that when the elevator moves downward with an acceleration of 1 m/s^2 the spring scale indicates a load of 60 N, determine (*a*) the weight of the packages, (*b*) the load indicated by the spring scale and the mass needed to balance the lever scale when the elevator moves upward with an acceleration of 1 m/s^2.

SOLUTION



Assume $$g = 9.81\text{ m/s}^2$$

$$m = \frac{W}{g}$$

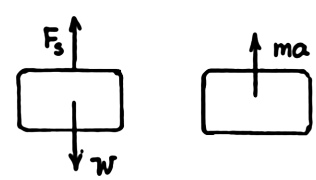
$$+\uparrow \Sigma F = ma: \quad F_s - W = -\frac{W}{g}a$$

$$W\left(1 - \frac{a}{g}\right) = F_s$$

or $$W = \frac{F_s}{1 - \frac{a}{g}} = \frac{60}{1 - \frac{1}{9.81}}$$

$W = 66.8\text{ N}$ ◀

(*b*)



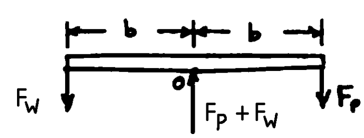
$$\Sigma F = ma: \quad F_s - W = \frac{W}{g}a$$

$$F_s = W\left(1 + \frac{a}{g}\right)$$

$$= 66.81\left(1 + \frac{1}{9.81}\right)$$

$F_s = 73.6\text{ N}$ ◀

For the balance system B,



$$\Sigma M_0 = 0: \quad bF_w - bF_p = 0$$

$$F_w = F_p$$

PROBLEM 12.4 (Continued)

But $$F_w = W_w\left(1+\frac{a}{g}\right)$$

and $$F_p = W_p\left(1+\frac{a}{g}\right)$$

so that $$W_w = W_p$$

and $$m_w = \frac{W_p}{g} = \frac{66.81}{9.81}$$

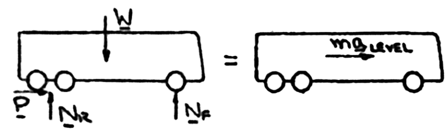
$m_w = 6.81\text{ kg}$ ◀

PROBLEM 12.5

In anticipation of a long 7° upgrade, a bus driver accelerates at a constant rate of 3 ft/s^2 while still on a level section of the highway. Knowing that the speed of the bus is 60 mi/h as it begins to climb the grade and that the driver does not change the setting of his throttle or shift gears, determine the distance traveled by the bus up the grade when its speed has decreased to 50 mi/h.

SOLUTION

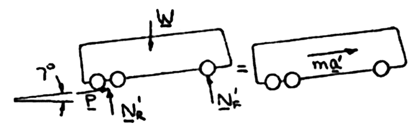
First consider when the bus is on the level section of the highway.



$$a_{\text{level}} = 3 \text{ ft/s}^2$$

We have $\xrightarrow{+} \Sigma F_x = ma$: $\quad P = \dfrac{W}{g} a_{\text{level}}$

Now consider when the bus is on the upgrade.



We have $+\nearrow \Sigma F_x = ma$: $\quad P - W \sin 7° = \dfrac{W}{g} a'$

Substituting for P $\quad \dfrac{W}{g} a_{\text{level}} - W \sin 7° = \dfrac{W}{g} a'$

or

$$\begin{aligned} a' &= a_{\text{level}} - g \sin 7° \\ &= (3 - 32.2 \sin 7°) \text{ ft/s}^2 \\ &= -0.92419 \text{ ft/s}^2 \end{aligned}$$

For the uniformly decelerated motion

$$v^2 = (v_0)^2_{\text{upgrade}} + 2a'(x_{\text{upgrade}} - 0)$$

Noting that $60 \text{ mi/h} = 88 \text{ ft/s}$, then when $v = 50 \text{ mi/h} \left(= \dfrac{5}{6} v_0 \right)$, we have

$$\left(\frac{5}{6} \times 88 \text{ ft/s} \right)^2 = (88 \text{ ft/s})^2 + 2(-0.92419 \text{ ft/s}^2) x_{\text{upgrade}}$$

or $\quad x_{\text{upgrade}} = 1280.16 \text{ ft}$

or $\quad x_{\text{upgrade}} = 0.242 \text{ mi}$ ◀

PROBLEM 12.6

A hockey player hits a puck so that it comes to rest 10 s after sliding 100 ft on the ice. Determine (*a*) the initial velocity of the puck, (*b*) the coefficient of friction between the puck and the ice.

SOLUTION

(*a*) Assume uniformly decelerated motion.

Then $v = v_0 + at$

At $t = 10$ s: $0 = v_0 + a(10)$

$$a = -\frac{v_0}{10}$$

Also $v^2 = v_0^2 + 2a(x - 0)$

At $t = 10$ s: $0 = v_0^2 + 2a(100)$

Substituting for a $0 = v_0^2 + 2\left(-\frac{v_0}{10}\right)(100) = 0$

$v_0 = 20.0$ ft/s or $v_0 = 20.0$ ft/s ◀

and $a = -\frac{20}{10} = -2\ \text{ft/s}^2$

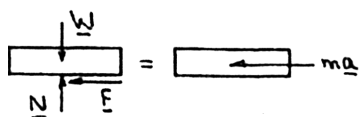
Alternate solution to part (*a*) $d = d_0 + v_0 t + \frac{1}{2}at^2$

$$d = v_0 t + \frac{1}{2}\left(-\frac{v_0}{t}\right)t^2$$

$$d = \frac{1}{2}v_0 t$$

$$v_0 = \frac{2d}{t}$$

(*b*) We have



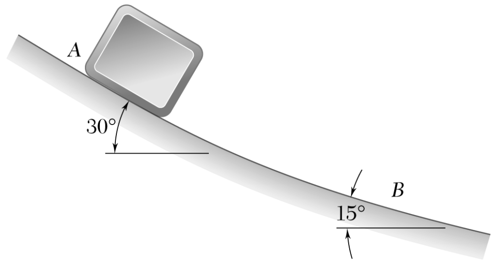
$+\uparrow \Sigma F_y = 0$: $N - W = 0$ $N = W = mg$

Sliding: $F = \mu_k N = \mu_k mg$

$\xrightarrow{+} \Sigma F_x = ma$: $-F = ma$ $-\mu_k mg = ma$

$\mu_k = -\frac{a}{g} = -\frac{-2.0\ \text{ft/s}^2}{32.2\ \text{ft/s}^2}$ $\mu_k = 0.0621$ ◀

PROBLEM 12.7

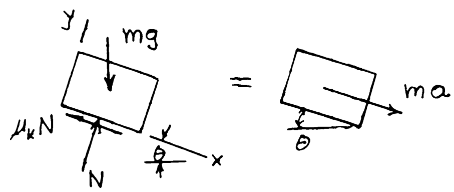


The acceleration of a package sliding at Point A is 3 m/s^2. Assuming that the coefficient of kinetic friction is the same for each section, determine the acceleration of the package at Point B.

SOLUTION

For any angle θ.

Use x and y coordinates as shown.



$$a_y = 0$$

$$+\nearrow \Sigma F_y = ma_y: \quad N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$$+\searrow \Sigma F_x = ma_x: \quad mg\sin\theta - \mu_k N = ma_x$$

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$

At Point A. $\quad \theta = 30°, \quad a_x = 3 \text{ m/s}^2$

$$\mu_k = \frac{g\sin 30° - a_x}{g\cos 30°}$$

$$= \frac{9.81\sin 30° - 3}{9.81\cos 30°}$$

$$= 0.22423$$

At Point B. $\quad \theta = 15°, \quad \mu_k = 0.22423$

$$a_x = 9.81(\sin 15° - 0.22423\cos 15°)$$

$$= 0.414 \text{ m/s}$$

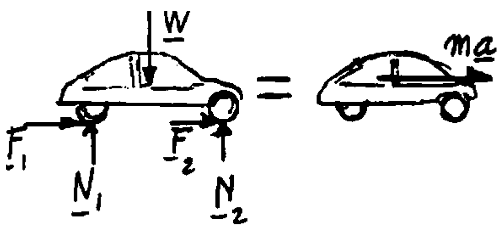
$\mathbf{a} = 0.414 \text{ m/s}^2 \searrow 15°$ ◀

PROBLEM 12.8

Determine the maximum theoretical speed that may be achieved over a distance of 60 m by a car starting from rest, knowing that the coefficient of static friction is 0.80 between the tires and the pavement and that 60 percent of the weight of the car is distributed over its front wheels and 40 percent over its rear wheels. Assume (*a*) four-wheel drive, (*b*) front-wheel drive, (*c*) rear-wheel drive.

SOLUTION

(*a*) Four-wheel drive



$$+\uparrow \Sigma F_y = 0: \quad N_1 + N_2 - W = 0$$

$$F_1 + F_2 = \mu N_1 + \mu N_2$$
$$= \mu(N_1 + N_2) = \mu w$$

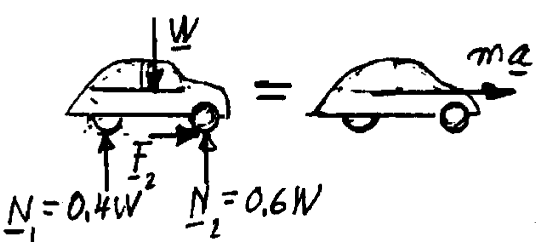
$$\xrightarrow{+} \Sigma F_x = ma: \quad F_1 + F_2 = ma$$
$$\mu w = ma$$

$$a = \frac{\mu W}{m} = \mu \frac{mg}{m} = \mu g = 0.80(9.81)$$
$$a = 7.848 \text{ m/s}^2$$

$$v^2 = 2ax = 2(7.848 \text{ m/s}^2)(60 \text{ m}) = 941.76 \text{ m}^2/\text{s}^2$$

$$v = 30.69 \text{ m/s} \qquad v = 110.5 \text{ km/h} \blacktriangleleft$$

(*b*) Front-wheel drive



$$F_2 = ma$$
$$\mu(0.6\ W) = ma$$

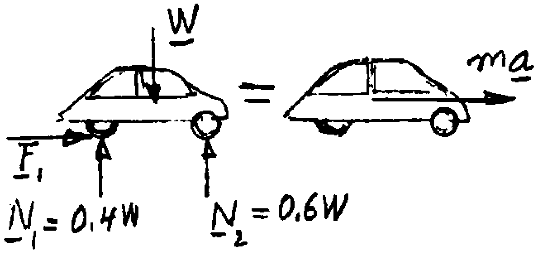
$$a = \frac{0.6\mu W}{m} = \frac{0.6\mu mg}{m}$$
$$= 0.6\mu g = 0.6(0.80)(9.81)$$
$$a = 4.709 \text{ m/s}^2$$

$$v^2 = 2ax = 2(4.709 \text{ m/s}^2)(60 \text{ m}) = 565.1 \text{ m}^2/\text{s}^2$$

$$v = 23.77 \text{ m/s} \qquad v = 85.6 \text{ km/h} \blacktriangleleft$$

PROBLEM 12.8 (Continued)

(*c*) Rear-wheel drive



$$F_1 = ma$$

$$\mu(0.4\ W) = ma$$

$$a = \frac{0.4\mu W}{m} = \frac{0.4\mu mg}{m}$$

$$= 0.4\mu g = 0.4(0.80)(9.81)$$

$$a = 3.139\ \text{m/s}^2$$

$$v^2 = 2ax = 2(3.139\ \text{m/s}^2)(60\ \text{m}) = 376.7\ \text{m}^2/\text{s}^2$$

$$v = 19.41\ \text{m/s} \qquad v = 69.9\ \text{km/h} \blacktriangleleft$$

PROBLEM 12.9

If an automobile's braking distance from 90 km/h is 45 m on level pavement, determine the automobile's braking distance from 90 km/h when it is (*a*) going up a 5° incline, (*b*) going down a 3-percent incline. Assume the braking force is independent of grade.

SOLUTION

Assume uniformly decelerated motion in all cases.

For braking on the level surface,

$$v_0 = 90 \text{ km/h} = 25 \text{ m/s}, \quad v_f = 0$$

$$x_f - x_0 = 45 \text{ m}$$

$$v_f^2 = v_0^2 + 2a(x_f - x_0)$$

$$a = \frac{v_f^2 - v_0^2}{2(x_f - x_0)} = \frac{0 - (25)^2}{(2)(45)} = -6.9444 \text{ m/s}^2$$

Braking force.

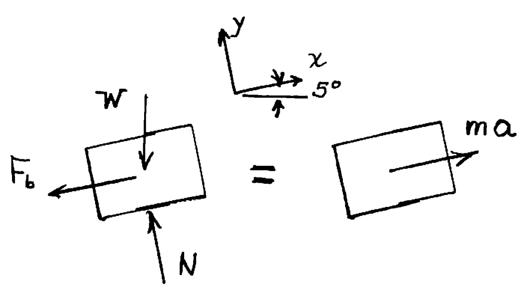
$$F_b = ma = \frac{W}{g}a = -\frac{6.944}{9.81}W = -0.70789W$$

(*a*) Going up a 5° incline.

$$\nearrow \Sigma F = ma$$

$$-F_b - W\sin 5° = \frac{W}{g}a$$

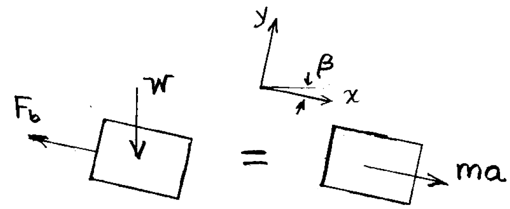
$$a = -\frac{F_b + W\sin 5°}{W}g = -(0.70789 + \sin 5°)(9.81) = -7.79944 \text{ m/s}^2$$



$$x_f - x_0 = \frac{v_f^2 - v_0^2}{2a} = \frac{0 - (25)^2}{(2)(-7.79944)}$$

$$x_f - x_0 = 40.1 \text{ m} \blacktriangleleft$$

(*b*) Going down a 3 percent incline.



$$\tan\beta = \frac{3}{100} \qquad \beta = 1.71835°$$

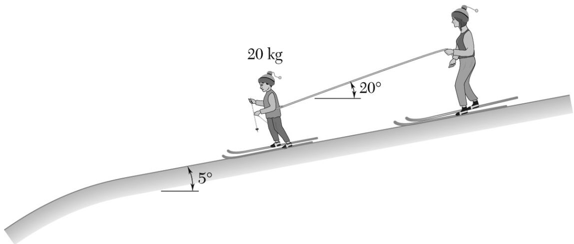
$$-F_b + W\sin\beta = \frac{W}{g}a$$

$$a = -(0.70789 - \sin\beta)(9.81)$$

$$= -6.65028 \text{ m/s}$$

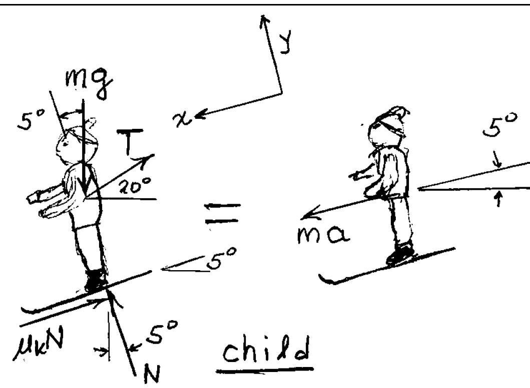
$$x_f = x_0 = \frac{0-(25)^2}{(2)(-6.65028)}$$

$$x_f - x_0 = 47.0 \text{ m} \blacktriangleleft$$

	**PROBLEM 12.10** A mother and her child are skiing together, and the mother is holding the end of a rope tied to the child's waist. They are moving at a speed of 7.2 km/h on a gently sloping portion of the ski slope when the mother observes that they are approaching a steep descent. She pulls on the rope with an average force of 7 N. Knowing the coefficient of friction between the child and the ground is 0.1 and the angle of the rope does not change, determine (*a*) the time required for the child's speed to be cut in half, (*b*) the distance traveled in this time.

SOLUTION

Draw free body diagram of child.



$\Sigma \mathbf{F} = m\mathbf{a}$:

x-direction: $mg \sin 5° - \mu_k N - T \cos 15° = ma$

y-direction: $N - mg \cos 5° + T \sin 15° = 0$

From *y*-direction,

$$N = mg \cos 5° - T \sin 15° = (20 \text{ kg})(9.81 \text{ m/s}^2)\cos 5° - (7 \text{ N})\sin 15° = 193.64 \text{ N}$$

From *x*-direction,

$$a = g \sin 5° - \frac{\mu_k N}{m} - \frac{T \cos 15°}{m}$$

$$= (9.81 \text{ m/s}^2)\sin 5° - \frac{(0.1)(193.64 \text{ N})}{20 \text{ kg}} - \frac{(7 \text{ N}) \cos 15°}{20 \text{ kg}}$$

$$= -0.45128 \text{ m/s}^2 \qquad (\text{in } x\text{-direction.})$$

$$v_0 = 7.2 \text{ km/h} = 2 \text{ m/s} \qquad x_0 = 0$$

$$v_f = \frac{1}{2} v_0 = 1 \text{ m/s}$$

$$v_f = v_0 + at \qquad t = \frac{v_f - v_0}{a} = \frac{-1 \text{ m/s}}{-0.45128 \text{ m/s}^2} = 2.2159 \text{ s}$$

PROBLEM 12.10 (Continued)

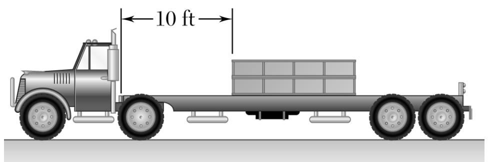
(*a*) *Time elapsed.* $t = 2.22 \text{ s}$ ◀

(*b*) *Corresponding distance.*

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$= 0 + (2 \text{ m/s})(2.2159 \text{ s}) + \frac{1}{2}(-0.45128 \text{ m/s}^2)(2.2159 \text{ s})^2$$

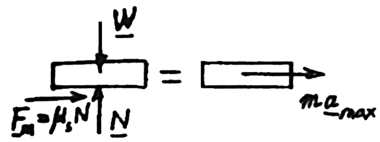
$x = 3.32 \text{ m}$ ◀



PROBLEM 12.11

The coefficients of friction between the load and the flat-bed trailer shown are $\mu_s = 0.40$ and $\mu_k = 0.30$. Knowing that the speed of the rig is 72 km/h, determine the shortest distance in which the rig can be brought to a stop if the load is not to shift.

SOLUTION



Load: We assume that sliding of load relative to trailer is impending:

$$F = F_m$$
$$= \mu_s N$$

Deceleration of load is same as deceleration of trailer, which is the maximum allowable deceleration $\mathbf{a}_{\max}$.

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \quad N = W$$

$$F_m = \mu_s N = 0.40\,W$$

$$\xrightarrow{+} \Sigma F_x = ma: \quad F_m = ma_{\max}$$

$$0.40\ W = \frac{W}{g} a_{\max} \qquad a_{\max} = 3.924\ \text{m/s}^2$$

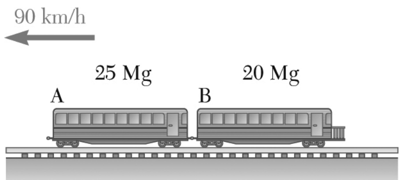
$$\mathbf{a}_{\max} = 3.92\ \text{m/s}^2 \longrightarrow$$

<u>Uniformly accelerated motion</u>.

$$v^2 = v_0^2 + 2ax \quad \text{with} \quad v = 0 \qquad\qquad v_0 = 72\ \text{km/h} = 20\ \text{m/s}$$

$$a = -a_{\max} = 3.924\ \text{m/s}^2$$

$$0 = (20)^2 + 2(-3.924)x \qquad\qquad x = 51.0\ \text{m} \blacktriangleleft$$



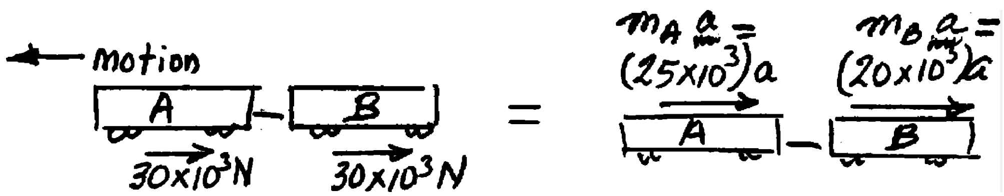
PROBLEM 12.12

A light train made up of two cars is traveling at 90 km/h when the brakes are applied to both cars. Knowing that car A has a mass of 25 Mg and car B a mass of 20 Mg, and that the braking force is 30 kN on each car, determine (a) the distance traveled by the train before it comes to a stop, (b) the force in the coupling between the cars while the train is showing down.

SOLUTION

$$v_0 = 90 \text{ km/h} = 90/3.6 = 25 \text{ m/s}$$

(a) Both cars:

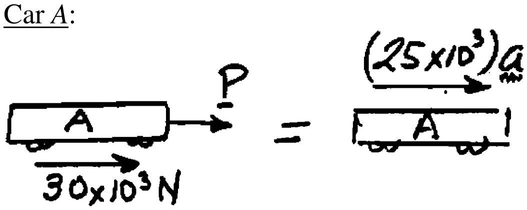


$\xrightarrow{+}$ $\Sigma F_x = \Sigma ma$: $\quad 60\times10^3 \text{ N} = (45\times10^3 \text{ kg})a$ $\qquad a = 1.333 \text{ m/s}^2 \longrightarrow$

$$v^2 = v_0^1 + 2ax: \quad 0 = (25)^2 + 2(-1.333)x$$

Stopping distance: $\qquad x = 234 \text{ m}$ ◀

(b) Car A:

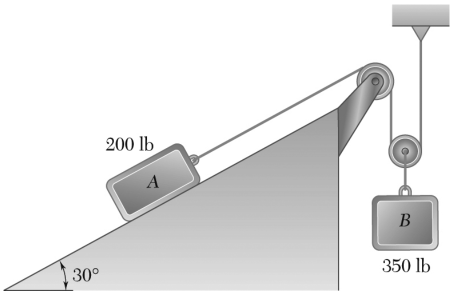


$\xrightarrow{+}$ $\Sigma F_x = ma$: $\quad 30\times10^3 + P = (25\times10^3)a$

$$P = (25\times10^3)(1.333) - 30\times10^3$$

Coupling force: $\qquad P = +3332 \text{ N} \qquad P = 3.33 \text{ kN (tension)}$ ◀

PROBLEM 12.13

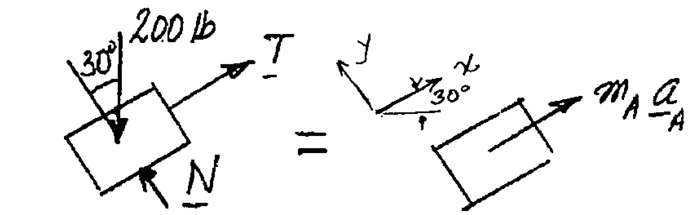


The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the incline, determine (*a*) the acceleration of each block, (*b*) the tension in the cable.

SOLUTION

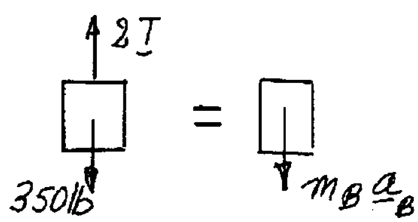
(*a*) We note that $a_B = \frac{1}{2} a_A$.

<u>Block *A*</u>



$$\nearrow + \Sigma F_x = m_A a_A: \quad T - (200 \text{ lb})\sin 30° = \frac{200}{32.2} a_A \tag{1}$$

<u>Block *B*</u>



$$+\downarrow \Sigma F_y = m_B a_B: \quad 350 \text{ lb} - 2T = \frac{350}{32.2}\left(\frac{1}{2} a_A\right) \tag{2}$$

(*a*) Multiply Eq. (1) by 2 and add Eq. (2) in order to eliminate T:

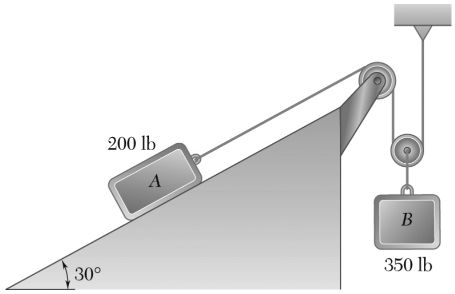
$$-2(200)\sin 30° + 350 = 2\frac{200}{32.2} a_A + \frac{350}{32.2}\left(\frac{1}{2} a_A\right)$$

$$150 = \frac{575}{32.2} a_A \qquad \mathbf{a}_A = 8.40 \text{ ft/s}^2 \measuredangle 30° \blacktriangleleft$$

$$a_B = \frac{1}{2} a_A = \frac{1}{2}(8.40 \text{ ft/s}^2), \qquad \mathbf{a}_B = 4.20 \text{ ft/s}^2 \downarrow \blacktriangleleft$$

(*b*) From Eq. (1), $T - (200)\sin 30° = \frac{200}{32.2}(8.40)$ $\qquad T = 152.2 \text{ lb} \blacktriangleleft$

PROBLEM 12.14

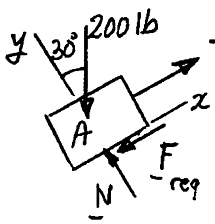


Solve Problem 12.13, assuming that the coefficients of friction between block A and the incline are $\mu_s = 0.25$ and $\mu_k = 0.20$.

PROBLEM 12.13 The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the incline, determine (a) the acceleration of each block, (b) the tension in the cable.

SOLUTION

We first determine whether the blocks move by computing the friction force required to maintain block A in equilibrium. $T = 175$ lb. When B in equilibrium,



$\overset{+}{\nearrow} \Sigma F_x = 0:$

$$+\nearrow \Sigma F_x = 0: \quad 175 - 200\sin 30° - F_{\text{req}} = 0$$

$$F_{\text{req}} = 75.0 \text{ lb}$$

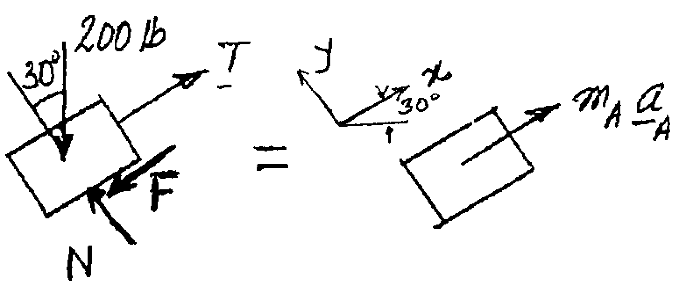
$$+\nwarrow \Sigma F_y = 0: \quad N - 200\cos 30° = 0 \quad N = 173.2 \text{ lb}$$

$$F_M = \mu_s N = 0.25(173.2 \text{ lb}) = 43.3 \text{ lb}$$

Since $F_{\text{req}} > \mathbf{F}_m$, blocks will move ($A$ up and B down).

We note that $a_B = \dfrac{1}{2} a_A$.

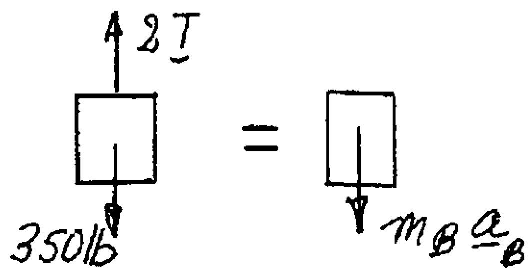
<u>Block A</u>



$$F = \mu_k N = (0.20)(173.2) = 34.64 \text{ lb.}$$

$$+\nearrow \Sigma F_x = m_A 0_A: \quad -200\sin 30° - 34.64 + T = \frac{200}{32.2} a_A \qquad (1)$$

Block B



$$+\downarrow \Sigma F_y = m_B a_B: \quad 350\,\text{lb} - 2T = \frac{350}{32.2}\left(\frac{1}{2}a_A\right) \qquad (2)$$

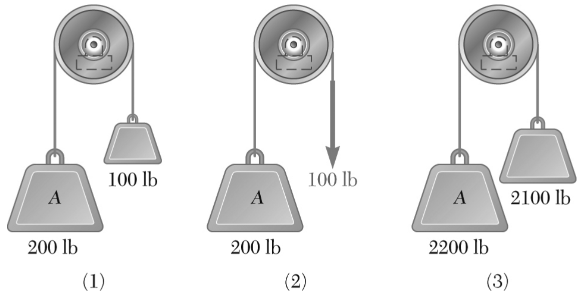
(a) Multiply Eq. (1) by 2 and add Eq. (2) in order to eliminate T:

$$-2(200)\sin 30° - 2(34.64) + 350 = 2\frac{200}{32.2}a_A + \frac{350}{32.2}\left(\frac{1}{2}a_A\right)$$

$$81.32 = \frac{575}{32.2}a_A \qquad a_A = 4.55\ \text{ft/s}^2 \measuredangle 30° \blacktriangleleft$$

$$a_B = \frac{1}{2}a_A = \frac{1}{2}(4.52\ \text{ft/s}^2), \qquad a_B = 2.28\ \text{ft/s}^2 \downarrow \blacktriangleleft$$

(b) From Eq. (1), $T - (200)\sin 30° - 34.64 = \frac{200}{32.2}(4.52)$ $\qquad T = 162.9\ \text{lb} \blacktriangleleft$



PROBLEM 12.15

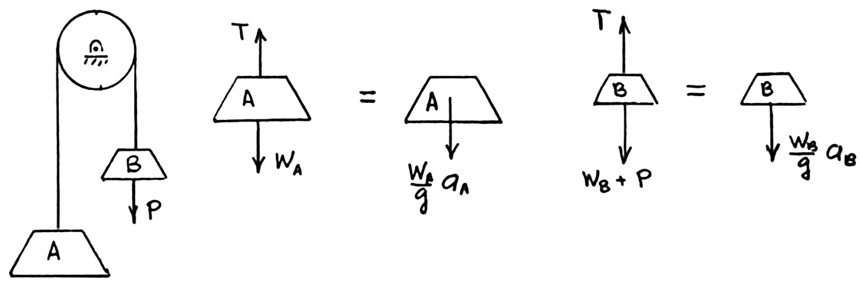
Each of the systems shown is initially at rest. Neglecting axle friction and the masses of the pulleys, determine for each system (*a*) the acceleration of block *A*, (*b*) the velocity of block *A* after it has moved through 10 ft, (*c*) the time required for block *A* to reach a velocity of 20 ft/s.

SOLUTION

Let *y* be positive downward for both blocks.

Constraint of cable: $y_A + y_B = \text{constant}$

$$a_A + a_B = 0 \qquad \text{or} \qquad a_B = -a_A$$



For blocks *A* and *B*, $+\!\downarrow \Sigma F = ma$:

Block *A*: $$W_A - T = \frac{W_A}{g}a_A \qquad \text{or} \qquad T = W_A - \frac{W_A}{g}a_A$$

Block *B*: $$P + W_B - T = \frac{W_B}{g}a_B = -\frac{W_B}{g}a_A$$

$$P + W_B - W_A + \frac{W_A}{g}a_A = -\frac{W_B}{g}a_A$$

Solving for a_A, $$a_A = \frac{W_A - W_B - P}{W_A + W_B}g \qquad (1)$$

$$v_A^2 - (v_A)_0^2 = 2a_A[y_A - (y_A)_0] \quad \text{with} \quad (v_A)_0 = 0$$

$$v_A = \sqrt{2a_A[y_A - (y_A)_0]} \qquad (2)$$

$$v_A - (v_A)_0 = a_A t \quad \text{with} \quad (v_A)_0 = 0$$

$$t = \frac{v_A}{a_A} \qquad (3)$$

(*a*) Acceleration of block *A*.

System (1): $W_A = 200 \text{ lb}, \quad W_B = 100 \text{ lb}, \quad P = 0$

By formula (1), $(a_A)_1 = \dfrac{200-100}{200+100}(32.2)$ $\qquad (\mathbf{a}_A)_1 = 10.73 \text{ ft/s}^2 \downarrow$ ◀

System (2): $W_A = 200 \text{ lb}, \quad W_B = 0, \quad P = 50 \text{ lb}$

By formula (1), $(a_A)_2 = \dfrac{200-100}{200}(32.2)$ $\qquad (\mathbf{a}_A)_2 = 16.10 \text{ ft/s}^2 \downarrow$ ◀

System (3): $W_A = 2200 \text{ lb}, \quad W_B = 2100 \text{ lb}, \quad P = 0$

By formula (1), $(a_A)_3 = \dfrac{2200-2100}{2200+2100}(32.2)$ $\qquad (\mathbf{a}_A)_3 = 0.749 \text{ ft/s}^2 \downarrow$ ◀

(*b*) v_A at $y_A - (y_A)_0 = 10$ ft. Use formula (2).

System (1): $(v_A)_1 = \sqrt{(2)(10.73)(10)}$ $\qquad (v_A)_1 = 14.65 \text{ ft/s} \downarrow$ ◀

System (2): $(v_A)_2 = \sqrt{(2)(16.10)(10)}$ $\qquad (v_A)_2 = 17.94 \text{ ft/s} \downarrow$ ◀

System (3): $(v_A)_3 = \sqrt{(2)(0.749)(10)}$ $\qquad (v_A)_3 = 3.87 \text{ ft/s} \downarrow$ ◀

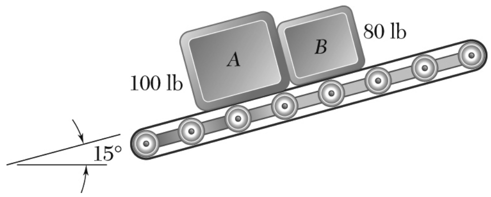
(*c*) Time at $v_A = 20$ ft/s. Use formula (3).

System (1): $t_1 = \dfrac{20}{10.73}$ $\qquad t_1 = 1.864 \text{ s}$ ◀

System (2): $t_2 = \dfrac{20}{16.10}$ $\qquad t_2 = 1.242 \text{ s}$ ◀

System (3): $t_3 = \dfrac{20}{0.749}$ $\qquad t_3 = 26.7 \text{ s}$ ◀

PROBLEM 12.16



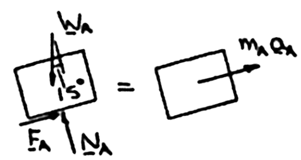
Boxes A and B are at rest on a conveyor belt that is initially at rest. The belt is suddenly started in an upward direction so that slipping occurs between the belt and the boxes. Knowing that the coefficients of kinetic friction between the belt and the boxes are $(\mu_k)_A = 0.30$ and $(\mu_k)_B = 0.32$, determine the initial acceleration of each box.

SOLUTION

Assume that $a_B > a_A$ so that the normal force N_{AB} between the boxes is zero.

A: $+\nwarrow \Sigma F_y = 0: \quad N_A - W_A \cos 15° = 0$

A:



or $N_A = W_A \cos 15°$

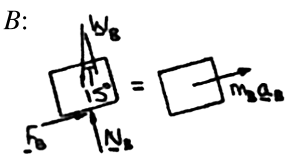
Slipping:
$$F_A = (\mu_k)_A N_A$$
$$= 0.3 W_A \cos 15°$$

$$+\nearrow \Sigma F_x = m_A a_A: \quad F_A - W_A \sin 15° = m_A a_A$$

or $$0.3W_A \cos 15° - W_A \sin 15° = \frac{W_A}{g} a_A$$

or $$a_A = (32.2 \text{ ft/s}^2)(0.3 \cos 15° - \sin 15°)$$
$$= 0.997 \text{ ft/s}^2$$

B:



B: $+\nwarrow \Sigma F_y = 0: \quad N_B - W_B \cos 15° = 0$

or $N_B = W_B \cos 15°$

Slipping:
$$F_B = (\mu_k)_B N_B$$
$$= 0.32 W_B \cos 15°$$

$$+\nearrow \Sigma F_x = m_B a_B: \quad F_B - W_B \sin 15° = m_B a_B$$

or $$0.32W_B \cos 15° - W_B \sin 15° = \frac{W_B}{g} a_B$$

or $$a_B = (32.2 \text{ ft/s}^2)(0.32 \cos 15° - \sin 15°) = 1.619 \text{ ft/s}^2$$

$a_B > a_A \Rightarrow$ assumption is correct

$\mathbf{a}_A = 0.997 \text{ ft/s}^2 \measuredangle 15°$ ◀

$\mathbf{a}_B = 1.619 \text{ ft/s}^2 \measuredangle 15°$ ◀

PROBLEM 12.16 (Continued)

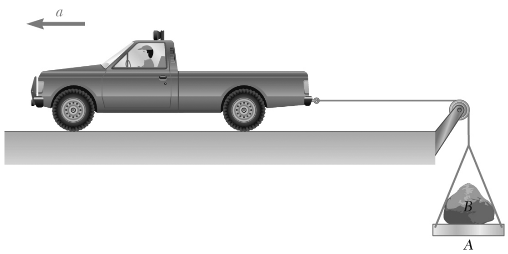
Note: If it is assumed that the boxes remain in contact $(N_{AB} \neq 0)$, then assuming N_{AB} to be compression,

$$a_A = a_B \quad \text{and find} \quad (\Sigma F_x = ma) \text{ for each box.}$$

A:
$$0.3W_A \cos 15° - W_A \sin 15° - N_{AB} = \frac{W_A}{g} a$$

B:
$$0.32W_B \cos 15° - W_B \sin 15° + N_{AB} = \frac{W_B}{g} a$$

Solving yields $a = 1.273 \text{ ft/s}^2$ and $N_{AB} = -0.859 \text{ lb}$, which contradicts the assumption.



PROBLEM 12.17

A 5000-lb truck is being used to lift a 1000 lb boulder B that is on a 200 lb pallet A. Knowing the acceleration of the truck is 1 ft/s^2, determine (*a*) the horizontal force between the tires and the ground, (*b*) the force between the boulder and the pallet.

SOLUTION

Kinematics:

$$\mathbf{a}_T = 1\text{ m/s}^2 \longleftarrow$$

$$\mathbf{a}_A = \mathbf{a}_B = 0.5\text{ m/s}^2 \uparrow$$

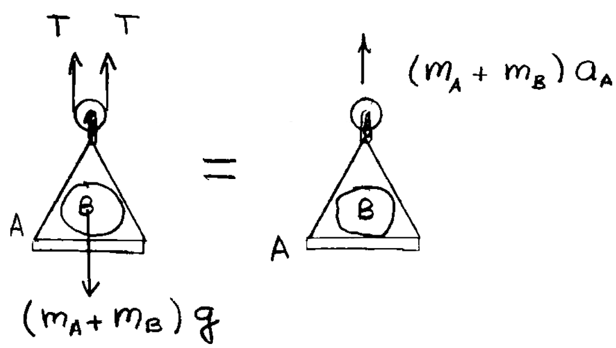
Masses:

$$m_T = \frac{5000}{32.2} = 155.28\text{ slugs}$$

$$m_A = \frac{200}{32.2} = 6.211\text{ slugs}$$

$$m_B = \frac{1000}{32.2} = 31.056\text{ slugs}$$

Let T be the tension in the cable. Apply Newton's second law to the lower pulley, pallet and boulder.

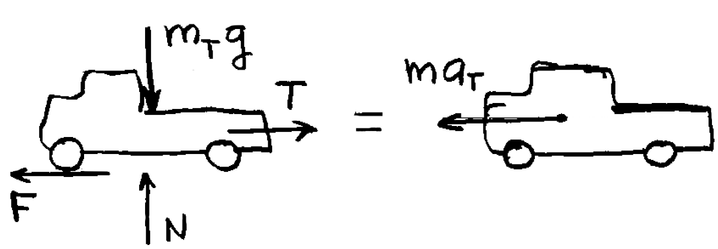


Vertical components $+\uparrow$:

$$2T - (m_A + m_B)g = (m_A + m_B)a_A$$
$$2T - (37.267)(32.2) = (37.267)(0.5)$$

$$T = 609.32\text{ lb}$$

Apply Newton's second law to the truck.



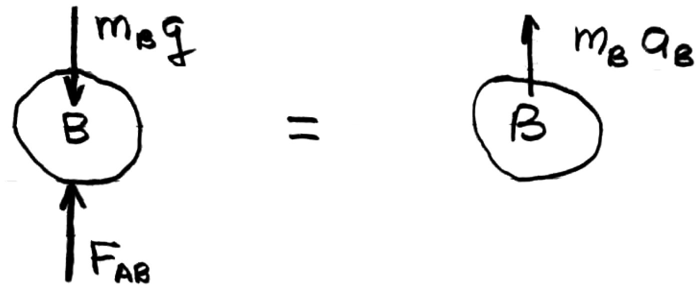
PROBLEM 12.17 (Continued)

Horizontal components $\overset{+}{\longleftarrow}$: $F - T = m_T a_T$

(*a*) Horizontal force between lines and ground.

$F = T + m_T a_T = 609.32 + (155.28)(1.0)$ $\qquad F = 765 \text{ lb}$ ◀

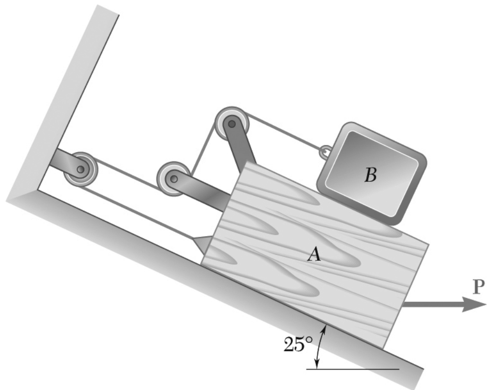
Apply Newton's second law to the boulder.



Vertical components $+\uparrow$: $\qquad F_{AB} - m_B g = m_B a_B$

$$F_{AB} = m_B(g + a) = 31.056(32.2 + 0.5)$$

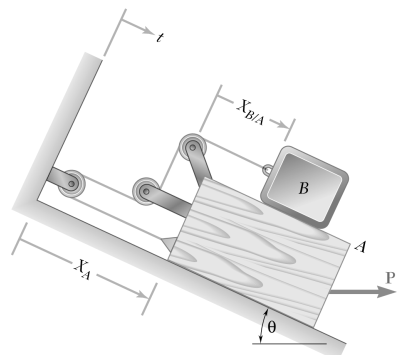
(*b*) Contact force: $\qquad F_{AB} = 1016 \text{ lb}$ ◀



PROBLEM 12.18

Block A has a mass of 40 kg, and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 0$, determine (a) the acceleration of block B, (b) the tension in the cord.

SOLUTION



From the constraint of the cord:

$$2x_A + x_{B/A} = \text{constant}$$

Then $$2v_A + v_{B/A} = 0$$

and $$2a_A + a_{B/A} = 0$$

Now $$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Then $$a_B = a_A + (-2a_A)$$

or $$a_B = -a_A \qquad (1)$$

First we determine if the blocks will move for the given value of θ. Thus, we seek the value of θ for which the blocks are in impending motion, with the impending motion of A down the incline.

B:

B: $$+\nearrow \Sigma F_y = 0: \quad N_{AB} - W_B \cos\theta = 0$$

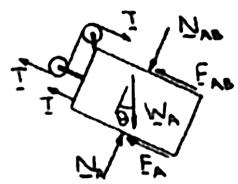
or $$N_{AB} = m_B g \cos\theta$$

Now $$F_{AB} = \mu_s N_{AB}$$
$$= 0.2 m_B g \cos\theta$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T + F_{AB} + W_B \sin\theta = 0$$

or $$T = m_B g(0.2\cos\theta + \sin\theta)$$

A:



A: $$+\nearrow \Sigma F_y = 0: \quad N_A - N_{AB} - W_A \cos\theta = 0$$

or $$N_A = (m_A + m_B) g \cos\theta$$

Now $$F_A = \mu_s N_A$$
$$= 0.2(m_A + m_B) g \cos\theta$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T - F_A - F_{AB} + W_A \sin\theta = 0$$

PROBLEM 12.18 (Continued)

or $$T = m_A g\sin\theta - 0.2(m_A + m_B)g\cos\theta - 0.2m_B g\cos\theta$$
$$= g[m_A\sin\theta - 0.2(m_A + 2m_B)\cos\theta]$$

Equating the two expressions for T

$$m_B g(0.2\cos\theta + \sin\theta) = g[m_A\sin\theta - 0.2(m_A + 2m_B)\cos\theta]$$

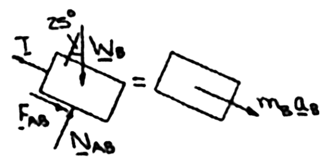
or $$8(0.2 + \tan\theta) = [40\tan\theta - 0.2(40 + 2\times 8)]$$

or $$\tan\theta = 0.4$$

or $\theta = 21.8°$ for impending motion. Since $\theta < 25°$, the blocks will move. Now consider the motion of the blocks.

(*a*) $+\nearrow \Sigma F_y = 0$: $N_{AB} - W_B\cos 25° = 0$

B:



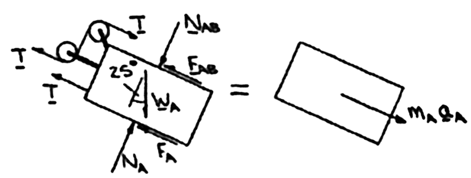
or $$N_{AB} = m_B g\cos 25°$$

Sliding: $$F_{AB} = \mu_k N_{AB} = 0.15 m_B g\cos 25°$$

$$\overset{+}{\searrow}\Sigma F_x = m_B a_B:\quad -T + F_{AB} + W_B\sin 25° = m_B a_B$$

or $$T = m_B[g(0.15\cos 25° + \sin 25°) - a_B]$$
$$= 8[9.81(0.15\cos 25° + \sin 25°) - a_B]$$
$$= 8(5.47952 - a_B) \qquad \text{(N)}$$

A:



$$+\nearrow \Sigma F_y = 0:\quad N_A - N_{AB} - W_A\cos 25° = 0$$

or $$N_A = (m_A + m_B)g\cos 25°$$

Sliding: $$F_A = \mu_k N_A = 0.15(m_A + m_B)g\cos 25°$$

$$\overset{+}{\searrow}\Sigma F_x = m_A a_A:\quad -T - F_A - F_{AB} + W_A\sin 25° = m_A a_A$$

Substituting and using Eq. (1)

$$T = m_A g\sin 25° - 0.15(m_A + m_B)g\cos 25°$$
$$-0.15 m_B g\cos 25° - m_A(-a_B)$$
$$= g[m_A\sin 25° - 0.15(m_A + 2m_B)\cos 25°] + m_A a_B$$
$$= 9.81[40\sin 25° - 0.15(40 + 2\times 8)\cos 25°] + 40a_B$$
$$= 91.15202 + 40a_B \qquad \text{(N)}$$

Equating the two expressions for T

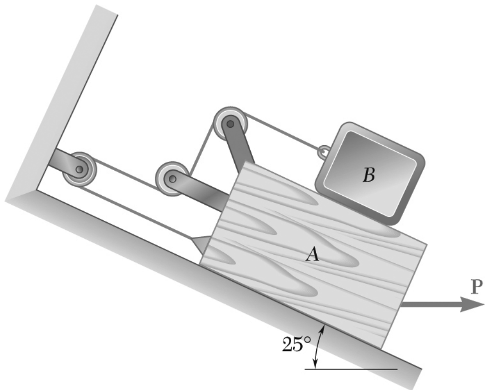
$$8(5.47952 - a_B) = 91.15202 + 40a_B$$

or $$a_B = -0.98575\ \text{m/s}^2$$

$$\mathbf{a}_B = 0.986\ \text{m/s}^2 \searrow 25° \blacktriangleleft$$

(*b*) We have $$T = 8[5.47952 - (-0.98575)]$$

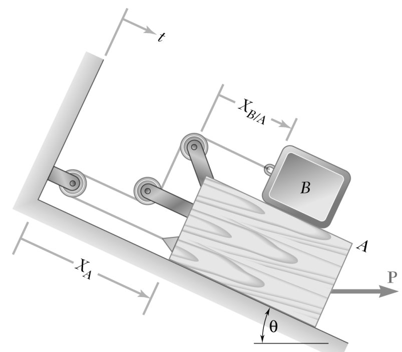
or $$T = 51.7\ \text{N} \blacktriangleleft$$



PROBLEM 12.19

Block A has a mass of 40 kg, and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 40$ N $\longrightarrow$, determine (*a*) the acceleration of block B, (*b*) the tension in the cord.

SOLUTION



From the constraint of the cord.

$$2x_A + x_{B/A} = \text{constant}$$

Then $$2v_A + v_{B/A} = 0$$

and $$2a_A + a_{B/A} = 0$$

Now $$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Then $$a_B = a_A + (-2a_A)$$

or $$a_B = -a_A \qquad (1)$$

First we determine if the blocks will move for the given value of P. Thus, we seek the value of P for which the blocks are in impending motion, with the impending motion of a down the incline.

B:

B: $\quad +\nearrow \Sigma F_y = 0$: $\quad N_{AB} - W_B \cos 25° = 0$

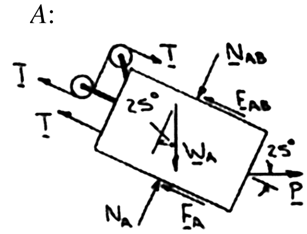
or $$N_{AB} = m_B g \cos 25°$$

Now $$F_{AB} = \mu_s N_{AB}$$
$$= 0.2\, m_B g \cos 25°$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T + F_{AB} + W_B \sin 25° = 0$$

or $$T = 0.2\, m_B g \cos 25° + m_B g \sin 25°$$
$$= (8 \text{ kg})(9.81 \text{ m/s}^2)(0.2 \cos 25° + \sin 25°)$$
$$= 47.39249 \text{ N}$$

A:



A: $\quad +\nearrow \Sigma F_y = 0$: $\quad N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$

or $$N_A = (m_A + m_B) g \cos 25° - P \sin 25°$$

Now $\qquad F_A = \mu_s N_A$

or $\qquad F_A = 0.2[(m_A + m_B)g \cos 25° - P \sin 25°]$

$$\searrow^{+} \Sigma F_x = 0: \quad -T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = 0$$

or $\qquad -T - 0.2[(m_A + m_B)g \cos 25° - P \sin 25°] - 0.2 m_B g \cos 25° + m_A g \sin 25° + P \cos 25° = 0$

or $\qquad P(0.2 \sin 25° + \cos 25°) = T + 0.2[(m_A + 2m_B)g \cos 25°] - m_A g \sin 25°$

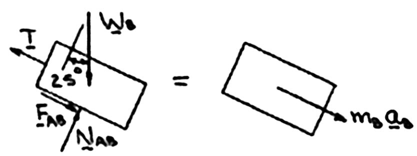
Then $\qquad P(0.2 \sin 25° + \cos 25°) = 47.39249 \text{ N} + 9.81 \text{ m/s}^2 \{0.2[(40 + 2 \times 8)\cos 25° - 40 \sin 25°] \text{ kg}\}$

or $\qquad P = -19.04 \text{ N}$ for impending motion.

