out

The design for website is to consist of 4 colors, 3 fonts & 3 positions for an image, how many different designs are possible?

colors   fonts   positions $= 4 \times 3 \times 3 = 36$

permutations

The num of permutations of n different element is $n!$

$n! = n \times (n-1) \times (n-2) \ldots$

Exp: The num of permutations of four letters a, b, c, d

$4! = 4 \times 3 \times 2 \times 1 = 24$

← لما بدي ارتب المجموعة كاملة بستخدم $n!$. less than or equal n.

← لما بدي ارتب عدد عناصر معين من المجموعة $P^n_r = n \times (n-1)(n-2)\ldots(n-r+1)$

$$P^n_r = \frac{n!}{(n-r)!}$$

Exp: consider $S = \{a, b, c, d, e\}$ what is the number of permutation of subsets of 3 elements selected from S?

$$P^5_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2!}{2!} = 60$$

← لما يكون عندي مجموعة مكونة من n مقسومة لمجموعتين

$$\frac{n!}{\underset{\text{المجموعة الثانية}}{!}\ \underset{\text{المجموعة الأولى}}{!}}$$

Combinations

Exp: how many possible selections of 3 balls from box contain 10 colored balls?

$$\frac{10!}{3!\,7!} = 120$$

chapter 3:

$p(X=x, Y=y, R=r, \dots)$

$p(X=x, Y=y)$

↑ joint – pMF

Ex: let X & Y be two R.Vs with the following Joint pMF

$$p(X=x) = \begin{cases} 1/8, & x=-1, y=0 \\ 1/8, & x=-1, y=1 \\ 1/4, & x=0, y=0 \\ 1/8, & x=1, y=0 \\ K, & x=1, y=2 \\ 0, & O.W \end{cases}$$

ما بنحط تقاطع لانه التقاطع بكون بين 2 events

مجموع كل الـ joint pro = 1

[a] Determine the value of constant K?

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} p(X=x, Y=y) = 1$$

$p(X=-1, Y=0) + p(X=-1, Y=1) + \dots = 1$

$\frac{1}{8} + \frac{1}{8} + \frac{1}{4} + \frac{1}{8} + K = 1$

$K = 1 - \frac{5}{8} = \frac{3}{8}$

X \ Y	0	1	2
-1	1/8	1/8	0
0	1/4	0	0
1	1/8	0	3/8

الاصفار بتكون عبارة عن O.W

لازم تتحقق الشرطين

[b] $p(X \le 0, Y \le 0) = ?$

$p(X=-1, Y=0) + p(X=0, Y=0)$

$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

properties of joint - pmf

1. $p(X=x, Y=y) \geq 0$

2. $\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} p(X=x, Y=y) = 1$

c. $p(X \leq 0, y \geq 1) = ?$

$p(x=0, y=1) = \frac{1}{8}$

d. $p(X \leq 0 / y \geq 1) = ?$

$\frac{p(X \leq 0, y \geq 1)}{p(y \geq 1)} = \frac{\frac{1}{8}}{\frac{4}{8}} = \frac{1}{4}$ التقاطع هون معناها يتحقق الشرطين

e. $p(X > 0) = ?$ مدام فش شرط لل y باخذ y كلها.

$\frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

f. $F_{x,y}(0,1) = ?$ joint CDF of x and y

الرقم الأول لل x ←

$= p(X \leq 0, y \leq 1) = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

g. $F_{y,x}(0,1) = ?$

$p(y \leq 0, X \leq 1)$ صدفة طلعو زي بعض f, g

$= \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{1}{2}$

h. $F_{x,y}(3, -2) = ?$

$p(X \leq 3, Y \leq -2)$ → فش عندي فش موجود

$= 0$

i. $F_{x,y}(3,3) = ?$

$p(X \leq 3, y \leq 3)$ كل الجدول .

$= 1$

j. $F_{x,y}(-3,-2)$

$= p(X \leq -3, y \leq -2)$

$= 0$

K. $F_{x,y}(-\infty,-\infty)$

$= p(X \leq -\infty, y \leq -\infty) = 0$

l. $F_{x,y}(\infty,\infty)$

$= p(X \leq \infty, y \leq \infty) = 1$

m. $F_{x,y}(\infty,-\infty) = ?$

$F_{x,y}(\infty,-\infty) = p(X \leq \infty, y \leq -\infty) = 0$

n. Determine the marginal PMF of X? كأنه بطلب pdf من x

$p(X=-1) = p(X=-1, -\infty < y < \infty)$ و فــ بـنرجع على y بكل القيم

$= \sum_{y=-\infty}^{\infty} p(X=-1, Y=y)$

$= p(X=-1, y=0) + p(X=-1, y=1)$

$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

prob of X   بكونه معطى عندي joint PMF وبدي أطلع منه

$p(X=0) = ?$

$$\sum_{y=-\infty}^{\infty} (X=0, Y=y)$$

$$= \frac{1}{4}$$

Result

$$p(X=x) = \begin{cases} 1/4 & , x=-1 \\ 1/4 & , x=0 \\ 1/2 & , x=1 \\ 0 & , o.w \end{cases}$$

ممنوع تنسيها

$p(X=1) = ?$

$$\sum_{y=-\infty}^{\infty} p(X=x)$$

$$= \frac{1}{8} + \frac{3}{8} \neq \frac{1}{4}$$

[o] PMF of y?

$$p(y=3) = \sum_{x=-\infty}^{\infty} p(y=3, X=x) = 0$$

$$p(y=2) = \sum_{x=-\infty}^{\infty} p(y=2, X=x) = \frac{3}{8}$$

$$p(y=1) = \sum_{x=-\infty}^{\infty} p(y=1, X=x) = \frac{1}{8}$$

$$p(y=0) = \sum_{x=-\infty}^{\infty} p(y=0, X=x) = \frac{4}{8}$$

$$p(y=y) = \begin{cases} 4/8 & , y=0 \\ 1/8 & , y=1 \\ 3/8 & , y=2 \\ 0 & , y=3 \\ 0 & , o.w \end{cases}$$

Note: x and y are said to be S. Independent if $p(X=x, Y=y) = p(X=x)\, p(Y=y)$

p. Are x and y S.Independent?