Since $P, < 40$ N, the blocks will move. Now consider the motion of the blocks.

(*a*) $\quad +\nearrow \Sigma F_y = 0: \quad N_{AB} - W_B \cos 25° = 0$

B:



or $\qquad N_{AB} = m_B g \cos 25°$

Sliding: $\qquad F_{AB} = \mu_k N_{AB}$

$\qquad = 0.15\, m_B g \cos 25°$

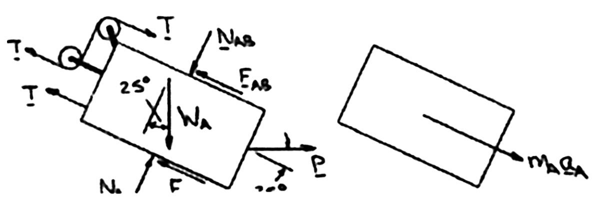
$$\searrow^{+} \Sigma F_x = m_B a_B: \quad -T + F_{AB} + W_B \sin 25° = m_B a_B$$

or $\qquad T = m_B[g(0.15 \cos 25° + \sin 25°) - a_B]$

$\qquad = 8[9.81(0.15 \cos 25° + \sin 25°) - a_B]$

$\qquad = 8(5.47952 - a_B) \qquad$ (N)

A:



$$+\nearrow \Sigma F_y = 0: \quad N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$$

or $\qquad N_A = (m_A + m_B)g \cos 25° - P \sin 25°$

Sliding: $\qquad F_A = \mu_k N_A$

$\qquad = 0.15[(m_A + m_B)g \cos 25° - P \sin 25°]$

$$\searrow^{+} \Sigma F_x = m_A a_A: \quad -T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = m_A a_A$$

Substituting and using Eq. (1)

$$\begin{aligned}T &= m_A g \sin 25° - 0.15[(m_A + m_B) g \cos 25° - P \sin 25°] \\ &\quad - 0.15\, m_B g \cos 25° + P \cos 25° - m_A(-a_B) \\ &= g[m_A \sin 25° - 0.15(m_A + 2m_B)\cos 25°] \\ &\quad + P(0.15 \sin 25° + \cos 25°) + m_A a_B \\ &= 9.81[40 \sin 25° - 0.15(40 + 2\times 8)\cos 25°] \\ &\quad + 40(0.15 \sin 25° + \cos 25°) + 40 a_B \\ &= 129.94004 + 40 a_B\end{aligned}$$

Equating the two expressions for T

$$8(5.47952 - a_B) = 129.94004 + 40 a_B$$

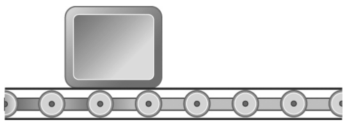
or $$a_B = -1.79383 \text{ m/s}^2$$

$$\mathbf{a}_B = 1.794 \text{ m/s}^2 \searrow 25° \blacktriangleleft$$

(*b*) We have $$T = 8[5.47952 - (-1.79383)]$$

or $$T = 58.2 \text{ N} \blacktriangleleft$$

PROBLEM 12.20



A package is at rest on a conveyor belt which is initially at rest. The belt is started and moves to the right for 1.3 s with a constant acceleration of 2 m/s^2. The belt then moves with a constant deceleration $\mathbf{a}_2$ and comes to a stop after a total displacement of 2.2 m. Knowing that the coefficients of friction between the package and the belt are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine (*a*) the deceleration $\mathbf{a}_2$ of the belt, (*b*) the displacement of the package relative to the belt as the belt comes to a stop.

SOLUTION

(*a*) Kinematics of the belt. $v_o = 0$

1. Acceleration phase with $\mathbf{a}_1 = 2 \text{ m/s}^2 \longrightarrow$

$$v_1 = v_o + a_1 t_1 = 0 + (2)(1.3) = 2.6 \text{ m/s}$$

$$x_1 = x_o + v_o t_1 + \frac{1}{2} a_1 t_1^2 = 0 + 0 + \frac{1}{2}(2)(1.3)^2 = 1.69 \text{ m}$$

2. Deceleration phase: $v_2 = 0$ since the belt stops.

$$v_2^2 - v_1^2 = 2a_2(x_2 - x_1)$$

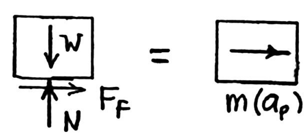
$$a_2 = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{0 - (2.6)^2}{2(2.2 - 1.69)} = -6.63$$

$\mathbf{a}_2 = 6.63 \text{ m/s}^2 \longleftarrow$ ◀

$$t_2 - t_1 = \frac{v_2 - v_1}{a_2} = \frac{0 - 2.6}{-6.63} = 0.3923 \text{ s}$$

(*b*) Motion of the package.

1. Acceleration phase. Assume no slip. $(\mathbf{a}_p)_1 = 2 \text{ m/s}^2 \longrightarrow$



$$\Sigma F_y = 0: \quad N - W = 0 \quad \text{or} \quad N = W = mg$$

$$\xrightarrow{+} \Sigma F_x = ma: \quad F_f = m(a_p)_1$$

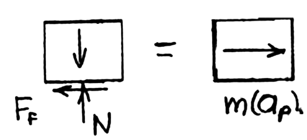
The required friction force is F_f.

The available friction force is $\mu_s N = 0.35W = 0.35mg$

$$\frac{F_f}{m} = (a_p)_1 < \frac{\mu_s N}{m} = \mu_s g = (0.35)(9.81) = 3.43 \text{ m/s}^2$$

Since $2.0 \text{ m/s}^2 < 3.43 \text{ m/s}^2$, the package does not slip.

$$(v_p)_1 = v_1 = 2.6 \text{ m/s} \quad \text{and} \quad (x_p)_1 = 1.69 \text{ m.}$$



2. Deceleration phase. Assume no slip. $(a_p)_2 = -11.52\text{ m/s}^2$

$$\xrightarrow{+} \Sigma F_x = ma: \quad -F_f = m(a_p)_2$$

$$\frac{F_f}{m} = (a_p)_2 = -6.63\text{ m/s}^2$$

$$\frac{\mu_s N}{m} = \frac{\mu_s mg}{m} = \mu_s g = 3.43\text{ m/s}^2 < 6.63\text{ m/s}^2$$

Since the available friction force $\mu_s N$ is less than the required friction force F_f for no slip, the package does slip.

$$(a_p)_2 < 6.63\text{ m/s}^2, \quad F_f = \mu_k N$$

$$\xrightarrow{+} \Sigma F_x = m(a_p)_2: \quad -\mu_k N = m(a_p)_2$$

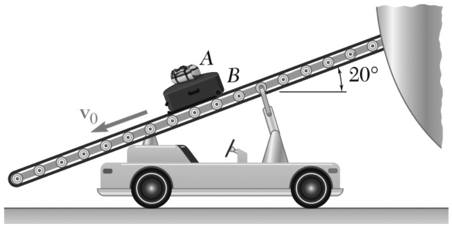
$$\begin{aligned}
(a_p)_2 &= -\frac{\mu_k N}{m} = -\mu_k g \\
&= -(0.25)(9.81) \\
&= -2.4525\text{ m/s}^2 \\
(v_p)_2 &= (v_p)_1 + (a_p)_2(t_2 - t_1) \\
&= 2.6 + (-2.4525)(0.3923) \\
&= 1.638\text{ m/s}^2 \\
(x_p)_2 &= (x_p)_1 + (v_p)_1(t_2 - t_1) + \frac{1}{2}(a_p)_2(t_2 - t_1)^2 \\
&= 1.69 + (2.6)(0.3923) + \frac{1}{2}(-2.4525)(0.3923)^2 \\
&= 2.521\text{ m}
\end{aligned}$$

Position of package relative to the belt

$$(x_p)_2 - x_2 = 2.521 - 2.2 = 0.321$$

$x_{p/\text{belt}} = 0.321\text{ m} \longrightarrow$ ◀

PROBLEM 12.21



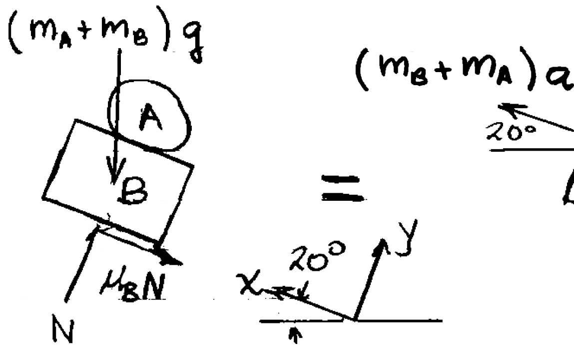
A baggage conveyor is used to unload luggage from an airplane. The 10-kg duffel bag *A* is sitting on top of the 20-kg suitcase *B*. The conveyor is moving the bags down at a constant speed of 0.5 m/s when the belt suddenly stops. Knowing that the coefficient of friction between the belt and *B* is 0.3 and that bag *A* does not slip on suitcase *B*, determine the smallest allowable coefficient of static friction between the bags.

SOLUTION

Since bag *A* does not slide on suitcase *B*, both have the same acceleration.

$$\mathbf{a} = a \searrow 20°$$

Apply Newton's second law to the bag *A* – suitcase *B* combination treated as a single particle.



$$+\nearrow \Sigma F_y = ma_y: \quad -(m_B + m_A)g\cos 20° + N = 0$$

$$N = (m_A + m_B)g \cos 30° = (30)(9.81)\cos 20° = 276.55 \text{ N}$$

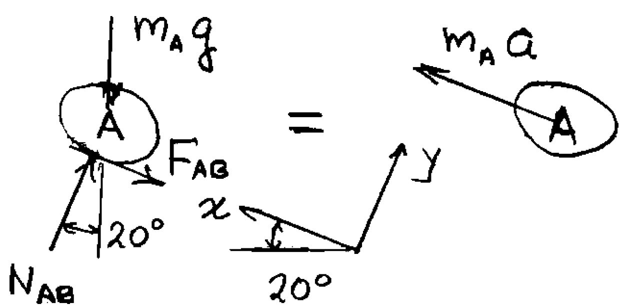
$$\mu_B N = (0.3)(276.55) = 82.965 \text{ N}$$

$$+\nwarrow \Sigma F_x = ma_x: \quad \mu_B N + (m_A + m_B)g\sin 20° = (m_A + m_B)a$$

$$a = g\sin 20° + \frac{\mu_B N}{m_A m_B} = 9.81\sin 20° - \frac{82.965}{30}$$

$$a = 0.58972 \text{ m/s}^2 \qquad \mathbf{a} = 0.58972 \text{ m/s}^2 \nearrow 20°$$

Apply Newton's second law to bag *A* alone.



$$+\nearrow \Sigma F_y = ma_y: \quad N_{AB} - m_A g\cos 20° = 0$$

$$N_{AB} = m_a g\sin 20° = (10)(9.81)\cos 20° = 92.184 \text{ N}$$

$$+\nwarrow ZF_x = ma_x: \quad M_A g\sin 20° - F_{AB} = m_A a$$

$$F_{AB} = m_A(g\sin 20° - a) = (10)(9.81\sin 20° - 0.58972)$$

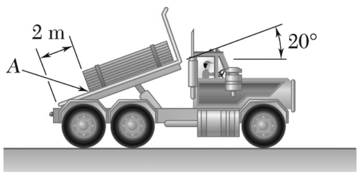
$$= 27.655 \text{ N}$$

Since bag A does not slide on suitcase B,

$$\mu_s > \frac{F_{AB}}{N_{AB}} = \frac{27.655}{92.184} = 0.300$$

$\mu_s > 0.300$ ◀

PROBLEM 12.22



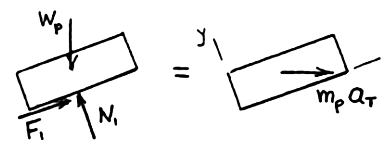
To unload a bound stack of plywood from a truck, the driver first tilts the bed of the truck and then accelerates from rest. Knowing that the coefficients of friction between the bottom sheet of plywood and the bed are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (*a*) the smallest acceleration of the truck which will cause the stack of plywood to slide, (*b*) the acceleration of the truck which causes corner *A* of the stack to reach the end of the bed in 0.9 s.

SOLUTION

Let $\mathbf{a}_P$ be the acceleration of the plywood, $\mathbf{a}_T$ be the acceleration of the truck, and $\mathbf{a}_{P/T}$ be the acceleration of the plywood relative to the truck.

(*a*) Find the value of $\mathbf{a}_T$ so that the relative motion of the plywood with respect to the truck is impending.

$$a_P = a_T \text{ and } F_1 = \mu_s N_1 = 0.40 N_1$$



$$+\nwarrow \Sigma F_y = m_P a_y: \quad N_1 - W_P \cos 20° = -m_P a_T \sin 20°$$
$$N_1 = m_P(g \cos 20° - a_T \sin 20°)$$

$$+\nearrow \Sigma F_x = m a_x: \quad F_1 - W_P \sin 20° = m_P a_T \cos 20°$$
$$F_1 = m_P(g \sin 20° + a_T \cos 20°)$$

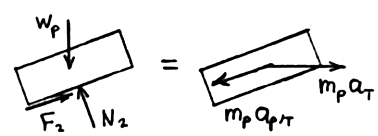
$$m_P(g \sin 20° + a_T \cos 20°) = 0.40\, m_P(g \cos 20° - a_T \sin 20°)$$

$$a_T = \frac{(0.40 \cos 20° - \sin 20°)}{\cos 20° + 0.40 \sin 20°} g$$
$$= (0.03145)(9.81)$$
$$= 0.309$$

$$\mathbf{a}_T = 0.309 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) $$x_{P/T} = (x_{P/T})_o + (v_{P/T})t + \frac{1}{2} a_{P/T} t^2 = 0 + 0 + \frac{1}{2} a_{P/T} t^2$$

$$a_{P/T} = \frac{2 x_{P/T}}{t^2} = \frac{(2)(2)}{(0.9)^2} = 4.94 \text{ m/s}^2$$



$$\mathbf{a}_{P/T} = 4.94 \text{ m/s}^2 \swarrow 20°$$

$$\mathbf{a}_P = \mathbf{a}_T + \mathbf{a}_{P/T} = (a_T \rightarrow) + (4.94 \text{ m/s}^2 \swarrow 20°)$$

$$+\nwarrow F_y = m_P a_y: \quad N_2 - W_P \cos 20° = -m_P a_T \sin 20°$$
$$N_2 = m_P(g \cos 20° - a_T \sin 20°)$$

PROBLEM 12.22 (Continued)

$$+\nearrow \Sigma F_x = \Sigma m a_x: \quad F_2 - W_P \sin 20° = m_P a_T \cos 20° - m_P a_{P/T}$$

$$F_2 = m_P (g \sin 20° + a_T \cos 20° - a_{P/T})$$

For sliding with friction $\quad F_2 = \mu_k N_2 = 0.30 N_2$

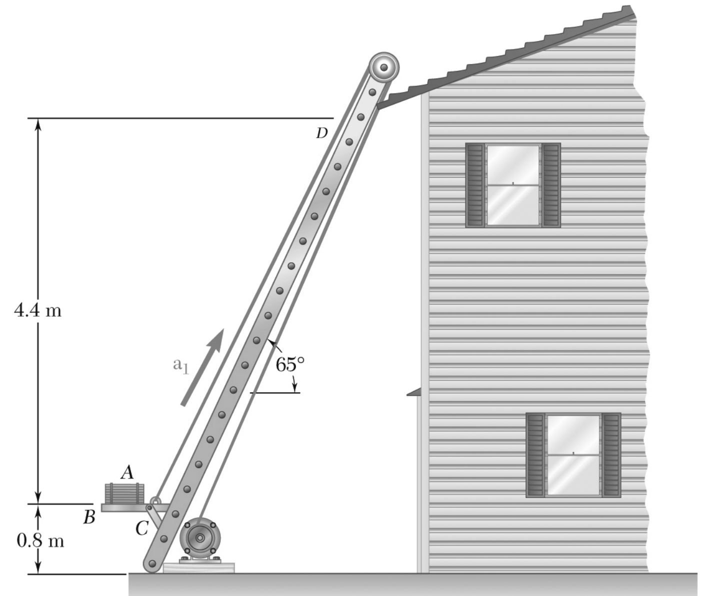
$$m_P (g \sin 20° + a_T \cos 20° - a_{P/T}) = 0.30 m_P (g \cos 20° + a_T \sin 20°)$$

$$a_T = \frac{(0.30 \cos 20° - \sin 20°) g + a_{P/T}}{\cos 20° + 0.30 \sin 20°}$$

$$= (-0.05767)(9.81) + (0.9594)(4.94)$$

$$= 4.17$$

$\mathbf{a}_T = 4.17 \text{ m/s}^2 \longrightarrow$ ◀

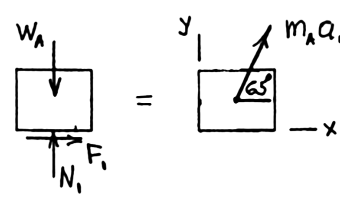


PROBLEM 12.23

To transport a series of bundles of shingles A to a roof, a contractor uses a motor-driven lift consisting of a horizontal platform BC which rides on rails attached to the sides of a ladder. The lift starts from rest and initially moves with a constant acceleration $\mathbf{a}_1$ as shown. The lift then decelerates at a constant rate $\mathbf{a}_2$ and comes to rest at D, near the top of the ladder. Knowing that the coefficient of static friction between a bundle of shingles and the horizontal platform is 0.30, determine the largest allowable acceleration $\mathbf{a}_1$ and the largest allowable deceleration $\mathbf{a}_2$ if the bundle is not to slide on the platform.

SOLUTION

Acceleration $\mathbf{a}_1$: Impending slip. $\quad F_1 = \mu_s N_1 = 0.30 N_1$



$$\Sigma F_y = m_A a_y: \qquad N_1 - W_A = m_A a_1 \sin 65°$$

$$N_1 = W_A + m_A a_1 \sin 65°$$

$$= m_A (g + a_1 \sin 65°)$$

$$\xrightarrow{+} \Sigma F_x = m_A a_x: \quad F_1 = m_A a_1 \cos 65°$$

$$F_1 = \mu_s N$$

or

$$m_A a_1 \cos 65° = 0.30 m_A (g + a_1 \sin 65°)$$

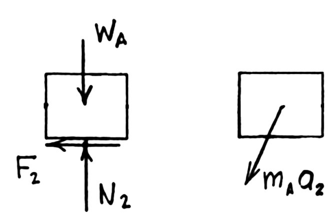
$$a_1 = \frac{0.30g}{\cos 65° - 0.30 \sin 65°}$$

$$= (1.990)(9.81)$$

$$= 19.53 \text{ m/s}^2$$

$$\mathbf{a}_1 = 19.53 \text{ m/s}^2 \measuredangle 65° \blacktriangleleft$$

Deceleration $\mathbf{a}_2$: Impending slip. $\quad F_2 = \mu_s N_2 = 0.30 N_2$



$$\Sigma F_y = m a_y: \quad N_1 - W_A = -m_A a_2 \sin 65°$$

$$N_1 = W_A - m_A a_2 \sin 65°$$

$$\xleftarrow{+} \Sigma F_x = m a_x: \qquad F_2 = m_A a_2 \cos 65°$$

$$F_2 = \mu_s N_2$$

PROBLEM 12.23 (Continued)

or $$m_A a_2 \cos 65° = 0.30\, m_A (g - a_2 \cos 65°)$$

$$a_2 = \frac{0.30g}{\cos 65° + 0.30 \sin 65°}$$

$$= (0.432)(9.81)$$

$$= 4.24 \text{ m/s}^2$$

$\mathbf{a}_2 = 4.24 \text{ m/s}^2 \measuredangle\, 65°$ ◀

PROBLEM 12.24

An airplane has a mass of 25 Mg and its engines develop a total thrust of 40 kN during take-off. If the drag **D** exerted on the plane has a magnitude $D = 2.25v^2$, where v is expressed in meters per second and D in newtons, and if the plane becomes airborne at a speed of 240 km/h, determine the length of runway required for the plane to take off.

SOLUTION

$$F = ma: \quad 40\times10^3 \text{ N} - 2.25v^2 = (25\times10^3 \text{ kg})a$$

Substituting

$$a = v\frac{dv}{dx}: \quad 40\times10^3 - 2.25\ v^2 = (25\times10^3)\ v\frac{dv}{dx}$$

$$\int_0^{x_1} dx = \int_0^{v_1} \frac{(25\times10^3)vdv}{40\times10^3 - 2.25v^2}$$

$$x_1 = -\frac{25\times10^3}{2(2.25)}[\ln(40\times10^3 - 2.25v^2)]_0^{v_1}$$

$$= \frac{25\times10^3}{4.5}\ln\frac{40\times10^3}{40\times10^3 - 2.25v_1^2}$$

For $v_1 = 240 \text{ km/h} = 66.67 \text{ m/s}$

$$x_1 = \frac{25\times10^3}{4.5}\ln\frac{40\times10^3}{40\times10^3 - 2.25(66.67)^2} = 5.556\ln 1.333$$

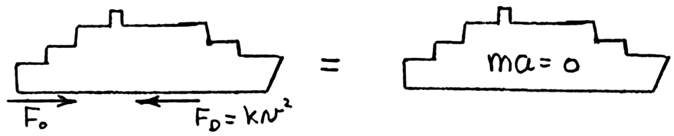
$$= 1.5982\times10^3 \text{ m}$$

$x_1 = 1.598 \text{ km}$ ◀

PROBLEM 12.25

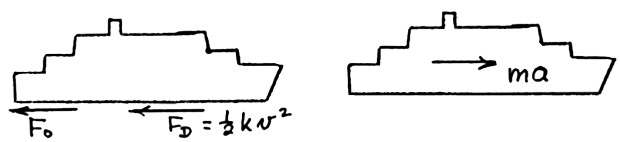
The propellers of a ship of weight W can produce a propulsive force $\mathbf{F}_0$; they produce a force of the same magnitude but of opposite direction when the engines are reversed. Knowing that the ship was proceeding forward at its maximum speed v_0 when the engines were put into reverse, determine the distance the ship travels before coming to a stop. Assume that the frictional resistance of the water varies directly with the square of the velocity.

SOLUTION



At maximum speed $a = 0$. $\qquad F_0 = kv_0^2 = 0 \qquad k = \frac{F_0}{v_0^2}$

When the propellers are reversed, F_0 is reversed.



$$\xrightarrow{+} \Sigma F_x = ma: \quad -F_0 - kv^2 = ma$$

$$-F_0 - F_0\frac{v^2}{v_0^2} = ma \qquad a - \frac{F_0}{mv_0^2}\left(v_0^2 + v^2\right)$$

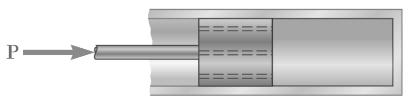
$$dx = \frac{vdv}{a} = \frac{mv_0^2 vdv}{F_0\left(v_0^2 + v^2\right)}$$

$$\int_0^x dx = -\frac{mv_0^2}{F_0}\int_{v_0}^{0}\frac{vdv}{v_0^2 + v^2}$$

$$x = -\frac{mv_0^2}{F_0}\frac{1}{2}\ln\left(v_0^2 + v^2\right)\Big|_{v_0}^{0}$$

$$= -\frac{mv_0^2}{2F_0}\left[\ln v_0^2 - \ln\left(2v_0^2\right)\right] = \frac{mv_0^2}{2F_0}\ln 2 \qquad x = 0.347\frac{m_0 v_0^2}{F_0} \blacktriangleleft$$

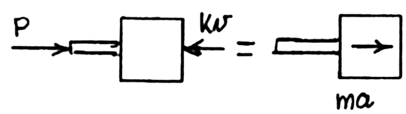
PROBLEM 12.26



A constant force **P** is applied to a piston and rod of total mass m to make them move in a cylinder filled with oil. As the piston moves, the oil is forced through orifices in the piston and exerts on the piston a force of magnitude kv in a direction opposite to the motion of the piston. Knowing that the piston starts from rest at $t = 0$ and $x = 0$, show that the equation relating x, v, and t, where x is the distance traveled by the piston and v is the speed of the piston, is linear in each of these variables.

SOLUTION

$$\xrightarrow{+} \Sigma F = ma: \quad P - kv = ma$$



$$\frac{dv}{dt} = a = \frac{P - kv}{m}$$

$$\int_0^t dt = \int_0^v \frac{m\,dv}{P - kv}$$

$$= -\frac{m}{k}\ln(P - kv)\Big|_0^v$$

$$= -\frac{m}{k}[\ln(P - kv) - \ln P]$$

$$t = -\frac{m}{k}\ln\frac{P - kv}{P} \qquad \text{or} \qquad \ln\frac{P - kv}{m} = -\frac{kt}{m}$$

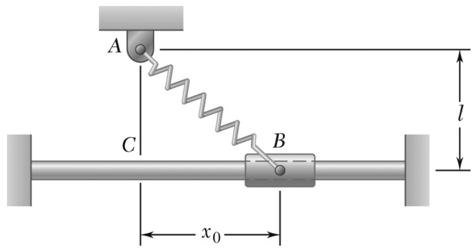
$$\frac{P - kv}{m} = e^{-kt/m} \qquad \text{or} \qquad v = \frac{P}{k}(1 - e^{-kt/m})$$

$$x = \int_0^t v\,dt = \frac{Pt}{k}\Bigg|_0^t - \frac{P}{k}\left(-\frac{k}{m}e^{-kt/m}\right)\Bigg|_0^t$$

$$= \frac{Pt}{k} + \frac{P}{m}(e^{-kt/m} - 1) = \frac{Pt}{k} - \frac{P}{m}(1 - e^{-kt/m})$$

$$x = \frac{Pt}{k} - \frac{kv}{m}, \quad \text{which is linear.} \qquad \blacktriangleleft$$

PROBLEM 12.27



A spring *AB* of constant *k* is attached to a support at *A* and to a collar of mass *m*. The unstretched length of the spring is ℓ. Knowing that the collar is released from rest at $x = x_0$ and neglecting friction between the collar and the horizontal rod, determine the magnitude of the velocity of the collar as it passes through Point *C*.

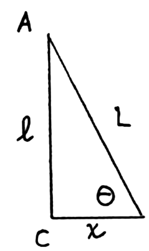
SOLUTION

Choose the origin at Point *C* and let *x* be positive to the right. Then *x* is a position coordinate of the slider *B* and x_0 is its initial value. Let *L* be the stretched length of the spring. Then, from the right triangle

$$L = \sqrt{\ell^2 + x^2}$$

The elongation of the spring is $e = L - \ell$, and the magnitude of the force exerted by the spring is

$$F_s = ke = k(\sqrt{\ell^2 + x^2} - \ell)$$

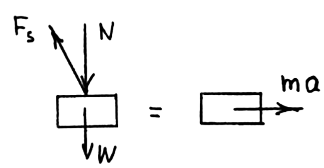


By geometry,

$$\cos\theta = \frac{x}{\sqrt{\ell^2 + x^2}}$$

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad -F_s \cos\theta = ma$$

$$-k(\sqrt{\ell^2 + x^2} - \ell)\frac{x}{\sqrt{\ell^2 + x^2}} = ma$$



$$a = -\frac{k}{m}\left(x - \frac{\ell x}{\sqrt{\ell^2 + x^2}}\right)$$

$$\int_0^v v\,dv = \int_{x_0}^0 a\,dx$$

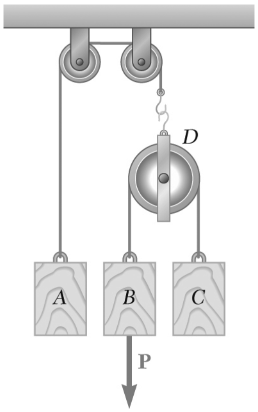
$$\frac{1}{2}v^2\Big|_0^v = -\frac{k}{m}\int_{x_0}^0 \left(x - \frac{\ell x}{\sqrt{\ell^2 + x^2}}\right)dx = -\frac{k}{m}\left(\frac{1}{2}x^2 - \ell\sqrt{\ell^2 + x^2}\right)\Bigg|_{x_0}^0$$

$$\frac{1}{2}v^2 = -\frac{k}{m}\left(0 - \ell^2 - \frac{1}{2}x_0^2 + \ell\sqrt{\ell^2 + x_0^2}\right)$$

$$v^2 = \frac{k}{m}\left(2\ell^2 + x_0^2 - 2\ell\sqrt{\ell^2 + x_0^2}\right)$$

$$= \frac{k}{m}\left[\left(\ell^2 + x_0^2\right) - 2\ell\sqrt{\ell^2 + x_0^2} + \ell^2\right]$$

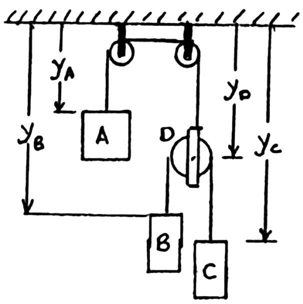
answer: $v = \sqrt{\frac{k}{m}}\left(\sqrt{\ell^2 + x_0^2} - \ell\right)$ ◀



PROBLEM 12.28

Block A has a mass of 10 kg, and blocks B and C have masses of 5 kg each. Knowing that the blocks are initially at rest and that B moves through 3 m in 2 s, determine (*a*) the magnitude of the force **P**, (*b*) the tension in the cord AD. Neglect the masses of the pulleys and axle friction.

SOLUTION



Let the position coordinate y be positive downward.

Constraint of cord AD: $\quad y_A + y_D = \text{constant}$

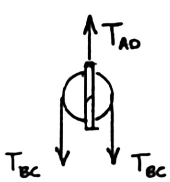
$$v_A + v_D = 0, \quad a_A + a_D = 0$$

Constraint of cord BC: $\quad (y_B - y_D) + (y_C - y_D) = \text{constant}$

$$v_B + v_C - 2v_D = 0, \quad a_B + a_C - 2a_D = 0$$

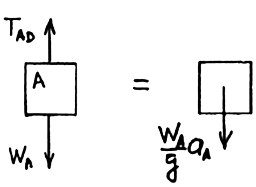
Eliminate a_D. $\quad 2a_A + a_B + a_C = 0 \qquad (1)$

We have uniformly accelerated motion because all of the forces are constant.



$$y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2}a_B t^2, \quad (v_B)_0 = 0$$

$$a_B = \frac{2[y_B - (y_B)_0]}{t^2} = \frac{(2)(3)}{(2)^2} = 1.5 \text{ m/s}^2$$

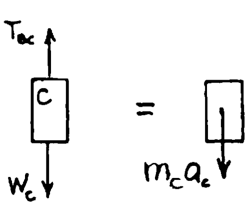


Pulley D: $\quad +\!\downarrow \Sigma F_y = 0: \quad 2T_{BC} - T_{AD} = 0$

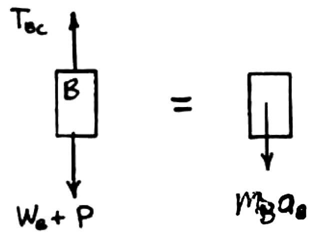
$$T_{AD} = 2T_{BC}$$

Block A: $\quad +\!\downarrow \Sigma F_y = ma_y: \quad W_A - T_{AD} = m_A a_A$

or $\quad a_A = \dfrac{W_A - T_{AD}}{m_A} = \dfrac{W_A - 2T_{BC}}{m_A} \qquad (2)$



PROBLEM 12.28 (Continued)



Block C: $+\!\downarrow \Sigma F_y = ma_y$: $W_C - T_{BC} = m_C a_C$

or $$a_C = \frac{W_C - T_{BC}}{m_C} \qquad (3)$$

Substituting the value for a_B and Eqs. (2) and (3) into Eq. (1), and solving for T_{BC},

$$2\left(\frac{W_A - 2T_{BC}}{m_A}\right) + a_B + \left(\frac{W_C - T_{BC}}{m_C}\right) = 0$$

$$2\left(\frac{m_A g - 2T_{BC}}{m_A}\right) + a_B + \left(\frac{m_C g - T_{BC}}{m_C}\right) = 0$$

$$\left(\frac{4}{m_A} + \frac{1}{m_C}\right) T_{BC} = 3g + a_B$$

$$\left(\frac{4}{10} + \frac{1}{5}\right) T_{BC} = 3(9.81) + 1.5 \quad \text{or} \quad T_{BC} = 51.55 \text{ N}$$

Block B: $+\!\downarrow \Sigma F_y = ma_y$: $P + W_B - T_{BC} = m_B a_B$

(*a*) <u>Magnitude of *P*</u>.

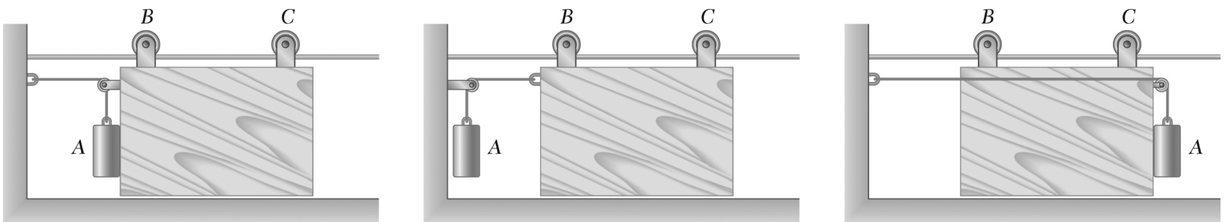
$$P = T_{BC} - W_B + m_B a_B$$
$$= 51.55 - 5(9.81) + 5(1.5) \qquad P = 10.00 \text{ N} \blacktriangleleft$$

(*b*) <u>Tension in cord *AD*</u>.

$$T_{AD} = 2T_{BC} = (2)(51.55) \qquad T_{AD} = 103.1 \text{ N} \blacktriangleleft$$

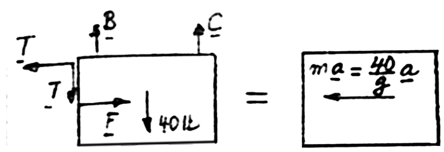
PROBLEM 12.29

A 40-lb sliding panel is supported by rollers at B and C. A 25-lb counterweight A is attached to a cable as shown and, in cases a and c, is initially in contact with a vertical edge of the panel. Neglecting friction, determine in each case shown the acceleration of the panel and the tension in the cord immediately after the system is released from rest.



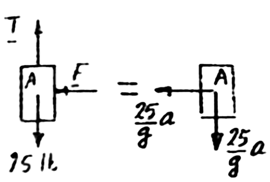
SOLUTION

(a) Panel: F = Force exerted by counterweight



$\xleftarrow{+} \Sigma F_x = ma$: $\quad T - F = \dfrac{40}{g}a \qquad (1)$

Counterweight A: Its acceleration has two components



$$\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P} = a \rightarrow + a \downarrow$$

$\xleftarrow{+} \Sigma F_x = ma_x$: $\quad F = \dfrac{25}{g}a \qquad (2)$

$+\!\downarrow \Sigma F_g = ma_g$: $\quad 25 - T = \dfrac{25}{g}a \qquad (3)$

Adding (1), (2), and (3):

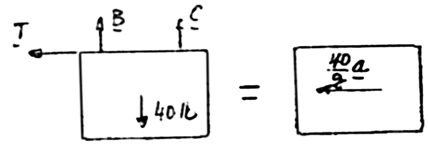
$$\cancel{T} - F + F + 25 - \cancel{T} = \frac{40+25+25}{g}a$$

$$a = \frac{25}{90}g = \frac{25}{90}(32.2) \qquad \mathbf{a} = 8.94 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$

Substituting for a into (3):

$$25 - T = \frac{25}{g}\left(\frac{25}{90}g\right) \quad T = 25 - \frac{625}{90} \qquad T = 18.06 \text{ lb} \blacktriangleleft$$

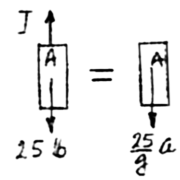
(*b*) Panel:



$\overset{+}{\leftarrow} \Sigma F_y = ma$: $$T = \frac{40}{g}a \qquad (1)$$

Counterweight A:

$+\!\downarrow \Sigma F_y = ma$: $$25 - T = \frac{25}{g}a \qquad (2)$$



Adding (1) and (2): $$\cancel{T} + 25 - \cancel{T} = \frac{40+25}{g}a$$

$$a = \frac{25}{65}g$$

$\mathbf{a} = 12.38\ \text{ft/s}^2 \longleftarrow$ ◀

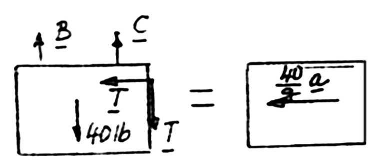
Substituting for a into (1):

$$T = \frac{40}{g}\left(\frac{25}{65}g\right) = \frac{1000}{65}$$

$T = 15.38$ lb ◀

(*c*) Since panel is accelerated to the left, there is no force exerted by panel on counterweight and vice versa.

Panel:



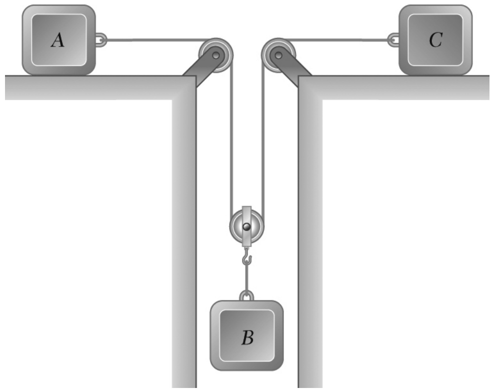
$\overset{+}{\leftarrow} \Sigma F_x = ma$: $$T = \frac{40}{g}a \qquad (1)$$

Counterweight A: Same free body as in Part (*b*):

$+\!\downarrow \Sigma F_y = ma$: $$25 - T = \frac{25}{g}a \qquad (2)$$

Since Eqs. (1) and (2) are the same as in (*b*), we get the same answers:

$\mathbf{a} = 12.38\ \text{ft/s}^2 \longleftarrow$; $T = 15.38$ lb ◀



PROBLEM 12.30

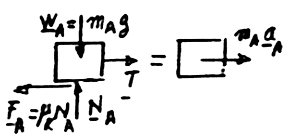
The coefficients of friction between blocks A and C and the horizontal surfaces are $\mu_s = 0.24$ and $\mu_k = 0.20$. Knowing that $m_A = 5$ kg, $m_B = 10$ kg, and $m_C = 10$ kg, determine (a) the tension in the cord, (b) the acceleration of each block.

SOLUTION

We first check that static equilibrium is not maintained:

$$\begin{aligned}(F_A)_m + (F_C)_m &= \mu_s(m_A + m_C)g \\ &= 0.24(5+10)g \\ &= 3.6g\end{aligned}$$

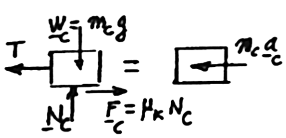
Since $W_B = m_B g = 10\text{g} > 3.6\text{g}$, equilibrium is *not* maintained.



Block A: ΣF_y: $N_A = m_A g$

$$F_A = \mu_k N_A = 0.2 m_A g$$

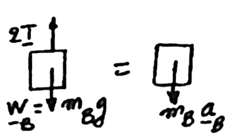
$$\xrightarrow{+} \Sigma F_\lambda = m_A a_A: \quad T - 0.2 m_A g = m_A a_A \tag{1}$$



Block C: ΣF_y: $N_C = m_C g$

$$F_C = \mu_k N_C = 0.2 m_C g$$

$$\xleftarrow{+} \Sigma F_x = m_C a_C: \quad T - 0.2 m_C g = m_C a_C \tag{2}$$



Block B: $+\!\downarrow \Sigma F_y = m_B a_B$

$$m_B g - 2T = m_B a_B \tag{3}$$

From kinematics:

$$a_B = \frac{1}{2}(a_A + a_C) \tag{4}$$

(a) Tension in cord. Given data: $m_A = 5$ kg

$m_B = m_C = 10$ kg

$$\text{Eq. (1):} \quad T - 0.2(5)g = 5a_A \qquad a_A = 0.2T - 0.2g \tag{5}$$

$$\text{Eq. (2):} \quad T - 0.2(10)g = 10a_C \qquad a_C = 0.1T - 0.2g \tag{6}$$

$$\text{Eq. (3):} \quad 10g - 2T = 10a_B \qquad a_B = g - 0.2T \tag{7}$$

Substitute into (4):

$$g - 0.2T = \frac{1}{2}(0.2T - 0.2g + 0.1T - 0.2g)$$

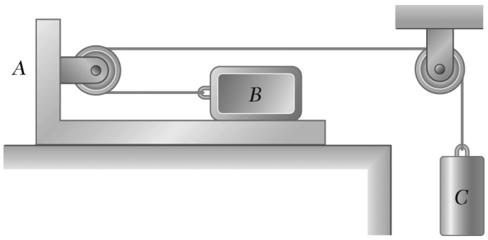
$$1.2g = 0.35T \qquad T = \frac{24}{7}g = \frac{24}{7}(9.81 \text{ m/s}^2) \qquad T = 33.6 \text{ N} \blacktriangleleft$$

(*b*) Substitute for T into (5), (7), and (6):

$$a_A = 0.2\left(\frac{24}{7}g\right) - 0.2g = 0.4857(9.81 \text{ m/s}^2) \qquad \mathbf{a}_A = 4.76 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

$$a_B = g - 0.2\left(\frac{24}{7}g\right) = 0.3143(9.81 \text{ m/s}^2) \qquad \mathbf{a}_B = 3.08 \text{ m/s}^2 \downarrow \blacktriangleleft$$

$$a_C = 0.1\left(\frac{24}{7}g\right) - 0.2g = 0.14286(9.81 \text{ m/s}^2) \qquad \mathbf{a}_C = 1.401 \text{ m/s}^2 \longleftarrow \blacktriangleleft$$

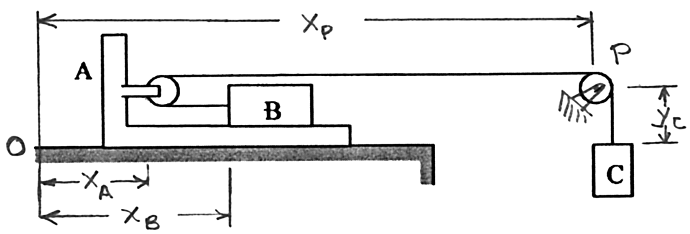


PROBLEM 12.31

A 10-lb block B rests as shown on a 20-lb bracket A. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$ between block B and bracket A, and there is no friction in the pulley or between the bracket and the horizontal surface. (a) Determine the maximum weight of block C if block B is not to slide on bracket A. (b) If the weight of block C is 10% larger than the answer found in a determine the accelerations of A, B and C.

SOLUTION

Kinematics. Let x_A and x_B be horizontal coordinates of A and B measured from a fixed vertical line to the left of A and B. Let y_C be the distance that block C is below the pulley. Note that y_C increases when C moves downward. See figure.



The cable length L is fixed.

$$L = (x_B - x_A) + (x_P - x_A) + y_C + \text{constant}$$

Differentiating and noting that $\dot{x}_P = 0$,

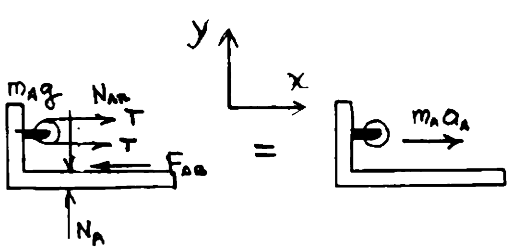
$$v_B - 2v_A + v_C = 0$$

$$-2a_A + a_B + a_C = 0 \qquad (1)$$

Here, a_A and a_B are positive to the right, and a_C is positive downward.

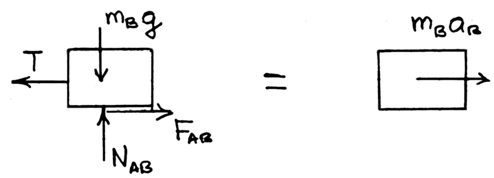
Kinetics. Let T be the tension in the cable and F_{AB} be the friction force between blocks A and B. The free body diagrams are:

Bracket A:

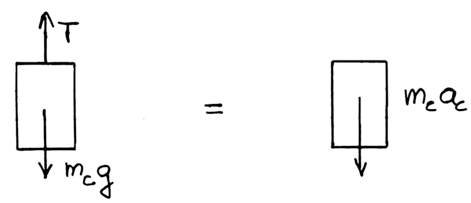


PROBLEM 12.31 (Continued)

Block B:



Block C:



Bracket A: $\quad \xrightarrow{+} \Sigma F_x = ma_x: \quad 2T - F_{AB} = \dfrac{W_A}{g} a_A \qquad (2)$

Block B: $\quad \xrightarrow{+} \Sigma F_x = ma_x: \quad F_{AB} - T = \dfrac{W_B}{g} a_B \qquad (3)$

$$+\uparrow \Sigma F_y = ma_y: \quad N_{AB} - W_B = 0$$

or

$$N_{AB} = W_B$$

Block C: $\quad +\downarrow \Sigma F_y = ma_y: \quad m_C - T = \dfrac{W_C}{g} a_C \qquad (4)$

Adding Eqs. (2), (3), and (4), and transposing,

$$\frac{W_A}{g} a_A + \frac{W_B}{g} a_B + \frac{W_C}{g} a_C = W_C \qquad (5)$$

Subtracting Eq. (4) from Eq. (3) and transposing,

$$\frac{W_B}{g} a_B - \frac{W_C}{g} a_C = F_{AB} - W_C \qquad (6)$$

(a) No slip between A and B. $\quad a_B = a_A$

From Eq. (1), $\quad a_A = a_B = a_C = a$

From Eq. (5), $\quad a = \dfrac{W_C g}{W_A + W_B + W_C}$

For impending slip, $\quad F_{AB} = \mu_s N_{AB} = \mu_s W_B$

PROBLEM 12.31 (Continued)

Substituting into Eq. (6),

$$\frac{(W_B - W_C)(W_C g)}{W_A + W_B + W_C} = \mu_s W_B - W_C$$

Solving for W_C,

$$W_C = \frac{\mu_s W_B (W_A + W_B)}{W_A + 2W_B - \mu_s W_B}$$
$$= \frac{(0.30)(10)(20+10)}{20 + (2)(10) - (0.30)(10)}$$

$W_C = 2.43 \text{ lbs}$ ◀

(*b*) W_C increased by 10%. $\quad W_C = 2.6757 \text{ lbs}$

Since slip is occurring, $\quad F_{AB} = \mu_k N_{AB} = \mu_k W_B$

Eq. (6) becomes $\quad \dfrac{W_B}{g} a_B - \dfrac{W_C}{g} a_C = \mu_k W_B - W_C$

or $\quad 10a_B - 2.6757 a_C = [(0.25)(10) - 2.6757](32.2) \qquad (7)$

With numerical data, Eq. (5) becomes

$$20a_A + 10a_B + 2.6757 a_C = (2.6757)(32.2) \qquad (8)$$

Solving Eqs. (1), (7), and (8) gives

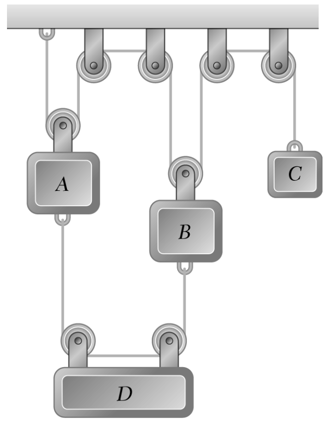
$$a_A = 3.144 \text{ ft/s}^2, \quad a_B = 0.881 \text{ ft/s}^2, \quad a_C = 5.407 \text{ ft/s}^2$$

$\mathbf{a}_A = 3.14 \text{ ft/s}^2 \longrightarrow$ ◀

$\mathbf{a}_B = 0.881 \text{ ft/s}^2 \longrightarrow$ ◀

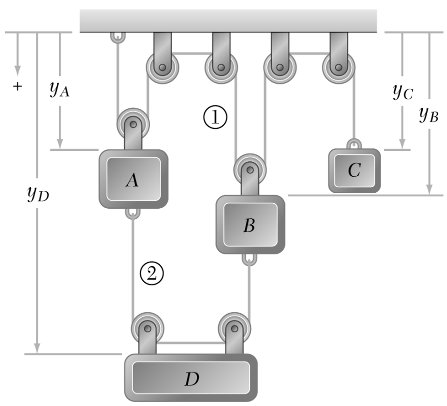
$\mathbf{a}_C = 5.41 \text{ ft/s}^2 \downarrow$ ◀

PROBLEM 12.32



The masses of blocks A, B, C and D are 9 kg, 9 kg, 6 kg and 7 kg, respectively. Knowing that a downward force of magnitude 120 N is applied to block D, determine (a) the acceleration of each block, (b) the tension in cord ABC. Neglect the weights of the pulleys and the effect of friction.

SOLUTION



Note: As shown, the system is in equilibrium.

From the diagram:

Cord 1: $$2y_A + 2y_B + y_C = \text{constant}$$

Then $$2v_A + 2v_B + v_C = 0$$

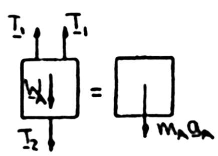
and $$2a_A + 2a_B + a_C = 0 \qquad (1)$$

Cord 2: $$(y_D - y_A) + (y_D - y_B) = \text{constant}$$

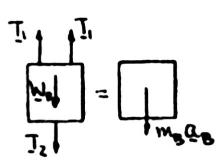
Then $$2v_D - v_A - v_B = 0$$

and $$2a_D - a_A - a_B = 0 \qquad (2)$$

A:



B:



(a) $+\downarrow \Sigma F_y = m_A a_A$: $\quad m_A g - 2T_1 + T_2 = m_A a_A$

or $$9(9.81) - 2T_1 + T_2 = 9a_A \qquad (3)$$

$+\downarrow \Sigma F_y = m_B a_B$: $\quad m_B g - 2T_1 + T_2 = m_B a_B$

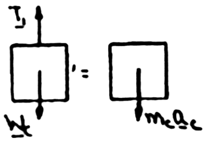
or $$9(9.81) - 2T_1 + T_2 = 9a_B \qquad (4)$$

Note: Eqs. (3) and (4) $\Rightarrow \mathbf{a}_A = \mathbf{a}_B$

Then Eq. (1) $\Rightarrow a_C = -4a_A$

Eq. (2) $\Rightarrow a_D = a_A$

C:



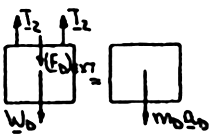
$$+\downarrow \Sigma F_y = m_C a_C: \quad m_C g - T_1 = m_C a_C$$

or $$T_1 = m_C(g - a_C) = 6(g + 4a_A) \qquad (5)$$

$$+\downarrow \Sigma F_y = m_D a_D: \quad m_D g - 2T_2 + (F_D)_{\text{ext}} = m_D a_D$$

or $$T_2 = \frac{1}{2}[m_D(g - a_D) + 120] = 94.335 - \frac{1}{2}(7a_A) \qquad (6)$$

D:



Substituting for T_1 [Eq. (5)] and T_2 [Eq. (6)] in Eq. (3)

$$9(9.81) - 2\times 6(g + 4a_A) + 94.335 - \frac{1}{2}(7a_A) = 9a_A$$

or $$a_A = \frac{9(9.81) - 2\times 6(9.81) + 94.335}{48 + 3.5 + 9} = 1.0728 \text{ m/s}^2$$

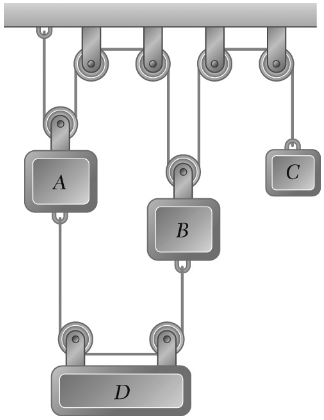
$$\mathbf{a}_A = \mathbf{a}_B = \mathbf{a}_D = 1.073 \text{ m/s}^2 \downarrow \blacktriangleleft$$

and $$a_C = -4(1.0728 \text{ m/s}^2) \qquad \text{or} \qquad \mathbf{a}_C = 4.29 \text{ m/s}^2 \uparrow \blacktriangleleft$$

(*b*) Substituting into Eq. (5)

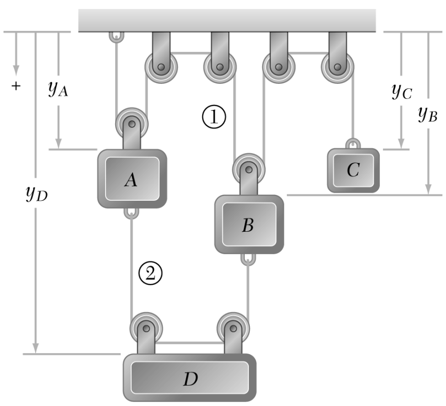
$$T_1 = 6\big(9.81 + 4(1.0728)\big) \qquad \text{or} \quad T_1 = 84.6 \text{ N} \blacktriangleleft$$

PROBLEM 12.33



The masses of blocks A, B, C and D are 9 kg, 9 kg, 6 kg and 7 kg, respectively. Knowing that a downward force of magnitude 50 N is applied to block B and that the system starts from rest, determine at $t = 3$ s the velocity (a) of D relative to A, (b) of C relative to D. Neglect the weights of the pulleys and the effect of friction.

SOLUTION



Note: As shown, the system is in equilibrium.

From the constraint of the two cords,

Cord 1: $$2y_A + 2y_B + y_C = \text{constant}$$

Then $$2v_A + 2v_B + v_C = 0$$

and $$2a_A + 2a_B + a_C = 0 \qquad (1)$$

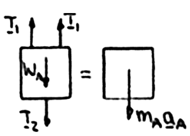
Cord 2: $$(y_D - y_A) + (y_D - y_B) = \text{constant}$$

Then $$2v_D - v_A - v_B = 0$$

and $$2a_D - a_A - a_B = 0 \qquad (2)$$

We determine the accelerations of blocks A, C, and D using the blocks as free bodies.

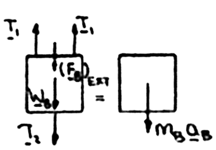
A:



$$+\!\downarrow \Sigma F_y = m_A a_A: \quad W_A - 2T_1 + T_2 = \frac{W_A}{g} a_A$$

or $$m_A g - 2T_1 + T_2 = m_A a_A \qquad (3)$$

B:



$$+\!\downarrow \Sigma F_y = m_B a_B: \quad W_B - 2T_1 + T_2 + (F_B)_{\text{ext}} = \frac{W_B}{g} a_B$$

or $$m_B g - 2T_1 + T_2 + (F_B)_{\text{ext}} = m_B a_B \qquad (4)$$

Forming $$(3) - (4) \Rightarrow -(F_B)_{\text{ext}} = 9(a_A - a_B)$$

or $$a_B = a_A + \frac{(F_B)_{\text{ext}}}{m_B}$$

PROBLEM 12.33 (Continued)

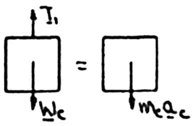
Then Eq. (1): $$2a_A + 2\left(a_A + \frac{(F_B)_{\text{ext}}}{m_g}\right) + a_C = 0$$

or $$a_C = -4a_A - \frac{2(F_B)_{\text{ext}}}{m_B}$$

Eq. (2): $$2a_D - a_A - \left(a_A + \frac{(F_B)_{\text{ext}}}{m_B}\right) = 0$$

or $$a_D = a_A + \frac{(F_B)_{\text{ext}}}{2m_B}$$

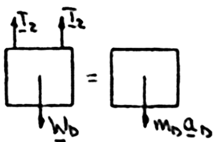
C:



$$+\!\downarrow \Sigma F_y = m_C a_C: \quad W_C - T_1 = m_C a_C = m_C\left(-4a_A - \frac{2(F_B)_{\text{ext}}}{m_B}\right)$$

or $$T_1 = m_C g + 4m_C a_A + \frac{2m_C(F_B)_{\text{ext}}}{m_B} \qquad (5)$$

D:



$$+\!\downarrow \Sigma F_y = m_D a_D: \quad W_D - 2T_2 = m_D a_D$$

or $$T_2 = \frac{1}{2} \times m_D\left[g - a_A - \frac{(F_B)_{\text{ext}}}{2m_B}\right] \qquad (6)$$

Substituting for T_1 [Eq. (5)] and T_2 [Eq. (6)] in Eq. (3)

$$m_A g - 2\left[m_C g + 4m_C a_A + \frac{2m_C(F_B)_{\text{ext}}}{m_B}\right] + \frac{1}{2} \times m_D\left[g - a_A - \frac{(F_B)_{\text{ext}}}{2m_B}\right] = m_A a_A$$

or $$a_A = \frac{m_A g - 2m_C g - \frac{4m_C(F_B)_{\text{ext}}}{m_B} - \frac{m_D(F_B)_{\text{ext}}}{4m_B} + \frac{m_D g}{2}}{m_A + 8m_C + \frac{m_D}{2}} = -2.2835 \text{ m/s}^2$$

Then $$a_C = -4(-2.2835 \text{ m/s}^2) - \frac{2(50)}{9} = -1.9771 \text{ m/s}^2$$

$$a_D = -2.2835 \text{ m/s}^2 + \frac{(50)}{2(9)} = 0.4943 \text{ m/s}^2$$

Note: We have uniformly accelerated motion, so that

$$v = 0 + at$$

(*a*) We have $$\mathbf{v}_{D/A} = \mathbf{v}_D - \mathbf{v}_A$$

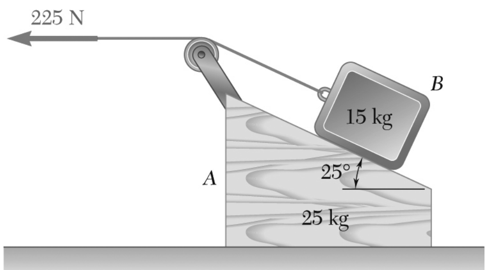
or $$\mathbf{v}_{D/A} = a_D t - a_A t = [0.4943 - (-2.2835)] \text{ m/s}^2 \times 3 \text{ s}$$

or $$\mathbf{v}_{D/A} = 8.33 \text{ m/s}\downarrow \blacktriangleleft$$

(*b*) And $$\mathbf{v}_{C/D} = \mathbf{v}_C = \mathbf{v}_D$$

or $$\mathbf{v}_{C/D} = a_C t - a_D t = (-1.9771 - 0.4943) \text{ m/s}^2 \times 3 \text{ s}$$

or $$\mathbf{v}_{C/D} = 7.41 \text{ m/s}\uparrow \blacktriangleleft$$



PROBLEM 12.34

The 15-kg block B is supported by the 25-kg block A and is attached to a cord to which a 225-N horizontal force is applied as shown. Neglecting friction, determine (a) the acceleration of block A, (b) the acceleration of block B relative to A.

SOLUTION

(a) First we note $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed along the inclined surface of A.

B: 

B: $\quad \nwarrow^{+} \Sigma F_x = m_B a_x: \quad P - W_B \sin 25° = m_B a_A \cos 25° + m_B a_{B/A}$

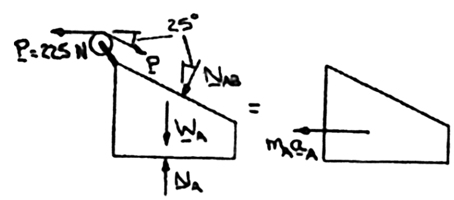
or $\quad 225 - 15g \sin 25° = 15(a_A \cos 25° + a_{B/A})$

or $\quad 15 - g \sin 25° = a_A \cos 25° + a_{B/A} \qquad (1)$

$+\nearrow \Sigma F_y = m_B a_y: \quad N_{AB} - W_B \cos 25° = -m_B a_A \sin 25°$

or $\quad N_{AB} = 15(g \cos 25° - a_A \sin 25°)$

A: $\quad \overset{+}{\longleftarrow} \Sigma F_{x'} = m_A a_A: \quad P - P\cos 25° + N_{AB} \sin 25° = m_A a_A$



or $\quad N_{AB} = [25a_A - 225(1 - \cos 25°)]/\sin 25°$

Equating the two expressions for N_{AB}

$$15(g \cos 25° - a_A \sin 25°) = \frac{25a_A - 225(1-\cos 25°)}{\sin 25°}$$

or

$$a_A = \frac{3(9.81)\cos 25° \sin 25° + 45(1 - \cos 25°)}{5 + 3\sin^2 25°}$$

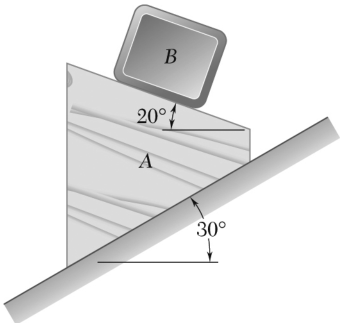
$$= 2.7979 \text{ m/s}^2$$

$\mathbf{a}_A = 2.80 \text{ m/s}^2 \longleftarrow$ ◀

(b) From Eq. (1)

$$a_{B/A} = 15 - (9.81)\sin 25° - 2.7979 \cos 25°$$

or $\quad \mathbf{a}_{B/A} = 8.32 \text{ m/s}^2 \searrow 25°$ ◀

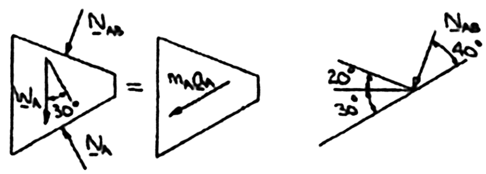


PROBLEM 12.35

Block B of mass 10-kg rests as shown on the upper surface of a 22-kg wedge A. Knowing that the system is released from rest and neglecting friction, determine (*a*) the acceleration of B, (*b*) the velocity of B relative to A at $t = 0.5$ s.

SOLUTION

A:

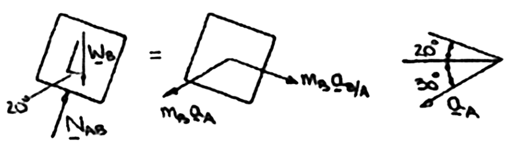


(*a*) $\quad +\swarrow \Sigma F_x = m_A a_A: \quad W_A \sin 30° + N_{AB} \cos 40° = m_A a_A$

or
$$N_{AB} = \frac{22\left(a_A - \frac{1}{2}g\right)}{\cos 40°}$$

Now we note: $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed along the top surface of A.

B:



$+\nearrow \Sigma F_{y'} = m_B a_{y'}: \quad N_{AB} - W_B \cos 20° = -m_B a_A \sin 50°$

or
$$N_{AB} = 10\,(g \cos 20° - a_A \sin 50°)$$

Equating the two expressions for N_{AB}

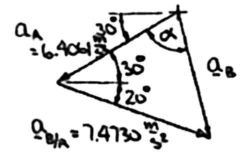
$$\frac{22\left(a_A - \frac{1}{2}g\right)}{\cos 40°} = 10(g \cos 20° - a_A \sin 50°)$$

or
$$a_A = \frac{(9.81)(1.1 + \cos 20° \cos 40°)}{2.2 + \cos 40° \sin 50°} = 6.4061 \text{ m/s}^2$$

$\overset{+}{\searrow} \Sigma F_{x'} = m_B a_{x'}: \quad W_B \sin 20° = m_B a_{B/A} - m_B a_A \cos 50°$

or
$$\begin{aligned} a_{B/A} &= g \sin 20° + a_A \cos 50° \\ &= (9.81 \sin 20° + 6.4061 \cos 50°) \text{ m/s}^2 \\ &= 7.4730 \text{ m/s}^2 \end{aligned}$$

Finally $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$



We have $a_B^2 = 6.4061^2 + 7.4730^2 - 2(6.4061 \times 7.4730)\cos 50°$

or $a_B = 5.9447 \text{ m/s}^2$

and $\dfrac{7.4730}{\sin\alpha} = \dfrac{5.9447}{\sin 50°}$

or $\alpha = 74.4°$

$\mathbf{a}_B = 5.94 \text{ m/s}^2 \measuredangle\ 75.6°$ ◀

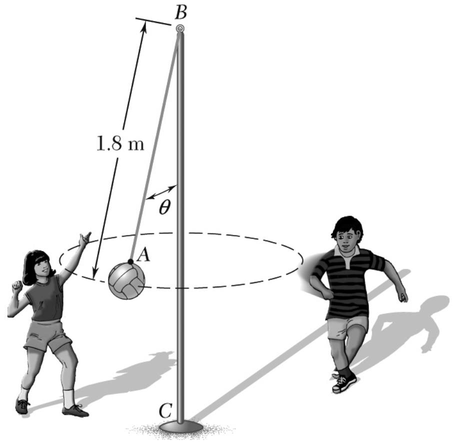
(*b*) *Note:* We have uniformly accelerated motion, so that

$$v = 0 + at$$

Now $\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = \mathbf{a}_B t - \mathbf{a}_A t = \mathbf{a}_{B/A} t$

At $t = 0.5$ s: $v_{B/A} = 7.4730 \text{ m/s}^2 \times 0.5 \text{ s}$

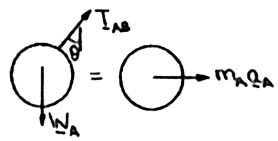
or $\mathbf{v}_{B/A} = 3.74 \text{ m/s} \measuredangle\ 20°$ ◀



PROBLEM 12.36

A 450-g tetherball A is moving along a horizontal circular path at a constant speed of 4 m/s. Determine (a) the angle θ that the cord forms with pole BC, (b) the tension in the cord.

SOLUTION



First we note $$a_A = a_n = \frac{v_A^2}{\rho}$$

where $$\rho = l_{AB}\sin\theta$$

(a) $+\uparrow \Sigma F_y = 0$: $T_{AB}\cos\theta - W_A = 0$

or $$T_{AB} = \frac{m_A g}{\cos\theta}$$

$\xrightarrow{+} \Sigma F_x = m_A a_A$: $T_{AB}\sin\theta = m_A \dfrac{v_A^2}{\rho}$

Substituting for T_{AB} and ρ

$$\frac{m_A g}{\cos\theta}\sin\theta = m_A \frac{v_A^2}{l_{AB}\sin\theta} \qquad \sin^2\theta = 1 - \cos^2\theta$$

$$1 - \cos^2\theta = \frac{(4 \text{ m/s})^2}{1.8 \text{ m} \times 9.81 \text{ m/s}^2}\cos\theta$$

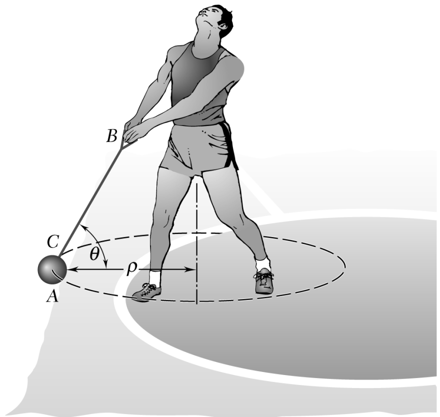
or $$\cos^2\theta + 0.906105\cos\theta - 1 = 0$$

Solving $$\cos\theta = 0.64479$$

or $$\theta = 49.9° \blacktriangleleft$$

(b) From above $$T_{AB} = \frac{m_A g}{\cos\theta} = \frac{0.450 \text{ kg} \times 9.81 \text{ m/s}^2}{0.64479}$$

or $$T_{AB} = 6.85 \text{ N} \blacktriangleleft$$

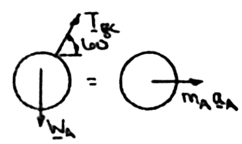


PROBLEM 12.37

During a hammer thrower's practice swings, the 7.1-kg head A of the hammer revolves at a constant speed v in a horizontal circle as shown. If $\rho = 0.93$ m and $\theta = 60°$, determine (*a*) the tension in wire BC, (*b*) the speed of the hammer's head.

SOLUTION

First we note $$a_A = a_n = \frac{v_A^2}{\rho}$$



(*a*) $+\uparrow \Sigma F_y = 0$: $T_{BC} \sin 60° - W_A = 0$

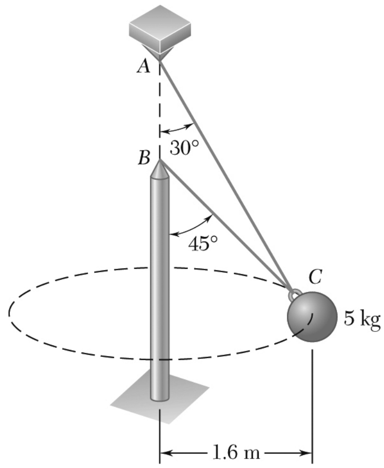
or $$T_{BC} = \frac{7.1 \text{ kg} \times 9.81 \text{ m/s}^2}{\sin 60°}$$
$$= 80.426 \text{ N}$$

$$T_{BC} = 80.4 \text{ N} \blacktriangleleft$$

(*b*) $\xrightarrow{+} \Sigma F_x = m_A a_A$: $T_{BC} \cos 60° = m_A \dfrac{v_A^2}{\rho}$

or $$v_A^2 = \frac{(80.426 \text{ N}) \cos 60° \times 0.93 \text{ m}}{7.1 \text{ kg}}$$

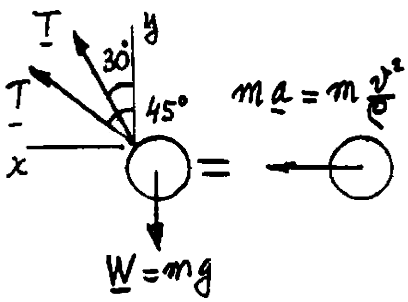
or $$v_A = 2.30 \text{ m/s} \blacktriangleleft$$



PROBLEM 12.38

A single wire ACB passes through a ring at C attached to a sphere which revolves at a constant speed v in the horizontal circle shown. Knowing that the tension is the same in both portions of the wire, determine the speed v.

SOLUTION



$$\overset{+}{\leftarrow} \Sigma F_x = ma: \quad T(\sin 30° + \sin 45°) = \frac{mv^2}{\rho} \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad T(\cos 30° + \cos 45°) - mg = 0$$

$$T(\cos 30° + \cos 45°) = mg \tag{2}$$

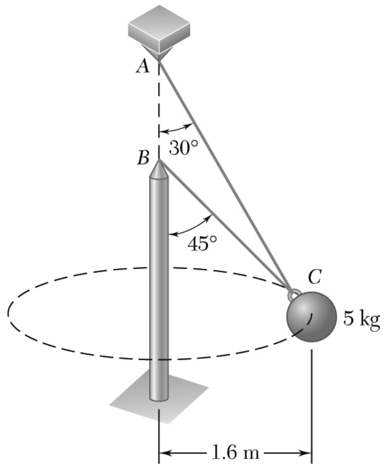
Divide Eq. (1) by Eq. (2):

$$\frac{\sin 30° + \sin 45°}{\cos 30° + \cos 45°} = \frac{v^2}{\rho g}$$

$$v^2 = 0.76733\rho g = 0.76733\,(1.6\text{ m})(9.81\text{ m/s}^2) = 12.044\text{ m}^2/\text{s}^2$$

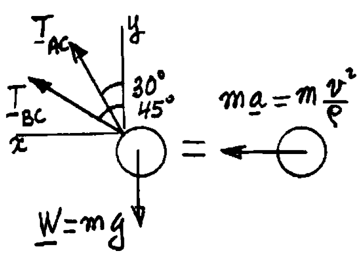
$v = 3.47$ m/s ◀

PROBLEM 12.39



Two wires AC and BC are tied at C to a sphere which revolves at a constant speed v in the horizontal circle shown. Determine the range of values of v for which both wires remain taut.

SOLUTION



$$\stackrel{+}{\leftarrow} \Sigma F_x = ma: \quad T_{AC} \sin 30° + T_{BC} \sin 45° = \frac{mv^2}{\rho} \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad T_{AC} \cos 30° + T_{BC} \cos 45° - mg = 0$$

$$T_{AC} \cos 30° + T_{BC} \cos 45° = mg \tag{2}$$

Divide Eq (1) by Eq. (2):

$$\frac{T_{AC} \sin 30° + T_{BC} \sin 45°}{T_{AC} \cos 30° + T_{BC} \cos 45°} = \frac{v^2}{\rho g} \tag{3}$$

When AC is slack, $T_{AC} = 0$.

Eq. (3) yields $\quad v_1^2 = \rho g \tan 45° = (1.6 \text{ m})\,(9.81 \text{ m/s}^2) \tan 45° = 15.696 \text{ m}^2/\text{s}^2$

Wire AC will remain taut if $v \le v_1$, that is, if $\quad v_1 = 3.96$ m/s $\quad v \le 3.96$ m/s ◁

When BC is slack, $T_{BC} = 0$.

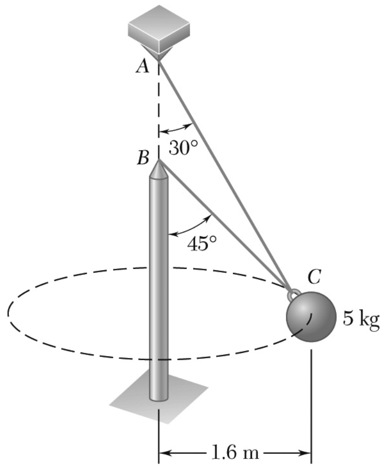
Eq. (3) yields $\quad v_2^2 = \rho g \tan 30° = (1.6 \text{ m})(9.81 \text{ m/s}^2) \tan 30° = 9.0621 \text{ m}^2/\text{s}^2$

Wire BC will remain taut if $v \ge v_2$, that is, if $\quad v_2 = 3.01$ m/s $\quad v \ge 3.01$ m/s ◀

Combining the results obtained, we conclude that both wires remain taut for

$3.01 \text{ m/s} \le v \le 3.96 \text{ m/s}$ ◀

PROBLEM 12.40



Two wires AC and BC are tied at C to a sphere which revolves at a constant speed v in the horizontal circle shown. Determine the range of the allowable values of v if both wires are to remain taut and if the tension in either of the wires is not to exceed 60 N.

SOLUTION

From the solution of Problem 12.39, we find that both wires remain taut for $3.01 \text{ m/s} \le v \le 3.96 \text{ m/s}$ ◁

To determine the values of v for which the tension in either wire will not exceed 60 N, we recall Eqs. (1) and (2) from Problem 12.39:

$$T_{AC} \sin 30° + T_{BC} \sin 45° = \frac{mv^2}{\rho} \quad (1)$$

$$T_{AC} \cos 30° + T_{BC} \cos 45° = mg \quad (2)$$

Subtract Eq. (1) from Eq. (2). Since $\sin 45° = \cos 45°$, we obtain

$$T_{AC}(\cos 30° - \sin 30°) = mg - \frac{mv^2}{\rho} \quad (3)$$

Multiply Eq. (1) by cos 30°, Eq. (2) by sin 30°, and subtract:

$$T_{BC}(\sin 45° \cos 30° - \cos 45° \sin 30°) = \frac{mv^2}{\rho} \cos 30° - mg \sin 30°$$

$$T_{BC} \sin 15° = \frac{mv^2}{\rho} \cos 30° - mg \sin 30° \quad (4)$$

Making $T_{AC} = 60$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81 \text{ m/s}^2$ in Eq. (3), we find the value v_1 of v for which

$T_{AC} = 60$ N: $\quad 60(\cos 30° - \sin 30°) = 5(9.81) - \dfrac{5v_1^2}{1.6}$

$$21.962 = 49.05 - \frac{v_1^2}{0.32} \qquad v_1^2 = 8.668, \qquad v_1 = 2.94 \text{ m/s}$$

We have $T_{AC} \le 60 \text{ N}$ for $v \ge v_1$, that is, for $v \ge 2.94 \text{ m/s}$ ◁

PROBLEM 12.40 (Continued)

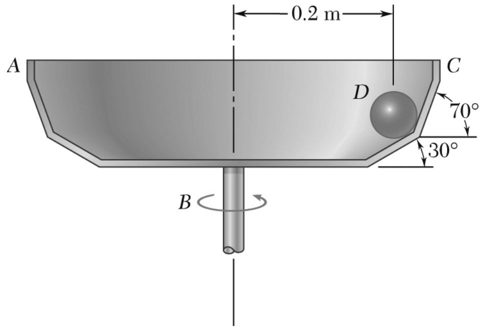
Making $T_{BC} = 60$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81$ m/s^2 in Eq. (4), we find the value v_2 of v for which

$T_{BC} = 60$ N: $$60 \sin 15° = \frac{5v_2^2}{1.6} \cos 30° - 5(9.81) \sin 30°$$

$$15.529 = 2.7063 v_2^2 - 24.523 \qquad v_2^2 = 14.80, \qquad v_2 = 3.85 \text{ m/s}$$

We have $T_{BC} \leq 60$ N for $v \leq v_2$, that is, for $v \leq 3.85$ m/s ◁

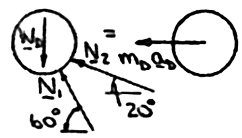
Combining the results obtained, we conclude that the range of allowable value is $3.01 \text{ m/s} \leq v \leq 3.85 \text{ m/s}$ ◀



PROBLEM 12.41

A 100-g sphere D is at rest relative to drum ABC which rotates at a constant rate. Neglecting friction, determine the range of the allowable values of the velocity v of the sphere if neither of the normal forces exerted by the sphere on the inclined surfaces of the drum is to exceed 1.1 N.

SOLUTION



First we note $a_D = a_n = \dfrac{v_D^2}{\rho}$

where $\rho = 0.2$ m

$$\overset{+}{\leftarrow} \Sigma F_x = m_D a_D: \quad N_1 \cos 60° + N_2 \cos 20° = m_D \frac{v_D^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_1 \sin 60° + N_2 \sin 20° - W_D = 0$$

or $$N_1 \sin 60° + N_2 \sin 20° = m_D g \qquad (2)$$

Case 1: N_1 is maximum.

Let $N_1 = 1.1$ N

Eq. (2) $(1.1 \text{ N}) \sin 60° + N_2 \sin 20° = (0.1 \text{ kg})\ (9.81 \text{ m/s}^2)$

or $N_2 = 0.082954$ N

$$(N_2)_{(N_1)_{\max}} < 1.1 \text{ N} \quad \text{OK}$$

Eq. (1) $$(v_D^2)_{(N_1)_{\max}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}}(1.1 \cos 60° + 0.082954 \cos 20°) \text{ N}$$

or $(v_D)_{(N_1)_{\max}} = 1.121$ m/s

Now we form $(\sin 20°) \times [\text{Eq. (1)}] - (\cos 20°) \times [\text{Eq. (2)}]$

$$N_1 \cos 60° \sin 20° - N_1 \sin 60° \cos 20° = m_D \frac{v_D^2}{\rho} \sin 20° - m_D g \cos 20°$$

or $$-N_1 \sin 40° = m_D \frac{v_D^2}{\rho} \sin 20° - m_D g \cos 20°$$

$(v_D)_{\min}$ occurs when $N_1 = (N_1)_{\max}$

$(v_D)_{\min} = 1.121$ m/s

PROBLEM 12.41 (Continued)

Case 2: N_2 is maximum.

Let $$N_2 = 1.1 \text{ N}$$

Eq. (2) $$N_1 \sin 60° + (1.1 \text{ N}) \sin 20° = (0.1 \text{ kg})(9.81 \text{ m/s}^2)$$

or $$N_1 = 0.69834 \text{ N}$$

$$(N_1)_{(N_2)_{\max}} \leq 1.1 \text{ N} \qquad \text{OK}$$

Eq. (1) $$(v_D^2)_{(N_2)_{\max}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}}(0.69834 \cos 60° + 1.1 \cos 20°) \text{ N}$$

or $$(v_D)_{(N_2)_{\max}} = 1.663 \text{ m/s}$$

Now we form $$(\sin 60°) \times [\text{Eq. (1)}] - (\cos 60°) \times [\text{Eq. (2)}]$$

$$N_2 \cos 20° \sin 60° - N_2 \sin 20° \cos 60° = m_D \frac{v_D^2}{\rho} \sin 60° - m_D g \cos 60°$$

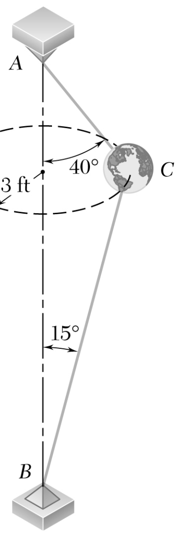
or $$N_2 \cos 40° = m_D \frac{v_D^2}{\rho} \sin 60° - m_D g \cos 60°$$

$(v_D)_{\max}$ occurs when $N_2 = (N_2)_{\max}$

$(v_D)_{\max} = 1.663$ m/s

For $N_1 \leq N_2 < 1.1$ N

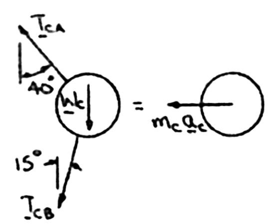
$$1.121 \text{ m/s} < v_D < 1.663 \text{ m/s} \blacktriangleleft$$



PROBLEM 12.42*

As part of an outdoor display, a 12-lb model C of the earth is attached to wires AC and BC and revolves at a constant speed v in the horizontal circle shown. Determine the range of the allowable values of v if both wires are to remain taut and if the tension in either of the wires is not to exceed 26 lb.

SOLUTION



First note $\qquad a_C = a_n = \dfrac{v_C^2}{\rho}$

where $\qquad \rho = 3 \text{ ft}$

$$\xleftarrow{+} \Sigma F_x = m_C a_C: \quad T_{CA} \sin 40° + T_{CB} \sin 15° = \frac{W_C}{g} \frac{v_C^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad T_{CA} \cos 40° - T_{CB} \cos 15° - W_C = 0 \qquad (2)$$

Note that Eq. (2) implies that

(a) when $\qquad T_{CB} = (T_{CB})_{\max}, \quad T_{CA} = (T_{CA})_{\max}$

(b) when $\qquad T_{CB} = (T_{CB})_{\min}, \quad T_{CA} = (T_{CA})_{\min}$

Case 1: T_{CA} is maximum.

Let $\qquad T_{CA} = 26 \text{ lb}$

Eq. (2) $\qquad (26 \text{ lb}) \cos 40° - T_{CB} \cos 15° - (12 \text{ lb}) = 0$

or $\qquad T_{CB} = 8.1964 \text{ lb}$

$$(T_{CB})_{(T_{CA})_{\max}} < 26 \text{ lb} \qquad \text{OK}$$

$$[(T_{CB})_{\max} = 8.1964 \text{ lb}]$$

PROBLEM 12.42* (Continued)

Eq. (1)

$$(v_C^2)_{(T_{CA})_{\max}} = \frac{(32.2 \text{ ft/s}^2)(3 \text{ ft})}{12 \text{ lb}}(26\sin 40° + 8.1964\sin 15°) \text{ lb}$$

or $$(v_C)_{(T_{CA})_{\max}} = 12.31 \text{ ft/s}$$

Now we form $$(\cos 15°)(\text{Eq. } 1) + (\sin 15°)(\text{Eq. } 2)$$

$$T_{CA}\sin 40°\cos 15° + T_{CA}\cos 40°\sin 15° = \frac{W_C}{g}\frac{v_C^2}{\rho}\cos 15° + W_C\sin 15°$$

or $$T_{CA}\sin 55° = \frac{W_C}{g}\frac{v_C^2}{\rho}\cos 15° + W_C\sin 15° \qquad (3)$$

$(v_c)_{\max}$ occurs when $T_{CA} = (T_{CA})_{\max}$

$(v_C)_{\max} = 12.31$ ft/s

Case 2: T_{CA} is minimum.

Because $(T_{CA})_{\min}$ occurs when $T_{CB} = (T_{CB})_{\min}$,

let $T_{CB} = 0$ (note that wire BC will not be taut).

Eq. (2) $$T_{CA}\cos 40° - (12 \text{ lb}) = 0$$

or $$T_{CA} = 15.6649 \text{ lb}, \; 26 \text{ lb} \quad \text{OK}$$

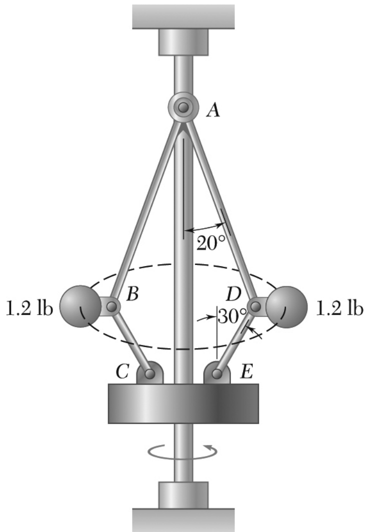
Note: Eq. (3) implies that when $T_{CA} = (T_{CA})_{\min}$, $v_C = (v_C)_{\min}$. Then

Eq. (1) $$(v_C^2)_{\min} = \frac{(32.2 \text{ ft/s}^2)(3 \text{ ft})}{12 \text{ lb}}(15.6649 \text{ lb})\sin 40°$$

or $$(v_C)_{\min} = 9.00 \text{ ft/s}$$

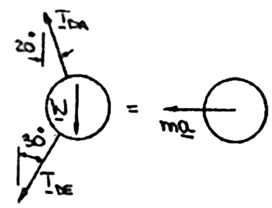
$0 < T_{CA} \leq T_{CB} < 6$ lb when $\qquad 9.00 \text{ ft/s} < v_C < 12.31 \text{ ft/s}$ ◀

PROBLEM 12.43*



The 1.2-lb flyballs of a centrifugal governor revolve at a constant speed v in the horizontal circle of 6-in. radius shown. Neglecting the weights of links AB, BC, AD, and DE and requiring that the links support only tensile forces, determine the range of the allowable values of v so that the magnitudes of the forces in the links do not exceed 17 lb.

SOLUTION



First note $$a = a_n = \frac{v^2}{\rho}$$

where $$\rho = 0.5 \text{ ft}$$

$$\overset{+}{\longleftarrow} \Sigma F_x = ma: \quad T_{DA} \sin 20° + T_{DE} \sin 30° = \frac{W}{g}\frac{v^2}{\rho} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad T_{DA} \cos 20° - T_{DE} \cos 30° - W = 0 \qquad (2)$$

Note that Eq. (2) implies that

(*a*) when $T_{DE} = (T_{DE})_{\max}, \quad T_{DA} = (T_{DA})_{\max}$

(*b*) when $T_{DE} = (T_{DE})_{\min}, \quad T_{DA} = (T_{DA})_{\min}$

Case 1: T_{DA} is maximum.

Let $T_{DA} = 17 \text{ lb}$

Eq. (2) $(17 \text{ lb}) \cos 20° - T_{DE} \cos 30° - (1.2 \text{ lb}) = 0$

or $T_{DE} = 17.06 \text{ lb}$ unacceptable (> 17 lb)

Now let $T_{DE} = 17 \text{ lb}$

Eq. (2) $T_{DA} \cos 20° - (17 \text{ lb}) \cos 30° - (1.2 \text{ lb}) = 0$

or $T_{DA} = 16.9443 \text{ lb}$ OK (≤ 17 lb)

$$(T_{DA})_{\max} = 16.9443 \text{ lb}$$
$$(T_{DE})_{\max} = 17 \text{ lb}$$

Eq. (1) $$(v^2)_{(T_{DA})_{\max}} = \frac{(32.2 \text{ ft/s}^2)(0.5 \text{ ft})}{1.2 \text{ lb}}(16.9443 \sin 20° + 17 \sin 30°) \text{ lb}$$

or $$v_{(T_{DA})_{\max}} = 13.85 \text{ ft/s}$$

Now form $$(\cos 30°) \times [\text{Eq. (1)}] + (\sin 30°) \times [\text{Eq. (2)}]$$

$$T_{DA} \sin 20° \cos 30° + T_{DA} \cos 20° \sin 30° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W \sin 30°$$

or $$T_{DA} \sin 50° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W \sin 30° \qquad (3)$$

$v_{\max}$ occurs when $T_{DA} = (T_{DA})_{\max}$

$v_{\max} = 13.85$ ft/s

Case 2: T_{DA} is minimum.

Because $(T_{DA})_{\min}$ occurs when $T_{DE} = (T_{DE})_{\min}$,

let $T_{DE} = 0$.

Eq. (2) $$T_{DA} \cos 20° - (1.2 \text{ lb}) = 0$$

or $$T_{DA} = 1.27701 \text{ lb}, \ 17 \text{ lb} \qquad \text{OK}$$

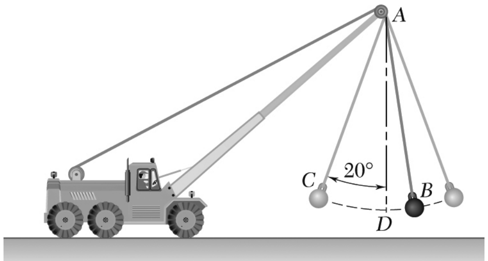
Note: Eq. (3) implies that when $T_{DA} = (T_{DA})_{\min}$, $v = v_{\min}$. Then

Eq. (1) $$(v^2)_{\min} = \frac{(32.2 \text{ ft/s}^2)(0.5 \text{ ft})}{1.2 \text{ lb}}(1.27701 \text{ lb}) \sin 20°$$

or $$v_{\min} = 2.42 \text{ ft/s}$$

$0 < T_{AB}, T_{BC}, T_{AD}, T_{DE} < 17$ lb

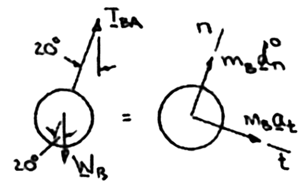
when $$2.42 \text{ ft/s} < v < 13.85 \text{ ft/s} \blacktriangleleft$$



PROBLEM 12.44

A 130-lb wrecking ball B is attached to a 45-ft-long steel cable AB and swings in the vertical arc shown. Determine the tension in the cable (*a*) at the top C of the swing, (*b*) at the bottom D of the swing, where the speed of B is 13.2 ft/s.

SOLUTION



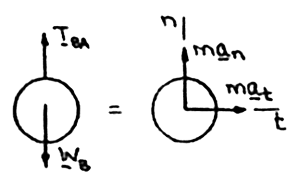
(*a*) At C, the top of the swing, $v_B = 0$; thus

$$a_n = \frac{v_B^2}{L_{AB}} = 0$$

$$+\nearrow \Sigma F_n = 0: \quad T_{BA} - W_B \cos 20° = 0$$

or $$T_{BA} = (130 \text{ lb}) \times \cos 20°$$

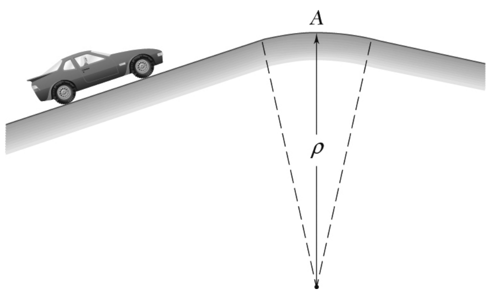
or $$T_{BA} = 122.2 \text{ lb} \blacktriangleleft$$



(*b*) $$+\uparrow \Sigma F_n = ma_n: \quad T_{BA} - W_B = m_B \frac{(v_B)_D^2}{L_{AB}}$$

or $$T_{BA} = (130 \text{ lb}) + \left[\left(\frac{130 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{(13.2 \text{ ft/s})^2}{45 \text{ ft}}\right)\right]$$

or $$T_{BA} = 145.6 \text{ lb} \blacktriangleleft$$

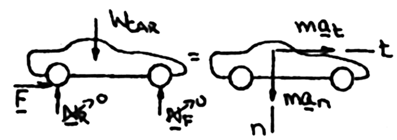


PROBLEM 12.45

During a high-speed chase, a 2400-lb sports car traveling at a speed of 100 mi/h just loses contact with the road as it reaches the crest A of a hill. (*a*) Determine the radius of curvature ρ of the vertical profile of the road at A. (*b*) Using the value of ρ found in part *a*, determine the force exerted on a 160-lb driver by the seat of his 3100-lb car as the car, traveling at a constant speed of 50 mi/h, passes through A.

SOLUTION

(*a*) *Note:* $100 \text{ mi/h} = 146.667 \text{ ft/s}$

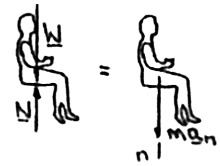


$$+\!\downarrow \Sigma F_n = ma_n: \quad W_{\text{car}} = \frac{W_{\text{car}}}{g}\frac{v_A^2}{\rho}$$

or $$\rho = \frac{(146.667 \text{ ft/s})^2}{32.2 \text{ ft/s}^2} = 668.05 \text{ ft}$$

or $$\rho = 668 \text{ ft} \blacktriangleleft$$

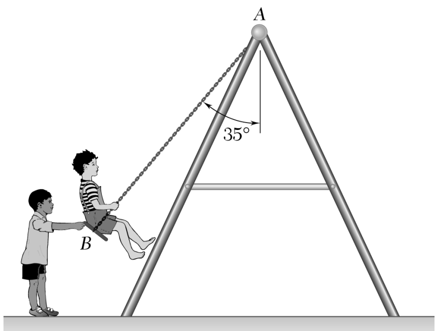
(*b*) *Note:* v is constant $\Rightarrow a_t = 0$; $50 \text{ mi/h} = 73.333 \text{ ft/s}$



$$+\!\downarrow \Sigma F_n = ma_n: \quad W - N = \frac{W}{g}\frac{v_A^2}{\rho}$$

or $$N = (160 \text{ lb})\left[1 - \frac{(73.333 \text{ ft/s})^2}{(32.2 \text{ ft/s}^2)(668.05 \text{ ft})}\right]$$

or $$\mathbf{N} = 120.0 \text{ lb} \uparrow \blacktriangleleft$$



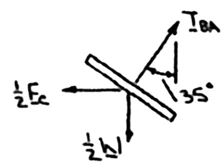
PROBLEM 12.46

A child having a mass of 22 kg sits on a swing and is held in the position shown by a second child. Neglecting the mass of the swing, determine the tension in rope *AB* (*a*) while the second child holds the swing with his arms outstretched horizontally, (*b*) immediately after the swing is released.

SOLUTION

Note: The factors of "$\frac{1}{2}$" are included in the following free-body diagrams because there are two ropes and only one is considered.

(*a*) For the swing at rest

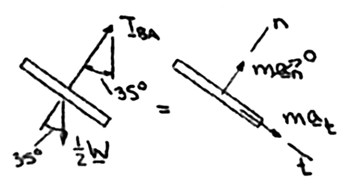


$$\Sigma F_y = 0: \quad T_{BA}\cos 35° - \frac{1}{2}W = 0$$

or $$T_{BA} = \frac{22 \text{ kg} \times 9.81 \text{ m/s}^2}{2\cos 35°}$$

or $$T_{BA} = 131.7 \text{ N} ◀$$

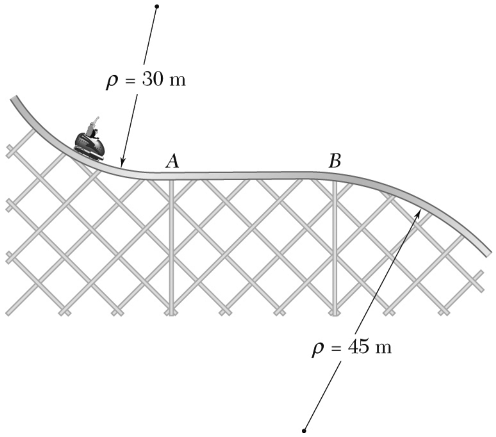
(*b*) At $t = 0$, $v = 0$, so that $$a_n = \frac{v^2}{\rho} = 0$$



$$+\nearrow \Sigma F_n = 0: \quad T_{BA} - \frac{1}{2}W\cos 35° = 0$$

or $$T_{BA} = \frac{1}{2}(22 \text{ kg})(9.81 \text{ m/s}^2)\cos 35°$$

or $$T_{BA} = 88.4 \text{ N} ◀$$



PROBLEM 12.47

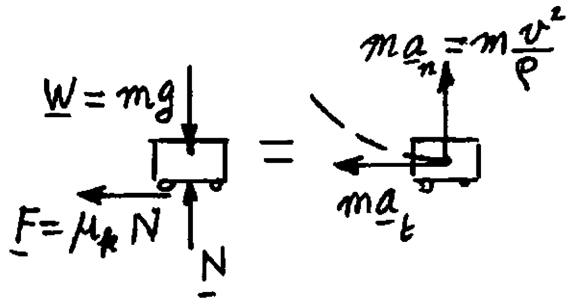
The roller-coaster track shown is contained in a vertical plane. The portion of track between A and B is straight and horizontal, while the portions to the left of A and to the right of B have radii of curvature as indicated. A car is traveling at a speed of 72 km/h when the brakes are suddenly applied, causing the wheels of the car to slide on the track ($\mu_k = 0.20$). Determine the initial deceleration of the car if the brakes are applied as the car (a) has almost reached A, (b) is traveling between A and B, (c) has just passed B.

SOLUTION

(a) $\quad +\uparrow \Sigma F_n = ma_n: \quad N - mg = m\dfrac{v^2}{\rho}$

$$N = m\left(g + \frac{v^2}{\rho}\right)$$

$$F = \mu_k N = \mu_k m\left(g + \frac{v^2}{\rho}\right)$$



$$\overset{+}{\longleftarrow} \Sigma F_t = ma_t: \quad F = ma_t$$

$$a_t = \frac{F}{m} = \mu_k\left(g + \frac{v^2}{\rho}\right)$$

Given data: $\quad \mu_k = 0.20, \; v = 72 \text{ km/h} = 20 \text{ m/s}$

$$g = 9.81 \text{ m/s}^2, \quad \rho = 30 \text{ m}$$

$$a_t = 0.20\left[9.81 + \frac{(20)^2}{30}\right]$$

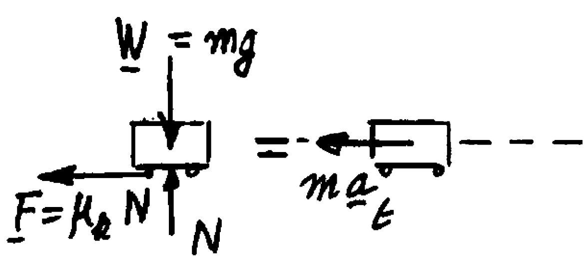
$a_t = 4.63 \text{ m/s}^2$ ◀

(b) $\quad a_n = 0 \quad +\uparrow \Sigma F_n = ma_n = 0: \quad N - mg = 0$

$$N = mg$$

$$F = \mu_k N = \mu_k mg$$

$$\overset{+}{\longleftarrow} \Sigma F_t = ma_t: \quad F = ma_t$$



$$a_t = \frac{F}{m} = \mu_k g = 0.20(9.81)$$

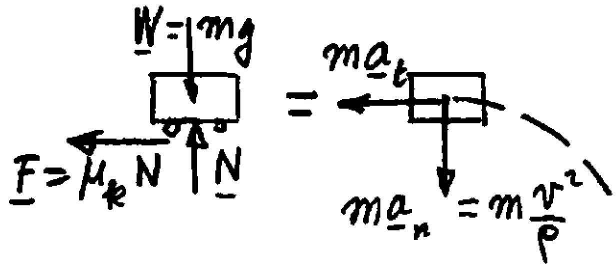
$a_t = 1.962 \text{ m/s}^2$ ◀

PROBLEM 12.47 (Continued)

(c) $+\downarrow \Sigma F_n = ma_n: \quad mg - N = \dfrac{mv^2}{\rho}$

$$N = m\left(g - \frac{v^2}{\rho}\right)$$

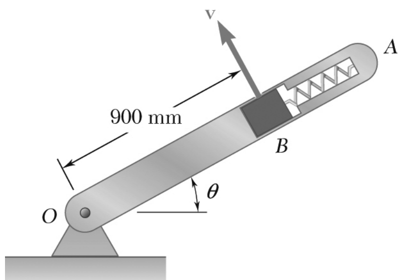
$$F = \mu_k N = \mu_k m\left(g - \frac{v^2}{\rho}\right)$$



$\overset{+}{\leftarrow} \Sigma F_t = ma_t: \quad F = ma_t$

$$a_t = \frac{F}{m} = \mu_k\left(g - \frac{v^2}{\rho}\right) = 0.20\left[9.81 - \frac{(20)^2}{45}\right]$$

$a_t = 0.1842 \text{ m/s}^2$ ◀

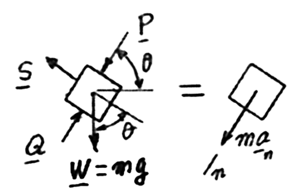


PROBLEM 12.48

A 250-g block fits inside a small cavity cut in arm OA, which rotates in the vertical plane at a constant rate such that $v = 3$ m/s. Knowing that the spring exerts on block B a force of magnitude $P = 1.5$ N and neglecting the effect of friction, determine the range of values of θ for which block B is in contact with the face of the cavity closest to the axis of rotation O.