$p(X=-1, y=0) \overset{?}{=} p(x=-1)\, p(y=0)$

$$\frac{1}{8} = \frac{1}{4} \times \frac{4}{8} = \frac{1}{8}$$

$p(x=-1, y=-1) \overset{?}{=} p(x=-1)\, p(y=1)$

$$\frac{1}{8} \overset{?}{=} \frac{1}{4} \cdot \frac{1}{8}$$

So x and y are not statistically independent

Q $p(x=0/y=+1) = ?$

$= \frac{p(x=0, y=+1)}{p(y=+1)} = \frac{1/8}{1/8} = 1$

R $p(\underbrace{x\geq 0/y\leq 0}_{\text{cond 1}}, \underbrace{x\leq 0}_{\text{cond 2}}) = ?$

x = 0 فقط مشتركة

$= \frac{p(x\geq 0, x\leq 0, y\leq 0)}{p(y\leq 0, x\leq 0)} = \frac{p(x=0, y\leq 0)}{p(y\leq 0, x\leq 0)} = \frac{1/4}{(1/8 + 1/4)} = \frac{1/4}{3/8}$

$= \frac{2/8}{3/8} = \frac{2}{3}$

Lec 2 From online :

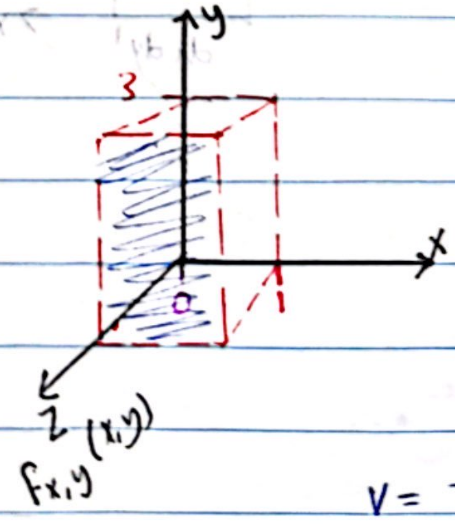
Exp: Let x & y be two R.Vs with the following joint pdf

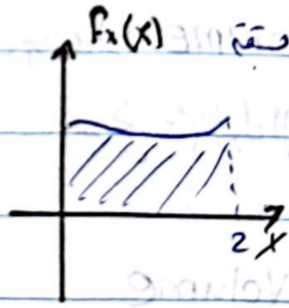
لازم تكون دائماً موجبة عشان ما تطلع سالبة

$$f_{x,y}(x,y) = \begin{cases} K & 0\leq x\leq 1, 0\leq y\leq 3 \\ 0 & \text{o.w} \end{cases}$$

a. Determine the value of the constant K?

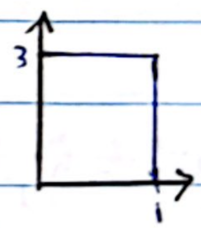
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x,y)\, dy\, dx = 1$$





المعلومات السابقة

$\int^{2} f_x(x)\, dx$



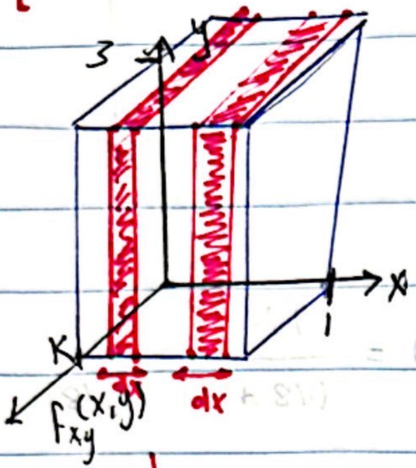
V = مساحة القاعدة × ع

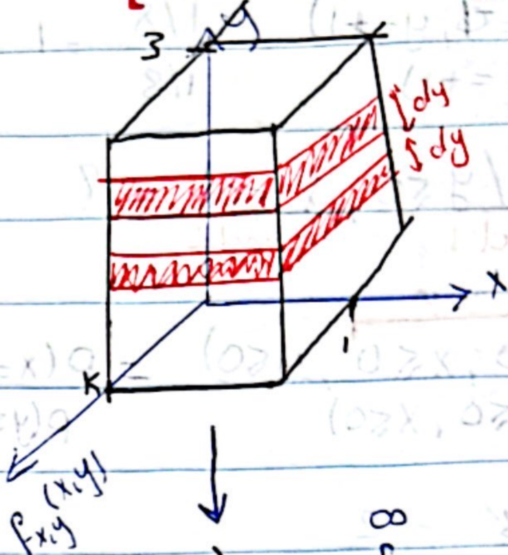
نقدر نستخدمها لأن الارتفاع ← $V = 1\times 3\times K = 3K$

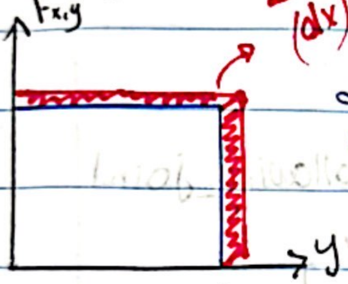
ثابت $V = 1 = 3K = 1 \quad K = \frac{1}{3}$

بعمل هاد أول

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f_{x,y}(x,y)\,dy\right]dx = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f_{x,y}(x,y)\,dx\right]dy$$

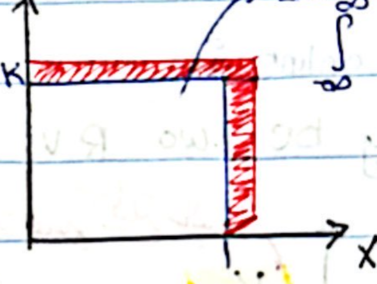






الهاعدة معين (dx)

كاني اخدت شريحة من الزهر.