SOLUTION

$$+\swarrow \Sigma F_n = ma_n: \quad P + mg\sin\theta - Q = m\frac{v^2}{\rho}$$



To have contact with the specified surface, we need $Q \geq 0$,

or $$Q = P + mg\sin\theta - \frac{mv^2}{\rho} > 0$$

$$\sin\theta > \frac{1}{g}\left(\frac{v^2}{\rho} - \frac{P}{m}\right) \qquad (1)$$

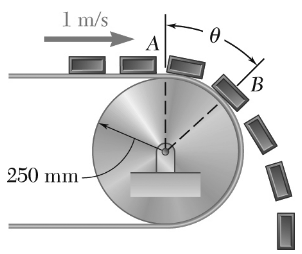
Data: $\quad m = 0.250$ kg, $\quad v = 3$ m/s, $\quad P = 1.5$ N, $\quad \rho = 0.9$ m

Substituting into (1):

$$\sin\theta > \frac{1}{9.81}\left[\frac{(3)^2}{0.9} - \frac{1.5}{0.25}\right]$$

$$\sin\theta > 0.40775$$

$24.1° < \theta < 155.9°$ ◀



PROBLEM 12.49

A series of small packages, each with a mass of 0.5 kg, are discharged from a conveyor belt as shown. Knowing that the coefficient of static friction between each package and the conveyor belt is 0.4, determine (*a*) the force exerted by the belt on a package just after it has passed Point *A*, (*b*) the angle θ defining the Point *B* where the packages first *slip* relative to the belt.

SOLUTION

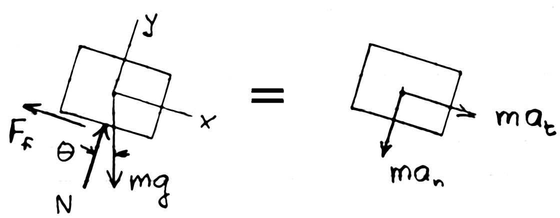
Assume package does not slip.

$$a_t = 0, \quad F_f \le \mu_s N$$

On the curved portion of the belt

$$a_n = \frac{v^2}{\rho} = \frac{(1 \text{ m/s})^2}{0.250 \text{ m}} = 4 \text{ m/s}^2$$

For any angle θ



$$+\nearrow \Sigma F_y = ma_y: \quad N - mg\cos\theta = -ma_n = -\frac{mv^2}{\rho}$$

$$N = mg\cos\theta - \frac{mv^2}{\rho} \tag{1}$$

$$+\searrow \Sigma F_x = ma_x: \quad -F_f + mg\sin\theta = ma_t = 0$$

$$F_f = mg\sin\theta \tag{2}$$

(*a*) At Point *A*, $\theta = 0°$

$$N = (0.5)(9.81)(1.000) - (0.5)(4)$$

$N = 2.905$ N ◀

(*b*) At Point *B*, $F_f = \mu_s N$

$$mg\sin\theta = \mu_s(mg\cos\theta - ma_n)$$

$$\sin\theta = \mu_s\left(\cos\theta - \frac{a_n}{g}\right) = 0.40\left[\cos\theta - \frac{4}{9.81}\right]$$

PROBLEM 12.49 (Continued)

Squaring and using trigonometic identities,

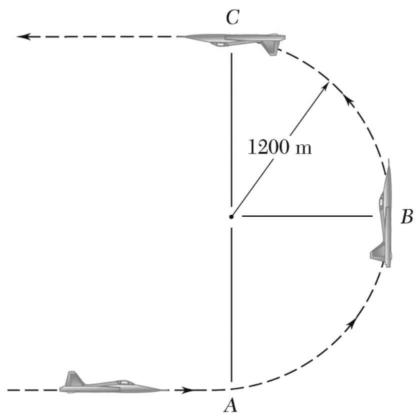
$$1-\cos^2\theta = 0.16\cos^2\theta - 0.130479\cos\theta + 0.026601$$

$$1.16\cos^2\theta - 0.130479\cos\theta - 0.97340 = 0$$

$$\cos\theta = 0.97402 \qquad \theta = 13.09° \blacktriangleleft$$

Check that package does not separate from the belt.

$$N = \frac{F_f}{\mu_s} = \frac{mg\sin\theta}{\mu_s} \qquad N > 0.$$



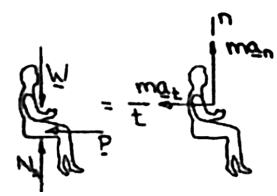
PROBLEM 12.50

A 54-kg pilot flies a jet trainer in a half vertical loop of 1200-m radius so that the speed of the trainer decreases at a constant rate. Knowing that the pilot's apparent weights at Points A and C are 1680 N and 350 N, respectively, determine the force exerted on her by the seat of the trainer when the trainer is at Point B.

SOLUTION

First we note that the pilot's apparent weight is equal to the vertical force that she exerts on the seat of the jet trainer.

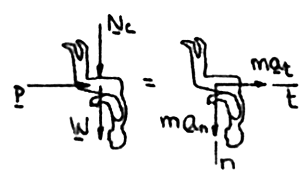
At A: $+\uparrow \Sigma F_n = ma_n$: $N_A - W = m\dfrac{v_A^2}{\rho}$



or $$v_A^2 = (1200 \text{ m})\left(\frac{1680 \text{ N}}{54 \text{ kg}} - 9.81 \text{ m/s}^2\right)$$

$$= 25{,}561.3 \text{ m}^2/\text{s}^2$$

At C: $+\downarrow \Sigma F_n = ma_n$: $N_C + W = m\dfrac{v_C^2}{\rho}$



or $$v_C^2 = (1200 \text{ m})\left(\frac{350 \text{ N}}{54 \text{ kg}} + 9.81 \text{ m/s}^2\right)$$

$$= 19{,}549.8 \text{ m}^2/\text{s}^2$$

Since $a_t = \text{constant}$, we have from A to C

$$v_C^2 = v_A^2 + 2a_t\ \Delta s_{AC}$$

or $$19{,}549.8 \text{ m}^2/\text{s}^2 = 25{,}561.3 \text{ m}^2/\text{s}^2 + 2a_t(\pi \times 1200 \text{ m})$$

or $$a_t = -0.79730 \text{ m/s}^2$$

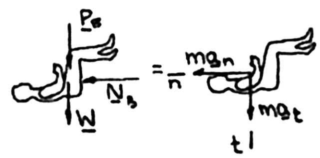
Then from A to B

$$v_B^2 = v_A^2 + 2a_t\ \Delta s_{AB}$$

$$= 25{,}561.3 \text{ m}^2/\text{s}^2 + 2(-0.79730 \text{ m/s}^2)\left(\frac{\pi}{2} \times 1200 \text{ m}\right)$$

$$= 22{,}555 \text{ m}^2/\text{s}^2$$

PROBLEM 12.50 (Continued)



At B: $\quad \overset{+}{\longleftarrow} \Sigma F_n = ma_n: \quad N_B = m\dfrac{v_B^2}{\rho}$

or $$N_B = 54\ \text{kg}\,\frac{22{,}555\ \text{m}^2/\text{s}^2}{1200\ \text{m}}$$

or $$\mathbf{N}_B = 1014.98\ \text{N} \longleftarrow$$

$$+\!\downarrow \Sigma F_t = ma_t: \quad W + P_B = m\,|a_t|$$

or $$P_B = (54\ \text{kg})(0.79730 - 9.81)\ \text{m/s}^2$$

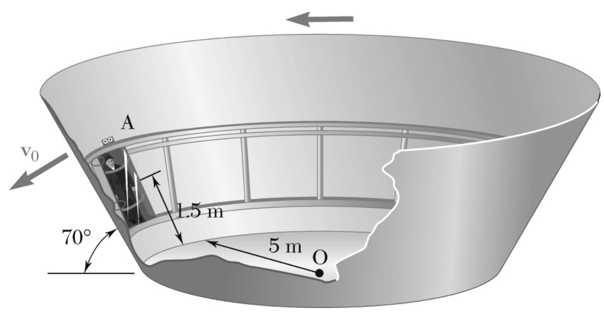
or $$\mathbf{P}_B = 486.69\ \text{N}\uparrow$$

Finally, $$(F_{\text{pilot}})_B = \sqrt{N_B^2 + P_B^2} = \sqrt{(1014.98)^2 + (486.69)^2}$$
$$= 1126\ \text{N}$$

or $$(\mathbf{F}_{\text{pilot}})_B = 1126\ \text{N} \measuredangle\ 25.6° \blacktriangleleft$$

PROBLEM 12.51

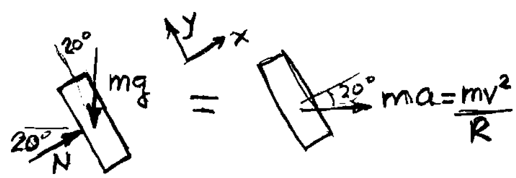
A carnival ride is designed to allow the general public to experience high acceleration motion. The ride rotates about Point O in a horizontal circle such that the rider has a speed v_0. The rider reclines on a platform A which rides on rollers such that friction is negligible. A mechanical stop prevents the platform from rolling down the incline. Determine (*a*) the speed v_0 at which the platform A begins to roll upwards, (*b*) the normal force experienced by an 80-kg rider at this speed.



SOLUTION

Radius of circle: $R = 5 + 1.5\cos 70° = 5.513$ m

$\Sigma \mathbf{F} = m\mathbf{a}$:



Components up the incline, ⦣70°:

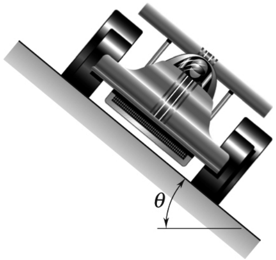
$$-m_A g \cos 20° = -\frac{mv_0^2}{R}\sin 20°$$

(*a*) Speed v_0: $v_0 = \left[\frac{g\,R}{\tan 20°}\right]^{\frac{1}{2}} = \left[\frac{(9.81 \text{ m/s})(5.513 \text{ m})}{\tan 20°}\right]^{\frac{1}{2}} = 12.1898$ m/s $\qquad v_0 = 12.19$ m/s ◀

Components normal to the incline, ⦣ 20°.

$$N - mg\sin 20° = \frac{mv_0^2}{R}\cos 20°.$$

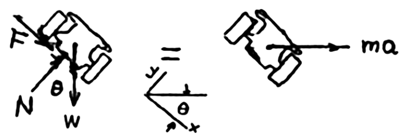
(*b*) Normal force: $N = (80)(9.81)\sin 20° + \dfrac{80(12.1898)^2}{5.513}\cos 20° = 2294$ N $\qquad N = 2290$ N ◀



PROBLEM 12.52

A curve in a speed track has a radius of 1000 ft and a rated speed of 120 mi/h. (See Sample Problem 12.6 for the definition of rated speed). Knowing that a racing car starts skidding on the curve when traveling at a speed of 180 mi/h, determine (*a*) the banking angle θ, (*b*) the coefficient of static friction between the tires and the track under the prevailing conditions, (*c*) the minimum speed at which the same car could negotiate that curve.

SOLUTION



Weight $\quad W = mg$

Acceleration $\quad a = \dfrac{v^2}{\rho}$

$$\Sigma F_x = ma_x: \quad F + W \sin\theta = ma \cos\theta$$

$$F = \frac{mv^2}{\rho}\cos\theta - mg\sin\theta \qquad (1)$$

$$\Sigma F_y = ma_y: \quad N - W\cos\theta = ma\sin\theta$$

$$N = \frac{mv^2}{\rho}\sin\theta + mg\cos\theta \qquad (2)$$

(*a*) Banking angle. Rated speed $v = 120 \text{ mi/h} = 176 \text{ ft/s}$. $F = 0$ at rated speed.

$$0 = \frac{mv^2}{\rho}\cos\theta - mg\sin\theta$$

$$\tan\theta = \frac{v^2}{\rho g} = \frac{(176)^2}{(1000)(32.2)} = 0.96199$$

$$\theta = 43.89°$$

$\theta = 43.9°$ ◀

(*b*) Slipping outward. $\quad v = 180 \text{ mi/h} = 264 \text{ ft/s}$

$$F = \mu N \quad \mu = \frac{F}{N} = \frac{v^2\cos\theta - \rho g\sin\theta}{v^2\sin\theta + \rho g\cos\theta}$$

$$\mu = \frac{(264)^2\cos 43.89° - (1000)(32.2)\sin 43.89°}{(264)^2\sin 43.89° + (1000)(32.2)\cos 43.89°}$$

$$= 0.39009$$

$\mu = 0.390$ ◀

PROBLEM 12.52 (Continued)

(*c*) Minimum speed. $F = -\mu N$

$$-\mu = \frac{v^2 \cos\theta - \rho g \sin\theta}{v^2 \sin\theta + \rho g \cos\theta}$$

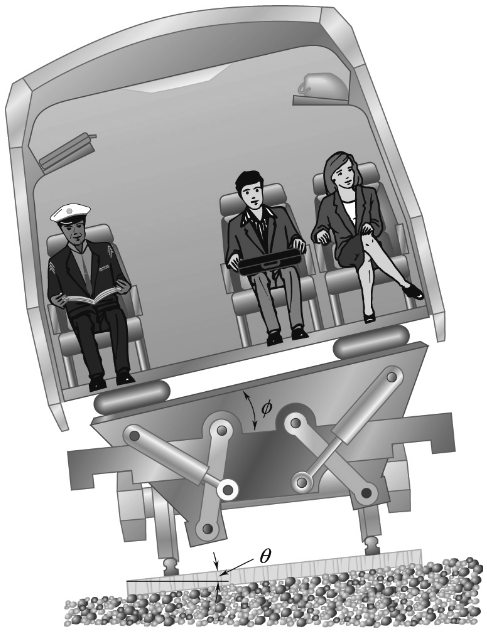
$$v^2 = \frac{\rho g\,(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu \sin\theta}$$

$$= \frac{(1000)(32.2)(\sin 43.89° - 0.39009 \cos 43.89°)}{\cos 43.89° + 0.39009 \sin 43.89°}$$

$$= 13.369 \text{ ft}^2/\text{s}^2$$

$$v = 115.62 \text{ ft/s}$$

$v = 78.8$ mi/h ◀



PROBLEM 12.53

Tilting trains, such as the *American Flyer* which will run from Washington to New York and Boston, are designed to travel safely at high speeds on curved sections of track which were built for slower, conventional trains. As it enters a curve, each car is tilted by hydraulic actuators mounted on its trucks. The tilting feature of the cars also increases passenger comfort by eliminating or greatly reducing the side force $\mathbf{F}_s$ (parallel to the floor of the car) to which passengers feel subjected. For a train traveling at 100 mi/h on a curved section of track banked through an angle $\theta = 6°$ and with a rated speed of 60 mi/h, determine (*a*) the magnitude of the side force felt by a passenger of weight W in a standard car with no tilt $(\phi = 0)$, (*b*) the required angle of tilt ϕ if the passenger is to feel no side force. (See Sample Problem 12.6 for the definition of rated speed.)

SOLUTION

Rated speed: $v_R = 60 \text{ mi/h} = 88 \text{ ft/s}, \; 100 \text{ mi/h} = 146.67 \text{ ft/s}$

From Sample Problem 12.6,

$$v_R^2 = g\rho \tan\theta$$

or

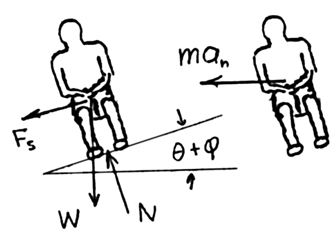
$$\rho = \frac{v_R^2}{g \tan\theta} = \frac{(88)^2}{32.2 \tan 6°} = 2288 \text{ ft}$$

Let the *x*-axis be parallel to the floor of the car.

$$+\swarrow \Sigma F_x = ma_x: \quad F_s + W\sin(\theta+\phi) = ma_n \cos(\theta+\phi)$$

$$= \frac{mv^2}{\rho}\cos(\theta+\phi)$$

(*a*) $\phi = 0.$



$$F_s = W\left[\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi)\right]$$

$$= W\left[\frac{(146.67)^2}{(32.2)(2288)}\cos 6° - \sin 6°\right]$$

$$= 0.1858W$$

$F_s = 0.1858W$ ◀

(*b*) For $F_s = 0$,

$$\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi) = 0$$

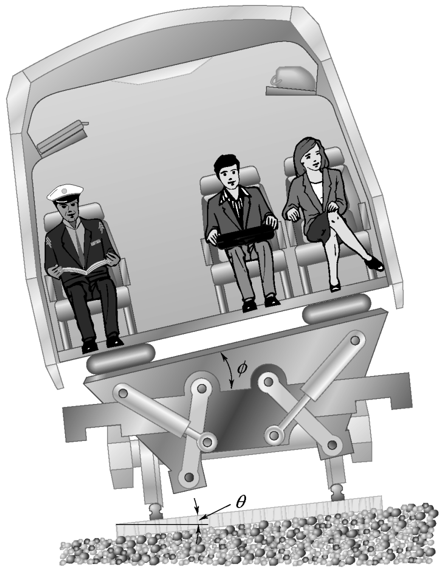
$$\tan(\theta+\phi) = \frac{v^2}{g\rho} = \frac{(146.67)^2}{(32.2)(2288)} = 0.29199$$

$$\theta+\phi = 16.28°$$

$$\phi = 16.28° - 6°$$

$\phi = 10.28°$ ◀

PROBLEM 12.54



Tests carried out with the tilting trains described in Problem 12.53 revealed that passengers feel queasy when they see through the car windows that the train is rounding a curve at high speed, yet do not feel any side force. Designers, therefore, prefer to reduce, but not eliminate, that force. For the train of Problem 12.53, determine the required angle of tilt ϕ if passengers are to feel side forces equal to 10% of their weights.

SOLUTION

Rated speed: $v_R = 60 \text{ mi/h} = 88 \text{ ft/s}, \;\; 100 \text{ mi/h} = 146.67 \text{ ft/s}$

From Sample Problem 12.6,

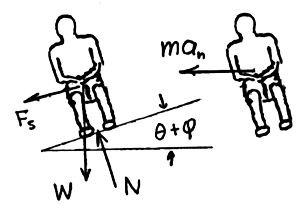
$$v_R^2 = g\rho \tan\theta$$

or $$\rho = \frac{v_R^2}{g \tan\theta} = \frac{(88)^2}{32.2 \tan 6^\circ} = 2288 \text{ ft}$$

Let the x-axis be parallel to the floor of the car.

$$+\swarrow \Sigma F_x = ma_x: \quad F_s + W\sin(\theta+\phi) = ma_n \cos(\theta+\phi)$$

$$= \frac{mv^2}{\rho}\cos(\theta+\phi)$$



Solving for F_s, $$F_s = W\left[\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi)\right]$$

Now $$\frac{v^2}{g\rho} = \frac{(146.67)^2}{(32.2)(2288)} = 0.29199 \quad \text{and} \quad F_s = 0.10W$$

So that $$0.10W = W[0.29199\cos(\theta+\phi) - \sin(\theta+\phi)]$$

Let $$u = \sin(\theta+\phi)$$

Then $$\cos(\theta+\phi) = \sqrt{1-u^2}$$

$$0.10 = 0.29199\sqrt{1-u^2} - u \quad \text{or} \quad 0.29199\sqrt{1-u^2} = 0.10 + u$$

Squaring both sides, $$0.08526(1-u^2) = 0.01 + 0.2u + u^2$$

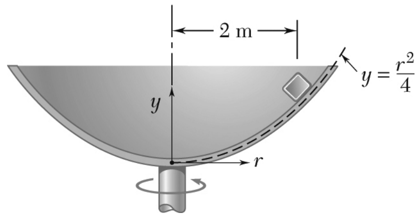
or $$1.08526u^2 + 0.2u - 0.07526 = 0$$

The positive root of the quadratic equation is $u = 0.18685$

Then, $$\theta + \phi = \sin^{-1} u = 10.77°$$

$$\phi = 10.77° - 6°$$

$\phi = 4.77°$ ◀



PROBLEM 12.55

A 3-kg block is at rest relative to a parabolic dish which rotates at a constant rate about a vertical axis. Knowing that the coefficient of static friction is 0.5 and that $r = 2$ m, determine the maximum allowable velocity v of the block.

SOLUTION

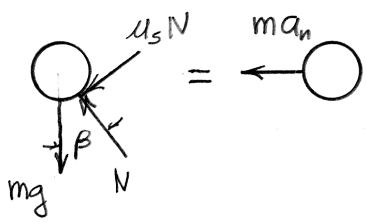
Let β be the slope angle of the dish. $\tan\beta = \frac{dy}{dr} = \frac{1}{2}r$

At $r = 2$ m, $\tan\beta = 1$ or $\beta = 45°$

Draw free body sketches of the sphere.

$$\Sigma F_y = 0: \quad N\cos\beta - \mu_S N\sin\beta - mg = 0$$

$$N = \frac{mg}{\cos\beta - \mu_S\sin\beta}$$

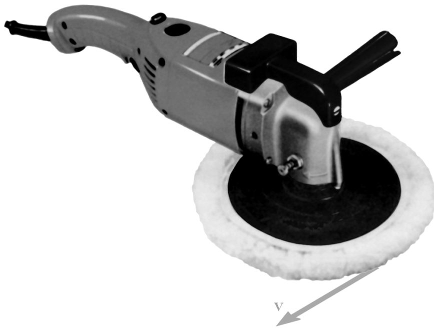


$$\xleftarrow{+} \Sigma F_n = ma_n: \quad N\sin\beta + \mu_S N\cos\beta = \frac{mv^2}{\rho}$$

$$\frac{mg(\sin\beta + \mu_S N\cos\beta)}{\cos\beta - \mu_S\sin\beta} = \frac{mv^2}{\rho}$$

$$v^2 = \rho g\frac{\sin\beta + \mu_S\cos\beta}{\cos\beta - \mu_S\sin\beta} = (2)(9.81)\frac{\sin 45° + 0.5\cos 45°}{\cos 45° - 0.5\sin 45°} = 58.86 \text{ m}^2/\text{s}^2$$

$v = 7.67$ m/s ◀



PROBLEM 12.56

Three seconds after a polisher is started from rest, small tufts of fleece from along the circumference of the 225-mm-diameter polishing pad are observed to fly free of the pad. If the polisher is started so that the fleece along the circumference undergoes a constant tangential acceleration of $4\ \text{m/s}^2$, determine (*a*) the speed v of a tuft as it leaves the pad, (*b*) the magnitude of the force required to free a tuft if the average mass of a tuft is 1.6 mg.

SOLUTION

(*a*) $a_t = \text{constant} \Rightarrow$ uniformly acceleration motion

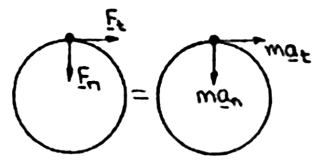
Then $\quad v = 0 + a_t t$

At $t = 3$ s: $\quad v = (4\ \text{m/s}^2)(3\ \text{s})$

or $\quad v = 12.00\ \text{m/s}$ ◀

(*b*) $\Sigma F_t = ma_t: \quad F_t = ma_t$

or
$$F_t = (1.6\times10^{-6}\,\text{kg})(4\ \text{m/s}^2) = 6.4\times10^{-6}\,\text{N}$$

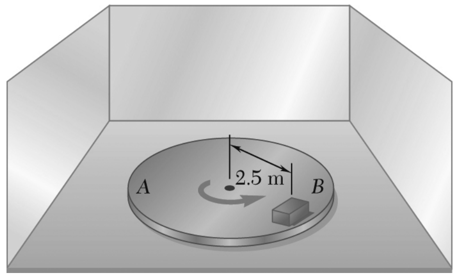


$$\Sigma F_n = ma_n: \quad F_n = m\frac{v^2}{\rho}$$

At $t = 3$ s:
$$F_n = (1.6\times10^{-6}\,\text{kg})\frac{(12\ \text{m/s})^2}{\left(\frac{0.225}{2}\,\text{m}\right)} = 2.048\times10^{-3}\,\text{N}$$

Finally,
$$F_{\text{tuft}} = \sqrt{F_t^2 + F_n^2} = \sqrt{(6.4\times10^{-6}\,\text{N})^2 + (2.048\times10^{-3}\,\text{N})^2}$$

or $\quad F_{\text{tuft}} = 2.05\times10^{-3}\,\text{N}$ ◀



PROBLEM 12.57

A turntable A is built into a stage for use in a theatrical production. It is observed during a rehearsal that a trunk B starts to slide on the turntable 10 s after the turntable begins to rotate. Knowing that the trunk undergoes a constant tangential acceleration of $0.24\ \text{m/s}^2$, determine the coefficient of static friction between the trunk and the turntable.

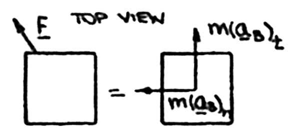
SOLUTION

First we note that $(a_B)_t = \text{constant}$ implies uniformly accelerated motion.

$$v_B = 0 + (a_B)_t t$$

At $t = 10$ s: $$v_B = (0.24\ \text{m/s}^2)(10\ \text{s}) = 2.4\ \text{m/s}$$

In the plane of the turntable

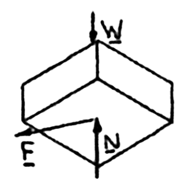


$$\Sigma \mathbf{F} = m_B \mathbf{a}_B: \quad \mathbf{F} = m_B(\mathbf{a}_B)_t + m_B(\mathbf{a}_B)_n$$

Then $$F = m_B\sqrt{(a_B)_t^2 + (a_B)_n^2}$$

$$= m_B\sqrt{(a_B)_t^2 + \left(\frac{v_B^2}{\rho}\right)^2}$$

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0$$



or $$N = m_B g$$

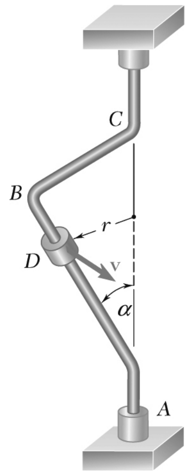
At $t = 10$ s: $$F = \mu_s N = \mu_s m_B g$$

Then $$\mu_s m_B g = m_B\sqrt{(a_B)_t^2 + \left(\frac{v_B^2}{\rho}\right)^2}$$

or $$\mu_s = \frac{1}{9.81\ \text{m/s}^2}\left\{(0.24\ \text{m/s}^2)^2 + \left[\frac{(2.4\ \text{m/s})^2}{2.5\ \text{m}}\right]^2\right\}^{1/2}$$

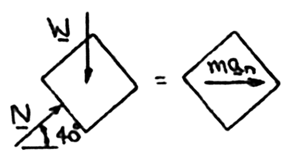
or $$\mu_s = 0.236 \blacktriangleleft$$

PROBLEM 12.58



A small, 300-g collar D can slide on portion AB of a rod which is bent as shown. Knowing that $\alpha = 40°$ and that the rod rotates about the vertical AC at a constant rate of 5 rad/s, determine the value of r for which the collar will not slide on the rod if the effect of friction between the rod and the collar is neglected.

SOLUTION



First note $$v_D = r\dot{\theta}_{ABC}$$

$$+\uparrow \Sigma F_y = 0: \quad N \sin 40° - W = 0$$

or $$N = \frac{mg}{\sin 40°}$$

$$\xrightarrow{+} \Sigma F_n = ma_n: \quad N \cos 40° = m\frac{v_D^2}{r}$$

or $$\frac{mg}{\sin 40°} \cos 40° = m\frac{(r\dot{\theta}_{ABC})^2}{r}$$

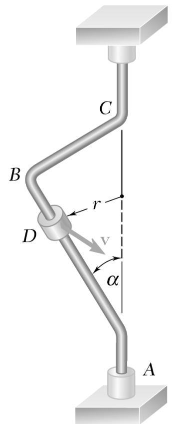
or $$r = \frac{g}{\dot{\theta}_{ABC}^2} \frac{1}{\tan 40°}$$

$$= \frac{9.81 \text{ m/s}^2}{(5 \text{ rad/s})^2} \frac{1}{\tan 40°}$$

$$= 0.468 \text{ m}$$

or $$r = 468 \text{ mm} \blacktriangleleft$$

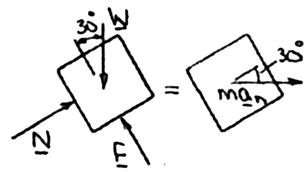
PROBLEM 12.59



A small, 200-g collar D can slide on portion AB of a rod which is bent as shown. Knowing that the rod rotates about the vertical AC at a constant rate and that $\alpha = 30°$ and $r = 600$ mm, determine the range of values of the speed v for which the collar will not slide on the rod if the coefficient of static friction between the rod and the collar is 0.30.

SOLUTION

Case 1: $v = v_{\text{min}}$, impending motion downward



$$+\nearrow \Sigma F_x = ma_x: \quad N - W\sin 30° = m\frac{v^2}{r}\cos 30°$$

or $$N = m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

$$+\nwarrow \Sigma F_y = ma_y: \quad F - W\cos 30° = -m\frac{v^2}{r}\sin 30°$$

or $$F = m\left(g\cos 30° - \frac{v^2}{r}\sin 30°\right)$$

Now $$F = \mu_s N$$

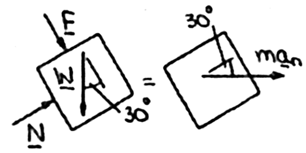
Then $$m\left(g\cos 30° - \frac{v^2}{r}\sin 30°\right) = \mu_s \times m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

or $$v^2 = gr\frac{1 - \mu_s \tan 30°}{\mu_s + \tan 30°}$$

$$= (9.81 \text{ m/s}^2)(0.6 \text{ m})\frac{1 - 0.3\tan 30°}{0.3 + \tan 30°}$$

or $$v_{\text{min}} = 2.36 \text{ m/s}$$

Case 2: $v = v_{max}$, impending motion upward



$$+\nearrow \Sigma F_x = ma_x: \quad N - W\sin 30° = m\frac{v^2}{r}\cos 30°$$

or
$$N = m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

$$+\searrow \Sigma F_y = ma_y: \quad F + W\cos 30° = m\frac{v^2}{r}\sin 30°$$

or
$$F = m\left(-g\cos 30° + \frac{v^2}{r}\sin 30°\right)$$

Now
$$F = \mu_s N$$

Then
$$m\left(-g\cos 30° + \frac{v^2}{r}\sin 30°\right) = \mu_s \times m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

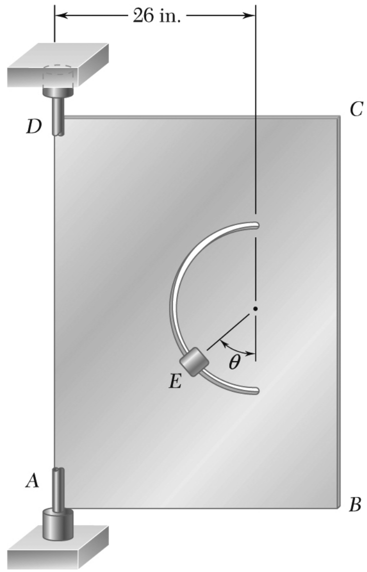
or
$$v^2 = gr\frac{1+\mu_s\tan 30°}{\tan 30° - \mu_s}$$

$$= (9.81\text{ m/s}^2)(0.6\text{ m})\frac{1+0.3\tan 30°}{\tan 30° - 0.3}$$

or
$$v_{max} = 4.99\text{ m/s}$$

For the collar not to slide

$$2.36\text{ m/s} < v < 4.99\text{ m/s} \blacktriangleleft$$



PROBLEM 12.60

A semicircular slot of 10-in. radius is cut in a flat plate which rotates about the vertical *AD* at a constant rate of 14 rad/s. A small, 0.8-lb block *E* is designed to slide in the slot as the plate rotates. Knowing that the coefficients of friction are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine whether the block will slide in the slot if it is released in the position corresponding to (*a*) $\theta = 80°$, (*b*) $\theta = 40°$. Also determine the magnitude and the direction of the friction force exerted on the block immediately after it is released.

SOLUTION

First note $$\rho = \frac{1}{12}(26 - 10\sin\theta)\ \text{ft}$$

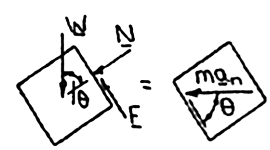
$$v_E = \rho\dot{\phi}_{ABCD}$$

Then $$a_n = \frac{v_E^2}{\rho} = \rho(\dot{\phi}_{ABCD})^2$$

$$= \left[\frac{1}{12}(26 - 10\sin\theta)\ \text{ft}\right](14\ \text{rad/s})^2$$

$$= \frac{98}{3}(13 - 5\sin\theta)\ \text{ft/s}^2$$

Assume that the block is at rest with respect to the plate.



$$+\swarrow \Sigma F_x = ma_x: \quad N + W\cos\theta = m\frac{v_E^2}{\rho}\sin\theta$$

or $$N = W\left(-\cos\theta + \frac{v_E^2}{g\rho}\sin\theta\right)$$

$$+\searrow \Sigma F_y = ma_y: \quad -F + W\sin\theta = -m\frac{v_E^2}{\rho}\cos\theta$$

or $$F = W\left(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta\right)$$

(*a*) We have $\theta = 80°$

Then

$$N = (0.8 \text{ lb})\left[-\cos 80° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 80°) \text{ ft/s}^2 \times \sin 80°\right]$$
$$= 6.3159 \text{ lb}$$

$$F = (0.8 \text{ lb})\left[\sin 80° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 80°) \text{ ft/s}^2 \times \cos 80°\right]$$
$$= 1.92601 \text{ lb}$$

Now $F_{max} = \mu_s N = 0.35(6.3159 \text{ lb}) = 2.2106 \text{ lb}$

The block does not slide in the slot, and

$$\mathbf{F} = 1.926 \text{ lb} \searrow 80° \blacktriangleleft$$

(*b*) We have $\theta = 40°$

Then

$$N = (0.8 \text{ lb})\left[-\cos 40° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 40°) \text{ ft/s}^2 \times \sin 40°\right]$$
$$= 4.4924 \text{ lb}$$

$$F = (0.8 \text{ lb})\left[\sin 40° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 40°) \text{ ft/s}^2 \times \cos 40°\right]$$
$$= 6.5984 \text{ lb}$$

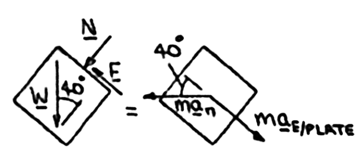
Now $F_{max} = \mu_s N$, from which it follows that

$$F > F_{max}$$

Block E will slide in the slot

and
$$\mathbf{a}_E = \mathbf{a}_n + \mathbf{a}_{E/\text{plate}}$$
$$= \mathbf{a}_n + (\mathbf{a}_{E/\text{plate}})_t + (\mathbf{a}_{E/\text{plate}})_n$$

At $t = 0$, the block is at rest relative to the plate, thus $(\mathbf{a}_{E/\text{plate}})_n = 0$ at $t = 0$, so that $\mathbf{a}_{E/\text{plate}}$ must be directed tangentially to the slot.



$$+\!\swarrow \Sigma F_x = ma_x: \quad N + W\cos 40° = m\frac{v_E^2}{\rho}\sin 40°$$

or
$$N = W\left(-\cos 40° + \frac{v_E^2}{g\rho}\sin 40°\right) \quad \text{(as above)}$$
$$= 4.4924 \text{ lb}$$

Sliding: $F = \mu_k N$

$= 0.25(4.4924 \text{ lb})$

$= 1.123 \text{ lb}$

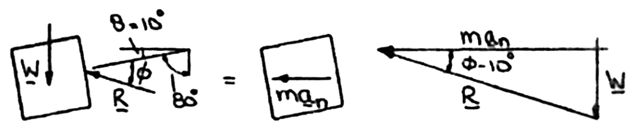
Noting that $\mathbf{F}$ and $\mathbf{a}_{E/\text{plane}}$ must be directed as shown (if their directions are reversed, then $\Sigma \mathbf{F}_x$ is ↘ while $m\mathbf{a}_x$ is ↖), we have

the block slides downward in the slot and $\mathbf{F} = 1.123 \text{ lb} \measuredangle 40° \blacktriangleleft$

Alternative solutions.

(*a*) Assume that the block is at rest with respect to the plate.

$$\Sigma \mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n$$



Then
$$\tan(\phi - 10°) = \frac{W}{ma_n} = \frac{W}{\frac{W}{g}\frac{v_E^2}{\rho}} = \frac{g}{\rho(\dot{\phi}_{ABCD})^2}$$

$$= \frac{32.2 \text{ ft/s}^2}{\frac{98}{3}(13 - 5\sin 80°) \text{ ft/s}^2} \qquad \text{(from above)}$$

or $\phi - 10° = 6.9588°$

and $\phi = 16.9588°$

Now $\tan\phi_s = \mu_s \quad \mu_s = 0.35$

so that $\phi_s = 19.29°$

$0 < \phi < \phi_s \Rightarrow$ Block does not slide and $\mathbf{R}$ is directed as shown.

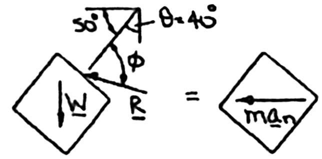
Now $F = R\sin\phi$ and $R = \dfrac{W}{\sin(\phi - 10°)}$

Then
$$F = (0.8 \text{ lb})\frac{\sin 16.9588°}{\sin 6.9588°}$$
$$= 1.926 \text{ lb}$$

The block does not slide in the slot and $\mathbf{F} = 1.926 \text{ lb} \measuredangle 80° \blacktriangleleft$

PROBLEM 12.60 (Continued)

(*b*) Assume that the block is at rest with respect to the plate.



$$\Sigma \mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n$$

From Part *a* (above), it then follows that

$$\tan(\phi - 50°) = \frac{g}{\rho(\dot{\phi}_{ABCD})^2} = \frac{32.2 \text{ ft/s}^2}{\frac{98}{3}(13 - 5\sin 40°) \text{ ft/s}^2}$$

or $\qquad \phi - 50° = 5.752°$

and $\qquad \phi = 55.752°$

Now $\qquad \phi_s = 19.29°$

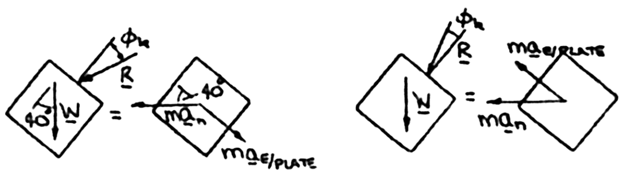
so that $\qquad \phi > \phi_s$

The block will slide in the slot and then

$$\phi = \phi_k, \quad \text{where} \quad \tan\phi_k = \mu_k \quad \mu_k = 0.25$$

or $\qquad \phi_k = 14.0362°$

To determine in which direction the block will slide, consider the free-body diagrams for the two possible cases.



SLIDING DOWNWARD $\qquad$ SLIDING UPWARD

Now $\qquad \Sigma \mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n + m\mathbf{a}_{E/\text{plate}}$

From the diagrams it can be concluded that this equation can be satisfied only if the block is sliding downward. Then

$$+\swarrow \Sigma F_x = ma_x: \quad W\cos 40° + R\cos\phi_k = m\frac{v_E^2}{\rho}\sin 40°$$

Now $\qquad F = R\sin\phi_k$

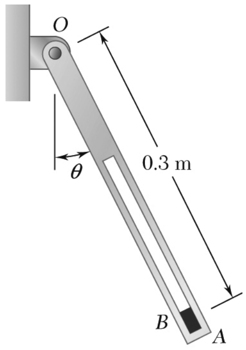
Then $\qquad W\cos 40° + \dfrac{F}{\tan\phi_k} = \dfrac{W}{g}\dfrac{v_E^2}{\rho}\sin 40°$

or

$$F = \mu_k W\left(-\cos 40° + \frac{v_E^2}{g\rho}\sin 40°\right)$$

$$= 1.123 \text{ lb} \qquad \text{(see the first solution)}$$

The block slides downward in the slot and $\qquad \mathbf{F} = 1.123 \text{ lb} \searrow 40°$ ◀

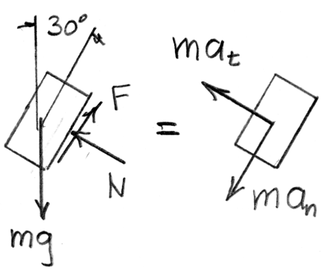


PROBLEM 12.61

A small block B fits inside a lot cut in arm OA which rotates in a vertical plane at a constant rate. The block remains in contact with the end of the slot closest to A and its speed is 1.4 m/s for $0 \le \theta \le 150°$. Knowing that the block begins to slide when $\theta = 150°$, determine the coefficient of static friction between the block and the slot.

SOLUTION

Draw the free body diagrams of the block B when the arm is at $\theta = 150°$.



$$\dot{v} = a_t = 0, \quad g = 9.81 \text{ m/s}^2$$

$$+\nwarrow \Sigma F_t = ma_t: \quad -mg\sin 30° + N = 0$$

$$N = mg\sin 30°$$

$$+\swarrow \Sigma F_n = ma_n: \quad mg\cos 30° - F = m\frac{v^2}{\rho}$$

$$F = mg\cos 30° - \frac{mv^2}{\rho}$$

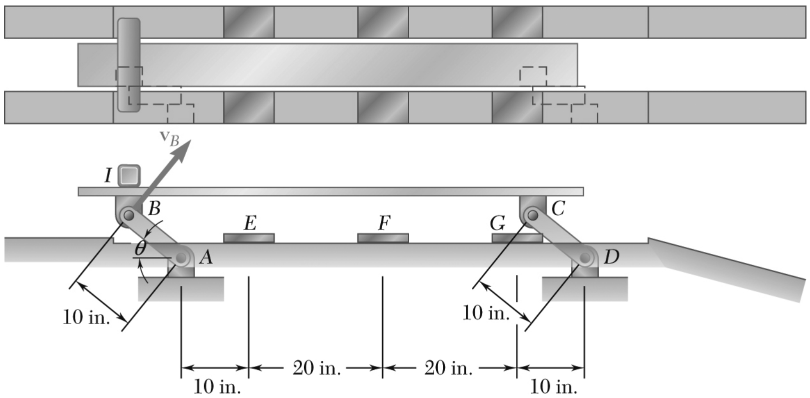
Form the ratio $\frac{F}{N}$, and set it equal to μ_s for impending slip.

$$\mu_s = \frac{F}{N} = \frac{g\cos 30° - v^2/\rho}{g\sin 30°} = \frac{9.81\cos 30° - (1.4)^2/0.3}{9.81\sin 30°}$$

$$\mu_s = 0.400 \blacktriangleleft$$

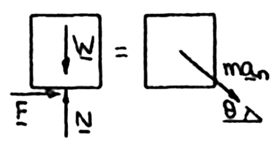
PROBLEM 12.62

The parallel-link mechanism *ABCD* is used to transport a component *I* between manufacturing processes at stations *E*, *F*, and *G* by picking it up at a station when $\theta = 0$ and depositing it at the next station when $\theta = 180°$. Knowing that member *BC* remains horizontal throughout its motion and that links *AB* and *CD* rotate at a constant rate in a vertical plane in such a way that $v_B = 2.2$ ft/s, determine (*a*) the minimum value of the coefficient of static friction between the component and *BC* if the component is not to slide on *BC* while being transferred, (*b*) the values of θ for which sliding is impending.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad F = \frac{W}{g}\frac{v_B^2}{\rho}\cos\theta$$



$$+\uparrow \Sigma F_y = ma_y: \quad N - W = -\frac{W}{g}\frac{v_B^2}{\rho}\sin\theta$$

or

$$N = W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

Now

$$F_{\max} = \mu_s N = \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

and for the component not to slide

$$F < F_{\max}$$

or

$$\frac{W}{g}\frac{v_B^2}{\rho}\cos\theta < \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

or

$$\mu_s > \frac{\cos\theta}{\frac{g\rho}{v_B^2} - \sin\theta}$$

PROBLEM 12.62 (Continued)

We must determine the values of θ which maximize the above expression. Thus

$$\frac{d}{d\theta}\left(\frac{\cos\theta}{\frac{g\rho}{v_B^2}-\sin\theta}\right)=\frac{-\sin\theta\left(\frac{g\rho}{v_B^2}-\sin\theta\right)-(\cos\theta)(-\cos\theta)}{\left(\frac{g\rho}{v_B^2}-\sin\theta\right)^2}=0$$

or
$$\sin\theta=\frac{v_B^2}{g\rho}\qquad\text{for}\qquad \mu_s=(\mu_s)_{\min}$$

Now
$$\sin\theta=\frac{(2.2\text{ ft/s})^2}{(32.2\text{ ft/s}^2)\left(\frac{10}{12}\text{ ft}\right)}=0.180373$$

or
$$\theta=10.3915°\quad\text{and}\quad\theta=169.609°$$

(*a*) From above,

$$(\mu_s)_{\min}=\frac{\cos\theta}{\frac{g\rho}{v_B^2}-\sin\theta}\quad\text{where}\quad\sin\theta=\frac{v_B^2}{g\rho}$$

$$(\mu_s)_{\min}=\frac{\cos\theta}{\frac{1}{\sin\theta}-\sin\theta}=\frac{\cos\theta\sin\theta}{1-\sin^2\theta}=\tan\theta$$

$$=\tan 10.3915°$$

or $(\mu_s)_{\min}=0.1834$ ◀

(*b*) We have impending motion

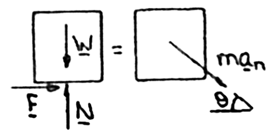
to the left for $\theta=10.39°$ ◀

to the right for $\theta=169.6°$ ◀

PROBLEM 12.63

Knowing that the coefficients of friction between the component *I* and member *BC* of the mechanism of Problem 12.62 are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine (*a*) the maximum allowable constant speed v_B if the component is not to slide on *BC* while being transferred, (*b*) the values of θ for which sliding is impending.

SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x: \quad F = \frac{W}{g}\frac{v_B^2}{\rho}\cos\theta$$

$$+\uparrow \Sigma F_y = ma_y: \quad N - W = -\frac{W}{g}\frac{v_B^2}{\rho}\sin\theta$$

or
$$N = W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

Now
$$F_{\max} = \mu_s N$$
$$= \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

and for the component not to slide

$$F < F_{\max}$$

or
$$\frac{W}{g}\frac{v_B^2}{\rho}\cos\theta < \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

or
$$v_B^2 < \mu_s \frac{g\rho}{\cos\theta + \mu_s \sin\theta} \qquad (1)$$

To ensure that this inequality is satisfied, $\left(v_B^2\right)_{\max}$ must be less than or equal to the minimum value of $\mu_s g\rho/(\cos\theta + \mu_s\sin\theta)$, which occurs when $(\cos\theta + \mu_s\sin\theta)$ is maximum. Thus

$$\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = -\sin\theta + \mu_s\cos\theta = 0$$

or
$$\tan\theta = \mu_s$$
$$\mu_s = 0.35$$

or
$$\theta = 19.2900°$$

(a) The maximum allowed value of v_B is then

$$\left(v_B^2\right)_{\max} = \mu_s \frac{g\rho}{\cos\theta + \mu_s \sin\theta} \quad \text{where} \quad \tan\theta = \mu_s$$

$$= g\rho \frac{\tan\theta}{\cos\theta + (\tan\theta)\sin\theta} = g\rho \sin\theta$$

$$= (32.2\text{ ft/s}^2)\left(\frac{10}{12}\text{ ft}\right)\sin 19.2900°$$

or $(v_B)_{\max} = 2.98$ ft/s ◀

(b) First note that for $90° < \theta < 180°$, Eq. (1) becomes

$$v_B^2 < \mu_s \frac{g\rho}{\cos\alpha + \mu_s \sin\alpha}$$

where $\alpha = 180° - \theta$. It then follows that the second value of θ for which motion is impending is

$$\theta = 180° - 19.2900°$$
$$= 160.7100°$$

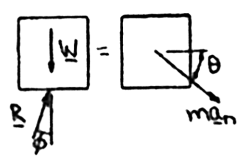
we have impending motion

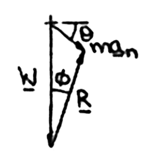
to the left for $\theta = 19.29°$ ◀

to the right for $\theta = 160.7°$ ◀

Alternative solution.

$$\Sigma\mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n$$

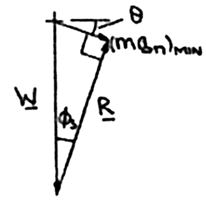




For impending motion, $\phi = \phi_s$. Also, as shown above, the values of θ for which motion is impending minimize the value of v_B, and thus the value of a_n is $\left(a_n = \frac{v_B^2}{\rho}\right)$. From the above diagram, it can be concluded that a_n is minimum when $m\mathbf{a}_n$ and $\mathbf{R}$ are perpendicular.

PROBLEM 12.63 (Continued)

Therefore, from the diagram



$$\theta = \phi_s = \tan^{-1} \mu_s \qquad \text{(as above)}$$

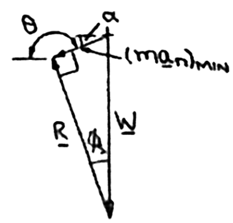
and $$ma_n = W \sin \phi_s$$

or $$m\frac{v_B^2}{\rho} = mg \sin \theta$$

or $$v_B^2 = g\rho \sin \theta \qquad \text{(as above)}$$

For $90° \le \theta \le 180°$, we have

from the diagram



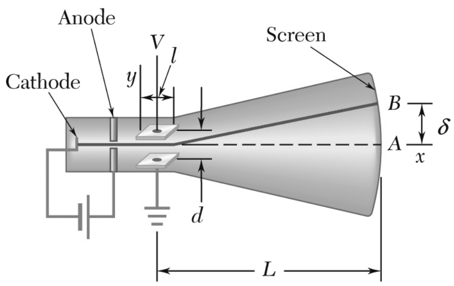
$$\alpha = 180° - \theta \qquad \text{(as above)}$$

$$\alpha = \phi_s$$

and $$ma_n = W \sin \phi_s$$

or $$v_B^2 = g\rho \sin \theta \qquad \text{(as above)}$$

PROBLEM 12.64



In the cathode-ray tube shown, electrons emitted by the cathode and attracted by the anode pass through a small hole in the anode and then travel in a straight line with a speed v_0 until they strike the screen at A. However, if a difference of potential V is established between the two parallel plates, the electrons will be subjected to a force **F** perpendicular to the plates while they travel between the plates and will strike the screen at Point B, which is at a distance δ from A. The magnitude of the force **F** is $F = eV/d$, where $-e$ is the charge of an electron and d is the distance between the plates. Derive an expression for the deflection d in terms of V, v_0, the charge $-e$ and the mass m of an electron, and the dimensions d, ℓ, and L.

SOLUTION

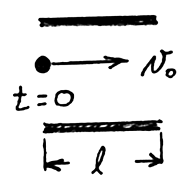
Consider the motion of one electron. For the horizontal motion, let $x = 0$ at the left edge of the plate and $x = \ell$ at the right edge of the plate. At the screen,

$$x = \frac{\ell}{2} + L$$

Horizontal motion: There are no horizontal forces acting on the electron so that $a_x = 0$.

Let $t_1 = 0$ when the electron passes the left edge of the plate, $t = t_1$ when it passes the right edge, and $t = t_2$ when it impacts on the screen. For uniform horizontal motion,

$$x = v_0 t,$$



so that $$t_1 = \frac{\ell}{v_0}$$

and $$t_2 = \frac{\ell}{2v_0} + \frac{L}{v_0}.$$

Vertical motion: The gravity force acting on the electron is neglected since we are interested in the deflection produced by the electric force. While the electron is between plates $(0 < t < t_1)$, the vertical force on the electron is $F_y = eV/d$. After it passes the plates $(t_1 < t < t_2)$, it is zero.

For $0 < t < t_1$, $$\Sigma F_y = ma_y: \quad a_y = \frac{F_y}{m} = \frac{eV}{md}$$

$$v_y = (v_y)_0 + a_y t = 0 + \frac{eVt}{md}$$

$$y = y_0 + (v_y)_0 t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{eVt^2}{2md}$$

PROBLEM 12.64 (Continued)

At $t = t_1$, $\qquad (v_y)_1 = \dfrac{eVt_1}{md} \quad$ and $\quad y_1 = \dfrac{eVt_1^2}{2md}$

For $t_1 < t < t_2$, $a_y = 0$

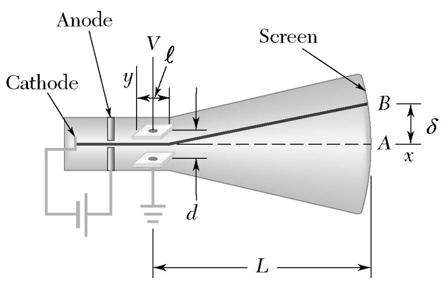
$$y = y_1 + (v_y)_1(t - t_1)$$

At $t = t_2$,

$$y_2 = \delta = y_1 + (v_y)_1(t_2 - t_1)$$

$$\delta = \frac{eVt_1^2}{2md} + \frac{eVt_1}{md}(t_2 - t_1) = \frac{eVt_1}{md}\left(t_2 - \frac{1}{2}t_1\right)$$

$$= \frac{eV\ell}{mdv_0}\left(\frac{\ell}{2v_0} + \frac{L}{v_0} - \frac{1}{2}\frac{\ell}{v_0}\right) \qquad \text{or} \qquad \delta = \frac{eV\ell L}{mdv_0^2} \blacktriangleleft$$



PROBLEM 12.65

In Problem 12.64, determine the smallest allowable value of the ratio d/ℓ in terms of e, m, v_0, and V if at $x=\ell$ the minimum permissible distance between the path of the electrons and the positive plate is $0.05d$.

Problem 12.64 In the cathode-ray tube shown, electrons emitted by the cathode and attracted by the anode pass through a small hole in the anode and then travel in a straight line with a speed v_0 until they strike the screen at A. However, if a difference of potential V is established between the two parallel plates, the electrons will be subjected to a force **F** perpendicular to the plates while they travel between the plates and will strike the screen at point B, which is at a distance δ from A. The magnitude of the force **F** is $F = eV/d$, where $-e$ is the charge of an electron and d is the distance between the plates. Derive an expression for the deflection d in terms of V, v_0, the charge $-e$ and the mass m of an electron, and the dimensions d, ℓ, and L.

SOLUTION

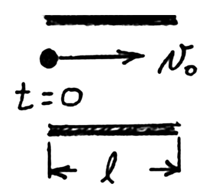
Consider the motion of one electron. For the horizontal motion, let $x=0$ at the left edge of the plate and $x=\ell$ at the right edge of the plate. At the screen,

$$x = \frac{\ell}{2} + L$$

Horizontal motion: There are no horizontal forces acting on the electron so that $a_x = 0$.

Let $t_1 = 0$ when the electron passes the left edge of the plate, $t = t_1$ when it passes the right edge, and $t = t_2$ when it impacts on the screen. For uniform horizontal motion,

$$x = v_0 t,$$



so that

$$t_1 = \frac{\ell}{v_0}$$

and

$$t_2 = \frac{\ell}{2v_0} + \frac{L}{v_0}.$$

Vertical motion: The gravity force acting on the electron is neglected since we are interested in the deflection produced by the electric force. While the electron is between the plates $(0 < t < t_1)$, the vertical force on the electron is $F_y = eV/d$. After it passes the plates $(t_1 < t < t_2)$, it is zero.

For $0 < t < t_1$,
$$\Sigma F_y = ma_y: \quad a_y = \frac{F_y}{m} = \frac{eV}{md}$$

PROBLEM 12.65 (Continued)

$$v_y = (v_y)_0 + a_y t = 0 + \frac{eVt}{md}$$

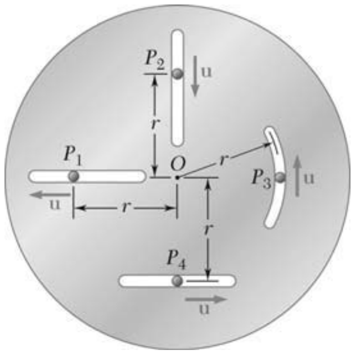
$$y = y_0 + (v_y)_0 t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{eVt^2}{2md}$$

At $t = t_1$, $\quad \frac{\ell}{v_0}, \quad y = \frac{eV\ell^2}{2mdv_0^2}$

But $\quad y < \frac{d}{2} - 0.05d = 0.450d$

so that $\quad \frac{eV\ell^2}{2mdv_0^2} < 0.450\,d$

$$\frac{d^2}{\ell^2} > \frac{1}{0.450}\frac{eV}{2mv_0^2} = 1.111\frac{eV}{mv_0^2} \qquad \frac{d}{\ell} > 1.054\sqrt{\frac{eV}{mv_0^2}} \blacktriangleleft$$

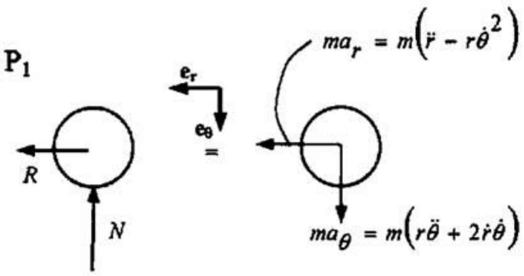


PROBLEM 12.F9

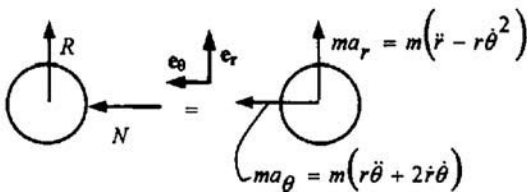
Four pins slide in four separate slots cut in a horizontal circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. Each pin has a mass m and maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω. Draw the FBDs and KDs to determine the forces on pins P_1 and P_2.

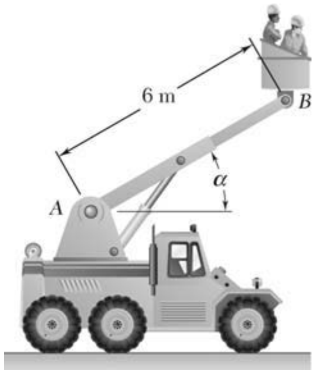
SOLUTION

Pin P_1



Pin P_2

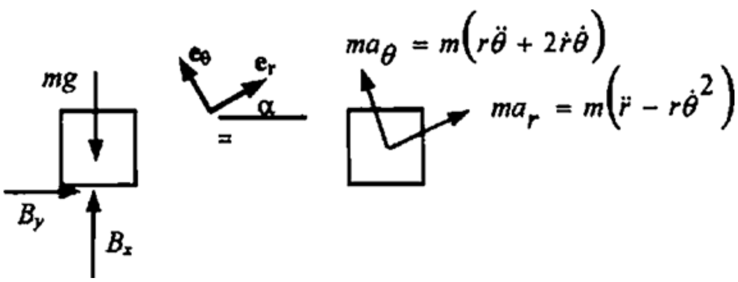




PROBLEM 12.F10

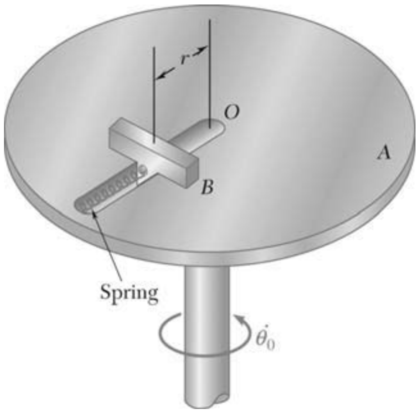
At the instant shown, the length of the boom AB is being *decreased* at the constant rate of 0.2 m/s, and the boom is being lowered at the constant rate of 0.08 rad/s. If the mass of the men and lift connected to the boom at Point B is m, draw the FBD and KD that could be used to determine the horizontal and vertical forces at B.

SOLUTION



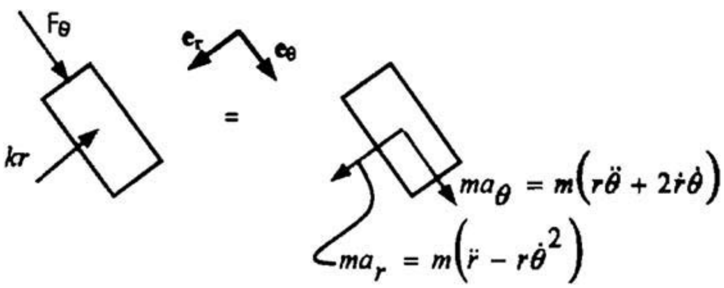
Where $r = 6$ m, $\dot{r} = -0.2$ m/s, $\ddot{r} = 0$, $\dot{\theta} = -0.08$ rad/s, $\ddot{\theta} = 0$

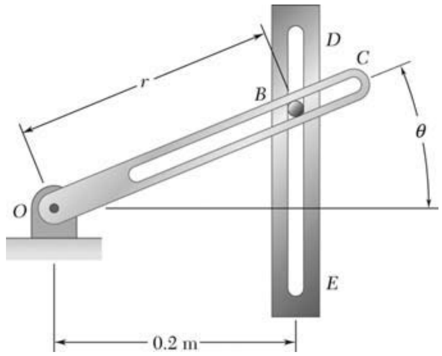
PROBLEM 12.F11



Disk A rotates in a horizontal plane about a vertical axis at the constant rate $\dot{\theta}_0$. Slider B has a mass m and moves in a frictionless slot cut in the disk. The slider is attached to a spring of constant k, which is undeformed when $r = 0$. Knowing that the slider is released with no radial velocity in the position $r = r_0$, draw a FBD and KD at an arbitrary distance r from O.

SOLUTION

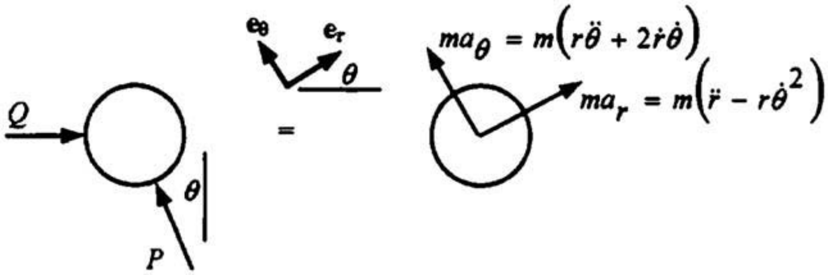


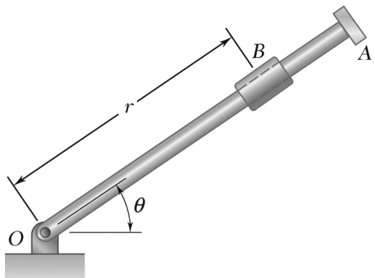


PROBLEM 12.F12

Pin B has a mass m and slides along the slot in the rotating arm OC and along the slot DE which is cut in a fixed horizontal plate. Neglecting friction and knowing that rod OC rotates at the constant rate $\dot{\theta}_0$, draw a FBD and KD that can be used to determine the forces **P** and **Q** exerted on pin B by rod OC and the wall of slot DE, respectively.

SOLUTION





PROBLEM 12.66

Rod OA rotates about O in a horizontal plane. The motion of the 0.5-lb collar B is defined by the relations $r = 10 + 6\cos\pi t$ and $\theta = \pi(4t^2 - 8t)$, where r is expressed in inches, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the collar when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

SOLUTION

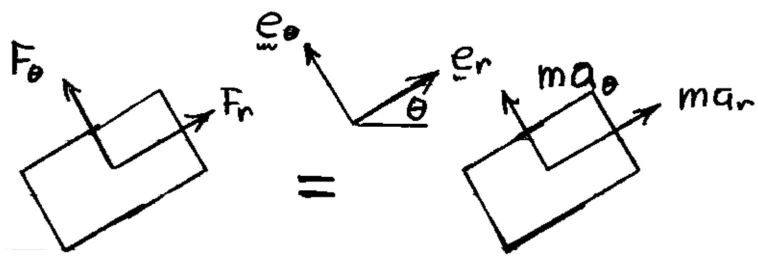
Use polar coordinates and calculate the derivatives of the coordinates r and θ with respect to time.

$$r = 10 + 6\cos\pi t \text{ in.} \qquad \theta = \pi(4t^2 - 8t) \text{ rad}$$

$$\dot{r} = -6\pi\sin\pi t \text{ in./s} \qquad \dot{\theta} = \pi(8t - 8) \text{ rad/s}$$

$$\ddot{r} = -6\pi^2\cos\pi t \text{ in./s}^2 \qquad \ddot{\theta} = 8\pi \text{ rad/s}^2$$

Mass of collar: $$m = \frac{0.5 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.015528 \text{ lb s}^2/\text{ft} = 1.294\times10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.}$$



(*a*) $t = 0$:

$r = 16$ in. $\qquad \theta = 0$

$\dot{r} = 0 \qquad \dot{\theta} = -8\pi = -25.1327$ rad/s

$\ddot{r} = -6\pi^2 = -59.218 \text{ in./s}^2 \qquad \ddot{\theta} = 8\pi = -25.1327 \text{ rad/s}^2$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -59.218 - (16)(-25.1327)^2 = -10165.6 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (16)(25.1327) + 0 = 402.12 \text{ in./s}^2$$

$$F_r = ma_r = (1.294\times10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.})(-10165.6 \text{ in./s}^2)$$ $F_r = -13.15$ lb ◀

$$F_\theta = ma_\theta = (1.294\times10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.})(402.12 \text{ in./s}^2)$$ $F_\theta = 0.520$ lb ◀

$\theta = 0$ ◀

(*b*) $t = 0.5$ s:

$r = 10 + 6\cos(0.5\pi) = 10$ in. $\qquad \theta = \pi[(4)(0.25) - (8)(0.5)] = -9.4248 \text{ rad} = -540° = 180°$

$\dot{r} = -6\pi\sin(0.5\pi) = -18.8496$ in./s $\qquad \dot{\theta} = \pi[(8)(0.5) - 8] = -12.5664$ rad/s

$\ddot{r} = -6\pi^2\cos(0.5\pi) = 0 \qquad \ddot{\theta} = 8\pi = 25.1327 \text{ rad/s}^2$

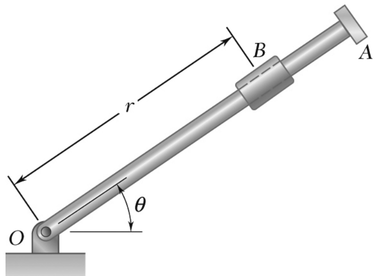
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (10)(-12.5664)^2 = -1579.14 \text{ in./s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (10)(25.1327) + (2)(-18.8496)(-12.5664) = 725.07 \text{ in./s}^2$

$F_r = ma_r = (1.294 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{in.})(-1579.14 \text{ in./s}^2)$ $\qquad F_r = -2.04 \text{ lb}$ ◀

$F_\theta = ma_\theta = (1.294 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{in.})(725.07 \text{ in./s}^2)$ $\qquad F_\theta = 0.938 \text{ lb}$ ◀

$\theta = 180°$ ◀



PROBLEM 12.67

Rod OA oscillates about O in a horizontal plane. The motion of the 2-lb collar B is defined by the relations $r = 6(1-e^{-2t})$ and $\theta = (3/\pi)(\sin \pi t)$, where r is expressed in inches, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the collar when (*a*) $t = 1$ s, (*b*) $t = 1.5$ s.

SOLUTION

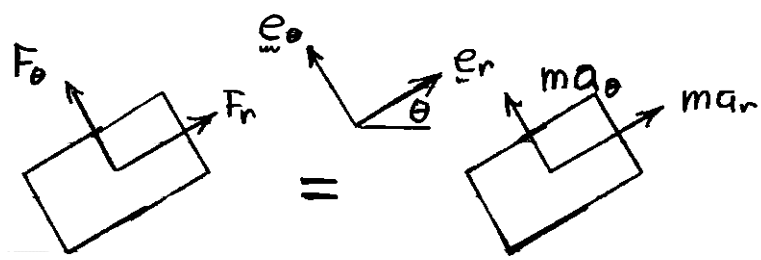
Use polar coordinates and calculate the derivatives of the coordinates r and θ with respect to time.

$$r = 6(1-e^{-2t}) \text{ in.} \qquad \theta = (3/\pi)\sin \pi t \text{ radians}$$

$$\dot{r} = 12e^{-2t} \text{ in./s} \qquad \dot{\theta} = 3\cos \pi t \text{ rad/s}$$

$$\ddot{r} = -24e^{-2t} \text{ in./s}^2 \qquad \ddot{\theta} = -3\pi \sin \pi t \text{ rad/s}^2$$

Mass of collar: $m = \dfrac{2 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.06211 \text{ lb}\cdot\text{s}^2/\text{ft} = 5.176\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.}$



(*a*) $t = 1$ s: $e^{-2t} = 0.13534, \quad \sin \pi t = 0, \quad \cos \pi t = -1$

$$r = 6(1-0.13534) = 5.188 \text{ in.} \qquad \theta = 0$$

$$\dot{r} = (12)(0.13534) = 1.62402 \text{ in./s} \qquad \dot{\theta} = -3.0 \text{ rad/s}$$

$$\ddot{r} = (-24)(0.13534) = -3.2480 \text{ in./s}^2 \qquad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.2480 - (5.188)(-3.0)^2 = -49.94 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + (2)(1.62402)(-3) = -9.744 \text{ in./s}^2$$

$$F_r = ma_r = (5.176\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.})(-49.94 \text{ in./s}^2)$$

$F_r = -0.258$ lb ◀

$$F_\theta = ma_\theta = (5.176\times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.})(-9.744 \text{ in./s}^2)$$

$F_\theta = -0.0504$ lb ◀

$\theta = 0$ ◀

(*b*) $t = 1.5$ s: $e^{-2t} = 0.049787, \quad \sin \pi t = -1, \quad \cos \pi t = 0$

$$r = 6(1 - 0.049787) = 5.7013 \text{ in.} \qquad \theta = (3/\pi)(-1) = -0.9549 \text{ rad} = -54.7°$$

$$\dot{r} = (12)(0.049787) = 0.59744 \text{ in./s}^2 \qquad \dot{\theta} = 0$$

$$\ddot{r} = -(24)(0.049787) = -1.19489 \text{ in./s}^2 \qquad \ddot{\theta} = -(3\pi)(-1) = 9.4248 \text{ rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -1.19489 - 0 = -1.19489 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (5.7013)(9.4248) + 0 = 53.733 \text{ in./s}^2$$

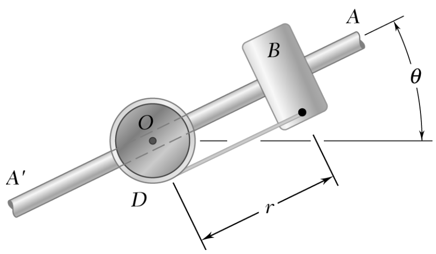
$$F_r = ma_r = (5.176 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{in.})(-1.19489 \text{ in./s}^2)$$

$F_r = -0.00618$ lb ◀

$$F_\theta = ma_\theta = (5.176 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{in.})(53.733 \text{ in./s}^2)$$

$F_\theta = 0.278$ lb ◀

$\theta = -54.7°$ ◀



PROBLEM 12.68

The 3-kg collar B slides on the frictionless arm AA'. The arm is attached to drum D and rotates about O in a horizontal plane at the rate $\dot{\theta} = 0.75t$, where $\dot{\theta}$ and t are expressed in rad/s and seconds, respectively. As the arm-drum assembly rotates, a mechanism within the drum releases cord so that the collar moves outward from O with a constant speed of 0.5 m/s. Knowing that at $t = 0$, $r = 0$, determine the time at which the tension in the cord is equal to the magnitude of the horizontal force exerted on B by arm AA'.

SOLUTION

<u>Kinematics</u>

We have $$\frac{dr}{dt} = \dot{r} = 0.5 \text{ m/s}$$

At $t = 0$, $r = 0$: $$\int_0^r dr = \int_0^t 0.5\, dt$$

or $$r = (0.5t) \text{ m}$$

Also, $$\ddot{r} = 0 \qquad \dot{\theta} = (0.75t) \text{ rad/s}$$
$$\ddot{\theta} = 0.75 \text{ rad/s}^2$$

Now $$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - [(0.5t) \text{ m}][(0.75t) \text{ rad/s}]^2 = -(0.28125t^3) \text{ m/s}^2$$

and $$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= [(0.5t) \text{ m}][0.75 \text{ rad/s}^2] + 2(0.5 \text{ m/s})[(0.75t) \text{ rad/s}]$$
$$= (1.125t) \text{ m/s}^2$$

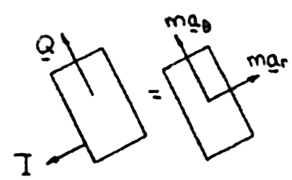
<u>Kinetics</u>

$$+\nearrow \Sigma F_r = ma_r: \quad -T = (3 \text{ kg})(-0.28125t^3) \text{ m/s}^2$$

or $$T = (0.84375t^3) \text{ N}$$

$$+\nwarrow \Sigma F_\theta = m_B a_\theta: \quad Q = (3 \text{ kg})(1.125t) \text{ m/s}^2$$

or $$Q = (3.375t) \text{ N}$$



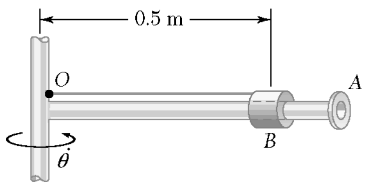
Now require that $$T = Q$$

or $$(0.84375t^3) \text{ N} = (3.375t) \text{ N}$$

or $$t^2 = 4.000$$

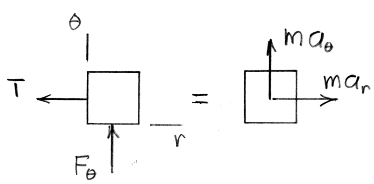
or $$t = 2.00 \text{ s} \blacktriangleleft$$

PROBLEM 12.69



The horizontal rod *OA* rotates about a vertical shaft according to the relation $\dot{\theta} = 10t$, where $\dot{\theta}$ and t are expressed in rad/s and seconds, respectively. A 250-g collar *B* is held by a cord with a breaking strength of 18 N. Neglecting friction, determine, immediately after the cord breaks, (*a*) the relative acceleration of the collar with respect to the rod, (*b*) the magnitude of the horizontal force exerted on the collar by the rod.

SOLUTION



$$\dot{\theta} = 10t \text{ rad/s}, \quad \ddot{\theta} = 10 \text{ rad/s}^2$$

$$m = 250 \text{ g} = 0.250 \text{ kg}$$

Before cable breaks: $F_r = -T$ and $\ddot{r} = 0$.

$$F_r = ma_r: \quad -T = m(\ddot{r} - r\dot{\theta}^2)$$

$$mr\dot{\theta}^2 = m\ddot{r} + T \qquad \text{or} \qquad \dot{\theta}^2 = \frac{m\ddot{r} + T}{mr} = \frac{0 - 18}{(0.25)(0.5)} = 144 \text{ rad}^2/\text{s}^2$$

$$\dot{\theta} = 12 \text{ rad/s}$$

Immediately after the cable breaks: $F_r = 0, \ \dot{r} = 0$

(*a*) Acceleration of *B* relative to the rod.

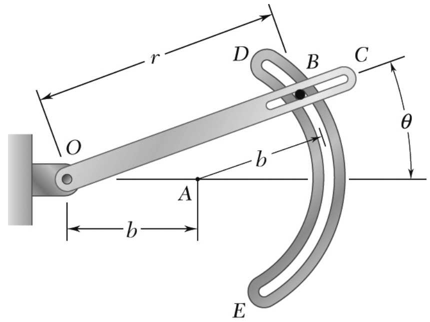
$$m(\ddot{r} - r\dot{\theta}^2) = 0 \qquad \text{or} \qquad \ddot{r} = r\dot{\theta}^2 = (0.5)(12)^2 = 72 \text{ m/s}^2$$

$$\mathbf{a}_{B/\text{rod}} = 72 \text{ m/s}^2 \text{ radially outward} \blacktriangleleft$$

(*b*) Transverse component of the force.

$$F_\theta = ma_\theta: \quad F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F_\theta = (0.250)[(0.5)(10) + (2)(0)(12)] = 1.25 \qquad F_\theta = 1.25 \text{ N} \blacktriangleleft$$



PROBLEM 12.70

Pin B weighs 4 oz and is free to slide in a horizontal plane along the rotating arm OC and along the circular slot DE of radius $b = 20$ in. Neglecting friction and assuming that $\dot{\theta} = 15$ rad/s and $\ddot{\theta} = 250\text{ rad/s}^2$ for the position $\theta = 20°$, determine for that position (*a*) the radial and transverse components of the resultant force exerted on pin B, (*b*) the forces **P** and **Q** exerted on pin B, respectively, by rod OC and the wall of slot DE.

SOLUTION

<u>Kinematics</u>.

From the geometry of the system, we have

Then $\quad r = 2b\cos\theta \qquad \dot{r} = -(2b\sin\theta)\dot{\theta} \qquad \ddot{r} = -2b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$

and $\quad a_r = \ddot{r} - r\dot{\theta}^2 = -2b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) - (2b\cos\theta)\dot{\theta}^2 = -2b(\ddot{\theta}\sin\theta + 2\dot{\theta}^2\cos\theta)$

Now $$= -2\left(\frac{20}{12}\text{ ft}\right)[(250\text{ rad/s}^2)\sin 20° + 2(15\text{ rad/s})^2\cos 20°] = -1694.56\text{ ft/s}^2$$

and $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (2b\cos\theta)\ddot{\theta} + 2(-2b\dot{\theta}\sin\theta)\dot{\theta} = 2b(\ddot{\theta}\cos\theta - 2\dot{\theta}^2\sin\theta)$

$$= 2\left(\frac{20}{12}\text{ ft}\right)[(250\text{ rad/s}^2)\cos 20° - 2(15\text{ rad/s})^2\sin 20°] = 270.05\text{ ft/s}^2$$

<u>Kinetics</u>.

(*a*) We have $\quad F_r = ma_r = \dfrac{\frac{1}{4}\text{ lb}}{32.2\text{ ft/s}^2} \times (-1694.56\text{ ft/s}^2) = -13.1565\text{ lb}$ $\qquad F_r = -13.16\text{ lb}$ ◀

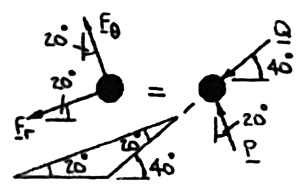
and $\quad F_\theta = ma_\theta = \dfrac{\frac{1}{4}\text{ lb}}{32.2\text{ ft/s}^2} \times (270.05\text{ ft/s}^2) = 2.0967\text{ lb}$ $\qquad F_\theta = 2.10\text{ lb}$ ◀

(*b*) $\quad +\nearrow\ \Sigma F_r\!: \quad -F_r = -Q\cos 20°$

or $$Q = \frac{1}{\cos 20°}(13.1565\text{ lb}) = 14.0009\text{ lb}$$

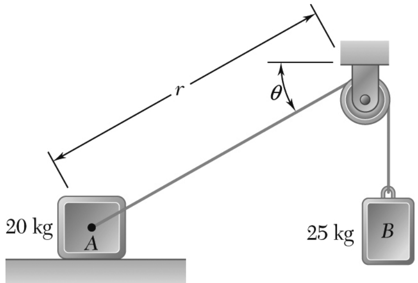
$+\nwarrow\ \Sigma F_\theta\!: \quad F_\theta = P - Q\sin 20°$

or $$P = (2.0967 + 14.0009\sin 20°)\text{ lb} = 6.89\text{ lb}$$



$\mathbf{P} = 6.89\text{ lb} \searrow 70°$ ◀

$\mathbf{Q} = 14.00\text{ lb} \nearrow 40°$ ◀



PROBLEM 12.71

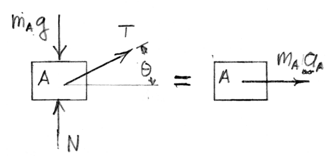
The two blocks are released from rest when $r = 0.8$ m and $\theta = 30°$. Neglecting the mass of the pulley and the effect of friction in the pulley and between block A and the horizontal surface, determine (*a*) the initial tension in the cable, (*b*) the initial acceleration of block A, (*c*) the initial acceleration of block B.

SOLUTION

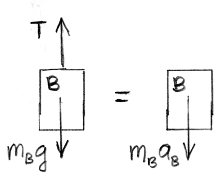
Let r and θ be polar coordinates of block A as shown, and let y_B be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block B.

Constraint of cable: $r + y_B = \text{constant},$

$$\dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \quad \text{or} \quad \ddot{r} = -a_B \qquad (1)$$



For block A, $\xrightarrow{+} \Sigma F_x = m_A a_A$: $T\cos\theta = m_A a_A$ or $T = m_A a_A \sec\theta \qquad (2)$



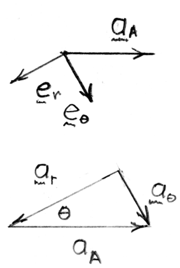
For block B, $+\downarrow \Sigma F_y = m_B a_B$: $m_B g - T = m_B a_B \qquad (3)$

Adding Eq. (1) to Eq. (2) to eliminate T, $m_B g = m_A a_A \sec\theta + m_B a_B \qquad (4)$

Radial and transverse components of $\mathbf{a}_A$.

Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.

$$\ddot{r} - r\dot{\theta}^2 = a_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A \cos\theta \qquad (5)$$



Noting that initially $\dot{\theta} = 0$, using Eq. (1) to eliminate $\ddot{r}$, and changing signs gives

$$a_B = a_A \cos\theta \qquad (6)$$

Substituting Eq. (6) into Eq. (4) and solving for a_A,

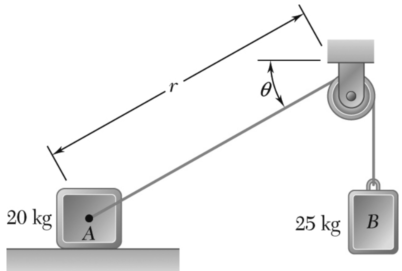
$$a_A = \frac{m_B g}{m_A \sec\theta + m_B \cos\theta} = \frac{(25)(9.81)}{20\sec 30° + 25\cos 30°} = 5.48 \text{ m/s}^2$$

From Eq. (6), $a_B = 5.48\cos 30° = 4.75 \text{ m/s}^2$

(*a*) From Eq. (2), $T = (20)(5.48)\sec 30° = 126.6$ $\qquad T = 126.6 \text{ N}$ ◀

(*b*) Acceleration of block A. $\qquad \mathbf{a}_A = 5.48 \text{ m/s}^2 \longrightarrow$ ◀

(*c*) Acceleration of block B. $\qquad \mathbf{a}_B = 4.75 \text{ m/s}^2 \downarrow$ ◀



PROBLEM 12.72

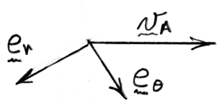
The velocity of block A is 2 m/s to the right at the instant when $r = 0.8$ m and $\theta = 30°$. Neglecting the mass of the pulley and the effect of friction in the pulley and between block A and the horizontal surface, determine, at this instant, (a) the tension in the cable, (b) the acceleration of block A, (c) the acceleration of block B.

SOLUTION

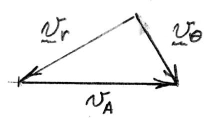
Let r and θ be polar coordinates of block A as shown, and let y_B be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block B.

Radial and transverse components of $\mathbf{v}_A$.

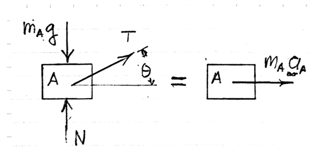
Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.



$$\dot{r} = v_r = \mathbf{v}_A \cdot \mathbf{e}_r = -v_A \cos 30°$$
$$= -2\cos 30° = -1.73205 \text{ m/s}$$



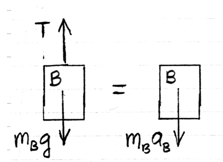
$$r\dot{\theta} = v_\theta = \mathbf{v}_A \cdot \mathbf{e}_\theta = -v_A \sin 30°$$
$$= 2\sin 30° = 1.000 \text{ m/s}^2$$



$$\dot{\theta} = \frac{v_\theta}{r} = \frac{1.000}{0.8} = 1.25 \text{ rad/s}$$

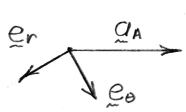
Constraint of cable: $r + y_B = \text{constant}$,

$$\dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \quad \text{or} \quad \ddot{r} = -a_B \qquad (1)$$



For block A, $\xrightarrow{+} \Sigma F_x = m_A a_A : T\cos\theta = m_A a_A$ or $T = m_A a_A \sec\theta$ (2)

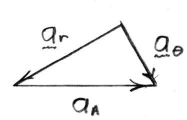
For block B, $+\!\downarrow \Sigma F_y = m_B a_B$: $m_B g - T = m_B a_B$ (3)



Adding Eq. (1) to Eq. (2) to eliminate T, $m_B g = m_A a_A \sec\theta + m_B a_B$ (4)

Radial and transverse components of $\mathbf{a}_A$.

Use a method similar to that used for the components of velocity.



$$\ddot{r} - r\dot{\theta}^2 = a_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A \cos\theta \qquad (5)$$

Using Eq. (1) to eliminate $\ddot{r}$ and changing signs gives

$$a_B = a_A \cos\theta - r\dot{\theta}^2 \qquad (6)$$

Substituting Eq. (6) into Eq. (4) and solving for a_A,

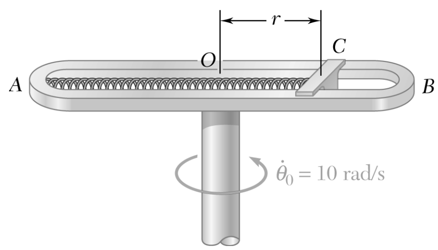
$$a_A = \frac{m_B\left(g + r\dot{\theta}^2\right)}{m_A \sec\theta + m_B \cos\theta} = \frac{(25)[9.81 + (0.8)(1.25)^2]}{20\sec 30° + 25\cos 30°} = 6.18 \text{ m/s}^2$$

From Eq. (6), $a_B = 6.18\cos 30° - (0.8)(1.25)^2 = 4.10 \text{ m/s}^2$

(*a*) From Eq. (2), $T = (20)(6.18)\sec 30° = 142.7$ $T = 142.7 \text{ N}$ ◀

(*b*) Acceleration of block *A*. $\mathbf{a}_A = 6.18 \text{ m/s}^2 \longrightarrow$ ◀

(*c*) Acceleration of block *B*. $\mathbf{a}_B = 4.10 \text{ m/s}^2 \downarrow$ ◀



PROBLEM 12.73*

Slider C has a weight of 0.5 lb and may move in a slot cut in arm AB, which rotates at the constant rate $\dot{\theta}_0 = 10$ rad/s in a horizontal plane. The slider is attached to a spring of constant $k = 2.5$ lb/ft, which is unstretched when $r = 0$. Knowing that the slider is released from rest with no radial velocity in the position $r = 18$ in. and neglecting friction, determine for the position $r = 12$ in. (*a*) the radial and transverse components of the velocity of the slider, (*b*) the radial and transverse components of its acceleration, (*c*) the horizontal force exerted on the slider by arm AB.

SOLUTION

Let l_0 be the radial coordinate when the spring is unstretched. Force exerted by the spring.

$$F_r = -k(r - l_0)$$

$$\Sigma F_r = ma_r: \quad -k(r - l_0) = m(\ddot{r} - r\dot{\theta}^2)$$

$$\ddot{r} = \left(\dot{\theta}^2 - \frac{k}{m}\right)r + \frac{kl_0}{m} \qquad (1)$$

But

$$\ddot{r} = \frac{d}{dt}(\dot{r}) = \frac{d\dot{r}}{dr}\frac{dr}{dt} = \dot{r}\frac{d\dot{r}}{dr}$$

$$\dot{r}d\dot{r} = \ddot{r}dr = \left[\left(\dot{\theta}^2 - \frac{k}{m}\right)r + \frac{kl_0}{m}\right]dr$$

Integrate using the condition $\dot{r} = \dot{r}_0$ when $r = r_0$.

$$\frac{1}{2}\dot{r}^2\Big|_{\dot{r}_0}^{\dot{r}} = \left[\frac{1}{2}\left(\dot{\theta}^2 - \frac{k}{m}\right)r^2 + \frac{kl_0}{m}r\right]\Bigg|_{r_0}^{r}$$

$$\frac{1}{2}\dot{r}^2 - \frac{1}{2}\dot{r}_0^2 = \frac{1}{2}\left(\dot{\theta}^2 - \frac{k}{m}\right)\left(r^2 - r_0^2\right) + \frac{kl_0}{m}(r - r_0)$$

$$\dot{r}^2 = \dot{r}_0^2 + \left(\dot{\theta}^2 - \frac{k}{m}\right)\left(r^2 - r_0^2\right) + \frac{2kl_0}{m}(r - r_0)$$

<u>Data</u>:

$$m = \frac{W}{g} = \frac{0.5 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.01553 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\dot{\theta} = 10 \text{ rad/s}, \quad k = 2.5 \text{ lb/ft}, \quad l_0 = 0$$

$$\dot{r}_0 = (v_r)_0 = 0, \quad r_0 = 18 \text{ in.} = 1.5 \text{ ft}, \quad r = 12 \text{ in.} = 1.0 \text{ ft}$$

(*a*) Components of velocity when $r = 12$ in.

$$\dot{r}^2 = 0 + \left(10^2 - \frac{2.5}{0.01553}\right)(1.0^2 - 1.5^2) + 0$$

$$= 76.223 \text{ ft}^2/\text{s}^2$$

$$v_r = \dot{r} = \pm 8.7306 \text{ ft/s}$$

Since r is decreasing, v_r is negative

$$\dot{r} = -8.7306 \text{ ft/s} \qquad v_r = -8.73 \text{ ft/s} \blacktriangleleft$$

$$v_\theta = r\dot{\theta} = (1.0)(10) \qquad v_\theta = 10.00 \text{ ft/s} \blacktriangleleft$$

(*b*) Components of acceleration.

$$F_r = -kr + kl_0 = -(2.5)(1.0) + 0 = -2.5 \text{ lb}$$

$$a_r = \frac{F_r}{m} = -\frac{2.5}{0.01553} \qquad a_r = 161.0 \text{ ft/s}^2 \blacktriangleleft$$

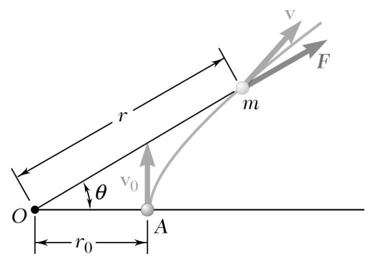
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + (2)(-8.7306)(10)$$

$$a_\theta = -174.6 \text{ ft/s}^2 \blacktriangleleft$$

(*c*) Transverse component of force.

$$F_\theta = ma_\theta = (0.01553)(-174.6) \qquad F_\theta = -2.71 \text{ lb} \blacktriangleleft$$

PROBLEM 12.74



A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force $\mathbf{F}$ directed away from the center of force O. Knowing that the particle follows a path defined by the equation $r = r_0/\sqrt{\cos 2\theta}$ and using Eq. (12.27), express the radial and transverse components of the velocity $\mathbf{v}$ of the particle as functions of θ.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27),
$$h = r^2\dot{\theta} = h_0 = r_0 v_0$$

or
$$\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0 \cos 2\theta}{r_0^2} = \frac{v_0}{r_0}\cos 2\theta$$

Radial component of velocity.

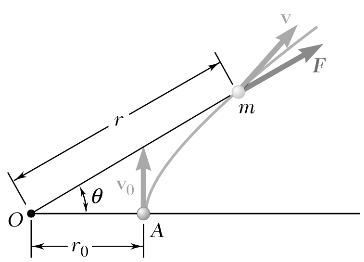
$$v_r = \dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(\frac{r_0}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = r_0 \frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta}$$

$$= r_0 \frac{\sin 2\theta}{(\cos 2\theta)^{3/2}} \frac{v_0}{r}\cos 2\theta \qquad v_r = v_0 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}} \blacktriangleleft$$

Transverse component of velocity.

$$v_\theta = \frac{h}{r} = \frac{r_0 v_0}{r_0}\sqrt{\cos 2\theta} \qquad v_\theta = v_0\sqrt{\cos 2\theta} \blacktriangleleft$$

PROBLEM 12.75



For the particle of Problem 12.74, show (*a*) that the velocity of the particle and the central force **F** are proportional to the distance r from the particle to the center of force O, (*b*) that the radius of curvature of the path is proportional to r^3.

PROBLEM 12.74 A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force **F** directed away from the center of force O. Knowing that the particle follows a path defined by the equation $r = r_0/\sqrt{\cos 2\theta}$ and using Eq. (12.27), express the radial and transverse components of the velocity **v** of the particle as functions of θ.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27),

$$h = r^2\dot{\theta} = h_0 = r_0 v_0 \quad \text{or} \quad \dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0 \cos 2\theta}{r_0{}^2} = \frac{v_0}{r_0}\cos 2\theta$$

Differentiating the expression for r with respect to time,

$$\dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(\frac{r_0}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = r_0 \frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta} = r_0 \frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\frac{v_0}{r_0}\cos 2\theta = v_0 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

Differentiating again,

$$\ddot{r} = \frac{d\dot{r}}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(v_0 \frac{\sin 2\theta}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = v_0 \frac{2\cos^2 2\theta + \sin^2 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta} = \frac{v_0{}^2}{r_0}\frac{2\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}}$$

(*a*) $\quad v_r = \dot{r} = v_0 \dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}} = \dfrac{v_0 r}{r_0}\sin 2\theta \qquad v_\theta = r\dot{\theta} = \dfrac{v_0 r}{r_0}\cos 2\theta$

$$v = \sqrt{(v_r)^2 + (v_\theta)^2} = \frac{v_0 r}{r_0}\sqrt{\sin^2 2\theta + \cos^2 2\theta} \qquad v = \frac{v_0 r}{r_0} \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{v_0{}^2}{r_0}\frac{2\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}} - \frac{r_0}{\sqrt{\cos 2\theta}}\frac{v_0{}^2}{r_0{}^2}\cos^2 2\theta$$

$$= \frac{v_0{}^2}{r_0}\frac{\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}} = \frac{v_0}{r_0\sqrt{\cos 2\theta}} = \frac{v_0{}^2 r}{r_0{}^2}$$

$$F_r = ma_r = \frac{mv_0^2 r}{r_0^2}: \qquad F_r = \frac{mv_0^2 r}{r_0^2} \blacktriangleleft$$

Since the particle moves under a central force, $a_\theta = 0.$

Magnitude of acceleration.

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \frac{v_0^{\,2} r}{r_0^2}$$

Tangential component of acceleration.

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{v_0 r}{r_0}\right) = \frac{v_0}{r_0}\dot{r} = \frac{v_0^{\,2} r}{r_0^2}\sin 2\theta$$

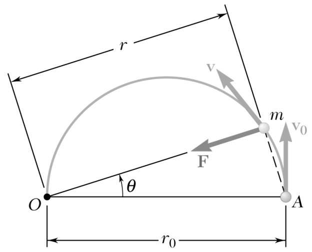
Normal component of acceleration.

$$a_t = \sqrt{a^2 - a_t^{\,2}} = \frac{v_0^{\,2} r}{r_0^2}\sqrt{1-\sin^2 2\theta} = \frac{v_0^{\,2} r\cos 2\theta}{r_0^2}$$

But $\quad \cos 2\theta = \left(\dfrac{r_0}{r}\right)^2$

Hence, $\quad a_n = \dfrac{v_0^{\,2}}{r}$

(*b*) But $a_n = \dfrac{v^2}{\rho}$ or $\quad \rho = \dfrac{v^2}{a_n} = \dfrac{v_0^{\,2} r^2}{r_0^{\,2}} \cdot \dfrac{r}{v_0^{\,2}}$ $\qquad \rho = \dfrac{r^3}{r_0^2}$ ◀



PROBLEM 12.76

A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force $\mathbf{F}$ along a semicircular path of diameter OA. Observing that $r = r_0 \cos\theta$ and using Eq. (12.27), show that the speed of the particle is $v = v_0/\cos^2\theta$.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27), $$h = r^2\dot{\theta} = h_0 = r_0 v_0$$

or $$\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{r_0^2 \cos^2\theta} = \frac{v_0}{r_0 \cos^2\theta}$$

Radial component of velocity.

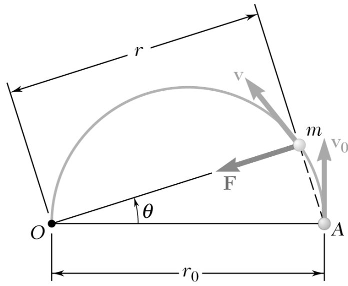
$$v_r = \dot{r} = \frac{d}{dt}(r_0 \cos\theta) = -(r_0 \sin\theta)\dot{\theta}$$

Transverse component of velocity.

$$v_\theta = r\dot{\theta} = (r_0 \cos\theta)\dot{\theta}$$

Speed. $$v = \sqrt{v_r^2 + v_\theta^2} = r_0\dot{\theta} = \frac{r_0 v_0}{r_0 \cos^2\theta}$$

$$v = \frac{v_0}{\cos^2\theta} \blacktriangleleft$$



PROBLEM 12.77

For the particle of Problem 12.76, determine the tangential component F_t of the central force **F** along the tangent to the path of the particle for (*a*) $\theta = 0$, (*b*) $\theta = 45°$.