$\int_{-\infty}^{\infty} K\,dx = \text{area}_{②}$

$\int_{-\infty}^{\infty} K\,dx\,dy = \text{area}_{②} * dy$

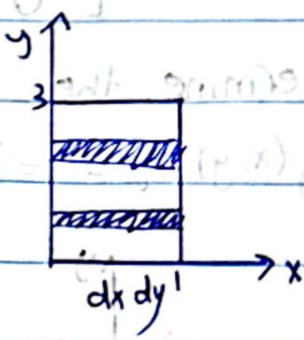
$= \text{Volume slice}_{②}$

$\int_{-\infty}^{\infty} K\,dy = \text{area}$

$\int_{-\infty}^{\infty} K\,dy\,dx = \text{area} * dx$

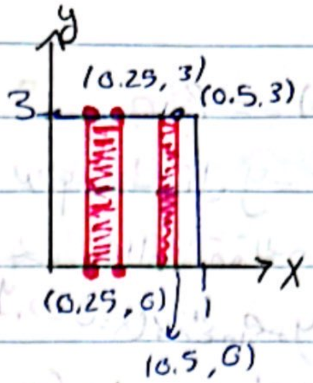
$= \text{Volume slice}_{①}$

dy dx → شرائح على طول ال y



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K\,dy\,dx = \text{Volume}$$

① $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x,y)\, dy\, dx$



$\int_0^1 \left[\int_0^3 K\, dy\right] dx$

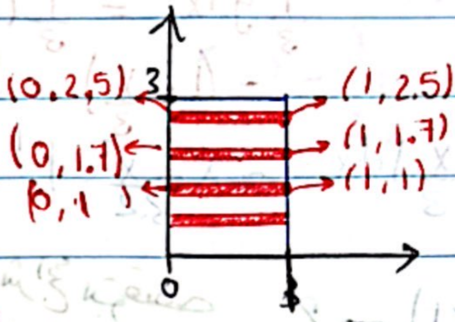
نشوف وين y بلشت ووين خلصت

$\int_0^1 Ky|_0^3\, dx = \int_0^1 K[3-0]\, dx = \int_0^1 3K\, dx$

$3Kx|_0^1 = 3K = 1$

$K = \frac{1}{3}$

② $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x,y)\, dx\, dy$

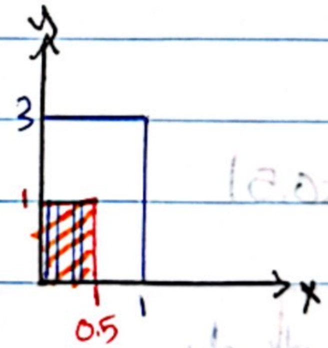


$\int_0^3 \int_0^1 K\, dx\, dy$

$= \int_0^3 Kx|_0^1\, dy = \int_0^3 K[1-0]\, dy$

$= \int_0^3 K\, dy = Ky|_0^3 = 3K = 1 \quad K = \frac{1}{3}$

$$\begin{cases} \frac{1}{3}, & 0 \le x \le 1 \quad 0 \le y \le 3 \\ 0 & \text{o.w} \end{cases}$$



b. $p(0 \le x \le 0.5,\ 0 \le y \le 1) = ?$

$\int_0^{0.5}\int_0^1 \frac{1}{3}\, dy\, dx$ → لما اني اخترت dydx لنرسم شرائح على طول dy

$\int_0^{0.5} \frac{1}{3} y|_0^1\, dx = \int_0^{0.5} \frac{1}{3}\, dx = \frac{1}{3} x|_0^{0.5} = \frac{0.5}{3} = \frac{1}{6}$

or مساحة هذا المستطيلات = م القاعدة × ع → نادراً ما يجيه الارتفاع رقم ثابت

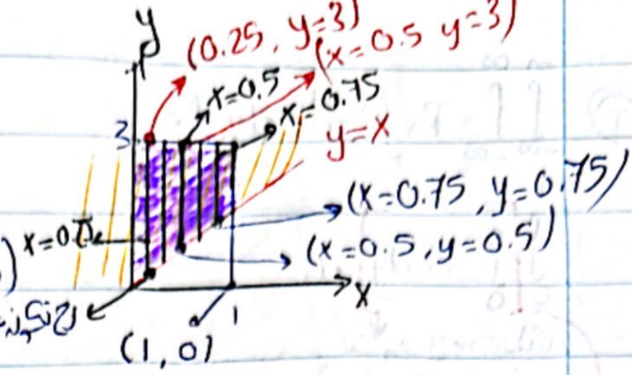
$\frac{1}{6} = \frac{1}{3} \times 0.5 =$

c. $p(x \le y) = ?$

برسم خط يمثل $x = y$

نجد إذا المنطقة يلي بدي ياها تحت أو فوق الخط

رح تكون عبارة $(x=0.25, y=0.25)$ لأنه المحور هو $x = y$



هل هون $(x=1 > y=0)$ لا إذا المنطقة اللي بدي ياها اللي فوق.

بهتم بالمساحة يلي بالبنفسجي لأنه يلي بالأصفر فاضي في مساحة.

$$p(x \le y) = \int_0^1 \left[\int_x^3 \frac{1}{3}\, dy\right] dx$$

من وين بتبلش x هي y

$$\frac{1}{3} y \Big|_x^3 = \frac{1}{3}(3-x)$$

$$= \left(1 - \frac{x}{3}\right)$$

$$\int_0^1 \left(1 - \frac{x}{3}\right) dx = x - \frac{x^2}{3 \cdot 2} \Big|_0^1 = 1 - \frac{1}{6} = \frac{5}{6}$$

d. $p(xy \le 1) = ?$

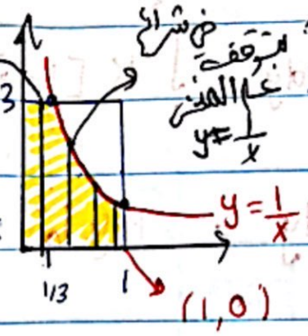
في شرائح بتوقف عند 3

في شرائح بتوقف عند المنحنى $y = \frac{1}{x}$

إذا نقسم التكامل لجزئين

$xy = 1$

$y = \frac{1}{x}$

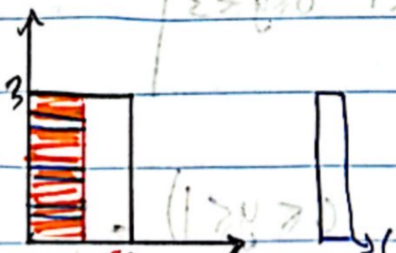


$$p(xy \le 1) = \int_0^{1/3}\int_0^3 \frac{1}{3}\, dy\, dx + \int_{\frac{1}{3}}^{1}\int_0^{\frac{1}{x}} dy\, dx$$

70% من الحل عبارة عن إيجاد حدود التكامل

$(1 \times 0 \le 1)$ yes

e. $p(x \le 0.5)$



$$\int_0^3\left[\int_0^{0.5} \frac{1}{3}\, dx\right] dy$$

$$\frac{x}{3}\Big|_0^{0.5} = \frac{1}{6} \qquad \int_0^3 \frac{1}{6}\, dy = \frac{y}{6}\Big|_0^3 = \frac{1}{2}$$