PROBLEM 12.76 A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force **F** along a semicircular path of diameter OA. Observing that $r = r_0 \cos\theta$ and using Eq. (12.27), show that the speed of the particle is $v = v_0/\cos^2\theta$.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$

Using Eq. (12.27),

$$h = r^2\dot{\theta} = h_0 = r_0 v_0$$

$$\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{r_0^2 \cos^2\theta} = \frac{v_0}{r_0 \cos^2\theta}$$

Radial component of velocity.

$$v_r = \dot{r} = \frac{d}{dt}(r_0 \cos\theta) = -(r_0 \sin\theta)\dot{\theta}$$

Transverse component of velocity.

$$v_\theta = r\dot{\theta} = (r_0 \cos\theta)\dot{\theta}$$

Speed.

$$v = \sqrt{v_r^2 + v_\theta^2} = r_0\dot{\theta} = \frac{r_0 v_0}{r_0 \cos^2\theta} = \frac{v_0}{\cos^2\theta}$$

Tangential component of acceleration.

$$a_t = \frac{dv}{dt} = v_0 \frac{(-2)(-\sin\theta)\dot{\theta}}{\cos^3\theta} = \frac{2v_0 \sin\theta}{\cos^3\theta} \cdot \frac{v_0}{r_0 \cos^2\theta}$$

$$= \frac{2v_0^2 \sin\theta}{r_0 \cos^5\theta}$$

Tangential component of force.

$$F_t = ma_t: \quad F_t = \frac{2mv_0^2 \sin\theta}{r_0 \cos^5\theta}$$

(*a*) $\theta = 0, \quad F_t = 0$ $\qquad F_t = 0$ ◀

(*b*) $\theta = 45°, \quad F_t = \dfrac{2mv_0 \sin 45°}{\cos^5 45°}$ $\qquad F_t = \dfrac{8mv_0^2}{r_0}$ ◀

PROBLEM 12.78

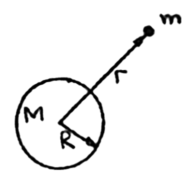
Determine the mass of the earth knowing that the mean radius of the moon's orbit about the earth is 238,910 mi and that the moon requires 27.32 days to complete one full revolution about the earth.

SOLUTION

We have $$F = G\frac{Mm}{r^2} \quad \text{[Eq. (12.28)]}$$

and $$F = F_n = ma_n = m\frac{v^2}{r}$$

Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$



or $$M = \frac{r}{G}v^2$$

Now $$v = \frac{2\pi r}{\tau}$$

so that $$M = \frac{r}{G}\left(\frac{2\pi r}{\tau}\right)^2 = \frac{1}{G}\left(\frac{2\pi}{\tau}\right)^2 r^3$$

Noting that $$\tau = 27.32\,\text{days} = 2.3604 \times 10^6\,\text{s}$$

and $$r = 238{,}910\text{ mi} = 1.26144 \times 10^9\,\text{ft}$$

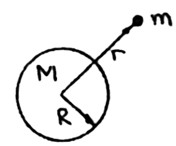
we have $$M = \frac{1}{34.4 \times 10^{-9}\,\text{ft}^4/\text{lb}\cdot\text{s}^4}\left(\frac{2\pi}{2.3604 \times 10^6\,\text{s}}\right)^2 (1.26144 \times 10^9\,\text{ft})^3$$

or $$M = 413 \times 10^{21}\,\text{lb}\cdot\text{s}^2/\text{ft} \blacktriangleleft$$

PROBLEM 12.79

Show that the radius r of the moon's orbit can be determined from the radius R of the earth, the acceleration of gravity g at the surface of the earth, and the time τ required for the moon to complete one full revolution about the earth. Compute r knowing that $\tau = 27.3$ days, giving the answer in both SI and U.S. customary units.

SOLUTION



We have $$F = G\frac{Mm}{r^2} \qquad \text{[Eq. (12.28)]}$$

and $$F = F_n = ma_n = m\frac{v^2}{r}$$

Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or $$v^2 = \frac{GM}{r}$$

Now $$GM = gR^2 \qquad \text{[Eq. (12.30)]}$$

so that $$v^2 = \frac{gR^2}{r} \quad \text{or} \quad v = R\sqrt{\frac{g}{r}}$$

For one orbit, $$\tau = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$$

or $$r = \left(\frac{g\tau^2 R^2}{4\pi^2}\right)^{1/3} \qquad \text{Q.E.D.}$$ ◀

Now $$\tau = 27.3 \text{ days} = 2.35872\times10^6 \text{ s}$$
$$R = 3960 \text{ mi} = 20.9088\times10^6 \text{ ft}$$

SI: $$r = \left[\frac{9.81 \text{ m/s}^2 \times (2.35872\times10^6 \text{ s})^2 \times (6.37\times10^6 \text{ m})^2}{4\pi^2}\right]^{1/3} = 382.81\times10^6 \text{ m}$$

or $$r = 383\times10^3 \text{ km}$$ ◀

U.S. customary units:

$$r = \left[\frac{32.2 \text{ ft/s}^2 \times (2.35872\times10^6 \text{ s})^2 \times (20.9088\times10^6 \text{ ft})^2}{4\pi^2}\right]^{1/3} = 1256.52\times10^6 \text{ ft}$$

or $$r = 238\times10^3 \text{ mi}$$ ◀

PROBLEM 12.80

Communication satellites are placed in a geosynchronous orbit, i.e., in a circular orbit such that they complete one full revolution about the earth in one sidereal day (23.934 h), and thus appear stationary with respect to the ground. Determine (*a*) the altitude of these satellites above the surface of the earth, (*b*) the velocity with which they describe their orbit. Give the answers in both SI and U.S. customary units.

SOLUTION

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let τ be the period time to complete one orbit.

But $\qquad v\tau = 2\pi r \qquad \text{or} \qquad v^2\tau^2 = \frac{GM\tau^2}{r} = 4\pi^2 r^2$

Then $\qquad r^3 = \frac{GM\tau^2}{4\pi^2} \qquad \text{or} \qquad r = \left(\frac{GM\tau^2}{4\pi^2}\right)^{1/3}$

<u>Data</u>: $\qquad \tau = 23.934 \text{ h} = 86.1624\times 10^3 \text{ s}$

(*a*) In SI units: $\qquad g = 9.81 \text{ m/s}^2, \; R = 6.37\times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)(6.37\times 10^6)^2 = 398.06\times 10^{12} \text{ m}^3/\text{s}^2$$

$$r = \left[\frac{(398.06\times 10^{12})(86.1624\times 10^3)^2}{4\pi^2}\right]^{1/3} = 42.145\times 10^6 \text{ m}$$

altitude $h = r - R = 35.775\times 10^6$ m $\qquad h = 35,800$ km ◀

In U.S. units: $\qquad g = 32.2 \text{ ft/s}^2, \; R = 3960 \text{ mi} = 20.909\times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)(20.909\times 10^6)^2 = 14.077\times 10^{15} \text{ ft}^3/\text{s}^2$$

$$r = \left[\frac{(14.077\times 10^{15})(86.1624\times 10^3)^2}{4\pi^2}\right]^{1/3} = 138.334\times 10^6 \text{ ft}$$

altitude $h = r - R = 117.425\times 10^6$ ft $\qquad h = 22,200$ mi ◀

PROBLEM 12.80 (Continued)

(*b*) In SI units:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{398.06\times10^{12}}{42.145\times10^{6}}} = 3.07\times10^{3} \text{ m/s} \qquad v = 3.07 \text{ km/s} \blacktriangleleft$$

In U.S. units:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{14.077\times10^{15}}{138.334\times10^{6}}} = 10.09\times10^{3} \text{ ft/s} \qquad v = 10.09\times10^{3} \text{ ft/s} \blacktriangleleft$$

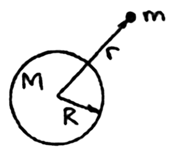
PROBLEM 12.81

Show that the radius r of the orbit of a moon of a given planet can be determined from the radius R of the planet, the acceleration of gravity at the surface of the planet, and the time τ required by the moon to complete one full revolution about the planet. Determine the acceleration of gravity at the surface of the planet Jupiter knowing that $R = 71{,}492$ km and that $\tau = 3.551$ days and $r = 670.9 \times 10^3$ km for its moon Europa.

SOLUTION

We have $$F = G\frac{Mm}{r^2} \qquad \text{[Eq. (12.28)]}$$

and $$F = F_n = ma_n = m\frac{v^2}{r}$$



Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or $$v^2 = \frac{GM}{r}$$

Now $$GM = gR^2 \qquad \text{[Eq. (12.30)]}$$

so that $$v^2 = \frac{gR^2}{r} \qquad \text{or} \qquad v = R\sqrt{\frac{g}{r}}$$

For one orbit, $$\tau = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$$

or $$r = \left(\frac{g\tau^2R^2}{4\pi^2}\right)^{1/3} \qquad \text{Q.E.D.} \blacktriangleleft$$

Solving for g, $$g = 4\pi^2\frac{r^3}{\tau^2R^2}$$

and noting that $\tau = 3.551\,\text{days} = 306{,}806$ s, then

$$g_{\text{Jupiter}} = 4\pi^2\frac{r_{\text{Eur}}^3}{\tau_{\text{Eur}}^2 R_{\text{Jup}}}$$

$$= 4\pi^2\frac{(670.9\times10^6\,\text{m})^3}{(306{,}806\text{ s})^2(71.492\times10^6\,\text{m})^2}$$

or $$g_{\text{Jupiter}} = 24.8\text{ m/s}^2 \blacktriangleleft$$

Note: $$g_{\text{Jupiter}} \approx 2.53g_{\text{Earth}}$$

PROBLEM 12.82

The orbit of the planet Venus is nearly circular with an orbital velocity of 126.5×10^3 km/h. Knowing that the mean distance from the center of the sun to the center of Venus is 108×10^6 km and that the radius of the sun is 695×10^3 km, determine (*a*) the mass of the sun, (*b*) the acceleration of gravity at the surface of the sun.

SOLUTION

Let M be the mass of the sun and m the mass of Venus.

For the circular orbit of Venus,

$$\frac{GMm}{r^2} = ma_n = \frac{mv^2}{r} \qquad GM = rv^2$$

where r is radius of the orbit.

Data: $$r = 108\times10^6 \text{ km} = 108\times10^9 \text{ m}$$

$$v = 126.5\times10^3 \text{ km/hr} = 35.139\times10^3 \text{ m/s}$$

$$GM = (108\times10^9)(35.139\times10^3)^2 = 1.3335\times10^{20} \text{ m}^3/\text{s}^2$$

(*a*) Mass of sun. $$M = \frac{GM}{G} = \frac{1.3335\times10^{20} \text{ m}^3/\text{s}^2}{66.73\times10^{-12}}$$ $$M = 1.998\times10^{30} \text{ kg} \blacktriangleleft$$

(*b*) At the surface of the sun, $$R = 695.5\times10^3 \text{ km} = 695.5\times10^6 \text{ m}$$

$$\frac{GMm}{R^2} = mg$$

$$g = \frac{GM}{R^2} = \frac{1.3335\times10^{20}}{(695.5\times10^6)^2}$$ $$g = 276 \text{ m/s}^2 \blacktriangleleft$$

PROBLEM 12.83

A satellite is placed into a circular orbit about the planet Saturn at an altitude of 2100 mi. The satellite describes its orbit with a velocity of 54.7×10^3 mi/h. Knowing that the radius of the orbit about Saturn and the periodic time of Atlas, one of Saturn's moons, are 85.54×10^3 mi and 0.6017 days, respectively, determine (*a*) the radius of Saturn, (*b*) the mass of Saturn. (The *periodic time* of a satellite is the time it requires to complete one full revolution about the planet.)

SOLUTION

Velocity of Atlas. $$v_A = \frac{2\pi r_A}{\tau_A}$$

where $$v_A = 85.54\times10^3 \text{ mi} = 451.651\times10^6 \text{ ft}$$

and $$\tau_A = 0.6017 \text{ days} = 51{,}987 \text{ s}$$

$$v_A = \frac{(2\pi)(451.651\times10^6)}{51{,}987} = 54.587\times10^3 \text{ ft/s}$$

Gravitational force. $$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

from which $$GM = rv^2 = \text{constant}$$

For the satellite, $$r_s v_s^2 = r_A v_A^2$$

$$r_s = \frac{r_A v_A^2}{v_s{}^2}$$

where $$v_s = 54.7\times10^3 \text{mi/h} = 80.227\times10^3 \text{ ft/s}$$

$$r_s = \frac{(451.651\times10^6)(54.587\times10^3)^2}{(80.227\times10^3)^2} = 209.09\times10^6 \text{ ft}$$

$$r_s = 39{,}600 \text{ mi}$$

(*a*) Radius of Saturn.

$$R = r_s - (\text{altitude}) = 39{,}600 - 2100 \qquad R = 37{,}500 \text{ mi} \blacktriangleleft$$

(*b*) Mass of Saturn.

$$M = \frac{r_A v_A^2}{G} = \frac{(451.651\times10^6)(54.587\times10^3)^2}{34.4\times10^{-9}}$$

$$M = 39.1\times10^{24} \text{ lb}\cdot\text{s}^2/\text{ft} \blacktriangleleft$$

PROBLEM 12.84

The periodic times (see Problem 12.83) of the planet Uranus's moons Juliet and Titania have been observed to be 0.4931 days and 8.706 days, respectively. Knowing that the radius of Juliet's orbit is 40,000 mi, determine (*a*) the mass of Uranus, (*b*) the radius of Titania's orbit.

SOLUTION

Velocity of Juliet. $$v_J = \frac{2\pi r_J}{\tau_J}$$

where $$r_J = 40{,}000 \text{ mi} = 2.112\times10^8 \text{ ft}$$

and $$\tau_J = 0.4931 \text{ days} = 42{,}604 \text{ s}$$

$$v_J = \frac{(2\pi)(2.112\times10^8 \text{ ft})}{42{,}604 \text{ s}} = 3.11476\times10^4 \text{ ft/s}$$

Gravitational force. $$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

from which $$GM = rv^2 = \text{constant}$$

(*a*) Mass of Uranus. $$M = \frac{r_J v_J^2}{G}$$

$$M = \frac{(2.112\times10^8)(3.11476\times10^4)^2}{34.4\times10^{-9}}$$
$$= 5.95642\times10^{24} \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$M = 5.96\times10^{24} \text{ lb}\cdot\text{s}^2/\text{ft} \blacktriangleleft$$

(*b*) Radius of Titania's orbit.

$$GM = r_T v_T^2 = \frac{4\pi^2 r_T^3}{\tau_T{}^2} = \frac{4\pi^2 r_J^3}{\tau_J{}^2}$$

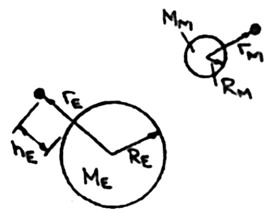
$$r_T^3 = r_J^3\left(\frac{\tau_T}{\tau_J}\right)^2 = (2.112\times10^8)^3\left(\frac{8.706}{0.4931}\right)^2 = 2.93663\times10^{27} \text{ ft}^3$$

$$r_T = 1.43202\times10^9 \text{ ft} = 2.71216\times10^5 \text{ mi} \qquad r_T = 2.71\times10^5 \text{ mi} \blacktriangleleft$$

PROBLEM 12.85

A 500 kg spacecraft first is placed into a circular orbit about the earth at an altitude of 4500 km and then is transferred to a circular orbit about the moon. Knowing that the mass of the moon is 0.01230 times the mass of the earth and that the radius of the moon is 1737 km, determine (*a*) the gravitational force exerted on the spacecraft as it was orbiting the earth, (*b*) the required radius of the orbit of the spacecraft about the moon if the periodic times (see Problem 12.83) of the two orbits are to be equal, (*c*) the acceleration of gravity at the surface of the moon.

SOLUTION



First note that $R_E = 6.37\times10^6$ m

Then
$$r_E = R_E + h_E = (6.37\times10^6 + 4.5\times10^6)\text{ m}$$
$$= 10.87\times10^6\text{ m}$$

(*a*) We have $F = \dfrac{GMm}{r^2}$ [Eq. (12.28)]

and $GM = gR^2$ [Eq. (12.29)]

Then
$$F = gR^2\frac{m}{r^2} = W\left(\frac{R}{r}\right)^2$$

For the earth orbit,
$$F = (500\text{ kg})(9.81\text{ m/s}^2)\left(\frac{6.37\times10^6\text{ m}}{10.87\times10^6\text{ m}}\right)^2$$

or $F = 1684$ N ◀

(*b*) From the solution to Problem 12.78, we have
$$M = \frac{1}{G}\left(\frac{2\pi}{\tau}\right)^2 r^3$$

Then
$$\tau = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Now
$$\tau_E = \tau_M \Rightarrow \frac{2\pi r_E^{3/2}}{\sqrt{GM_E}} = \frac{2\pi r_M^{3/2}}{\sqrt{GM_M}} \qquad (1)$$

or
$$r_M = \left(\frac{M_M}{M_E}\right)^{1/3} r_E = (0.01230)^{1/3}(10.87\times10^6\text{ m})$$

or $r_M = 2.509\times10^6$ m $\qquad r_M = 2510$ km ◀

PROBLEM 12.85 (Continued)

(*c*) We have $GM = gR^2$ [Eq.(12.29)]

Substituting into Eq. (1)

$$\frac{2\pi r_E^{3/2}}{R_E\sqrt{g_E}} = \frac{2\pi r_M^{3/2}}{R_M\sqrt{g_M}}$$

or

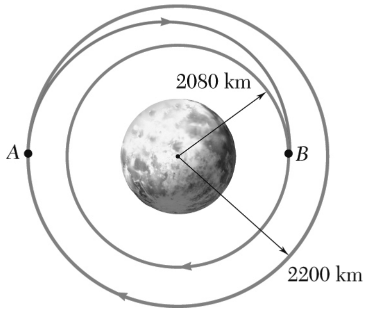
$$g_M = \left(\frac{R_E}{R_M}\right)^2\left(\frac{r_M}{r_E}\right)^3 g_E = \left(\frac{R_E}{R_M}\right)^2\left(\frac{M_M}{M_E}\right)g_E$$

using the results of Part (b). Then

$$g_M = \left(\frac{6370\text{ km}}{1737\text{ km}}\right)^2(0.01230)(9.81\text{ m/s}^2)$$

or $g_{\text{moon}} = 1.62\text{ m/s}^2$ ◀

Note: $g_{\text{moon}} \approx \frac{1}{6}g_{\text{earth}}$



PROBLEM 12.86

A space vehicle is in a circular orbit of 2200-km radius around the moon. To transfer it to a smaller circular orbit of 2080-km radius, the vehicle is first placed on an elliptic path AB by reducing its speed by 26.3 m/s as it passes through A. Knowing that the mass of the moon is 73.49×10^{21} kg, determine (*a*) the speed of the vehicle as it approaches B on the elliptic path, (*b*) the amount by which its speed should be reduced as it approaches B to insert it into the smaller circular orbit.

SOLUTION

For a circular orbit, $\Sigma F_n = ma_n$: $F = m\dfrac{v^2}{r}$

Eq. (12.28): $$F = G\frac{Mm}{r^2}$$

Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or $$v^2 = \frac{GM}{r}$$

Then $$(v_A)^2_{\text{circ}} = \frac{66.73\times10^{-12}\ \text{m}^3/\text{kg}\cdot\text{s}^2 \times 73.49\times10^{21}\ \text{kg}}{2200\times10^3\ \text{m}}$$

or $$(v_A)_{\text{circ}} = 1493.0\ \text{m/s}$$

and $$(v_B)^2_{\text{circ}} = \frac{66.73\times10^{-12}\ \text{m}^3/\text{kg}\cdot\text{s}^2 \times 73.49\times10^{21}\ \text{kg}}{2080\times10^3\ \text{m}}$$

or $$(v_B)_{\text{circ}} = 1535.5\ \text{m/s}$$

(*a*) We have
$$(v_A)_{TR} = (v_A)_{\text{circ}} + \Delta v_A$$
$$= (1493.0 - 26.3)\ \text{m/s}$$
$$= 1466.7\ \text{m/s}$$

Conservation of angular momentum requires that
$$r_A m(v_A)_{TR} = r_B m(v_B)_{TR}$$

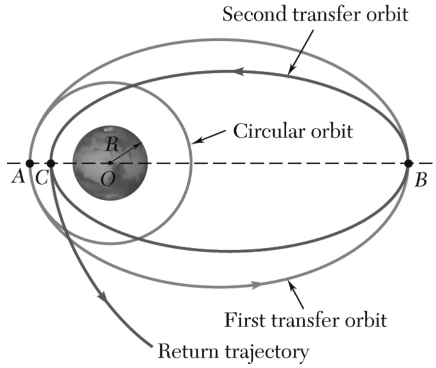
or $$(v_B)_{TR} = \frac{2200\ \text{km}}{2080\ \text{km}} \times 1466.7\ \text{m/s}$$
$$= 1551.3\ \text{m/s}$$

or $$(v_B)_{TR} = 1551\ \text{m/s} \blacktriangleleft$$

(*b*) Now $$(v_B)_{\text{circ}} = (v_B)_{TR} + \Delta v_B$$

or $$\Delta v_B = (1535.5 - 1551.3)\ \text{m/s}$$

or $$\Delta v_B = -15.8\ \text{m/s} \blacktriangleleft$$



PROBLEM 12.87

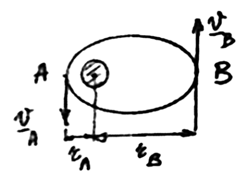
Plans for an unmanned landing mission on the planet Mars called for the earth-return vehicle to first describe a circular orbit at an altitude d_A = 2200 km above the surface of the planet with a velocity of 2771 m/s. As it passed through Point A, the vehicle was to be inserted into an elliptic transfer orbit by firing its engine and increasing its speed by $\Delta v_A = 1046$ m/s. As it passed through Point B, at an altitude d_B = 100,000 km, the vehicle was to be inserted into a second transfer orbit located in a slightly different plane, by changing the direction of its velocity and reducing its speed by $\Delta v_B = -22.0$ m/s. Finally, as the vehicle passed through Point C, at an altitude d_C = 1000 km, its speed was to be increased by $\Delta v_C = 660$ m/s to insert it into its return trajectory. Knowing that the radius of the planet Mars is $R = 3400$ km, determine the velocity of the vehicle after completion of the last maneuver.

SOLUTION

$$r_A = 3400 + 2200 = 5600 \text{ km} = 5.60\times10^6 \text{ m}$$
$$r_B = 3400 + 100{,}000 = 103{,}400 \text{ km} = 103.4\times10^6 \text{ m}$$
$$r_C = 3400 + 1000 = 4400 \text{ km} = 4.40\times10^6 \text{ m}$$

<u>First transfer orbit</u>.

$$v_A = 2771 \text{ m/s} + 1046 \text{ m/s} = 3817 \text{ m/s}$$

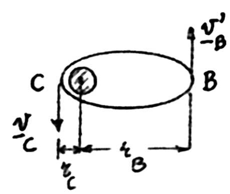


Conservation of angular momentum:

$$r_A m v_A = r_B m v_B$$
$$(5.60\times10^6)(3817) = (103.4\times10^6)v_B$$
$$v_B = 206.7 \text{ m/s}$$

<u>Second transfer orbit</u>.

$$v_B' = v_B + \Delta v_B$$
$$= 206.7 - 22.0 = 184.7 \text{ m/s}$$



Conservation of angular momentum:

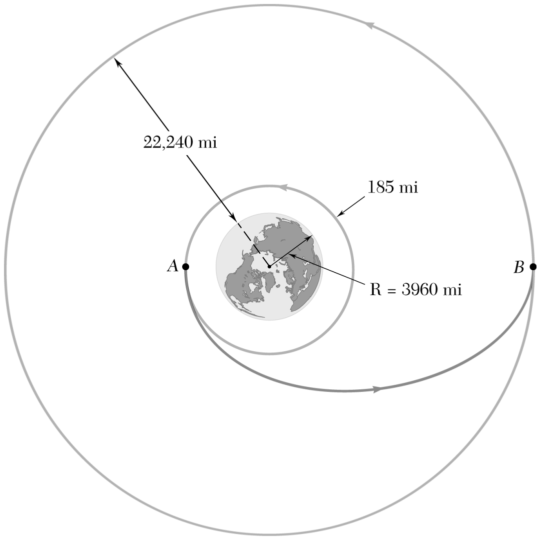
$$r_B m v_B' = r_C m v_C$$
$$(103.4\times10^6)(184.7) = (4.40\times10^6)v_C$$
$$v_C = 4340 \text{ m/s}$$

<u>After last maneuver</u>.

$$v = v_C + \Delta v_C = 4340 + 660$$

$v = 5000$ m/s ◀

PROBLEM 12.88



To place a communications satellite into a geosynchronous orbit (see Problem 12.80) at an altitude of 22,240 mi above the surface of the earth, the satellite first is released from a space shuttle, which is in a circular orbit at an altitude of 185 mi, and then is propelled by an upper-stage booster to its final altitude. As the satellite passes through *A*, the booster's motor is fired to insert the satellite into an elliptic transfer orbit. The booster is again fired at *B* to insert the satellite into a geosynchronous orbit. Knowing that the second firing increases the speed of the satellite by 4810 ft/s, determine (*a*) the speed of the satellite as it approaches *B* on the elliptic transfer orbit, (*b*) the increase in speed resulting from the first firing at *A*.

SOLUTION

For earth, $R = 3960 \text{ mi} = 20.909\times10^6 \text{ ft}$

$$GM = gR^2 = (32.2)(20.909\times10^6)^2 = 14.077\times10^{15} \text{ ft}^3/\text{s}^2$$

$$r_A = 3960 + 185 = 4145 \text{ mi} = 21.8856\times10^6 \text{ ft}$$

$$r_B = 3960 + 22,240 = 26,200 \text{ mi} = 138.336\times10^6 \text{ ft}$$

Speed on circular orbit through *A*.

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077\times10^{15}}{21.8856\times10^6}} = 25.362\times10^3 \text{ ft/s}$$

Speed on circular orbit through *B*.

$$(v_B)_{\text{circ}} = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{14.077\times10^{15}}{138.336\times10^6}} = 10.088\times10^3 \text{ ft/s}$$

PROBLEM 12.88 (Continued)

(*a*) Speed on transfer trajectory at *B*.

$$(v_B)_{\text{tr}} = 10.088\times 10^3 - 4810$$
$$= 5.278\times 10^3$$

5280 ft/s ◀

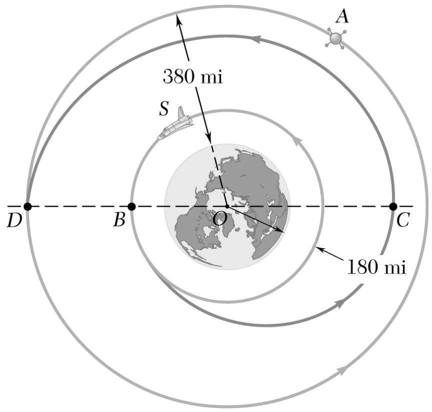
Conservation of angular momentum for transfer trajectory.

$$r_A(v_A)_{\text{tr}} = r_B(v_B)_{\text{tr}}$$
$$(v_A)_{\text{tr}} = \frac{r_B(v_B)_{\text{tr}}}{r_A}$$
$$= \frac{(138.336\times 10^6)(5278)}{21.8856\times 10^6}$$
$$= 33.362\times 10^3 \text{ ft/s}$$

(*b*) Change in speed at *A*.

$$\Delta v_A = (v_A)_{\text{tr}} - (v_A)_{\text{circ}}$$
$$= 33.362\times 10^3 - 25.362\times 10^3$$
$$= 8.000\times 10^3$$

$\Delta v_A = 8000$ ft/s ◀



PROBLEM 12.89

A space shuttle S and a satellite A are in the circular orbits shown. In order for the shuttle to recover the satellite, the shuttle is first placed in an elliptic path BC by increasing its speed by $\Delta v_B = 280$ ft/s as it passes through B. As the shuttle approaches C, its speed is increased by $\Delta v_C = 260$ ft/s to insert it into a second elliptic transfer orbit CD. Knowing that the distance from O to C is 4289 mi, determine the amount by which the speed of the shuttle should be increased as it approaches D to insert it into the circular orbit of the satellite.

SOLUTION

First note
$$R = 3960 \text{ mi} = 20.9088\times10^6 \text{ ft}$$
$$r_A = (3960+380) \text{ mi} = 4340 \text{ mi} = 22.9152\times10^6 \text{ ft}$$
$$r_B = (3960+180) \text{ mi} = 4140 \text{ mi} = 21.8592\times10^6 \text{ ft}$$

For a circular orbit,
$$\Sigma F_n = ma_n: \qquad F = m\frac{v^2}{r}$$

Eq. (12.28):
$$F = G\frac{Mm}{r^2}$$

Then
$$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or
$$v^2 = \frac{GM}{r} = \frac{gR^2}{r} \qquad \text{using Eq. (12.29).}$$

Then
$$(v_A)^2_{\text{circ}} = \frac{32.2 \text{ ft/s}^2 \times (20.9088\times10^6 \text{ ft})^2}{22.9152\times10^6 \text{ ft}}$$

or
$$(v_A)_{\text{circ}} = 24{,}785 \text{ ft/s}$$

and
$$(v_B)^2_{\text{circ}} = \frac{32.2 \text{ ft/s}^2 \times (20.9088\times10^6 \text{ ft})^2}{21.8592\times10^6 \text{ ft}}$$

or
$$(v_B)_{\text{circ}} = 25{,}377 \text{ ft/s}$$

We have
$$(v_B)_{TR_{BC}} = (v_B)_{\text{circ}} + \Delta v_B = (25{,}377+280) \text{ ft/s}$$
$$= 25{,}657 \text{ ft/s}$$

PROBLEM 12.89 (Continued)

Conservation of angular momentum requires that

$$BC: \quad r_B\, m(v_B)_{TR_{BC}} = r_C\, m(v_C)_{TR_{BC}} \tag{1}$$

$$CD: \quad r_C\, m(v_C)_{TR_{CD}} = r_A\, m(v_D)_{TR_{CD}} \tag{2}$$

From Eq. (1)
$$(v_C)_{TR_{BC}} = \frac{r_B}{r_C}(v_B)_{TR_{BC}} = \frac{4140 \text{ mi}}{4289 \text{ mi}} \times 25{,}657 \text{ ft/s}$$
$$= 24{,}766 \text{ ft/s}$$

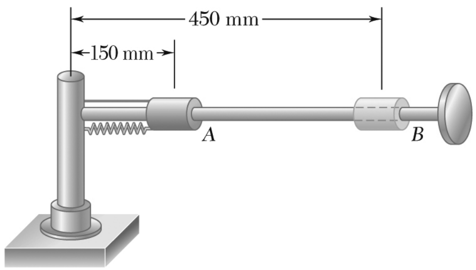
Now
$$(v_C)_{TR_{CD}} = (v_C)_{TR_{BC}} + \Delta v_C = (24{,}766 + 260) \text{ ft/s}$$
$$= 25{,}026 \text{ ft/s}$$

From Eq. (2)
$$(v_D)_{TR_{CD}} = \frac{r_C}{r_A}(v_C)_{TR_{CD}} = \frac{4289 \text{ mi}}{4340 \text{ mi}} \times 25{,}026 \text{ ft/s}$$
$$= 24{,}732 \text{ ft/s}$$

Finally,
$$(v_A)_{\text{circ}} = (v_D)_{TR_{CD}} + \Delta v_D$$

or
$$\Delta v_D = (24{,}785 - 24{,}732) \text{ ft/s}$$

or
$$\Delta v_D = 53 \text{ ft/s} \blacktriangleleft$$

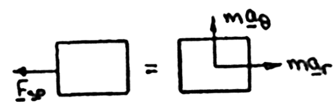


PROBLEM 12.90

A 1 kg collar can slide on a horizontal rod, which is free to rotate about a vertical shaft. The collar is initially held at *A* by a cord attached to the shaft. A spring of constant 30 N/m is attached to the collar and to the shaft and is undeformed when the collar is at *A*. As the rod rotates at the rate $\dot{\theta} = 16$ rad/s, the cord is cut and the collar moves out along the rod. Neglecting friction and the mass of the rod, determine (*a*) the radial and transverse components of the acceleration of the collar at *A*, (*b*) the acceleration of the collar relative to the rod at *A*, (*c*) the transverse component of the velocity of the collar at *B*.

SOLUTION

First note $\qquad F_{sp} = k(r - r_A)$



(*a*) $\quad F_\theta = 0$ and at *A*, $\qquad F_r = -F_{sp} = 0$

$$(a_A)_r = 0 \blacktriangleleft$$

$$(a_A)_\theta = 0 \blacktriangleleft$$

(*b*) $\quad \xrightarrow{+} \Sigma F_r = ma_r: \qquad -F_{sp} = m(\ddot{r} - r\dot{\theta}^2)$

Noting that $\qquad a_{\text{collar/rod}} = \ddot{r}, \quad$ we have at *A*

$$0 = m[a_{\text{collar/rod}} - (150 \text{ mm})(16 \text{ rad/s})^2]$$

$$a_{\text{collar/rod}} = 38400 \text{ mm/s}^2$$

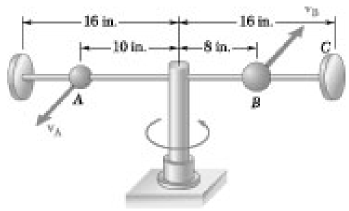
or $\qquad (a_{\text{collar/rod}})_A = 38.4 \text{ m/s}^2 \blacktriangleleft$

(*c*) After the cord is cut, the only horizontal force acting on the collar is due to the spring. Thus, angular momentum about the shaft is conserved.

$$r_A\, m(v_A)_\theta = r_B\, m(v_B)_\theta \quad \text{where} \quad (v_A)_\theta = r_A\,\dot{\theta}_0$$

Then $\qquad (v_B)_\theta = \dfrac{150 \text{ mm}}{450 \text{ mm}}[(150 \text{ mm})(16 \text{ rad/s})] = 800 \text{ mm/s}$

or $\qquad (v_B)_\theta = 0.800 \text{ m/s} \blacktriangleleft$



PROBLEM 12.91

A 1-lb ball A and a 2-lb ball B are mounted on a horizontal rod which rotates freely about a vertical shaft. The balls are held in the positions shown by pins. The pin holding B is suddenly removed and the ball moves to position C as the rod rotates. Neglecting friction and the mass of the rod and knowing that the initial speed of A is $v_A = 8$ ft/s, determine (a) the radial and transverse components of the acceleration of ball B immediately after the pin is removed, (b) the acceleration of ball B relative to the rod at that instant, (c) the speed of ball A after ball B has reached the stop at C.

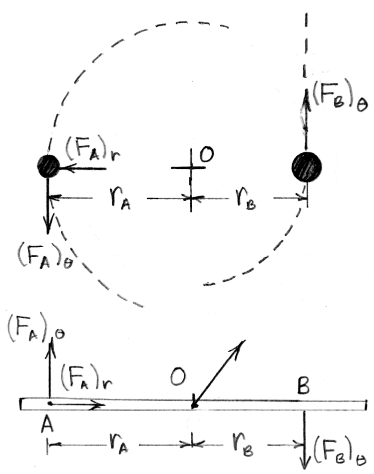
SOLUTION

Let r and θ be polar coordinates with the origin lying at the shaft.

Constraint of rod: $\theta_B = \theta_A + \pi$ radians; $\dot{\theta}_B = \dot{\theta}_A = \dot{\theta}$; $\ddot{\theta}_B = \ddot{\theta}_A = \ddot{\theta}$.

(a) Components of acceleration

Sketch the free body diagrams of the balls showing the radial and transverse components of the forces acting on them. Owing to frictionless sliding of B along the rod, $(F_B)_r = 0$.



Radial component of acceleration of B.

$$F_r = m_B(a_B)_r: \qquad (a_B)_r = 0 \blacktriangleleft$$

Transverse components of acceleration.

$$(a_A)_\theta = r_A\ddot{\theta} + 2\dot{r}_A\dot{\theta} = ra\ddot{\theta}$$

$$(a_B)_\theta = r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta} \qquad (1)$$

Since the rod is massless, it must be in equilibrium. Draw its free body diagram, applying Newton's 3rd Law.

$$+\circlearrowleft \Sigma M_0 = 0: \quad r_A(F_A)_\theta + r_B(F_B)_\theta = r_A m_A(a_A)_\theta + r_B m_B(a_B)_\theta = 0$$

$$r_A m_A r_A\ddot{\theta} + r_B m_B(r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta}) = 0$$

$$\ddot{\theta} = \frac{-2m_B\dot{r}_B\dot{\theta}}{m_A r_A^2 + m_B r_B^2}$$

At $t = 0$, $\quad \dot{r}_B = 0 \quad$ so that $\quad \ddot{\theta} = 0.$

From Eq. (1), $\qquad (a_B)_\theta = 0 \blacktriangleleft$

(*b*) Acceleration of *B* relative to the rod.

At $t = 0$, $(v_A)_\theta = 8 \text{ ft/s} = 96 \text{ in./s}$, $\dot{\theta} = \dfrac{(v_A)_\theta}{r_A} = \dfrac{96}{10} = 9.6 \text{ rad/s}$

$$\ddot{r}_B - r_B\dot{\theta}^2 = (a_B)_r = 0$$

$$\ddot{r}_B = r_B\dot{\theta}^2 = (8)(9.6)^2 = 737.28 \text{ in./s}^2$$

$$\ddot{r}_B = 61.4 \text{ ft/s}^2 \blacktriangleleft$$

(*c*) Speed of *A*.

Substituting $\dfrac{d}{dt}(mr^2\dot{\theta})$ for rF_θ in each term of the moment equation gives

$$\frac{d}{dt}\left(m_A r_A{}^2\dot{\theta}\right) + \frac{d}{dt}\left(m_B r_B{}^2\dot{\theta}\right) = 0$$

Integrating with respect to time,

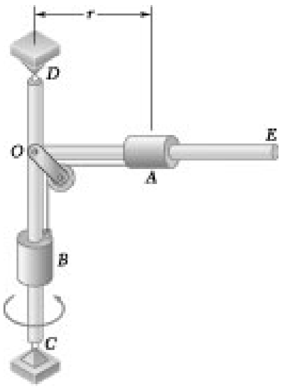
$$m_A r_A{}^2\dot{\theta} + m_B r_B{}^2\dot{\theta} = \left(m_A r_A{}^2\dot{\theta}\right)_0 + \left(m_B r_B{}^2\dot{\theta}\right)_0$$

Applying to the final state with ball *B* moved to the stop at *C*,

$$\left(\frac{W_A}{g}r_A^2 + \frac{W_B}{g}r_C^2\right)\dot{\theta}_f = \left[\frac{W_A}{g}r_A^2 + \frac{W_B}{g}(r_B)_0^2\right]\dot{\theta}_0$$

$$\dot{\theta}_f = \frac{W_A r_A{}^2 + W_B(r_B)_0^2}{W_A r_A{}^2 + W_B r_C{}^2}\dot{\theta}_0 = \frac{(1)(10)^2 + (2)(8)^2}{(1)(10)^2 + (2)(16)^2}(9.6) = 3.5765 \text{ rad/s}$$

$$(v_A)_f = r_A\dot{\theta}_f = (10)(3.5765) = 35.765 \text{ in./s} \qquad (v_A)_f = 2.98 \text{ ft/s} \blacktriangleleft$$



PROBLEM 12.92

Two 2.6-lb collars A and B can slide without friction on a frame, consisting of the horizontal rod OE and the vertical rod CD, which is free to rotate about CD. The two collars are connected by a cord running over a pulley that is attached to the frame at O and a stop prevents collar B from moving. The frame is rotating at the rate $\dot{\theta} = 12$ rad/s and $r = 0.6$ ft when the stop is removed allowing collar A to move out along rod OE. Neglecting friction and the mass of the frame, determine, for the position $r = 1.2$ ft, (*a*) the transverse component of the velocity of collar A, (*b*) the tension in the cord and the acceleration of collar A relative to the rod OE.

SOLUTION

Masses: $m_A = m_B = \dfrac{2.6}{32.2} = 0.08075\ \text{lb}\cdot\text{s}^2/\text{ft}$

(*a*) Conservation of angular momentum of collar A: $(H_0)_2 = (H_0)_1$

$$m_A r_1 (v_\theta)_1 = m_A r_2 (v_\theta)_2$$

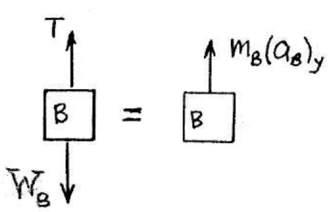
$$(v_\theta)_2 = \frac{r_1 (v_\theta)_1}{r_2} = \frac{r_1^2 \dot{\theta}_1}{r_2} = \frac{(0.6)^2(12)}{1.2} = 3.6$$

$$(v_\theta)_2 = 3.60\ \text{ft/s} \ ◀$$

$$\dot{\theta}_2 = \frac{(v_\theta)_2}{r_A} = \frac{3.6}{1.2} = 3.00\ \text{rad/s}$$

(*b*) Let y be the position coordinate of B, positive upward with origin at O.

Constraint of the cord: $r - y = \text{constant}$ or $\ddot{y} = \ddot{r}$



Kinematics:

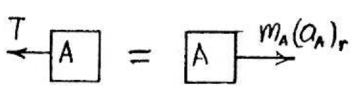
$$(a_B)_y = \ddot{y} = \ddot{r} \quad \text{and} \quad (a_A)_r = \ddot{r} - r\dot{\theta}^2$$

Collar B: $\Sigma F_y = m_B a_B$: $T - W_B = m_B \ddot{y} = m_B \ddot{r}$ (1)

Collar A: $\xrightarrow{+} \Sigma F_r = m_A (a_A)_r$: $-T = m_A(\ddot{r} - r\dot{\theta}^2)$ (2)

Adding (1) and (2) to eliminate T,

$$-W_B = (m_A + m_B)\ddot{r} + m_A r\dot{\theta}^2$$

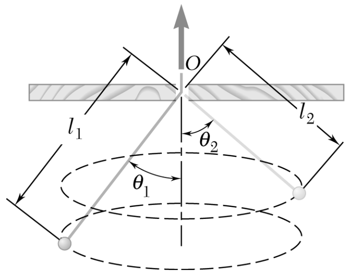


$$a_{A/\text{rod}} = \ddot{r} = \frac{m_A r\dot{\theta}^2 - W_B}{m_A + m_B} = \frac{(0.08075)(1.2)(3.00)^2 - (2.6)}{0.08075 + 0.08075} = -10.70\ \text{ft/s}^2$$

$T = m_B(\ddot{r} + g) = (0.08075)(-10.70 + 32.2)$ $\quad T = 1.736\ \text{lb}$ ◀

$a_{A/\text{rod}} = 10.70\ \text{ft/s}^2$ radially inward. ◀

PROBLEM 12.93



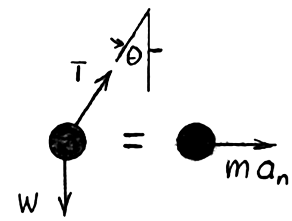
A small ball swings in a horizontal circle at the end of a cord of length l_1, which forms an angle θ_1 with the vertical. The cord is then slowly drawn through the support at O until the length of the free end is l_2. (*a*) Derive a relation among l_1, l_2, θ_1, and θ_2. (*b*) If the ball is set in motion so that initially $l_1 = 0.8$ m and $\theta_1 = 35°$, determine the angle θ_2 when $l_2 = 0.6$ m.

SOLUTION

(*a*) For state 1 or 2, neglecting the vertical component of acceleration,

$$+\uparrow \Sigma F_y = 0: \quad T\cos\theta - W = 0$$

$$T = W\cos\theta$$



$$\xrightarrow{+} \Sigma F_x = ma_n: \quad T\sin\theta = W\sin\theta\cos\theta = \frac{mv^2}{\rho}$$

But $\rho = \ell\sin\theta$ so that

$$v^2 = \frac{\rho W}{m}\sin^2\theta\cos\theta = \ell\, g\,\sin\theta\tan\theta$$

$$v_1 = \sqrt{\ell_1 g\,\sin\theta_1\tan\theta_1}$$

and
$$v_2 = \sqrt{\ell_2\, g\,\sin\theta_2\tan\theta_2}$$

$$\Sigma M_y = 0: \quad H_y = \text{constant}$$

$$r_1 m v_1 = r_2 m v_2 \qquad \text{or} \qquad v_1\ell_1\sin\theta_1 = v_2\ell_2\sin\theta_2$$

$$\ell_1^{3/2} g\,\sin\theta_1\sqrt{\sin\theta_1\tan\theta_1} = \ell_2^{3/2}\,\sin\theta_2\sqrt{\sin\theta_2\tan\theta_2}$$

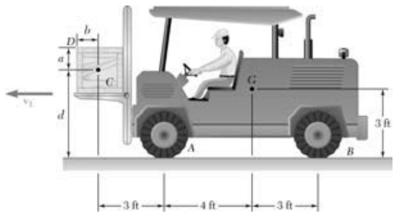
$$\ell_1^3\sin^3\theta_1\tan\theta_1 = \ell_2^3\sin^3\theta_2\tan\theta_2 \quad \blacktriangleleft$$

(*b*) With $\theta_1 = 35°$, $\ell_1 = 0.8$ m, and $\ell_2 = 0.6$ m

$$(0.8)^3\sin^3 35°\tan 35° = (0.6)^3\sin^3\theta_2\tan\theta_2$$

$$\sin^3\theta_2\tan\theta_2 - 0.31320 = 0 \qquad \theta_2 = 43.6° \quad \blacktriangleleft$$

PROBLEM 12.CQ6



A uniform crate C with mass m_C is being transported to the left by a forklift with a constant speed v_1. What is the magnitude of the angular momentum of the crate about Point D, that is, the upper left corner of the crate?

(a) 0

(b) mv_1a

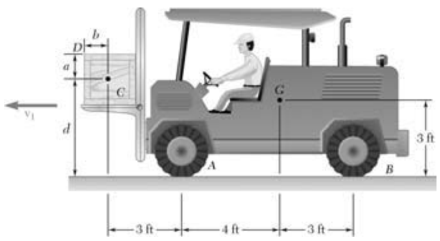
(c) mv_1b

(d) $mv_1\sqrt{a^2+b^2}$

SOLUTION

Answers: (b) The angular momentum is the moment of the momentum, so simply take the linear momentum, mv_1, and multiply it by the perpendicular distance from the line of action of mv_1 and Point D.

PROBLEM 12.CQ7



A uniform crate C with mass m_C is being transported to the left by a forklift with a constant speed v_1. What is the magnitude of the angular momentum of the crate about Point A, that is, the point of contact between the front tire of the forklift and the ground?

(*a*) 0

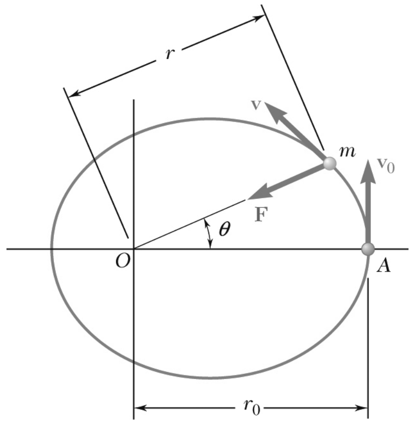
(*b*) mv_1d

(*c*) $3mv_1$

(*d*) $mv_1\sqrt{3^2 + d^2}$

SOLUTION

Answer: (*b*)



PROBLEM 12.94

A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to OA and moves under a central force $\mathbf{F}$ along an elliptic path defined by the equation $r = r_0/(2-\cos\theta)$. Using Eq. (12.37), show that $\mathbf{F}$ is inversely proportional to the square of the distance r from the particle to the center of force O.

SOLUTION

$$u = \frac{1}{r} = \frac{2-\cos\theta}{r_0}, \quad \frac{du}{d\theta} = \frac{\sin\theta}{r_0}, \quad \frac{d^2u}{d\theta^2} = \frac{\cos\theta}{r_0}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{2}{r_0} = \frac{F}{mh^2u^2} \quad \text{by Eq. (12.37).}$$

Solving for F,

$$F = \frac{2mh^2u^2}{r_0} = \frac{2mh^2}{r_0 r^2}$$

Since m, h, and r_0 are constants, F is proportional to $\frac{1}{r^2}$, or inversely proportional to r^2.

PROBLEM 12.95

A particle of mass m describes the logarithmic spiral $r = r_0 e^{b\theta}$ under a central force **F** directed toward the center of force O. Using Eq. (12.37) show that **F** is inversely proportional to the cube of the distance r from the particle to O.

SOLUTION

$$u = \frac{1}{r} = \frac{1}{r_0} e^{-b\theta}$$

$$\frac{du}{d\theta} = -\frac{b}{r_0} e^{-b\theta}$$

$$\frac{d^2u}{d\theta^2} = \frac{b^2}{r_0} e^{-b\theta}$$

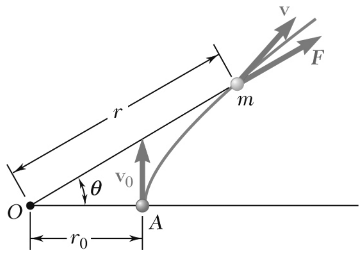
$$\frac{d^2u}{d\theta^2} + u = \frac{b^2 + 1}{r_0} e^{-b\theta} = \frac{F}{mh^2u^2}$$

$$F = \frac{(b^2 + 1)mh^2u^2}{r_0} e^{-b\theta}$$

$$= \frac{(b^2 + 1)mh^2u^2}{r} = \frac{(b^2 + 1)mh^2}{r^3}$$

Since b, m, and h are constants, **F** is proportional to $\frac{1}{r^3}$, or inversely proportional to r^3.

PROBLEM 12.96



For the particle of Problem 12.74, and using Eq. (12.37), show that the central force **F** is proportional to the distance r from the particle to the center of force O.

PROBLEM 12.74 A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force **F** directed away from the center of force O. Knowing that the particle follows a path defined by the equation $r = r_0/\sqrt{\cos 2\theta}$ and using Eq. (12.27), express the radial and transverse components of the velocity **v** of the particle as functions of θ .