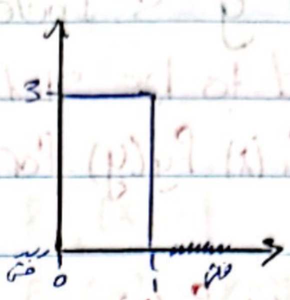
← بنوخذ الحدود الأصلية من السؤال

f. determine the marginal pdf of X ? pdf → x

بنكامل بالنسبة لل y

$$f(x) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\, dy \rightarrow \text{marginal pdf of } X$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\, dy$$



case 1

$x < 0 \quad \int_{-\infty}^{\infty} 0\, dy = 0$

case 2

$0 < x < 1 \quad f_x(x) = \int_0^3 \frac{1}{3}\, dy = 1$

المساواة بتعتمد حسب اذا كان موجود بالفترة الأصلية أولا.

$$f_x(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{o.w} \end{cases}$$

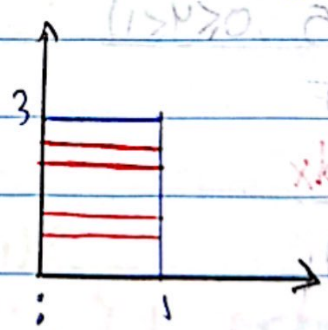
case 3

$1 < x \quad f_x(x) = \int_{-\infty}^{\infty} 0\, dy = 0$

g. determine the marginal pdf of y.

$$f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\, dx \rightarrow \text{marginal pdf of } y$$

$$f_y(y) = \int_{-\infty}^{\infty} f_{x,y}(x,y)\, dx$$



case 1 $\quad y < 0$

$f_y(y) = \int_{-\infty}^{\infty} 0\, dx = 0$

case 2 $\quad 0 < y \leq 3$

$\int_0^1 \frac{1}{3}\, dx = \frac{x}{3}\Big|_0^1 = \frac{1}{3}$

case 3 $\quad y > 3$

$\int_{-\infty}^{\infty} 0\, dx = 0$

$$f_y(y) = \begin{cases} 1/3 & 0 \le y \le 3 \\ 0 & \text{O.W} \end{cases}$$

h. Are x and y S. Independent?

X & y are said to be statistically Independent if $f_{x,y}(x,y) = f_x(x)\, f_y(y)$ for all values of x & y.

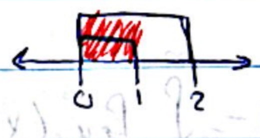
$$f_{x,y}(x,y) \stackrel{?}{=} f_x(x)\, f_y(y).$$

$$\begin{cases} \frac{1}{3} & 0 \le x \le 1,\ 0 \le y \le 3 \\ 0 & \text{O.W} \end{cases} \stackrel{?}{=} \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{O.W} \end{cases} \cdot \begin{cases} \frac{1}{3} & 0 \le y \le 3 \\ 0 & \text{O.W} \end{cases}$$

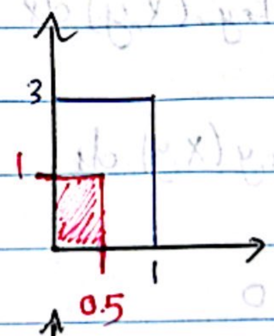
x & y are S. Independent

i. $P(0 \le X \le 0.5,\ 0 \le y \le 1 \,/\, y \le 2)$

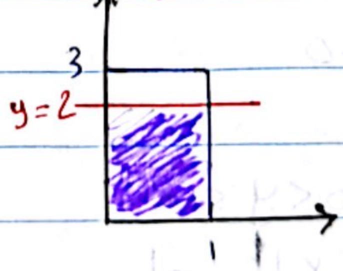
$$= \frac{P(0 \le X \le 0.5,\ 0 \le y \le 1,\ y \le 2)}{P(y \le 2)}$$



$$= \frac{P(0 \le X \le 0.5,\ 0 \le y \le 1)}{P(y \le 2)}$$



$$= \frac{\int_0^{0.5}\int_0^{1} \frac{1}{3}\, dy\, dx}{\int_0^{1}\int_0^{2} \frac{1}{3}\, dy\, dx}$$



و ما برنم أحل نفس الفرع يلي قبل

d. $p(y \le 1 / x = 0.5) = ?$ — بما انه cont ما بزبط اسأل عن نقطة معينة، مش فترة على فترة

← لهيك بنفي معادلة بنجاب conditional pdf

Conditional pdf of x: $f_{y/x=0.5}(y) = \frac{f_{x,y}(x,y)}{f_x(x)}\Big|_{x=0.5}$ (given x)

Conditional pdf of y: $f_{x/y}(x) = \frac{f_{x,y}(x,y)}{f_y(y)}\Big|_{y=y}$

solution

$f_{y/x=0.5}(y) = \frac{f_{x,y}(x,y)}{f_x(x)}\Big|_{x=0.5}$ (→ pdf of x) $\begin{cases} K = \frac{1}{3} & 0 \le x \le 1, 0 \le y \le 3 \\ 0 & \text{o.w} \end{cases}$

$= \frac{\frac{1}{3}}{1} = \frac{1}{3}$ $= \begin{cases} \frac{1/3}{1} & 0 \le y \le 3 \\ \frac{0}{1} & \text{o.w} \end{cases}$

$f_{y/x=0.5}(y) = \begin{cases} \frac{1}{3} & 0 \le y \le 3 \\ 0 & \text{o.w} \end{cases}$ عبارة عن pdf لل y

$p(y \le 1 / x = 0.5) = \int_{-\infty}^{1} f_{y/x=0.5}(y)\, dy$

$= \int_{0}^{1} \frac{1}{3}\, dy = \frac{1}{3}$

K. $p(0.5 \le x \le 0.75 / y = 1)$?

$f_{x/y=1}(x) = \begin{cases} \frac{\frac{1}{3}}{\frac{1}{3}} \cdot \frac{1}{30}... & 0 \le x \le 1 \\ 0 & \text{o.w} \end{cases} = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{o.w} \end{cases}$

غالباً x & y S.I ال cond طلع هو نفسه pdf لل x

$p(0.5 \le x \le 0.75 / y = 1) = \int_{0.5}^{0.75} f_{x/y=1}(x)\, dx = \int_{0.5}^{0.75} 1\, dx = 0.25$

1. $p(0.5 \leq x \leq 0.75 \mid y=4) = ?$

$f_{x/y=4}(x) = \frac{0}{0} = ?$ مش لازم يسأل لأنه أنا فوق قلت انه

$p(0.5 \leq x \leq 0.75 \mid y=4) = 0$. لما $y=4$ صحيح أنا هو احتمالي صفر

lec 3:

Exp: x & y are two R.V with the following joint pMF.

x \ y	-1	0	1
-1	1/8	1/2	0
1	0	1/4	1/8

a. $E\{xy\} = ?$

$E\{g(x,y)\} = \sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} g(x,y)\, p(X=x, y=y)$

$g(x,y) = xy$

$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} xy \; p(X=x, Y=y)$

$= \underset{(x=-1)}{(-1)} \underset{y=-1}{(-1)} p(x=-1, y=-1) + \underset{x=-1}{(-1)} \underset{y=0}{(0)} p(x=-1, y=0) +$

$= (-1)(-1)(1/8) + (-1)(0)(1/2) + (-1)(1)(0) + (1) \sim + (1)(1)(1/8)$

$= 2/8$

b. $E\{X^2Y\} = ?$

$$= \sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} x^2 y \; p(X=x, Y=y).$$

$(-1)^2 \cdot (-1) \cdot 1/8 + (-1)^2 (0)(1/2) + \_\_ + (1)^2 (1)(1/8)$

$= -1/8 + 1/8 = 0$

c. $E\{(X+1)Y\} = ?$

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} (x+1)y \; p(X=x, Y=y)$$

$(0)(-1) \cdot (1/8) + (0) + \_\_ + (1+1)(-1)(0) + (1+1)(1)(1/8)$

$= 2 \cdot \frac{1}{8} = 1/4$

Exp2: X & Y are two R.V with the following joint pdf

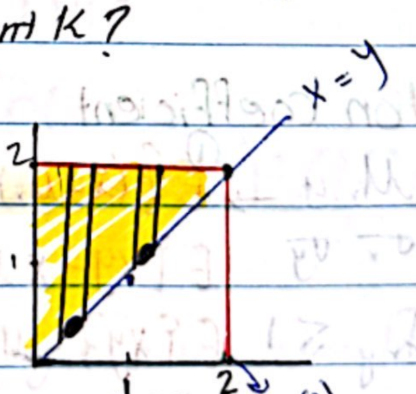
$$f_{X,Y}(x,y) = \begin{cases} Kx^2y & 0 \le x \le y \le 2 \\ 0 & \text{O.W} \end{cases}$$

معناها $0 \le x \le 2$, $0 \le y \le 2$, $x \le y$

a. Determine the value of the constant K?

نرجع للمساحة، نرسم خط $x=y$

نرسم هاي الفترة من هاد الاقتران



$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy\, dx = 1$$

$$\int_{0}^{2} \int_{x}^{2} Kx^2 y \, dy \, dx = \frac{Kx^2 y^2}{2} \Big|_x^2 = K\left[2x^2 - \frac{x^4}{2}\right]$$

$$\int_0^2 2Kx^2 dx - \int_0^2 \frac{x^4}{2} dx = \frac{2Kx^3}{3}\Big|_0^2 - \frac{x^5}{10}\Big|_0^2 = \frac{16K}{3} - \frac{32}{10} = 1$$

$\frac{16K}{3} = \frac{42}{10}$

$K = \frac{15}{32}$

b. $E\{(y+1)x\} = ?$

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} x(y+1)\, p(X=x, Y=y)\, dy\, dx$$

$$\int_0^2 \int_x^2 x(y+1)\; x^2 y \; dy\, dx = \int_0^2 \left[\int_x^2 (x^3 y^2 + x^3 y)\, dy \right] dx$$

الحدود تتكون نفس الحاجة يلي معطي عليها الفكرة

$$\int_0^2 \frac{x^3 y^3}{3} + \frac{x^3 y^2}{2} \Big|_x^2 dx = \frac{2 \times 8x^3}{2 \times 3} + \frac{3 \times 4x^3}{3 \times 2} = \frac{16 + 12}{6} x^3 -$$

$$\frac{28x^3}{6} - \left[\frac{2x^6}{2 \times 3} + \frac{3x^5}{3 \times 2} \right] = \int_0^2 \frac{28x^3}{6} - \frac{x^6}{3} - \frac{x^5}{2}\, dx$$

$$= \frac{28}{4} \frac{x^4}{6} - \frac{x^7}{3.7} - \frac{x^6}{2.6} = \frac{7}{6} x^4 - \frac{x^7}{21} - \frac{x^6}{12} \Big|_0^2$$

Notes:

1. $E\{x+y\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x+y)\, f_{x,y}(x,y)\, dy\, dx$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x\, f_{x,y}(x,y)\, dy\, dx + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y\, f_{x,y}(x,y)\, dy\, dx$$

2. $E\{a\, xy\} = a\, E\{x\}\, E\{y\}$ only if x & y are S.I

Corroleation Coefficient — يقيس الترابط بين 2R.V

$$\delta_{xy} = \frac{\mu_{x,y}}{\sigma_x \sigma_y} \rightarrow ① \; E\{(x-\mu_x)(y-\mu_y)\}$$ هل وزا واحد زاد الثاني بزيد

$E\{xy - x\mu_y - y\mu_x + \mu_x \mu_y\}$

① $-1 \le \delta_{x,y} \le 1$ — $E\{xy\} - \mu_y E\{x\} - \mu_x E\{y\} + \mu_x \mu_y$

② $\delta_{x,y} = 0$ — x & y are uncorrelated → لما عرفت x مش بحكيلك اشي عن y علاقة أضعف

③ $\delta_{x,y} = \mp 1$ — x & y are fully correlated

covariance ما بتعرف ② $\mu_{xy} = E\{xy\} - \mu_x \mu_y$

Exp: X & y are two R.Vs with the following Joint - pMF

x \ y	−1	1
−1	1/4	1/4
1	1/4	1/4

a. Determine $\rho_{xy} = ?$

$\rho_{xy} = \frac{\mu_{xy}}{\sigma_x \sigma_y}$; $\mu_{xy} = E[(x-\mu_x)(y-\mu_y)]$

$p(X=x) = \begin{cases} \frac{1}{2} & x=-1 \quad \rightarrow (\frac{1}{4}+\frac{1}{4}) \\ \frac{1}{2} & x=1 \\ 0 & \text{O.W} \end{cases}$

$\mu_x = \sum_{x=-\infty}^{\infty} x\, p(x=x)$
$= (-1)(1/2) + 1(\frac{1}{2})$
$= 0$

$p(Y=y) = \begin{cases} \frac{1}{2} & y=-1 \\ \frac{1}{2} & y=1 \\ 0 & \text{O.W} \end{cases}$

$\mu_y = \sum_{-\infty}^{\infty} y\, p(Y=y)$
$= (-1)(1/2) + (1)(1/2)$
$= 0$

$\mu_{xy} = E\{(x-\mu_x)(y-\mu_y)\} = E\{xy\}$

$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} xy\, p(X=x, Y=y)$

$= (-1)(-1)(1/4) + (-1)(1)(1/4) + (1/4)(1)(-1) + (1)(1)(1/4)$
$= 0$

$\rho_{xy} = \frac{0}{\sigma_x \sigma_y} = 0$ so x & y are uncorelated

b. Are x & y S. Independent

$p(X=x, Y=y) \overset{?}{=} p(X=x)\,p(Y=y)$ for all values of x & y.