SOLUTION

$$u = \frac{1}{r} = \frac{\sqrt{\cos 2\theta}}{r_0}, \quad \frac{du}{d\theta} = -\frac{\sin 2\theta}{r_0\sqrt{\cos 2\theta}}$$

$$\frac{d^2u}{d\theta} = -\frac{\sqrt{\cos 2\theta}(2\cos 2\theta) - \sin 2\theta(-\sin 2\theta/\sqrt{\cos 2\theta})}{r_0 \cos 2\theta}$$

$$= -\frac{2\cos^2 2\theta + \sin^2 2\theta}{r_0(\cos 2\theta)^{3/2}} = -\frac{(1+\cos^2 2\theta)}{r_0(\cos 2\theta)^{3/2}}$$

Eq. (12.37): $\dfrac{d^2u}{d\theta^2} + u = \dfrac{F}{mh^2u^2}$

Solving for F,

$$F = mh^2u^2\left(\frac{d^2u}{d\theta} + u\right)$$

$$= mh^2\frac{\cos 2\theta}{r_0^2}\left[-\frac{1+\cos^2 2\theta}{r_0(\cos 2\theta)^{3/2}} + \frac{\sqrt{\cos 2\theta}}{r_0}\right]$$

$$= mh^2\frac{\cos 2\theta}{r_0^2}\left[-\frac{1}{r_0(\cos 2\theta)^{3/2}} - \frac{\sqrt{\cos 2\theta}}{r_0} + \frac{\sqrt{\cos 2\theta}}{r_0}\right]$$

$$= -\frac{mh^2}{r_0^3\sqrt{\cos 2\theta}} = -\frac{mh^2}{r_0^4}\frac{r_0}{\sqrt{\cos 2\theta}} \qquad F = -\frac{mh^2 r}{r_0^4} \blacktriangleleft$$

The force F is proportional to r. The minus sign indicates that it is repulsive.

PROBLEM 12.97

A particle of mass m describes the path defined by the equation $r = r_0 \sin\theta$ under a central force **F** directed toward the center of force O. Using Eq. (12.37), show that **F** is inversely proportional to the fifth power of the distance r from the particle to O.

SOLUTION

We have $$\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2} \qquad \text{Eq. (12.37)}$$

where $$u = \frac{1}{r} \quad \text{and} \quad mh^2 = \text{constant}$$

$$F \times u^2\left(\frac{d^2u}{d\theta^2} + u\right)$$

Now $$u = \frac{1}{r} = \frac{1}{r_0 \sin\theta}$$

Then $$\frac{du}{d\theta} = \frac{1}{d\theta}\left(\frac{1}{r_0 \sin\theta}\right) = -\frac{1}{r_0}\frac{\cos\theta}{\sin^2\theta}$$

and $$\frac{d^2u}{d\theta^2} = -\frac{1}{r_0}\left[\frac{-\sin\theta(\sin^2\theta) - \cos\theta(2\sin\theta\cos\theta)}{\sin^4\theta}\right]$$

$$= \frac{1}{r_0}\frac{1+\cos^2\theta}{\sin^3\theta}$$

Then $$F \times \left(\frac{1}{r^2}\right)\left(\frac{1}{r_0}\frac{1+\cos^2\theta}{\sin^3\theta} + \frac{1}{r_0\sin\theta}\right)$$

$$= mh^2\frac{1}{r_0}\frac{1}{r^2}\left(\frac{1+\cos^2\theta}{\sin^3\theta} + \frac{\sin^2\theta}{\sin^3\theta}\right)$$

$$= mh^2\frac{2}{r_0}\frac{1}{r^2}\frac{1}{\sin^3\theta} \qquad \sin^3\theta = \left(\frac{r}{r_0}\right)^3$$

$$= mh^2\frac{2r_0^2}{r^5}$$

F is proportional to $\frac{1}{r^5}$ $\quad F \times \frac{1}{r^5}$ Q.E.D. ◀

Note: $F > 0$ implies that **F** is attractive.

PROBLEM 12.98

It was observed that during its second flyby of the earth, the Galileo spacecraft had a velocity of 14.1 km/s as it reached its minimum altitude of 303 km above the surface of the earth. Determine the eccentricity of the trajectory of the spacecraft during this portion of its flight.

SOLUTION

For earth, $R = 6.37 \times 10^6$ m

$$r_0 = 6.37 \times 10^6 + 303. \times 10^3 = 6.673 \times 10^6 \text{ m}$$

$$h = r_0 v_0 = (6.673 \times 10^6)(14.1 \times 10^3) = 94.09 \times 10^9 \text{ m}^2/\text{s}$$

$$GM = gR^2 = (9.81)(6.37 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$\frac{1}{r_0} = \frac{GM}{h^2}(1 + \varepsilon)$$

$$1 + \varepsilon = \frac{h^2}{r_0 GM} = \frac{(94.09 \times 10^9)^2}{(6.673 \times 10^6)(398.06 \times 10^{12})} = 3.33$$

$$\varepsilon = 3.33 - 1 \qquad \varepsilon = 2.33 \blacktriangleleft$$

PROBLEM 12.99

It was observed that during the Galileo spacecraft's first flyby of the earth, its maximum altitude was 600 mi above the surface of the earth. Assuming that the trajectory of the spacecraft was parabolic, determine the maximum velocity of Galileo during its first flyby of the earth.

SOLUTION

For the earth: $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)(20.909 \times 10^6)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For a parabolic trajectory, $\varepsilon = 1$.

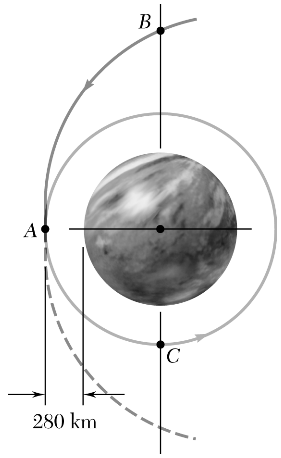
Eq. (12.39′): $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \cos\theta)$

At $\theta = 0$, $\dfrac{1}{r_0} = \dfrac{2GM}{h^2} = \dfrac{2GM}{r_0^2 v_0^2}$ or $v_0 = \sqrt{\dfrac{2GM}{r_0}}$

At $r_0 = 3960 + 600 = 4560 \text{ mi} = 24.077 \times 10^6 \text{ ft}$,

$$v_0 = \sqrt{\frac{(2)(14.077 \times 10^{15})}{24.077 \times 10^6}} = 34.196 \times 10^3 \text{ ft/s}$$

$v_0 = 6.48 \text{ mi/s}$ ◀



PROBLEM 12.100

As a space probe approaching the planet Venus on a parabolic trajectory reaches Point A closest to the planet, its velocity is decreased to insert it into a circular orbit. Knowing that the mass and the radius of Venus are 4.87×10^{24} kg and 6052 km, respectively, determine (*a*) the velocity of the probe as it approaches A, (*b*) the decrease in velocity required to insert it into the circular orbit.

SOLUTION

First note $\quad r_A = (6052+280)\text{ km} = 6332\text{ km}$

(*a*) From the textbook, the velocity at the point of closest approach on a parabolic trajectory is given by

$$v_0 = \sqrt{\frac{2GM}{r_0}}$$

Thus,

$$(v_A)_{\text{par}} = \left[\frac{2\times66.73\times10^{-12}\text{ m}^3/\text{kg}\cdot\text{s}^2\times4.87\times10^{24}\text{kg}}{6332\times10^3\text{ m}}\right]^{1/2}$$

$$=10{,}131.4\text{ m/s}$$

or $\quad (v_A)_{\text{par}} = 10.13\text{ km/s}$ ◀

(*b*) We have $\quad (v_A)_{\text{circ}} = (v_A)_{\text{par}} + \Delta v_A$

Now

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_0}} \qquad \text{Eq. (12.44)}$$

$$=\frac{1}{\sqrt{2}}(v_A)_{\text{par}}$$

Then

$$\Delta v_A = \frac{1}{\sqrt{2}}(v_A)_{\text{par}} - (v_A)_{\text{par}}$$

$$=\left(\frac{1}{\sqrt{2}}-1\right)(10.1314\text{ km/s})$$

$$=-2.97\text{ km/s}$$

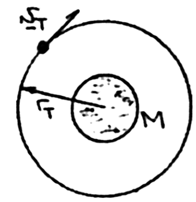
$|\Delta v_A| = 2.97\text{ km/s}$ ◀

PROBLEM 12.101

It was observed that as the Voyager I spacecraft reached the point of its trajectory closest to the planet Saturn, it was at a distance of 185×10^3 km from the center of the planet and had a velocity of 21.0 km/s. Knowing that Tethys, one of Saturn's moons, describes a circular orbit of radius 295×10^3 km at a speed of 11.35 km/s, determine the eccentricity of the trajectory of Voyager I on its approach to Saturn.

SOLUTION

For a circular orbit,



Eq. (12.44)

$$v = \sqrt{\frac{GM}{r}}$$

For the orbit of Tethys,

$$GM = r_T v_T^2$$

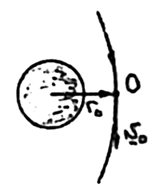
For Voyager's trajectory, we have

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$$

where $h = r_0 v_0$

At O,

$$r = r_0,\ \theta = 0$$



Then

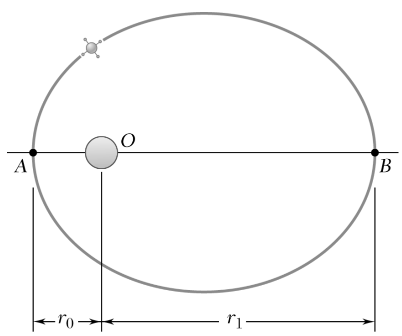
$$\frac{1}{r_0} = \frac{GM}{(r_0 v_0)^2}(1+\varepsilon)$$

or

$$\varepsilon = \frac{r_0 v_0^2}{GM} - 1 = \frac{r_0 v_0^2}{r_T v_T^2} - 1$$

$$= \frac{185\times10^3\text{ km}}{295\times10^3\text{ km}} \times \left(\frac{21.0\text{ km/s}}{11.35\text{ km/s}}\right)^2 - 1$$

or $\varepsilon = 1.147$ ◀



PROBLEM 12.102

A satellite describes an elliptic orbit about a planet of mass M. Denoting by r_0 and r_1, respectively, the minimum and maximum values of the distance r from the satellite to the center of the planet, derive the relation

$$\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2}$$

where h is the angular momentum per unit mass of the satellite.

SOLUTION

Using Eq. (12.39), $$\frac{1}{r_A}=\frac{GM}{h^2}+C\cos\theta_A$$

and $$\frac{1}{r_B}=\frac{GM}{h^2}+C\cos\theta_B.$$

But $$\theta_B=\theta_A+180°,$$

so that $$\cos\theta_A=-\cos\theta_B.$$

Adding, $$\frac{1}{r_A}+\frac{1}{r_B}=\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2}$$

PROBLEM 12.103

A space probe is describing a circular orbit about a planet of radius R. The altitude of the probe above the surface of the planet is αR and its speed is v_0. To place the probe in an elliptic orbit which will bring it closer to the planet, its speed is reduced from v_0 to βv_0, where $\beta < 1$, by firing its engine for a short interval of time. Determine the smallest permissible value of β if the probe is not to crash on the surface of the planet.

SOLUTION

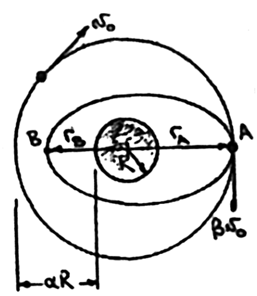
For the circular orbit, $$v_0 = \sqrt{\frac{GM}{r_A}} \quad \text{Eq. (12.44)},$$

where $$r_A = R + \alpha R = R(1+\alpha)$$

Then $$GM = v_0^2 R(1+\alpha)$$

From the solution to Problem 12.102, we have for the elliptic orbit,

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$



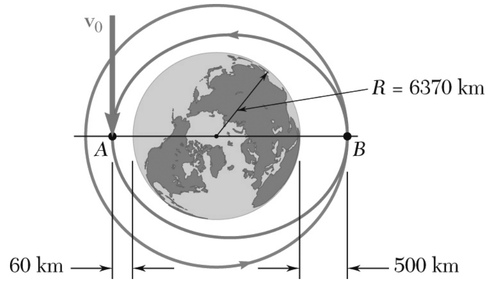
Now $$h = h_A = r_A (v_A)_{AB}$$
$$= [R(1+\alpha)](\beta v_0)$$

Then $$\frac{1}{R(1+\alpha)} + \frac{1}{r_B} = \frac{2v_0^2 R(1+\alpha)}{[R(1+\alpha)\beta v_0]^2}$$
$$= \frac{2}{\beta^2 R(1+\alpha)}$$

Now $\beta_{\min}$ corresponds to $r_B \to R$.

Then $$\frac{1}{R(1+\alpha)} + \frac{1}{R} = \frac{2}{\beta_{\min}^2 R(1+\alpha)}$$

or $$\beta_{\min} = \sqrt{\frac{2}{2+\alpha}} \blacktriangleleft$$



PROBLEM 12.104

At main engine cutoff of its thirteenth flight, the space shuttle Discovery was in an elliptic orbit of minimum altitude 60 km and maximum altitude 500 km above the surface of the earth. Knowing that at Point A the shuttle had a velocity $\mathbf{v}_0$ parallel to the surface of the earth and that the shuttle was transferred to a circular orbit as it passed through Point B, determine (a) the speed v_0 of the shuttle at A, (b) the increase in speed required at B to insert the shuttle into the circular orbit.

SOLUTION

For earth, $R = 6370 \text{ km} = 6370 \times 10^3 \text{ m}$

$$GM = gR^2 = (9.81)(6370 \times 10^3)^2 = 3.9806 \times 10^{14} \text{ m}^3/\text{s}^2$$

$$r_A = 6370 + 60 = 6430 \text{ km} = 6430 \times 10^3 \text{ m}$$

$$r_B = 6370 + 500 = 6870 \text{ km} = 6870 \times 10^3 \text{ m}$$

Elliptic trajectory.

Using Eq. (12.39), $\qquad \dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A \quad$ and $\quad \dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B.$

But $\qquad \theta_B = \theta_A + 180°, \quad$ so that $\quad \cos\theta_A = -\cos\theta_B$

Adding, $\qquad \dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h^2}$

$$h = \sqrt{\frac{2GMr_A r_B}{r_A + r_B}} = \sqrt{\frac{(2)(3.9806 \times 10^{14})(6430 \times 10^3)(6870 \times 10^3)}{6430 \times 10^3 + 6870 \times 10^3}} = 51.422 \times 10^9 \text{ m}^2/\text{s}$$

(a) Speed v_0 at A.

$$v_0 = v_A = \frac{h}{r_A} = \frac{51.422 \times 10^9}{6430 \times 10^3}$$

$v_0 = 8.00 \times 10^3$ m/s ◀

$$(v_B)_1 = \frac{h}{r_A} = \frac{51.422 \times 10^9}{6870 \times 10^3} = 7.48497 \times 10^3 \text{ m/s}$$

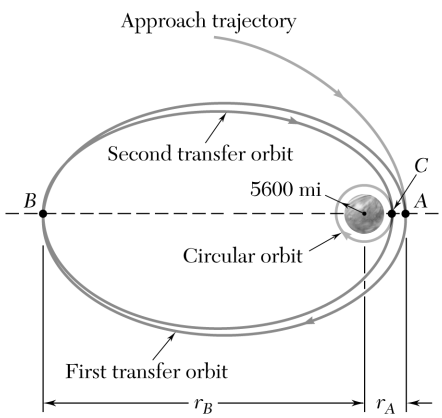
For a circular orbit through Point B,

$$(v_B)_{\text{circ}} = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{3.9806 \times 10^{14}}{6870 \times 10^3}} = 7.6119 \times 10^3 \text{ m/s}$$

(b) Increase in speed at Point B.

$$\Delta v_B = (v_B)_{\text{circ}} - (v_B)_1 = 126.97 \text{ m/s}$$

$\Delta v_B = 127$ m/s ◀



PROBLEM 12.105

A space probe is to be placed in a circular orbit of 5600 mi radius about the planet Venus in a specified plane. As the probe reaches A, the point of its original trajectory closest to Venus, it is inserted in a first elliptic transfer orbit by reducing its speed by Δv_A. This orbit brings it to Point B with a much reduced velocity. There the probe is inserted in a second transfer orbit located in the specified plane by changing the direction of its velocity and further reducing its speed by Δv_B. Finally, as the probe reaches Point C, it is inserted in the desired circular orbit by reducing its speed by Δv_C. Knowing that the mass of Venus is 0.82 times the mass of the earth, that $r_A = 9.3 \times 10^3$ mi and $r_B = 190 \times 10^3$ mi, and that the probe approaches A on a parabolic trajectory, determine by how much the velocity of the probe should be reduced (*a*) at A, (*b*) at B, (*c*) at C.

SOLUTION

For Earth, $\quad R = 3690 \text{ mi} = 20.9088 \times 10^6 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

$$GM_{\text{earth}} = gR^2 = (32.2)(20.9088 \times 10^6)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For Venus, $\quad GM = 0.82 GM_{\text{earth}} = 11.543 \times 10^{15} \text{ ft}^3/\text{s}^2$

For a parabolic trajectory with $\quad r_A = 9.3 \times 10^3 \text{ mi} = 49.104 \times 10^6 \text{ ft}$

$$(v_A)_1 = v_{\text{esc}} = \sqrt{\frac{2GM}{r_A}} = \sqrt{\frac{(2)(11.543 \times 10^{15})}{49.104 \times 10^6}} = 21.683 \times 10^3 \text{ ft/s}$$

<u>First transfer orbit *AB*</u>. $\quad r_B = 190 \times 10^3 \text{ mi} = 1003.2 \times 10^6 \text{ ft}$

At Point A, where $\theta = 180°$

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C \cos 180° = \frac{GM}{h_{AB}^2} - C \qquad (1)$$

At Point B, where $\theta = 0°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C \cos 0 = \frac{GM}{h_{AB}^2} + C \qquad (2)$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_B + r_A}{r_A r_B} = \frac{2GM}{h_{AB}^2}$$

Solving for h_{AB},

$$h_{AB} = \sqrt{\frac{2GMr_A r_B}{r_B + r_A}} = \sqrt{\frac{(2)(11.543\times10^{15})(49.104\times10^6)(1003.2\times10^6)}{1052.3\times10^6}} = 1.039575\times10^{12}\ \text{ft}^2/\text{s}$$

$$(v_A)_2 = \frac{h_{AB}}{r_A} = \frac{1.039575\times10^{12}}{49.104\times10^6} = 21.174\times10^3\ \text{ft/s}$$

$$(v_B)_1 = \frac{h_{AB}}{r_B} = \frac{1.039575\times10^{12}}{1003.2\times10^6} = 1.03626\times10^3\ \text{ft/s}$$

<u>Second transfer orbit *BC*</u>. $\quad r_C = 5600\ \text{mi} = 29.568\times10^6\ \text{ft}$

At Point *B*, where $\theta = 0$

$$\frac{1}{r_B} = \frac{GM}{h_{BC}^2} + C\cos 0 = \frac{GM}{h_{BC}^2} + C$$

At Point *C*, where $\theta = 180°$

$$\frac{1}{r_C} = \frac{GM}{h_{BC}^2} + C\cos 180° = \frac{GM}{h_{BC}^2} - C$$

Adding, $$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B + r_C}{r_B r_C} = \frac{2GM}{h_{BC}^2}$$

$$h_{BC} = \sqrt{\frac{2GMr_B r_C}{r_B + r_C}} = \sqrt{\frac{(2)(11.543\times10^{15})(1003.2\times10^6)(29.568\times10^6)}{1032.768\times10^6}} = 814.278\times10^9\ \text{ft}^2/\text{s}$$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{814.278\times10^9}{1003.2\times10^6} = 811.69\ \text{ft/s}$$

$$(v_C)_1 = \frac{h_{BC}}{r_C} = \frac{814.278\times10^9}{29.568\times10^6} = 27.539\times10^3\ \text{ft/s}$$

<u>Final circular orbit</u>. $\quad r_C = 29.568\times10^6\ \text{ft}$

$$(v_C)_2 = \sqrt{\frac{GM}{r_C}} = \sqrt{\frac{11.543\times10^{15}}{29.568\times10^6}} = 19.758\times10^3\ \text{ft/s}$$

<u>Speed reductions</u>.

(*a*) At *A*: $\quad (v_A)_1 - (v_A)_2 = 21.683\times10^3 - 21.174\times10^3 \quad$ $\Delta v_A = 509\ \text{ft/s}$ ◀

(*b*) At *B*: $\quad (v_B)_1 - (v_B)_2 = 1.036\times10^3 - 811.69 \quad$ $\Delta v_B = 224\ \text{ft/s}$ ◀

(*c*) At *C*: $\quad (v_C)_1 - (v_C)_2 = 27.539\times10^3 - 19.758\times10^3 \quad$ $\Delta v_C = 7.78\times10^3\ \text{ft/s}$ ◀

PROBLEM 12.106

For the space probe of Problem 12.105, it is known that $r_A = 9.3\times 10^3$ mi and that the velocity of the probe is reduced to $20,000$ ft/s as it passes through A. Determine (a) the distance from the center of Venus to Point B, (b) the amounts by which the velocity of the probe should be reduced at B and C, respectively.

SOLUTION

Data from Problem 12.105: $r_C = 29.568\times 10^6$ ft, $M = 0.82\, M_{\text{earth}}$

For Earth, $R = 3960$ mi $= 20.9088\times 10^6$ ft, $g = 32.2$ ft/s^2

$$GM_{\text{earth}} = gR^2 = (32.2)(20.9088\times 10^6)^2 = 14.077\times 10^{15}\ \text{m}^3/\text{s}^2$$

For Venus, $GM = 0.82GM_{\text{earth}} = 11.543\times 10^{15}\ \text{ft}^3/\text{s}^2$

Transfer orbit AB: $v_A = 20,000$ ft/s, $r_A = 9.3\times 10^3$ mi $= 49.104\times 10^6$ ft

$$h_{AB} = r_A v_A = (49.104\times 10^6)(20,000) = 982.08\times 10^9\ \text{ft}^2/\text{s}$$

At Point A, where $\theta = 180°$

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C\cos 180° = \frac{GM}{h_{AB}^2} - C$$

At Point B, where $\theta = 0°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C\cos 0 = \frac{GM}{h_{AB}^2} + C$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h_{AB}^2}$$

$$\frac{1}{r_B} = \frac{2GM}{h_{AB}^2} - \frac{1}{r_A}$$

$$= \frac{(2)(11.543\times 10^{15})}{(982.08\times 10^9)^2} - \frac{1}{49.104\times 10^6}$$

$$= 3.57125\times 10^{-9}\ \text{ft}^{-1}$$

(a) Radial coordinate r_B. $r_B = 280.01\times 10^6$ ft $\qquad r_B = 53.0\times 10^3$ mi ◀

$$(v_B)_1 = \frac{h_{AB}}{r_B} = \frac{982.08\times 10^9}{280.01\times 10^6} = 3.5073\times 10^3\ \text{ft/s}$$

PROBLEM 12.106 (Continued)

<u>Second transfer orbit *BC*</u>. $r_C = 5600 \text{ mi} = 29.568\times10^6 \text{ ft}$

At Point *B*, where $\theta = 0$

$$\frac{1}{r_B} = \frac{GM}{h_{BC}^2} + C\cos 0 = \frac{GM}{h_{BC}^2} + C$$

At Point *C*, where $\theta = 180°$

$$\frac{1}{r_C} = \frac{GM}{h_{BC}^2} + C\cos 180° = \frac{GM}{h_{BC}^2} - C$$

Adding,
$$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B + r_C}{r_B r_C} = \frac{2GM}{h_{BC}^2}$$

$$h_{BC} = \sqrt{\frac{2GMr_Br_C}{r_B + r_C}} = \sqrt{\frac{(2)(11.543\times10^{15})(280.01\times10^6)(29.568\times10^6)}{309.578\times10^6}}$$

$$= 785.755\times10^9 \text{ ft}^2/\text{s}$$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{785.755\times10^9}{280.01\times10^6} = 2.8062\times10^3 \text{ ft/s}$$

$$(v_C)_1 = \frac{h_{BC}}{r_C} = \frac{785.755\times10^9}{29.568\times10^6} = 26.575\times10^3 \text{ ft/s}$$

Circular orbit with $r_C = 29.568\times10^6 \text{ ft}$

$$(v_C)_2 = \sqrt{\frac{GM}{r_C}} = \sqrt{\frac{11.543\times10^{15}}{29.568\times10^6}} = 19.758\times10^3 \text{ ft/s}$$

(*b*) <u>Speed reductions at *B* and *C*</u>.

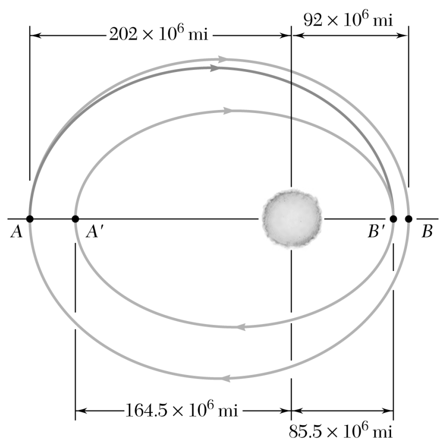
At *B*: $(v_B)_1 - (v_B)_2 = 3.5073\times10^3 - 2.8062\times10^3$

$$\Delta v_B = 701 \text{ ft/s} \blacktriangleleft$$

At *C*: $(v_C)_1 - (v_C)_2 = 26.575\times10^3 - 19.758\times10^3$

$$\Delta v_C = 6.82\times10^3 \text{ ft/s} \blacktriangleleft$$

PROBLEM 12.107



As it describes an elliptic orbit about the sun, a spacecraft reaches a maximum distance of 202×10^6 mi from the center of the sun at Point A (called the aphelion) and a minimum distance of 92×10^6 mi at Point B (called the perihelion). To place the spacecraft in a smaller elliptic orbit with aphelion at A' and perihelion at B', where A' and B' are located 164.5×10^6 mi and 85.5×10^6 mi, respectively, from the center of the sun, the speed of the spacecraft is first reduced as it passes through A and then is further reduced as it passes through B'. Knowing that the mass of the sun is 332.8×10^3 times the mass of the earth, determine (a) the speed of the spacecraft at A, (b) the amounts by which the speed of the spacecraft should be reduced at A and B' to insert it into the desired elliptic orbit.

SOLUTION

First note
$$R_{\text{earth}} = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$$
$$r_A = 202 \times 10^6 \text{ mi} = 1066.56 \times 10^9 \text{ ft}$$
$$r_B = 92 \times 10^6 \text{ mi} = 485.76 \times 10^9 \text{ ft}$$

From the solution to Problem 12.102, we have for any elliptic orbit about the sun

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{\text{sun}}}{h^2}$$

(a) For the elliptic orbit AB, we have

$$r_1 = r_A, \quad r_2 = r_B, \quad h = h_A = r_A v_A$$

Also,
$$GM_{\text{sun}} = G[(332.8 \times 10^3) M_{\text{earth}}]$$
$$= gR_{\text{earth}}^2 (332.8 \times 10^3) \qquad \text{using Eq. (12.30).}$$

Then
$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR_{\text{earth}}^2 (332.8 \times 10^3)}{(r_A v_A)^2}$$

or
$$v_A = \frac{R_{\text{earth}}}{r_A} \left(\frac{665.6 g \times 10^3}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2}$$
$$= \frac{3960 \text{ mi}}{202 \times 10^6 \text{ mi}} \left(\frac{665.6 \times 10^3 \times 32.2 \text{ ft/s}^2}{\frac{1}{1066.56 \times 10^9 \text{ ft}} + \frac{1}{485.76 \times 10^9 \text{ ft}}} \right)^{1/2}$$
$$= 52{,}431 \text{ ft/s}$$

or $$v_A = 52.4 \times 10^3 \text{ ft/s} \blacktriangleleft$$

(*b*) From Part (*a*), we have

$$2GM_{\text{sun}} = (r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)$$

Then, for any other elliptic orbit about the sun, we have

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{(r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)}{h^2}$$

For the elliptic transfer orbit AB', we have

$$r_1 = r_A, \quad r_2 = r_{B'}, \quad h = h_{\text{tr}} = r_A (v_A)_{\text{tr}}$$

Then $$\frac{1}{r_A} + \frac{1}{r_{B'}} = \frac{(r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)}{[r_A (v_A)_{\text{tr}}]^2}$$

or $$(v_A)_{\text{tr}} = v_A \left(\frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_A} + \frac{1}{r_{B'}}} \right)^{1/2} = v_A \left(\frac{1 + \frac{r_A}{r_B}}{1 + \frac{r_A}{r_{B'}}} \right)^{1/2}$$

$$= (52{,}431 \text{ ft/s}) \left(\frac{1 + \frac{202}{92}}{1 + \frac{202}{85.5}} \right)^{1/2}$$

$$= 51{,}113 \text{ ft/s}$$

Now $$h_{\text{tr}} = (h_A)_{\text{tr}} = (h_{B'})_{\text{tr}}: \quad r_A (v_A)_{\text{tr}} = r_{B'} (v_{B'})_{\text{tr}}$$

Then $$(v_{B'})_{\text{tr}} = \frac{202 \times 10^6 \text{ mi}}{85.5 \times 10^6 \text{ mi}} \times 51{,}113 \text{ ft/s} = 120{,}758 \text{ ft/s}$$

For the elliptic orbit $A'B'$, we have

$$r_1 = r_{A'}, \quad r_2 = r_{B'}, \quad h = r_{B'} v_{B'}$$

Then $$\frac{1}{r_{A'}} + \frac{1}{r_{B'}} = \frac{(r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)}{(r_{B'} v_{B'})^2}$$

or $$v_{B'} = v_A \frac{r_A}{r_{B'}} \left(\frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_{A'}} + \frac{1}{r_{B'}}} \right)^{1/2}$$

$$= (52{,}431 \text{ ft/s}) \frac{202 \times 10^6 \text{ mi}}{85.5 \times 10^6 \text{ mi}} \left(\frac{\frac{1}{202 \times 10^6} + \frac{1}{92 \times 10^6}}{\frac{1}{164.5 \times 10^6} + \frac{1}{85.5 \times 10^6}} \right)^{1/2}$$

$$= 116{,}862 \text{ ft/s}$$

PROBLEM 12.107 (Continued)

Finally, $$(v_A)_{\text{tr}} = v_A + \Delta v_A$$

or $$\Delta v_A = (51{,}113 - 52{,}431) \text{ ft/s}$$

or $$|\Delta v_A| = 1318 \text{ ft/s} \blacktriangleleft$$

and $$v_{B'} = (v_{B'})_{\text{tr}} + \Delta v_B$$

or $$\Delta v_{B'} = (116{,}862 - 120{,}758) \text{ ft/s}$$
$$= -3896 \text{ ft/s}$$

or $$|\Delta v_B| = 3900 \text{ ft/s} \blacktriangleleft$$

PROBLEM 12.108

Halley's comet travels in an elongated elliptic orbit for which the minimum distance from the sun is approximately $\frac{1}{2}r_E$, where $r_E = 150\times10^6$ km is the mean distance from the sun to the earth. Knowing that the periodic time of Halley's comet is about 76 years, determine the maximum distance from the sun reached by the comet.

SOLUTION

We apply Kepler's Third Law to the orbits and periodic times of earth and Halley's comet:

$$\left(\frac{\tau_H}{\tau_E}\right)^2 = \left(\frac{a_H}{a_E}\right)^3$$

Thus

$$a_H = a_E\left(\frac{\tau_H}{\tau_E}\right)^{2/3}$$

$$= r_E\left(\frac{76 \text{ years}}{1 \text{ year}}\right)^{2/3}$$

$$= 17.94 r_E$$

But

$$a_H = \frac{1}{2}(r_{\min} + r_{\max})$$

$$17.94 r_E = \frac{1}{2}\left(\frac{1}{2}r_E + r_{\max}\right)$$

$$r_{\max} = 2(17.94 r_E) - \frac{1}{2}r_E$$

$$= (35.88 - 0.5)r_E$$

$$= 35.38\, r_E$$

$$r_{\max} = (35.38)(150\times10^6 \text{ km})$$

$$r_{\max} = 5.31\times10^9 \text{ km} \blacktriangleleft$$

PROBLEM 12.109

Based on observations made during the 1996 sighting of comet Hyakutake, it was concluded that the trajectory of the comet is a highly elongated ellipse for which the eccentricity is approximately $\varepsilon = 0.999887$. Knowing that for the 1996 sighting the minimum distance between the comet and the sun was $0.230R_E$, where R_E is the mean distance from the sun to the earth, determine the periodic time of the comet.

SOLUTION

For Earth's orbit about the sun,

$$v_0 = \sqrt{\frac{GM}{R_E}}, \quad \tau_0 = \frac{2\pi R_E}{v_0} = \frac{2\pi R_E^{3/2}}{\sqrt{GM}} \quad \text{or} \quad \sqrt{GM} = \frac{2\pi R_E^{3/2}}{\tau_0} \tag{1}$$

For the comet Hyakutake,

$$\frac{1}{r_0} = \frac{GM}{h^2} = (1+\varepsilon), \quad \frac{1}{r_1} = \frac{GM}{h^2}(1+\varepsilon), \quad r_1 = \frac{1+\varepsilon}{1-\varepsilon} r_0$$

$$a = \frac{1}{2}(r_0 + r_1) = \frac{r_0}{1-\varepsilon}, \quad b = \sqrt{r_0 r_1} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\, r_0$$

$$h = \sqrt{GMr_0(1+\varepsilon)}$$

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi r_0^2 (1+\varepsilon)^{1/2}}{(1-\varepsilon)^{3/2}\sqrt{GMr_0(1+\varepsilon)}}$$

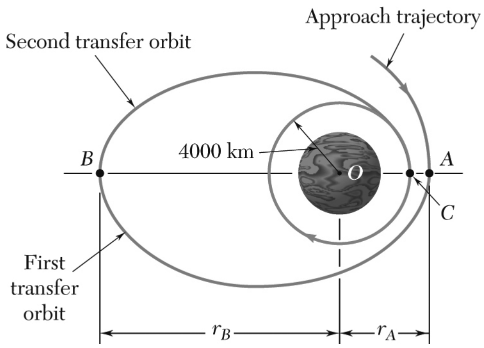
$$= \frac{2\pi r_0^{3/2}}{\sqrt{GM}\,(1-\varepsilon)^{3/2}} = \frac{2\pi r_0^{3/2}\tau_0}{2\pi R_E^3 (1-\varepsilon)^{3/2}}$$

$$= \left(\frac{r_0}{R_E}\right)^{3/2} \frac{1}{(1-\varepsilon)^{3/2}}\, \tau_0$$

$$= (0.230)^{3/2} \frac{1}{(1-0.999887)^{3/2}}\, \tau_0 = 91.8 \times 10^3 \tau_0$$

Since $\quad \tau_0 = 1 \text{ yr}, \quad \tau = (91.8 \times 10^3)(1.000) \qquad \tau = 91.8 \times 10^3 \text{ yr}$ ◀

PROBLEM 12.110



A space probe is to be placed in a circular orbit of radius 4000 km about the planet Mars. As the probe reaches A, the point of its original trajectory closest to Mars, it is inserted into a first elliptic transfer orbit by reducing its speed. This orbit brings it to Point B with a much reduced velocity. There the probe is inserted into a second transfer orbit by further reducing its speed. Knowing that the mass of Mars is 0.1074 times the mass of the earth, that r_A = 9000 km and r_B = 180,000 km, and that the probe approaches A on a parabolic trajectory, determine the time needed for the space probe to travel from A to B on its first transfer orbit.

SOLUTION

For earth, $R = 6373 \text{ km} = 6.373 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)(6.373 \times 10^6)^2 = 398.43 \times 10^{12} \text{ m}^3/\text{s}^2$$

For Mars, $GM = (0.1074)(398.43 \times 10^{12}) = 42.792 \times 10^{12} \text{ m}^3/\text{s}^2$

$$r_A = 9000 \text{ km} = 9.0 \times 10^6 \text{ m} \qquad r_B = 180000 \text{ km} = 180 \times 10^6 \text{ m}$$

For the parabolic approach trajectory at A,

$$(v_A)_1 = \sqrt{\frac{2GM}{r_A}} = \sqrt{\frac{(2)(42.792 \times 10^{12})}{9.0 \times 10^6}} = 3.0837 \times 10^3 \text{ m/s}$$

First elliptic transfer orbit AB.

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h_{AB}^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h_{AB}^2} + C\cos\theta_B.$

But $\theta_B = \theta_A + 180°,$ so that $\cos\theta_A = -\cos\theta_B.$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h_{AB}^2}$

$$h_{AB} = \sqrt{\frac{2GMr_A r_B}{r_A + r_B}} = \sqrt{\frac{(2)(42.792 \times 10^{12})(9.0 \times 10^6)(180 \times 10^6)}{189.0 \times 10^6}}$$

$$h_{AB} = 27.085 \times 10^9 \text{ m}^2/\text{s}$$

$$a = \frac{1}{2}(r_A + r_B) = \frac{1}{2}(9.0 \times 10^6 + 180 \times 10^6) = 94.5 \times 10^6 \text{ m}$$

$$b = \sqrt{r_A r_B} = \sqrt{(9.0 \times 10^6)(180 \times 10^6)} = 40.249 \times 10^6 \text{ m}$$

PROBLEM 12.110 (Continued)

Periodic time for full ellipse: $\tau = \dfrac{2\pi ab}{h}$

For half ellipse AB, $\tau_{AB} = \dfrac{1}{2}\tau = \dfrac{\pi ab}{h}$

$$\tau_{AB} = \frac{\pi(94.5\times 10^6)(40.249\times 10^6)}{27.085\times 10^9} = 444.81\times 10^3\ \text{s} \qquad \tau_{AB} = 122.6\ \text{h} \blacktriangleleft$$

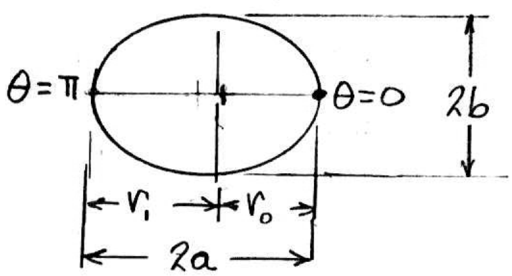
PROBLEM 12.111

A space shuttle is in an elliptic orbit of eccentricity 0.0356 and a minimum altitude of 300 km above the surface of the earth. Knowing that the radius of the earth is 6370 km, determine the periodic time for the orbit.

SOLUTION

For earth, $g = 9.81 \text{ m/s}^2, \quad R = 6370 \text{ km} = 6.370 \times 10^6 \text{ m}$

$$\text{GM} = gR^2 = (9.81)(6.370 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$



For the orbit, $r_0 = 6370 + 300 = 6670 \text{ km} = 6.670 \times 10^6 \text{ m}$

$$\frac{1}{r_0} = \frac{GM}{h^2}(1+\varepsilon) \qquad \frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon)$$

$$r_1 = r_0 \frac{1+\varepsilon}{1-\varepsilon} - (6.670 \times 10^6)\frac{1.0356}{0.9644} = 7.1624 \times 10^6 \text{ m}$$

$$a = \frac{1}{2}(r_0 + r_1) = 6.9162 \times 10^6 \text{ m}$$

$$b = \sqrt{r_0 r_1} = 6.9118 \times 10^6 \text{ m}$$

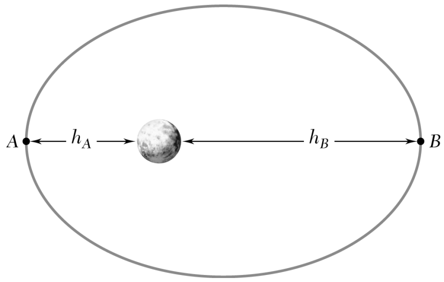
$$h = \sqrt{(1+\varepsilon)GMr_0} = \sqrt{(1.0356)(398.06 \times 10^{12})(6.670 \times 10^6)}$$

$$= 52.436400 \times 10^9 \text{ m}^2/\text{s}$$

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi(6.9118 \times 10^6)(6.9162 \times 10^6)}{52.436400 \times 10^9 \text{ m}^2/\text{s}}$$

$$= 5.7281 \times 10^3 \text{ s}$$

$\tau = 95.5 \text{ min}$ ◀



PROBLEM 12.112

The Clementine spacecraft described an elliptic orbit of minimum altitude $h_A = 400\text{ km}$ and a maximum altitude of $h_B = 2940\text{ km}$ above the surface of the moon. Knowing that the radius of the moon is 1737 km and that the mass of the moon is 0.01230 times the mass of the earth, determine the periodic time of the spacecraft.

SOLUTION

For earth, $$R = 6370 \text{ km} = 6.370\times 10^6 \text{ m}$$

$$GM = gR^2 = (9.81)(6.370\times 10^6)^2 = 398.06\times 10^{12} \text{ m}^3/\text{s}^2$$

For moon, $$GM = (0.01230)(398.06\times 10^{12}) = 4.896\times 10^{12} \text{ m}^3/\text{s}^2$$

$$r_A = 1737 + 400 = 2137 \text{ km} = 2.137\times 10^6 \text{ m}$$

$$r_B = 1737 + 2940 = 4677 \text{ km} = 4.677\times 10^6 \text{ m}$$

Using Eq. (12.39), $$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A \quad \text{and} \quad \frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$$

But $$\theta_B = \theta_A + 180°, \quad \text{so that} \quad \cos\theta_A = -\cos\theta_B.$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A + r_B}{r_A r_B} = \frac{2GM}{h_{AB}^2}$$

$$h_{AB} = \sqrt{\frac{2GMr_A r_B}{r_A + r_B}} = \sqrt{\frac{(2)(4.896\times 10^{12})(2.137\times 10^6)(4.677\times 10^6)}{6.814\times 10^6}}$$

$$= 3.78983\times 10^9 \text{ m}^2/\text{s}$$

$$a = \frac{1}{2}(r_A + r_B) = 3.402\times 10^6 \text{ m}$$

$$b = \sqrt{r_A r_B} = 3.16145\times 10^6 \text{ m}$$

Periodic time.

$$\tau = \frac{2\pi ab}{h_{AB}} = \frac{2\pi(3.402\times 10^6)(3.16145\times 10^6)}{3.78983\times 10^9} = 17.831\times 10^3 \text{ s}$$

$$\tau = 4.95 \text{ h} \blacktriangleleft$$

PROBLEM 12.113

Determine the time needed for the space probe of Problem 12.100 to travel from *B* to *C*.

SOLUTION

From the solution to Problem 12.100, we have

$$(v_A)_{\text{par}} = 10{,}131.4 \text{ m/s}$$

and $$(v_A)_{\text{circ}} = \frac{1}{\sqrt{2}}(v_A)_{\text{par}} = 7164.0 \text{ m/s}$$

Also, $$r_A = (6052 + 280) \text{ km} = 6332 \text{ km}$$

For the parabolic trajectory *BA*, we have

$$\frac{1}{r} = \frac{GM_v}{h_{BA}^2}(1 + \varepsilon\cos\theta) \qquad \text{[Eq. (12.39')]}$$

where $\varepsilon = 1$. Now

at A, $\theta = 0$: $$\frac{1}{r_A} = \frac{GM_v}{h_{BA}^2}(1+1)$$

or $$r_A = \frac{h_{BA}^2}{2GM_v}$$

at B, $\theta = -90°$: $$\frac{1}{r_B} = \frac{GM_v}{h_{BA}^2}(1+0)$$

or $$r_B = \frac{h_{BA}^2}{GM_v}$$

$$r_B = 2r_A$$

As the probe travels from *B* to *A*, the area swept out is the semiparabolic area defined by Vertex *A* and Point *B*. Thus,

$$(\text{Area swept out})_{BA} = A_{BA} = \frac{2}{3}r_A r_B = \frac{4}{3}r_A^2$$

Now $$\frac{dA}{dt} = \frac{1}{2}h$$

where $h = \text{constant}$

Then $$A = \frac{1}{2}ht \quad \text{or} \quad t_{BA} = \frac{2A_{BA}}{h_{BA}} \qquad h_{BA} = r_A v_A$$

$$t_{BA} = \frac{2 \times \frac{4}{3} r_A^2}{r_A v_A} = \frac{8}{3}\frac{r_A}{v_A}$$

$$= \frac{8}{3}\frac{6332 \times 10^3 \text{ m}}{10{,}131.4 \text{ m/s}}$$

$$= 1666.63 \text{ s}$$

For the circular trajectory AC,

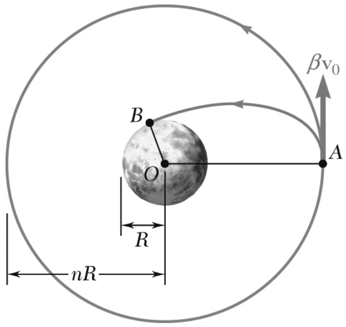
$$t_{AC} = \frac{\frac{\pi}{2} r_A}{(v_A)_{\text{circ}}} = \frac{\pi}{2}\frac{6332 \times 10^3 \text{ m}}{7164.0 \text{ m/s}} = 1388.37 \text{ s}$$

Finally, $$t_{BC} = t_{BA} + t_{AC}$$

$$= (1666.63 + 1388.37) \text{ s}$$

$$= 3055.0 \text{ s}$$

or $$t_{BC} = 50 \text{ min } 55 \text{ s} \blacktriangleleft$$



PROBLEM 12.114

A space probe is describing a circular orbit of radius nR with a velocity v_0 about a planet of radius R and center O. As the probe passes through Point A, its velocity is reduced from v_0 to βv_0, where $\beta < 1$, to place the probe on a crash trajectory. Express in terms of n and β the angle AOB, where B denotes the point of impact of the probe on the planet.

SOLUTION

For the circular orbit,

$$r_0 = r_A = nR$$

$$v_0 = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{GM}{nR}}$$

The crash trajectory is elliptic.

$$v_A = \beta v_0 = \sqrt{\frac{\beta^2 GM}{nR}}$$

$$h = r_A v_A = nRv_A = \sqrt{\beta^2 nGMR}$$

$$\frac{GM}{h^2} = \frac{1}{\beta^2 nR}$$

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta) = \frac{1+\varepsilon\cos\theta}{\beta^2 nR}$$

At Point A, $\theta = 180°$

$$\frac{1}{r_A} = \frac{1}{nR} = \frac{1-\varepsilon}{\beta^2 nR} \quad \text{or} \quad \beta^2 = 1-\varepsilon \quad \text{or} \quad \varepsilon = 1-\beta^2$$

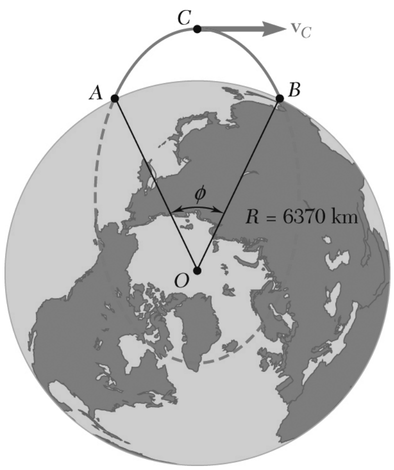
At impact Point B, $\theta = \pi - \phi$

$$\frac{1}{r_B} = \frac{1}{R}$$

$$\frac{1}{R} = \frac{1+\varepsilon\cos(\pi-\phi)}{\beta^2 nR} = \frac{1-\varepsilon\cos\phi}{\beta^2 nR}$$

$$\varepsilon\cos\phi = 1 - n\beta^2 \quad \text{or} \quad \cos\phi = \frac{1-n\beta^2}{\varepsilon} = \frac{1-n\beta^2}{1-\beta^2}$$

$$\phi = \cos^{-1}[(1-n\beta^2)/(1-\beta^2)] \blacktriangleleft$$



PROBLEM 12.115

A long-range ballistic trajectory between Points A and B on the earth's surface consists of a portion of an ellipse with the apogee at Point C. Knowing that Point C is 1500 km above the surface of the earth and the range $R\phi$ of the trajectory is 6000 km, determine (*a*) the velocity of the projectile at C, (*b*) the eccentricity ε of the trajectory.

SOLUTION

For earth, $R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)(6.37 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

For the trajectory, $r_C = 6370 + 1500 = 7870 \text{ km} = 7.87 \times 10^6 \text{ m}$

$$r_A = r_B = R = 6.37 \times 10^6 \text{ m}, \qquad \frac{r_C}{r_A} = \frac{7870}{6370} = 1.23548$$

Range A to B: $s_{AB} = 6000 \text{ km} = 6.00 \times 10^6 \text{ m}$

$$\varphi = \frac{s_{AB}}{R} = \frac{6.00 \times 10^6}{6.37 \times 10^6} = 0.94192 \text{ rad} = 53.968°$$

For an elliptic trajectory, $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \varepsilon \cos\theta)$

At A, $\theta = 180° - \dfrac{\varphi}{2} = 153.016°$, $\dfrac{1}{r_A} = \dfrac{GM}{h^2}(1 + \varepsilon \cos 153.016°)$ (1)

At C, $\theta = 180°$, $\dfrac{1}{r_C} = \dfrac{GM}{h^2}(1 - \varepsilon)$ (2)

Dividing Eq. (1) by Eq. (2),

$$\frac{r_C}{r_A} = \frac{1 + \varepsilon \cos 153.016°}{1 - \varepsilon} = 1.23548$$

$$\varepsilon = \frac{1.23548 - 1}{1.23548 + \cos 153.016°} = 0.68384$$

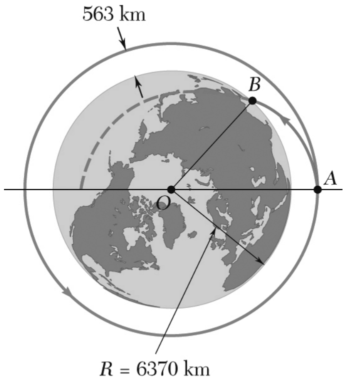
PROBLEM 12.115 (Continued)

From Eq. (2), $h = \sqrt{GM(1-\varepsilon)r_C}$

$$h = \sqrt{(398.06\times 10^{12})(0.31616)(7.87\times 10^{6})} = 31.471\times 10^{9}\ \text{m}^2/\text{s}$$

(*a*) Velocity at *C*. $v_C = \dfrac{h}{r_C} = \dfrac{31.471\times 10^{9}}{7.87\times 10^{6}} = 4.00\times 10^{3}\ \text{m/s}$ $v_C = 4\ \text{km/s}$ ◀

(*b*) Eccentricity of trajectory. $\varepsilon = 0.684$ ◀



PROBLEM 12.116

A space shuttle is describing a circular orbit at an altitude of 563 km above the surface of the earth. As it passes through Point A, it fires its engine for a short interval of time to reduce its speed by 152 m/s and begin its descent toward the earth. Determine the angle AOB so that the altitude of the shuttle at Point B is 121 km. (*Hint:* Point A is the apogee of the elliptic descent trajectory.)