$<x=-1, y=-1> \quad \frac{1}{4} \overset{?}{=} \frac{1}{2} * \frac{1}{2}$ ✓

$<x=-1, y=1> \quad \frac{1}{4} \overset{?}{=} \frac{1}{2} * \frac{1}{2}$ ✓

$<x=1, y=-1> \quad \frac{1}{4} \overset{?}{=} \frac{1}{2} * \frac{1}{2}$ ✓

$<x=1, y=1> \quad \frac{1}{4} \overset{?}{=} \frac{1}{2} * \frac{1}{2}$ ✓

so x & y are S. Indep.

if x & y S. In ⟶ covariance will be 0

$\rho_{xy} = 0$

Exp: Let x be a R.V with $\mu_x = 1$ & $\sigma_x^2 = 4$, y is another R.V with $\mu_y = -1$ & $\sigma_y^2 = 9$, $R = 2X - y \longrightarrow a_1 X + a_2 y$

$a_1 = 2 \quad a_2 = -1$

a. μ_R ?

$E\{R\} = \mu\{R\} = E\{2x-4\} = 2\mu_x - \mu_y$ انتبهي إذا معطيني

$= 2 \cdot 1 - -1 = 3$ or variance / standard deviation

b. $var\{R\} = ?$

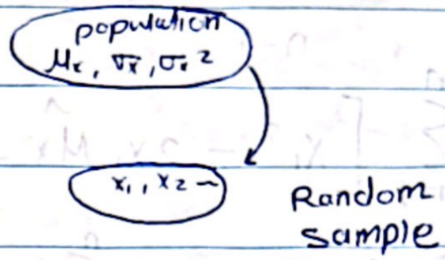
$\sqrt{\sigma_x^2}$ مش σ_x^2 ... $\sqrt{\sigma_y^2}$

$Var\{R\} = \sigma_R^2 = (a_1)^2 \sigma_x^2 + (a_2)^2 \sigma_y^2 + 2a_1a_2\sigma_x\sigma_y\rho_{xy}$

$= (2)^2 \cdot 4 + (-1)^2 \cdot 9 + 2(2)(-1)(\sqrt{4})(\sqrt{9})(0.5)$

Chapter 4:

population → جميع القيم اللي بتمثل الحالة اللي بدي بدرسه
↓ بنوخذ منه عينة عشوائية عشان نقدر نحسب اشي معين
μ_x مثلا



$\hat{\mu}_x, \hat{\sigma}_x^2$ size of sample
$\hat{\sigma}_x$, $\hat{b}$ is $\underline{n}$

sample mean $\hat{\mu}_x = \bar{x}$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i$$

Sample variance $= S_x^2 , \hat{\sigma}_x^2$

$$\hat{\sigma}_x^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_x)^2$$ μ_x : true Mean is known

$$= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \hat{\mu}_x)^2$$ $\hat{\mu}_x$: true Mean is unknown

بستخدم القانون حسب الـ mean المستخدم.

Sample standard deviation $= S_x , \hat{\sigma}_x$

$$S_x = \hat{\sigma}_x = \sqrt{S_x^2} = \sqrt{\hat{\sigma}_x^2}$$

Sample covariance $= \mu_{x,y} = C_{x,y}$

$$C_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y)$$

Sample correlation coefficient $= r_{xy}$

$$r_{xy} = \frac{C_{xy}}{S_x S_y}$$

$$S_x^2 = \hat{\sigma}_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \hat{\mu}_x)^2$$

$$= \frac{1}{n-1} \sum_{i=1}^{n} \left[x_i^2 - 2x_i \hat{\mu}_x + \hat{\mu}_x^2 \right]$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} x_i^2 - 2\hat{\mu}_x \sum_{i=1}^{n} x_i + \hat{\mu}_x^2 \sum_{i=1}^{n} 1 \right]$$

عبارة عن ثابت نخرجه برا المجموع

من قانون μ

لما نجمع الواحد n مرة بطلع عندي n

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} x_i^2 - 2\hat{\mu}_x \, n\hat{\mu}_x + \hat{\mu}_x^2 \, n \right]$$

بنطرح

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} x_i^2 - n\hat{\mu}_x^2 \right]$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} x_i^2 - n \frac{1}{n^2} \left(\sum_{i=1}^{n} x_i \right)^2 \right]$$ multiply by n

$$S_x^2 = \frac{1}{n(n-1)} \left[n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2 \right]$$

$$C_{xy} = \frac{1}{n(n-1)} \left[n \sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i \right) \left(\sum_{i=1}^{n} y_i \right) \right]$$

Regression Techniques

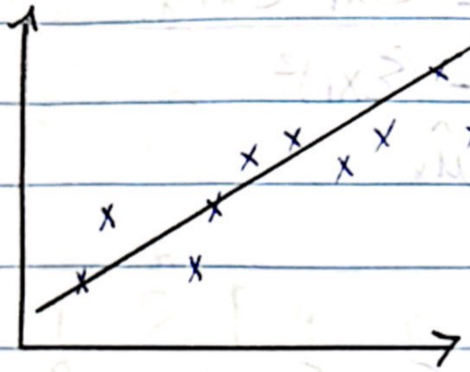
x_i	x_1	x_2	x_3	—	—	x_n
y_i	y_1	y_2	y_3	—	—	y_n
$y = g(x)$	$g(x_1)$	$g(x_2)$	$g(x_3)$			$g(x_n)$

random sample: $\langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle$

y_n → practical measured

$g(x_n)$ → theoratical → بحاول تكون اقرب شي للبراكتكال

Scatter plot



$g(x)$: $y = \underline{\alpha} x + \underline{\beta}$ بنمثل الأرقام
? ?

اذا كان صفر معناها كل النقاط جاية على الخط

Error ممكن يكون نتيجة علة القياس

Error function

$$E = y_i - g(x_i) \xrightarrow[\text{عشان كل النقاط}]{\text{ولكن}} \frac{1}{n}\sum_{i=1}^{n}\left(y_i - g(x_i)\right)^2$$ mean square error

$$E = \frac{1}{n}\sum_{i=1}^{n}(y_i - \alpha x_i - \beta)^2$$

(α ?, β ?)

بنوجد قيمة β, α اللي بتخلي قيمة الإيرور أقل ما يمكن

مرة بنشتقها بالنسبة $d\alpha$ ومرة بالنسبة $d\beta$

$$\frac{dE}{d\beta} = 0 \rightarrow \frac{1}{n}\sum_{i=1}^{n} 2(y_i - \alpha x_i - \beta)(-1) = 0$$

$$-\sum_{i=1}^{n} y_i + \alpha\sum_{i=1}^{n} x_i + \beta n = 0$$ (مشتقة ما بداخل القوس)

بجمعهن

$$\beta n + \alpha\sum_{i=1}^{n} x_i = \sum y_i$$

$$\frac{dE}{d\alpha} = 0$$

$$\frac{1}{n}\sum_{i=1}^{n} 2(y_i - \alpha x_i - \beta)(-x_i) \rightarrow -\sum_{i=1}^{n} y_i x_i + \alpha\sum_{i=1}^{n} x_i^2 + \beta\sum_{i=1}^{n} x_i = 0$$

$$\beta\sum_{i=1}^{n} x_i + \alpha\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i$$

$$\beta n + \alpha \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$y = \alpha x^1 + \beta x^0$$

order اول

$$\begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix} \begin{bmatrix} \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{bmatrix}$$

نحط بدل العمود الاول

$$\beta = \frac{\begin{vmatrix} \Sigma y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma x_i^2 \end{vmatrix}}{\begin{vmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{vmatrix}} = \frac{\Sigma y_i \Sigma x_i^2 - \Sigma x_i \Sigma x_i y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

الاصلية

$$\beta = \hat{\mu}_y - \alpha \hat{\mu}_x$$

$$\alpha = \frac{\begin{vmatrix} n & \Sigma y_i \\ \Sigma x_i & \Sigma x_i y_i \end{vmatrix}}{\begin{vmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{vmatrix}} = \frac{n \Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

$$\alpha = \frac{C_{xy}}{S_x^2}$$

Exp

$$\begin{bmatrix} 10 & 95 \\ 95 & 140 \end{bmatrix} \begin{bmatrix} \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} 115 \\ 210 \end{bmatrix}$$

$$\alpha = \frac{\begin{vmatrix} 10 & 115 \\ 95 & 210 \end{vmatrix}}{\begin{vmatrix} 10 & 95 \\ 95 & 140 \end{vmatrix}}$$

$$y = B_0 + B_1 x + B_2 x^2$$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$

1. $n B_0 + B_1 \sum x_i + B_2 \sum x_i^2 = \sum y_i$
2. $B_0 \sum x_i + B_1 \sum x_i^2 + B_2 \sum x_i^3 = \sum x_i y_i$
3. $B_0 \sum x_i^2 + B_1 \sum x_i^3 + B_2 \sum x_i^4 = \sum x_i^2 y_i$

$$E = \frac{1}{n} \sum_{i=1}^{n} \left[y_i - B_0 - B_1 x_i - B_2 x_i^2 \right]^2$$

Linearzation

$y = ae^{bx}$ → a? b? "اللي زعنڤير"

$\ln y = \ln a + \ln e^{bx}$ خذي ln ،

$\ln y = \ln a + bx$

↓ ↓ ↓

$y_{new} = \beta + \alpha x$

x			
y			
$y_{new} = \ln[y]$			

$\beta = \ln a \rightarrow a = e^{\beta}$

$\alpha = b$

$x \quad y \quad \ln\left[\frac{L-y}{y}\right]$

$$y = \frac{L}{1+e^{a+bx}}$$

$$y + ye^{a+bx} = L$$

$$ye^{a+bx} = L-y$$

$$e^{a+bx} = \frac{L-y}{y} \rightarrow \ln e^{a+bx} = \ln\left[\frac{L-y}{y}\right]$$

$$a + bx = y_{new}$$

central limit theorem

($\mu_x = 10$, $\sigma_x^2 = 16$) population

↓ Random sample of size n

($x_1, x_2, \ldots x_n$)

$$\hat{\mu}_x = \frac{1}{n}\sum_{i=1}^{n} x_i$$

كيف بقدر أحسب الـ prob معينة عن سامبل مأخوذ من popn

Note : $y = c_1X_1 + c_2X_2 + c_3X_3$

$$E\{y\} = c_1\mu_{x_1} + c_2\mu_{x_2} + c_3\mu_{x_3}$$

$$Var\{y\} = c_1^2\sigma_{x_1}^2 + c_2^2\sigma_{x_2}^2 + c_3^2\sigma_{x_3}^2$$

$$+ 2c_1c_2\,\sigma_{x_1}\sigma_{x_2}\,\delta_{x_1,x_2} \rightarrow x_1 \,\&\, x_2$$

$$+ 2c_1c_3\,\sigma_{x_1}\sigma_{x_3}\,\delta_{x_1,x_3} \rightarrow x_1 \,\&\, x_3$$

$$+ 2c_2c_3\,\sigma_{x_2}\sigma_{x_3}\,\delta_{x_2,x_3} \rightarrow x_2 \,\&\, x_3$$

if x_1, x_2, x_3 are S. Indep

$$\delta_{x_1,x_2} = \delta_{x_1,x_3} = \delta_{x_2,x_3} = 0$$

Exp (5-6);

$E\{y\} = 2\mu_{x_1} + 3\mu_{x_2} = 30$

$var\{y\} = a_1^2 \sigma_{x_1}^2 + a_2^2 \sigma_{x_2}^2 = (2)^2 \sigma_{x_1}^2 + (3)^2 \sigma_{x_2}^2 + 2(2)(3)\sqrt{\sigma_{x_1}^2}\sqrt{\sigma_{x_2}^2}\, \delta_{x}$

$= 4.4 + 9.4 + 4.6.2.2.,25 = 64$

$p(y < 35) = \varnothing\left(\frac{35-30}{\sqrt{64}}\right)$

Exp(5-7):

indep $\longrightarrow \delta_{x_1, x_2} = 0$

$\mu\{y\} = 2\mu_{x_1} + 3\mu_{x_2} = 30$

$\sigma_y^2 = 4.4 + 9(4)$

Exp(5-8);

($\mu_x = 330$, $\sigma_x = 1.5$) population cans $\rightarrow$ $n = 10$

$E\{x\} = E\{x_1\}$ $\leftarrow$ (x_1, x_2, x_n)

بتساوي M للمجتمع لأنهم مأخوذين من نفس المجتمع.