SOLUTION

$$GM = gR^2 = (9.81)(6.37\times 10^6)^2 = 398.06\times 10^{12}\ \text{m}^3/\text{s}^2$$

$$r_A = 6370 + 563 = 6933\ \text{km} = 6.933\times 10^6\ \text{m}$$

$$r_B = 6370 + 121 = 6491\ \text{km} = 6.491\times 10^6\ \text{m}$$

For the circular orbit through Point A,

$$v_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.06\times 10^{12}}{6.933\times 10^6}} = 7.5773\times 10^3\ \text{m/s}$$

For the descent trajectory,

$$v_A = v_{\text{circ}} + \Delta v = 7.5773\times 10^3 - 152 = 7.4253\times 10^3\ \text{m/s}$$

$$h = r_A v_A = (6.933\times 10^6)(7.4253\times 10^3) = 51.4795\times 10^9\ \text{m}^2/\text{s}$$

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$$

At Point A, $\quad \theta = 180°, \quad r = r_A$

$$\frac{1}{r_A} = \frac{GM}{h^2}(1-\varepsilon)$$

$$1-\varepsilon = \frac{h^2}{GM\ r_A} = \frac{(51.4795\times 10^9)^2}{(398.06\times 10^{12})(6.933\times 10^6)} = 0.96028$$

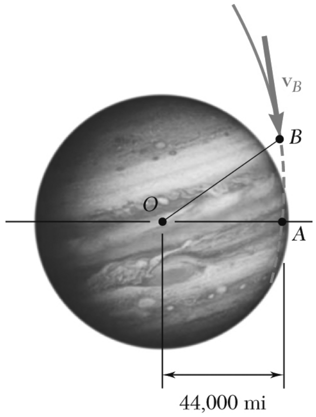
$$\varepsilon = 0.03972$$

$$\frac{1}{r_B} = \frac{GM}{h^2}(1+\varepsilon\cos\theta_B)$$

$$1+\varepsilon\cos\theta_B = \frac{h^2}{GM\ r_B} = \frac{(51.4795\times 10^9)^2}{(398.06\times 10^{12})(6.491\times 10^6)} = 1.02567$$

$$\cos\theta_B = \frac{1.02567 - 1}{\varepsilon} = 0.6463$$

$\theta_B = 49.7°$ $\qquad \measuredangle AOB = 180° - \theta_B = 130.3°$ $\qquad \measuredangle AOB = 130.3°$ ◀



PROBLEM 12.117

As a spacecraft approaches the planet Jupiter, it releases a probe which is to enter the planet's atmosphere at Point B at an altitude of 280 mi above the surface of the planet. The trajectory of the probe is a hyperbola of eccentricity $\varepsilon = 1.031$. Knowing that the radius and the mass of Jupiter are 44423 mi and 1.30×10^{26} slug, respectively, and that the velocity $\mathbf{v}_B$ of the probe at B forms an angle of 82.9° with the direction of OA, determine (*a*) the angle AOB, (*b*) the speed v_B of the probe at B.

SOLUTION

First we note $\quad r_B = (44.423\times10^3 + 280)\text{ mi} = 44.703\times10^3\text{ mi}$

(*a*) We have
$$\frac{1}{r} = \frac{GM_j}{h^2}(1+\varepsilon\cos\theta) \qquad \text{[Eq. (12.39′)]}$$

At A, $\theta = 0$:
$$\frac{1}{r_A} = \frac{GM_j}{h^2}(1+\varepsilon)$$

or
$$\frac{h^2}{GM_j} = r_A(1+\varepsilon)$$

At B, $\theta = \theta_B = \measuredangle AOB$:
$$\frac{1}{r_B} = \frac{GM_j}{h^2}(1+\varepsilon\cos\theta_B)$$

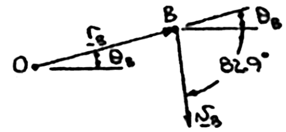
or
$$\frac{h^2}{GM_j} = r_B(1+\varepsilon\cos\theta_B)$$

Then
$$r_A(1+\varepsilon) = r_B(1+\varepsilon\cos\theta_B)$$

or
$$\cos\theta_B = \frac{1}{\varepsilon}\left[\frac{r_A}{r_B}(1+\varepsilon) - 1\right]$$
$$= \frac{1}{1.031}\left[\frac{44.0\times10^3\text{ mi}}{44.703\times10^3\text{ mi}}(1+1.031) - 1\right]$$
$$= 0.96902$$

or
$$\theta_B = 14.2988° \qquad \measuredangle AOB = 14.30° \blacktriangleleft$$

(*b*) From above $h^2 = GM_j r_B(1+\varepsilon\cos\theta_B)$



where $h = \dfrac{1}{m}|\mathbf{r}_B \times m\mathbf{v}_B| = r_B v_B \sin\phi$

$$\phi = (\theta_B + 82.9°) = 97.1988°$$

Then $(r_B v_B \sin\phi)^2 = GM_j r_B(1+\varepsilon\cos\theta_B)$

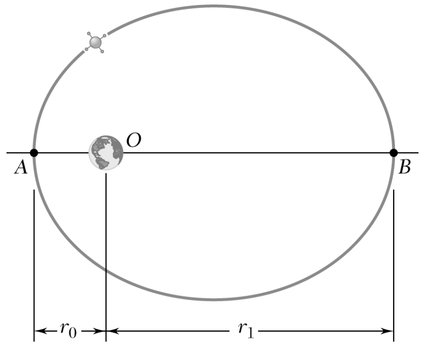
or

$$v_B = \frac{1}{\sin\phi}\left[\frac{GM_j}{r_B}(1+\varepsilon\cos\theta_B)\right]^{1/2}$$

$$= \frac{1}{\sin 97.1988°}\left\{\frac{34.4\times10^{-9}\ \text{ft}^4/\text{lb}\cdot\text{s}^4\times(1.30\times10^{26}\ \text{slug})}{236.03\times10^6\ \text{ft}}\times[1+(1.031)(0.96902)]\right\}^{1/2}$$

or $v_B = 196.2\ \text{ft/s}$ ◀

PROBLEM 12.118



A satellite describes an elliptic orbit about a planet. Denoting by r_0 and r_1 the distances corresponding, respectively, to the perigee and apogee of the orbit, show that the curvature of the orbit at each of these two points can be expressed as

$$\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right)$$

SOLUTION

Using Eq. (12.39),
$$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$$

and
$$\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$$

But
$$\theta_B = \theta_A + 180°,$$

so that
$$\cos\theta_A = -\cos\theta_B$$

Adding,
$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$

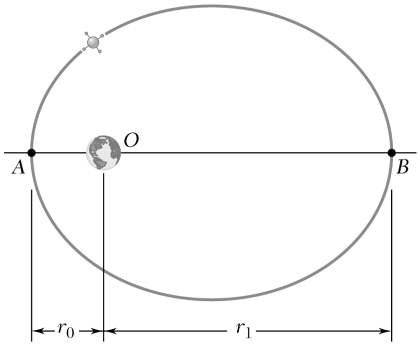
At Points A and B, the radial direction is normal to the path.

$$a_n = \frac{v^2}{\rho} = \frac{h^2}{r^2\rho}$$

But
$$F_n = \frac{GMm}{r^2} = ma_n = \frac{mh^2}{r^2\rho}$$

$$\frac{1}{\rho} = \frac{GM}{h^2} = \frac{1}{2}\left(\frac{1}{r_A} + \frac{1}{r_B}\right)$$

$$\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right) \blacktriangleleft$$



PROBLEM 12.119

(a) Express the eccentricity ε of the elliptic orbit described by a satellite about a planet in terms of the distances r_0 and r_1 corresponding, respectively, to the perigee and apogee of the orbit. (b) Use the result obtained in Part a and the data given in Problem 12.109, where $R_E = 149.6 \times 10^6$ km, to determine the approximate maximum distance from the sun reached by comet Hyakutake.

SOLUTION

(a) We have $$\frac{1}{r} = \frac{GM}{h^2}(1 + \varepsilon \cos \theta) \qquad \text{Eq. (12.39')}$$

At A, $\theta = 0$: $$\frac{1}{r_0} = \frac{GM}{h^2}(1 + \varepsilon)$$

or $$\frac{h^2}{GM} = r_0(1 + \varepsilon)$$

At B, $\theta = 180°$: $$\frac{1}{r_1} = \frac{GM}{h^2}(1 - \varepsilon)$$

or $$\frac{h^2}{GM} = r_1(1 - \varepsilon)$$

Then $$r_0(1 + \varepsilon) = r_1(1 - \varepsilon)$$

or $$\varepsilon = \frac{r_1 - r_0}{r_1 + r_0} \blacktriangleleft$$

(b) From above, $$r_1 = \frac{1 + \varepsilon}{1 - \varepsilon} r_0$$

where $$r_0 = 0.230 R_E$$

Then $$r_1 = \frac{1 + 0.999887}{1 - 0.999887} \times 0.230(149.6 \times 10^9 \text{ m})$$

or $$r_1 = 609 \times 10^{12} \text{ m} \blacktriangleleft$$

Note: $r_1 = 4070 R_E$ or $r_1 = 0.064$ lightyears.

PROBLEM 12.120

Derive Kepler's third law of planetary motion from Eqs. (12.39) and (12.45).

SOLUTION

For an ellipse, $$2a = r_A + r_B \quad \text{and} \quad b = \sqrt{r_A r_B}$$

Using Eq. (12.39), $$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$$

and $$\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$$

But $$\theta_B = \theta_A + 180°,$$

so that $$\cos\theta_A = -\cos\theta_B.$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A + r_B}{r_A r_B} = \frac{2a}{b^2} = \frac{2GM}{h^2}$$

$$h = b\sqrt{\frac{GM}{a}}$$

By Eq. (12.45),

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi ab\sqrt{a}}{b\sqrt{GM}} = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

$$\tau^2 = \frac{4\pi^2 a^3}{GM}$$

For Orbits 1 and 2 about the same large mass,

$$\tau_1^2 = \frac{4\pi^2 a_1^3}{GM}$$

and $$\tau_2^2 = \frac{4\pi^2 a_2^3}{GM}$$

Forming the ratio, $$\left(\frac{\tau_1}{\tau_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3 \blacktriangleleft$$

PROBLEM 12.121

Show that the angular momentum per unit mass h of a satellite describing an elliptic orbit of semimajor axis a and eccentricity ε about a planet of mass M can be expressed as

$$h = \sqrt{GMa(1-\varepsilon^2)}$$

SOLUTION

By Eq. (12.39′), $\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$

At A, $\theta = 0°$: $\frac{1}{r_A} = \frac{GM}{h^2} = (1+\varepsilon)$ or $r_A = \frac{h^2}{GM(1+\varepsilon)}$

At B, $\theta = 180°$: $\frac{1}{r_B} = \frac{GM}{h^2} = (1-\varepsilon)$ or $r_B = \frac{h^2}{GM(1-\varepsilon)}$

Adding, $r_A + r_B = \frac{h^2}{GM} = \left(\frac{1}{1+\varepsilon} + \frac{1}{1-\varepsilon}\right) = \frac{2h^2}{GM(1-\varepsilon^2)}$

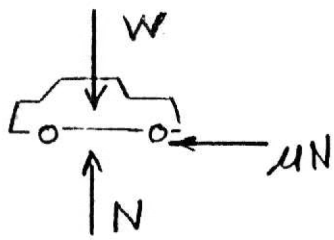
But for an ellipse, $r_A + r_B = 2a$

$$2a = \frac{2h^2}{GM(1-\varepsilon^2)} \qquad h = \sqrt{GMa(1-\varepsilon^2)} \blacktriangleleft$$

PROBLEM 12.122

In the braking test of a sports car its velocity is reduced from 70 mi/h to zero in a distance of 170 ft with slipping impending. Knowing that the coefficient of kinetic friction is 80 percent of the coefficient of static friction, determine (*a*) the coefficient of static friction, (*b*) the stopping distance for the same initial velocity if the car skids. Ignore air resistance and rolling resistance.

SOLUTION



(*a*) Coefficient of static friction.

$$\Sigma F_y = 0: \qquad N - W = 0 \qquad N = W$$

$$v_0 = 70 \text{ mi/h} = 102.667 \text{ ft/s}$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = a_t(s - s_0)$$

$$a_t = \frac{v^2 - v_0^2}{2(s - s_0)} = \frac{0 - (102.667)^2}{(2)(170)} = -31.001 \text{ ft/s}^2$$

For braking without skidding $\mu = \mu_s$, so that $\mu_s N = m|a_t|$

$$\xrightarrow{+} \Sigma F_t = ma_t: \quad -\mu_s N = ma_t$$

$$\mu_s = -\frac{ma_t}{W} = -\frac{a_t}{g} = \frac{31.001}{32.2} \qquad \mu_s = 0.963 \blacktriangleleft$$

(*b*) Stopping distance with skidding.

Use $\mu = \mu_k = (0.80)(0.963) = 0.770$

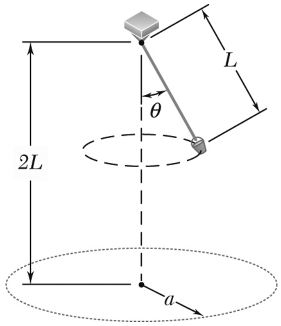
$$\xleftarrow{+} \Sigma F = ma_t: \quad \mu_k N = -ma_t$$

$$a_t = -\frac{\mu_k N}{m} = -\mu_k g = -24.801 \text{ ft/s}^2$$

Since acceleration is constant,

$$(s - s_0) = \frac{v^2 - v_0^2}{2a_t} = \frac{0 - (102.667)^2}{(2)(-24.801)}$$

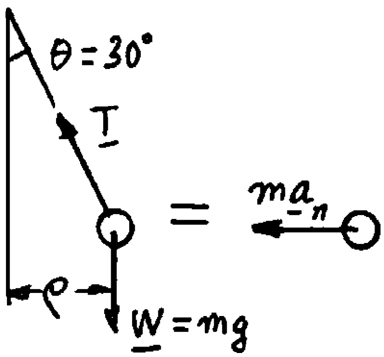
$$s - s_0 = 212 \text{ ft} \blacktriangleleft$$

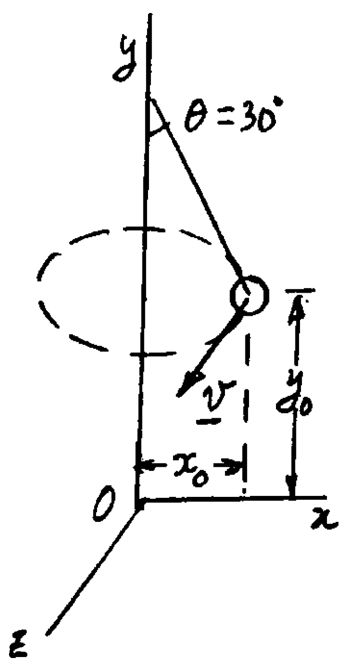


PROBLEM 12.123

A bucket is attached to a rope of length $L = 1.2$ m and is made to revolve in a horizontal circle. Drops of water leaking from the bucket fall and strike the floor along the perimeter of a circle of radius a. Determine the radius a when $\theta = 30°$.

SOLUTION





Initial velocity of drop = velocity of bucket

$$\Sigma F_y = 0: \qquad T\cos 30° = mg \tag{1}$$

$$\overset{+}{\longleftarrow}\ \Sigma F_x = ma_x: \qquad T\sin 30° = ma_n \tag{2}$$

Divide (2) by (1): $\quad \tan 30° = \dfrac{a_n}{g} = \dfrac{v^2}{\rho g}$

Thus $\quad v^2 = \rho g \tan 30°$

But $\quad \rho = L\sin 30° = (1.2 \text{ m}) \sin 30° = 0.6 \text{ m}$

Thus $\quad v^2 = 0.6(9.81)\tan 30° = 3.398 \text{ m}^2/\text{s}^2 \qquad v = 1.843 \text{ m/s}$

Assuming the bucket to rotate clockwise (when viewed from above), and using the axes shown, we find that the components of the initial velocity of the drop are

$$(v_0)_x = 0,\ (v_0)_y = 0,\ (v_0)_2 = 1.843 \text{ m/s}$$

<u>Free fall of drop</u>

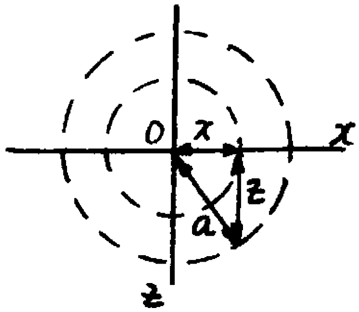
$$y = y_0 + (v_0)_y t - \frac{1}{2}gt^2 \qquad y = y_0 - \frac{1}{2}gt^2$$

When drop strikes floor:

$$y = 0 \qquad y_0 - \frac{1}{2}gt^2 = 0$$

But $\quad y_0 = 2L - L\cos 30° = 2(1.2) - 1.2\cos 30° = 1.361 \text{ m}$

Thus $\quad 1.361 - \dfrac{1}{2}(9.81)t^2 = 0 \qquad t = 0.5275$



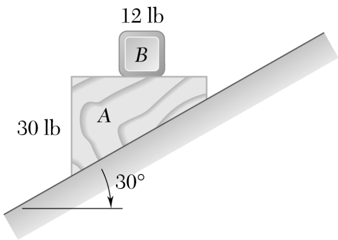
Projection on horizontal floor (uniform motion)

$$x = x_0 + (v_0)_z t = L\sin 30° + 0, \qquad x = 0.6 \text{ m}$$
$$z = z_0 + (v_0)_z t = 0 + 1.843(0.527) = 0.971 \text{ m}$$

Radius of circle: $a = \sqrt{x^2 + z^2}$

$$a = \sqrt{(0.6)^2 + (0.971)^2} \qquad a = 1.141 \text{ m} \blacktriangleleft$$

Note: The drop travels in a vertical plane parallel to the yz plane.



PROBLEM 12.124

A 12-lb block B rests as shown on the upper surface of a 30-lb wedge A. Neglecting friction, determine immediately after the system is released from rest (a) the acceleration of A, (b) the acceleration of B relative to A.

SOLUTION

Acceleration vectors:

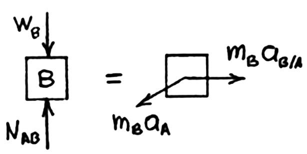
$$\mathbf{a}_A = a_A \swarrow 30^\circ, \quad \mathbf{a}_{B/A} = a_{B/A} \longrightarrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Block B:

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad m_B a_{B/A} - m_B a_A \cos 30^\circ = 0$$

$$a_{B/A} = a_A \cos 30^\circ \tag{1}$$

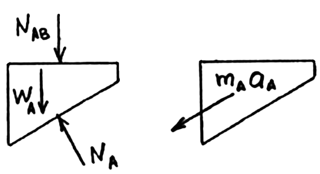


$$+\downarrow \Sigma F_y = ma_y: \quad N_{AB} - W_B = -m_B a_A \sin 30^\circ$$

$$N_{AB} = W_B - (W_B \sin 30^\circ)\frac{a_A}{g} \tag{2}$$

Block A:

$$+\swarrow \Sigma F = ma: \quad W_A \sin 30^\circ + N_{AB} \sin 30^\circ = W_A \frac{a_A}{g}$$



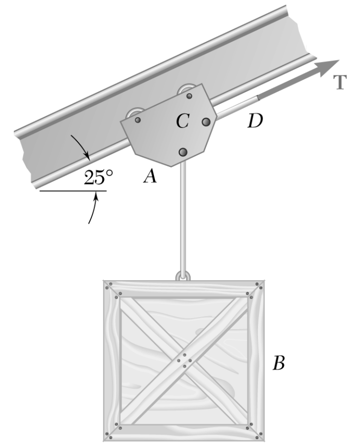
$$W_A \sin 30^\circ + W_B \sin 30^\circ - (W_B \sin^2 30^\circ)\frac{a_A}{g} = W_A \frac{a_A}{g}$$

$$a_A = \frac{(W_A + W_B)\sin 30^\circ}{W_A + W_B \sin^2 30^\circ} g = \frac{(30+12)\sin 30^\circ}{30 + 12\sin^2 30^\circ}(32.2) = 20.49 \text{ ft/s}^2$$

(a) $\mathbf{a}_A = 20.49 \text{ ft/s}^2 \swarrow 30^\circ$ ◀

$$a_{B/A} = (20.49)\cos 30^\circ = 17.75 \text{ ft/s}^2$$

(b) $\mathbf{a}_{B/A} = 17.75 \text{ ft/s}^2 \longrightarrow$ ◀



PROBLEM 12.125

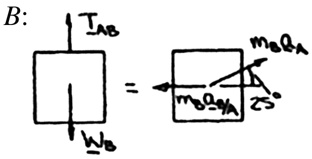
A 500-lb crate B is suspended from a cable attached to a 40-lb trolley A which rides on an inclined I-beam as shown. Knowing that at the instant shown the trolley has an acceleration of 1.2 ft/s^2 up and to the right, determine (a) the acceleration of B relative to A, (b) the tension in cable CD.

SOLUTION

(a) First we note: $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed perpendicular to cable AB.

$$\xrightarrow{+} \Sigma F_x = m_B a_x: \quad 0 = -m_B a_x + m_B a_A \cos 25°$$

B:



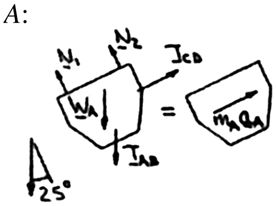
or $$a_{B/A} = (1.2 \text{ ft/s}^2)\cos 25°$$

or $$\mathbf{a}_{B/A} = 1.088 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$

(b) For crate B

$$+\!\uparrow \Sigma F_y = m_B a_y: \quad T_{AB} - W_B = \frac{W_B}{g} a_A \sin 25°$$

A:



or $$T_{AB} = (500 \text{ lb})\left[1 + \frac{(1.2 \text{ ft/s}^2)\sin 25°}{32.2 \text{ ft/s}^2}\right]$$

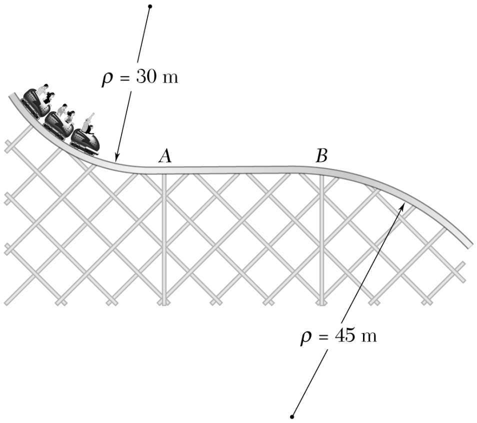
$$= 507.87 \text{ lb}$$

For trolley A

$$+\!\nearrow \Sigma F_x = m_A a_A: \quad T_{CD} - T_{AB}\sin 25° - W_A \sin 25° = \frac{W_A}{g} a_A$$

or $$T_{CD} = (507.87 \text{ lb})\sin 25° + (40 \text{ lb})\left(\sin 25° + \frac{1.2 \text{ ft/s}^2}{32.2 \text{ ft/s}^2}\right)$$

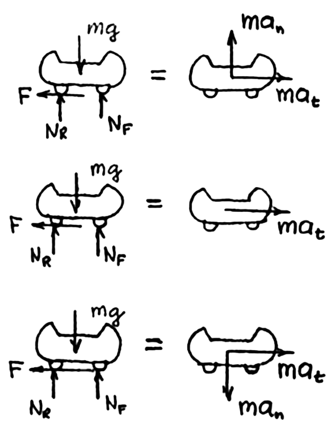
or $$T_{CD} = 233 \text{ lb} \blacktriangleleft$$



PROBLEM 12.126

The roller-coaster track shown is contained in a vertical plane. The portion of track between A and B is straight and horizontal, while the portions to the left of A and to the right of B have radii of curvature as indicated. A car is traveling at a speed of 72 km/h when the brakes are suddenly applied, causing the wheels of the car to slide on the track ($\mu_k = 0.25$). Determine the initial deceleration of the car if the brakes are applied as the car (a) has almost reached A, (b) is traveling between A and B, (c) has just passed B.

SOLUTION



$$v = 72 \text{ km/h} = 20 \text{ m/s}$$

(a) <u>Almost reached Point A</u>. $\rho = 30$ m

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{30} = 13.333 \text{ m/s}^2 \uparrow$$

$$\Sigma F_y = ma_y: \quad N_R + N_F - mg = ma_n$$

$$N_R + N_F = m(g + a_n)$$

$$F = \mu_k(N_R + N_F) = \mu_k m(g + a_n)$$

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad -F = ma_t$$

$$a_t = -\frac{F}{m} = -\mu_k(g + a_n)$$

$$|a_t| = \mu_k(g + a_n) = 0.25(9.81 + 13.33) \qquad |a_t| = 5.79 \text{ m/s}^2 \blacktriangleleft$$

(b) <u>Between A and B</u>. $\rho = \infty$

$$a_n = 0$$

$$|a_t| = \mu_k g = (0.25)(9.81) \qquad |a_t| = 2.45 \text{ m/s}^2 \blacktriangleleft$$

(c) <u>Just passed Point B</u>. $\rho = 45$ m

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{45} = 8.8889 \text{ m/s}^2 \downarrow$$

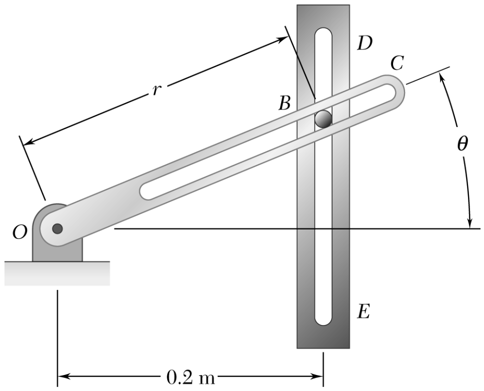
$$\Sigma F_y = ma_y: \quad N_R + N_F - mg = -ma_n$$

PROBLEM 12.126 (Continued)

or
$$N_R + N_F = m(g - a_n)$$
$$F = \mu_k (N_R + N_F) = \mu_k m(g - a_n)$$
$$\xrightarrow{+} \Sigma F_x = ma_x: \quad -F = ma_t$$
$$a_t = -\frac{F}{m} = -\mu_k (g - a_n)$$
$$|a_t| = \mu_k (g - a_n) = (0.25)(9.81 - 8.8889)$$

$|a_t| = 0.230 \text{ m/s}^2$ ◀

PROBLEM 12.127



The 100-g pin B slides along the slot in the rotating arm OC and along the slot DE which is cut in a fixed horizontal plate. Neglecting friction and knowing that rod OC rotates at the constant rate $\dot{\theta}_0 = 12$ rad/s, determine for any given value of θ (*a*) the radial and transverse components of the resultant force **F** exerted on pin B, (*b*) the forces **P** and **Q** exerted on pin B by rod OC and the wall of slot DE, respectively.

SOLUTION

<u>Kinematics</u>

From the drawing of the system, we have

$$r = \frac{0.2}{\cos\theta} \text{ m}$$

Then $$\dot{r} = \left(0.2\frac{\sin\theta}{\cos^2\theta}\dot{\theta}\right) \text{m/s} \qquad \dot{\theta} = 12 \text{ rad/s}$$

and $$\ddot{\theta} = 0$$

$$\ddot{r} = 0.2\frac{\cos\theta(\cos^2\theta) - \sin\theta(-2\cos\theta\sin\theta)}{\cos^4\theta}\dot{\theta}^2$$

$$= \left(0.2\frac{1+\sin^2\theta}{\cos^3\theta}\dot{\theta}^2\right)\text{m/s}^2$$

Substituting for $\dot{\theta}$

$$\dot{r} = 0.2\frac{\sin\theta}{\cos^2\theta}(12) = \left(2.4\frac{\sin\theta}{\cos^2\theta}\right)\text{m/s}$$

$$\ddot{r} = 0.2\frac{1+\sin^2\theta}{\cos^3\theta}(12)^2 = \left(28.8\frac{1+\sin^2\theta}{\cos^3\theta}\right)\text{m/s}^2$$

Now $$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= \left(28.8\frac{1+\sin^2\theta}{\cos^3\theta}\right) - \left(\frac{0.2}{\cos\theta}\right)(12)^2$$

$$= \left(57.6\frac{\sin^2\theta}{\cos^3\theta}\right)\text{m/s}^2$$

and $$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= 0 + 2\left(2.4\frac{\sin\theta}{\cos^2\theta}\right)(12)$$
$$= \left(57.6\frac{\sin\theta}{\cos^2\theta}\right)\text{m/s}^2$$

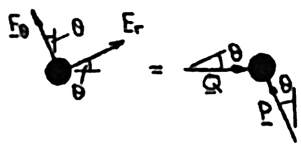
<u>Kinetics</u>

(*a*) We have $$F_r = m_B a_r = (0.1\text{ kg})\left[\left(57.6\frac{\sin^2\theta}{\cos^3\theta}\right)\text{m/s}^2\right]$$

or $$F_r = (5.76\text{ N})\tan^2\theta\sec\theta \blacktriangleleft$$

and $$F_\theta = m_B a_\theta = (0.1\text{ kg})\left[\left(57.6\frac{\sin\theta}{\cos^2\theta}\right)\text{m/s}^2\right]$$

or $$F_\theta = (5.76\text{ N})\tan\theta\sec\theta \blacktriangleleft$$



(*b*) Now $$+\uparrow \Sigma F_y: \quad F_\theta\cos\theta + F_r\sin\theta = P\cos\theta$$

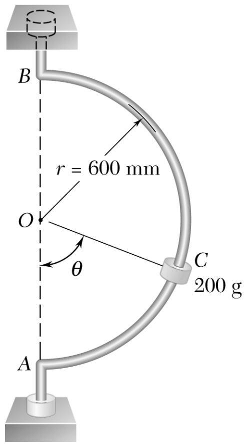
or $$P = 5.76\tan\theta\sec\theta + (5.76\tan^2\theta\sec\theta)\tan\theta$$

or $$\mathbf{P} = (5.76\text{ N})\tan\theta\sec^2\theta \;\theta \blacktriangleleft$$

$$+\nearrow \Sigma F_r: \quad F_r = Q\cos\theta$$

or $$Q = (5.76\tan^2\theta\sec\theta)\frac{1}{\cos\theta}$$

or $$\mathbf{Q} = (5.76\text{ N})\tan^2\theta\sec^2\theta \longrightarrow \blacktriangleleft$$



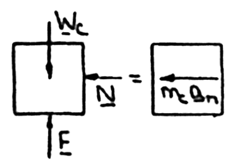
PROBLEM 12.128

A small 200-g collar C can slide on a semicircular rod which is made to rotate about the vertical AB at the constant rate of 6 rad/s. Determine the minimum required value of the coefficient of static friction between the collar and the rod if the collar is not to slide when (*a*) $\theta = 90°$, (*b*) $\theta = 75°$, (*c*) $\theta = 45°$. Indicate in each case the direction of the impending motion.

SOLUTION

First note
$$v_C = (r\sin\theta)\dot{\phi}_{AB}$$
$$= (0.6\text{ m})(6\text{ rad/s})\sin\theta$$
$$= (3.6\text{ m/s})\sin\theta$$

(*a*) With $\theta = 90°$,
$$v_C = 3.6\text{ m/s}$$



$$+\uparrow \Sigma F_y = 0: \quad F - W_C = 0$$

or
$$F = m_C g$$

Now
$$F = \mu_s N$$

or
$$N = \frac{1}{\mu_s} m_C g$$

$$\overset{+}{\longleftarrow} \Sigma F_n = m_C a_n: \quad N = m_C \frac{v_C^2}{r}$$

or
$$\frac{1}{\mu_s} m_C g = m_C \frac{v_C^2}{r}$$

or
$$\mu_s = \frac{gr}{v_C^2} = \frac{(9.81\text{ m/s}^2)(0.6\text{ m})}{(3.6\text{ m/s})^2}$$

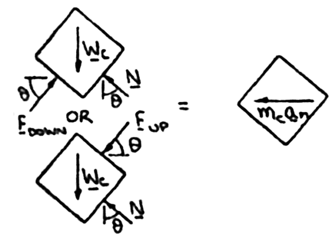
or
$$(\mu_s)_{\min} = 0.454 \blacktriangleleft$$

The direction of the impending motion is downward. ◀

(*b*) and (*c*)

First observe that for an arbitrary value of θ, it is not known whether the impending motion will be upward or downward. To consider both possibilities for each value of θ, let F_{down} correspond to impending motion downward, F_{up} correspond to impending motion upward, then with the "top sign" corresponding to F_{down}, we have

$$+\uparrow \Sigma F_y = 0: \quad N\cos\theta \pm F\sin\theta - W_C = 0$$



Now $$F = \mu_s N$$

Then $$N\cos\theta \pm \mu_s N\sin\theta - m_C g = 0$$

or $$N = \frac{m_C g}{\cos\theta \pm \mu_s \sin\theta}$$

and $$F = \frac{\mu_s m_C g}{\cos\theta \pm \mu_s \sin\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m_C a_n: \quad N\sin\theta \mp F\cos\theta = m_C \frac{v_C^2}{\rho} \qquad \rho = r\sin\theta$$

Substituting for N and F

$$\frac{m_C g}{\cos\theta \pm \mu_s \sin\theta}\sin\theta \mp \frac{\mu_s m_C g}{\cos\theta \pm \mu_s \sin\theta}\cos\theta = m_C \frac{v_C^2}{r\sin\theta}$$

or $$\frac{\tan\theta}{1 \pm \mu_s \tan\theta} \mp \frac{\mu_s}{1 \pm \mu_s \tan\theta} = \frac{v_C^2}{gr\sin\theta}$$

or $$\mu_s = \pm \frac{\tan\theta - \frac{v_C^2}{gr\sin\theta}}{1 + \frac{v_C^2}{gr\sin\theta}\tan\theta}$$

Now $$\frac{v_C^2}{gr\sin\theta} = \frac{[(3.6\text{ m/s})\sin\theta]^2}{(9.81\text{ m/s}^2)(0.6\text{ m})\sin\theta} = 2.2018\sin\theta$$

Then $$\mu_s = \pm \frac{\tan\theta - 2.2018\sin\theta}{1 + 2.2018\sin\theta\tan\theta}$$

(*b*) $\theta = 75°$

$$\mu_s = \pm \frac{\tan 75° - 2.2018\sin 75°}{1 + 2.2018\sin 75°\tan 75°} = \pm 0.1796$$

Then

downward: $\mu_s = +0.1796$

upward: $\mu_s < 0$ not possible

$(\mu_s)_{\min} = 0.1796$ ◀

The direction of the impending motion is downward. ◀

(c) $\theta = 45°$

$$\mu_s = \pm\frac{\tan 45° - 2.2018\sin 45°}{1 + 2.2018\sin 45°\tan 45°} = \pm(-0.218)$$

Then

downward: $\mu_s < 0$ not possible

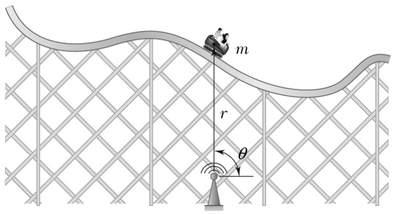
upward: $\mu_s = 0.218$ $(\mu_s)_{\min} = 0.218$ ◀

The direction of the impending motion is upward. ◀

Note: When $\tan\theta - 2.2018\sin\theta = 0$

or $\theta = 62.988°$,

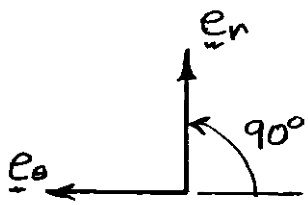
$\mu_s = 0.$ Thus, for this value of θ, friction is not necessary to prevent the collar from sliding on the rod.

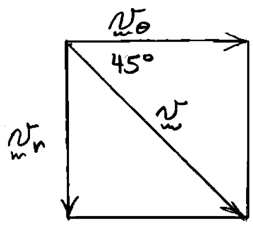


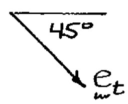
PROBLEM 12.129

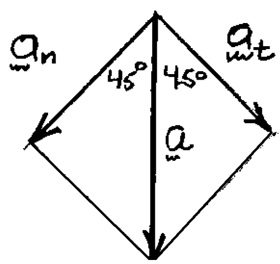
Telemetry technology is used to quantify kinematic values of a 200-kg roller coaster cart as it passes overhead. According to the system, $r = 25$ m, $\dot{r} = -10$ m/s, $\ddot{r} = -2$ m/s^2, $\theta = 90°$, $\dot{\theta} = -0.4$ rad/s, $\ddot{\theta} = -0.32$ rad/s^2. At this instant, determine (*a*) the normal force between the cart and the track, (*b*) the radius of curvature of the track.

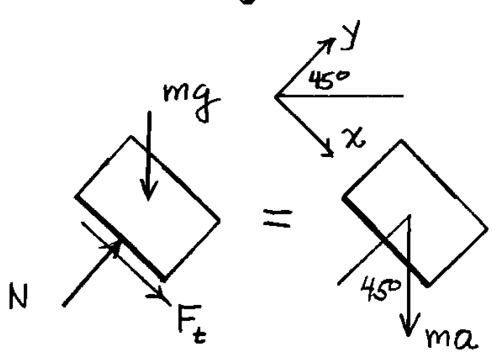
SOLUTION











Find the acceleration and velocity using polar coordinates.

$$v_r = \dot{r} = -10 \text{ m/s}$$

$$v_\theta = r\dot{\theta} = (25 \text{ m})(-0.4 \text{ rad/s}) = -10 \text{ m/s}$$

So the tangential direction is $\searrow$ 45° and $v = 10\sqrt{2}$ m/s.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2 \text{ m/s} - (25 \text{ m})(-0.4 \text{ rad/s})^2$$
$$= -6 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= (25 \text{ m})(-0.32) \text{ rad/s}^2| + 2(-10 \text{ m/s})(-0.4 \text{ rad/s})$$
$$= 0$$

So the acceleration is vertical and downward.

(*a*) To find the normal force use Newton's second law.

y-direction

$$N - mg\sin 45° = -ma\cos 45°$$
$$N = m(g\sin 45° - a\cos 45°)$$
$$= (200 \text{ kg})(9.81) \text{ m/s}^2 - 6 \text{ m/s}^2)(0.70711)$$
$$= 538.815 \text{ N}$$

$N = 539$ N ◀

(*b*) Radius *l* curvature of the track.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(10\sqrt{2})^2}{6\cos 45°}$$

$\rho = 47.1$ m ◀

PROBLEM 12.130

The radius of the orbit of a moon of a given planet is equal to twice the radius of that planet. Denoting by ρ the mean density of the planet, show that the time required by the moon to complete one full revolution about the planet is $(24\,\pi/G\rho)^{1/2}$, where G is the constant of gravitation.

SOLUTION

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let τ be the periodic time to complete one orbit.

$$v\tau = 2\pi r \qquad \text{or} \qquad \tau\sqrt{\frac{GM}{r}} = 2\pi r$$

Solving for τ,

$$\tau = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

But

$$M = \frac{4}{3}\pi R^3 \rho, \quad \text{hence,} \quad \sqrt{GM} = 2\sqrt{\frac{\pi}{3}G\rho}\; R^{3/2}$$

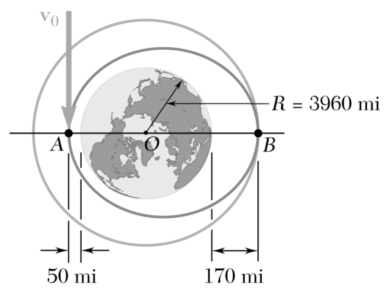
Then

$$\tau = \sqrt{\frac{3\pi}{G\rho}}\left(\frac{r}{R}\right)^{3/2}$$

Using $r = 2R$ as a given leads to

$$\tau = 2^{3/2}\sqrt{\frac{3\pi}{G\rho}} = \sqrt{\frac{24\pi}{G\rho}}$$

$\tau = (24\pi/G\rho)^{1/2}$ ◀



PROBLEM 12.131

At engine burnout on a mission, a shuttle had reached Point A at an altitude of 40 mi above the surface of the earth and had a horizontal velocity $\mathbf{v}_0$. Knowing that its first orbit was elliptic and that the shuttle was transferred to a circular orbit as it passed through Point B at an altitude of 170 mi, determine (a) the time needed for the shuttle to travel from A to B on its original elliptic orbit, (b) the periodic time of the shuttle on its final circular orbit.

SOLUTION

For Earth, $R = 3960 \text{ mi} = 20.909\times10^6 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

$$GM = gR^2 = (32.2)(20.909\times10^6)^2 = 14.077\times10^{15} \text{ ft}^3/\text{s}^2$$

(a) For the elliptic orbit,

$$r_A = 3960 + 40 = 4000 \text{ mi} = 21.12\times10^6 \text{ ft}$$
$$r_B = 3960 + 170 = 4130 \text{ mi} = 21.8064\times10^6 \text{ ft}$$

$$a = \frac{1}{2}(r_A + r_B) = 21.5032\times10^6 \text{ ft}$$
$$b = \sqrt{r_A r_B} = 21.4605\times10^6 \text{ ft}$$

Using Eq. 12.39, $$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$$

and $$\frac{1}{r_B} = \frac{GM}{r_B^2} + C\cos\theta_B$$

But $\theta_B = \theta_A + 180°,$ so that $\cos\theta_A = -\cos\theta_B$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A + r_B}{r_A r_B} = \frac{2a}{b^2} = \frac{2GM}{h^2}$$

or $$h = \sqrt{\frac{GMb^2}{a}}$$

Periodic time. $$\tau = \frac{2\pi ab}{h} = \frac{2\pi ab\sqrt{a}}{\sqrt{GMb^2}} = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

$$\tau = \frac{2\pi(21.5032\times10^6)^{3/2}}{\sqrt{14.077\times10^{15}}} = 5280.6 \text{ s} = 1.4668 \text{ h}$$

The time to travel from A to B is one half the periodic time

$$\tau_{AB} = 0.7334 \text{ h}$$ $\tau_{AB} = 44.0 \text{ min}$ ◀

PROBLEM 12.131 (Continued)

(*b*) For the circular orbit, $a = b = r_B = 21.8064 \times 10^6 \text{ ft}$

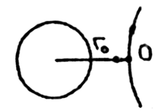
$$\tau_{\text{circ}} = \frac{2\pi a^{3/2}}{\sqrt{GM}} = \frac{2\pi(21.8064 \times 10^6)^{3/2}}{\sqrt{14.077 \times 10^{15}}} = 5393 \text{ s}$$

$\tau_{\text{circ}} = 1.498 \text{ h}$ $\tau_{\text{circ}} = 89.9 \text{ min}$ ◀

PROBLEM 12.132

It was observed that as the Galileo spacecraft reached the point on its trajectory closest to Io, a moon of the planet Jupiter, it was at a distance of 1750 mi from the center of Io and had a velocity of 49.4×10^3 ft/s. Knowing that the mass of Io is 0.01496 times the mass of the earth, determine the eccentricity of the trajectory of the spacecraft as it approached Io.

SOLUTION



First note $\quad r_0 = 1750 \text{ mi} = 9.24\times10^6 \text{ ft}$

$$R_{\text{earth}} = 3960 \text{ mi} = 20.9088\times10^6 \text{ ft}$$

We have $\quad \dfrac{1}{r} = \dfrac{GM}{h^2}(1+\varepsilon\cos\theta) \qquad$ Eq. (12.39)

At Point O, $\quad r = r_0, \quad \theta = 0, \quad h = h_0 = r_0 v_0$

Also, $\quad GM_{Io} = G(0.01496 M_{\text{earth}})$

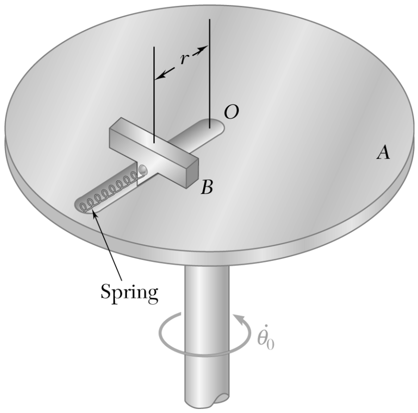
$$= 0.01496\, g R_{\text{earth}}^2 \qquad \text{using Eq. (12.30).}$$

Then $\quad \dfrac{1}{r_0} = \dfrac{0.01496\, g R_{\text{earth}}^2}{(r_0 v_0)^2}(1+\varepsilon)$

or $\quad \varepsilon = \dfrac{r_0 v_0^2}{0.01496\, g R_{\text{earth}}^2} - 1$

$$= \frac{(9.24\times10^6 \text{ ft})(49.4\times10^3 \text{ ft/s})^2}{0.01496(32.2 \text{ ft/s}^2)(20.9088\times10^6 \text{ ft})^2} - 1$$

or $\quad \varepsilon = 106.1$ ◀



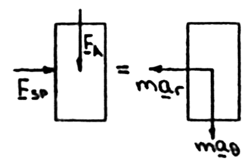
PROBLEM 12.133*

Disk A rotates in a horizontal plane about a vertical axis at the constant rate $\dot{\theta}_0 = 10$ rad/s. Slider B has mass 1 kg and moves in a frictionless slot cut in the disk. The slider is attached to a spring of constant k, which is undeformed when $r = 0$. Knowing that the slider is released with no radial velocity in the position $r = 500$ mm, determine the position of the slider and the horizontal force exerted on it by the disk at $t = 0.1$ s for (a) $k = 100$ N/m, (b) $k = 200$ N/m.

SOLUTION

First we note when $r = 0, \quad x_{sp} = 0 \Rightarrow F_{sp} = kr$

$$r_0 = 500 \text{ mm} = 0.5 \text{ m}$$



and $$\dot{\theta} = \dot{\theta}_0 = 12 \text{ rad/s}$$

then $$\ddot{\theta} = 0$$

$$\overset{+}{\leftarrow} \Sigma F_r = m_B a_r: \quad -F_{sp} = m_B(\ddot{r} - r\dot{\theta}_0^2)$$

$$\ddot{r} + \left(\frac{k}{m_B} - \dot{\theta}_0^2\right) r = 0 \tag{1}$$

$$+\!\downarrow \Sigma F_\theta = m_B a_\theta: \quad F_A = m_B(0 + 2\dot{r}\dot{\theta}_0) \tag{2}$$

(a) $k = 100$ N/m

Substituting the given values into Eq. (1)

$$\ddot{r} + \left[\frac{100 \text{ N/m}}{1 \text{ kg}} - (10 \text{ rad/s})^2\right] r = 0$$

$$\ddot{r} = 0$$

Then $\dfrac{d\dot{r}}{dt} = \ddot{r} = 0$ and at $t = 0, \ \dot{r} = 0$:

$$\int_0^{\dot{r}} d\dot{r} = \int_0^{0.1} (0)\, dt$$

$$\dot{r} = 0$$

and $$\frac{dr}{dt} = \dot{r} = 0 \text{ and at } t = 0,\ r_0 = 0.5 \text{ m}$$

$$\int_{r_0}^{r} dr = \int_{0}^{0.1} (0)\, dt$$

$$r = r_0$$

$$r = 0.5 \text{ m} \blacktriangleleft$$

Note: $\dot{r} = 0$ implies that the slider remains at its initial radial position.

With $\dot{r} = 0$, Eq. (2) implies

$$F_H = 0 \blacktriangleleft$$

(*b*) $k = 200$ N/m

Substituting the given values into Eq. (1)

$$\ddot{r} + \left[\frac{200}{1 \text{ kg}} - (10 \text{ rad/s})^2\right] r = 0$$

$$\ddot{r} + 100\, r = 0$$

Now $$\ddot{r} = \frac{d}{dt}(\dot{r}) \qquad \dot{r} = v_r \qquad \frac{d}{dt} = \frac{dr}{dt}\frac{d}{dr} = v_r \frac{d}{dr}$$

Then $$\ddot{r} = v_r \frac{dv_r}{dr}$$

so that $$v_r \frac{dv_r}{dr} + 100r = 0$$

At $t = 0,\ v_r = 0,\ r = r_0$: $$\int_0^{v_r} v_r d\, v_r = -100 \int_{r_0}^{r} r\, dr$$

$$v_r^2 = -100(r^2 - r_0^2)$$

$$v_r = 10\sqrt{r_0^2 - r^2}$$

Now $$v_r = \frac{dr}{dt} = 10\sqrt{r_0^2 - r^2}$$

At $t = 0,\ r = r_0$: $$\int_{r_0}^{r} \frac{dr}{\sqrt{r_0^2 - r^2}} = \int_0^t 10\, dt = 10t$$

Let $$r = r_0 \sin\phi, \qquad dr = r_0 \cos\phi\, d\phi$$

Then $$\int_{\pi/2}^{\sin^{-1}(r/r_0)} \frac{r_0 \cos\phi\, d\phi}{\sqrt{r_0^2 - r_0^2 \sin^2\phi}} = 10t$$

$$\int_{\pi/2}^{\sin^{-1}(r/r_0)} d\phi = 10t$$

$$\sin^{-1}\left(\frac{r}{r_0}\right) - \frac{\pi}{2} = 10t$$

$$r = r_0 \sin\left(10t + \frac{\pi}{2}\right) = r_0 \cos 10t = (0.5 \text{ ft})\cos 10t$$

Then $$\dot{r} = -(5 \text{ m/s})\sin 10t$$

Finally, at $t = 0.1$ s:

$$r = (0.5 \text{ ft})\cos(10 \times 0.1)$$

$$r = 0.270 \text{ m} \blacktriangleleft$$

Eq. (2) $$F_H = 1 \text{ kg} \times 2 \times [-(5 \text{ ft/s})\sin(10 \times 0.1)]\,(10 \text{ rad/s})$$

$$F_H = -84.1 \text{ N} \blacktriangleleft$$