$\hat{\mu}_x = y = \frac{1}{n} \sum x_i$

$= \frac{1}{n} x_1 + \frac{1}{n} x_2 + \frac{1}{n} x_3$ ——

$E\{\hat{\mu}_x\} = \frac{1}{n} E\{x_1\} + \frac{1}{n} E\{x_2\}$ ——

$= \frac{1}{n}\mu_x + \frac{1}{n}\mu_x +$

$= \frac{1}{n} . n \, \mu_x = \mu_x$

$p(\hat{\mu}_x \leq 328)$

$E\{\hat{\mu}_x\} = \mu_x = 330$

$var\{\mu_x\} = \left(\frac{1}{n}\right)^2 \sigma_x^2 + \left(\frac{1}{n}\right)^2 \sigma_x^2 + \sim$

$= \frac{1}{n^2} n \, \sigma_x^2 = \frac{\sigma_x^2}{n}$

$var\{\mu_x\} = \frac{\sigma_x^2}{2} = \frac{(1.5)^2}{2} = 0.225$

$p(\hat{\mu}_x \leq 328) = \emptyset\left(\frac{328-330}{\sqrt{0.225}}\right)$

↳ Gausian

$E\{\hat{\mu}_x\} = \mu_x$

$var\{\hat{\mu}_x\} = \frac{\sigma_x^2}{n}$

بنحل هيك لما يكون ال pop gausian ـ
وبنربط نكملها لو مش غاوسيان

Exp (5-10): true mean ← ($\mu_x = 100$, $\sigma_x = 10$) population resistors

→ $n = 25$

$\hat{\mu}_x = \frac{1}{n} \sum x_i$

$p(\hat{\mu}_x < 95) = \emptyset\left(\frac{95-100}{\sqrt{4}}\right)$

↳ gausian $E\{\hat{\mu}_x\} = \mu_x = 100$

$var\{\hat{\mu}_x\} = \frac{\sigma_x^2}{n} = \frac{10^2}{25} = 4$

Exp (5-11)

point Estimation :

$E\{\hat{\mu}_x\} = \mu_x \rightarrow \hat{\mu}_x$ is unbiased estimator for the mean μ_x.

$E\{\hat{p}\} = p \rightarrow \hat{p}$ is an unbiased estimator for the probability of sucess p.

$E\{x\} \overset{D}{=} \sum_{x=-\infty}^{\infty} x\, p(X=x)$ pmf

$\overset{C}{=} \sum_{-\infty}^{\infty} x\, f(x)\, dx$ pdf

Exp(5-16) :

$\mu_{x,1} = \frac{x_1 + x_2}{2} \rightarrow E\{\hat{\mu}_{x_1}\} = E\left[\frac{x_1 + x_2}{2}\right]$

$= E\{\frac{1}{2}x_1\} + E\{\frac{1}{2}x_2\} = \frac{1}{2}E\{x_1\} + \frac{1}{2}E\{x_2\}$

$= \frac{1}{2}\mu_x + \frac{1}{2}\mu_x = \mu_x$

$E\{\hat{\mu}_{x,1}\} = \mu_x$ $\mu_{x,1}$ is an unbiased estimate of μ_x

$\hat{\mu}_{x2} = \frac{x_1 + 2x_2}{3} \rightarrow E\{\hat{\mu}_{x2}\} = E\left[\frac{x_1 + 2x_2}{3}\right]$

$\frac{1}{3}E\{x_1\} + \frac{2}{3}E\{x_2\} = \frac{1}{3}\mu_x + \frac{2}{3}\mu_x = \mu_x$

μ_{x2} is an unbiased estimator of μ_x

حالة خارجية $\mu_{x3} = \frac{x_1 + x_2 + 1}{3}$ لو واحد فيهم طلع biased ما بنختاره.

$E\{\mu_{x3}\} = \frac{1}{3}E\{x_1\} + \frac{1}{3}E\{x_2\} + \frac{1}{3} = \frac{1}{3}\mu_x + \frac{1}{3}\mu_x + \frac{1}{5}$

$E\{\mu_{x3}\} = \frac{2}{3}\mu_x + \frac{1}{3} \neq \mu_x$ biased إذاً

$B = E\{\hat{\theta}\} - \theta$ مقدار ال biased

$= E\{\hat{\mu}_x\} - \mu_x$

$= \frac{2}{3}\mu_x + \frac{1}{3} - \mu_x = \frac{1}{3} - \frac{1}{3}\mu_x$

$B = \frac{1 - \mu_x}{3}$

The mean square error of an estimator

$MSE = E(\hat{\theta} - \theta)^2$

$MSE(\hat{\theta}) = Var(\hat{\theta}) + B^2$

Exp: Check whether the following estimator is biased or unbiased & try to modify the estimator to be unbiased if it is found to be biased.

$\hat{\sigma}_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2$

$E(\hat{\sigma}_x^2) = E\left\{\frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2\right\} = E\left\{\frac{1}{n}\sum_{i=1}^{n}\left[x_i^2 - 2x_i\hat{\mu}_x + \hat{\mu}_x^2\right]\right\}$

من السابقة

$\frac{1}{n}E\left\{\sum_{i=1}^{n}x_i^2 - 2\hat{\mu}_x\sum_{i=1}^{n}x_i + n\hat{\mu}_x^2\right\} = \frac{1}{n}E\left\{\sum_{i=1}^{n}x_i^2 - 2\hat{\mu}_x n\hat{\mu}_x + n\hat{\mu}_x\right\}$

$\frac{1}{n}E\left\{\sum_{i=1}^{n}x_i^2 - n\hat{\mu}_x^2\right\} = \frac{1}{n}\left[\sum_{i=1}^{n}E\{x^2\} - nE\{\hat{\mu}_x^2\}\right]$

$var\{\hat{\mu}_x\} = E\{\hat{\mu}_x^2\} - (E(\hat{\mu}_x))^2$

The Maximum likelihood Estimator (ML)

↳ pdf عبارة عن cont

Exp (6-3):

Gaussian μ_x, σ_x^2 → random sample x_1, x_2 size n

$$f_x(x_1) = \frac{1}{\sqrt{2\pi\sigma_{x_1}^2}} e^{-\frac{(x_1-\mu_{x_1})^2}{2\sigma_{x_1}^2}}$$

$$= \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x_1-\mu_x)^2}{2\sigma_x^2}}$$

→ ماخذ من population

$$f_x(x_2) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x_2-\mu_x)^2}{2\sigma_x^2}}$$

$$f_x(x_n) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x_n-\mu_x)^2}{2\sigma_x^2}}$$