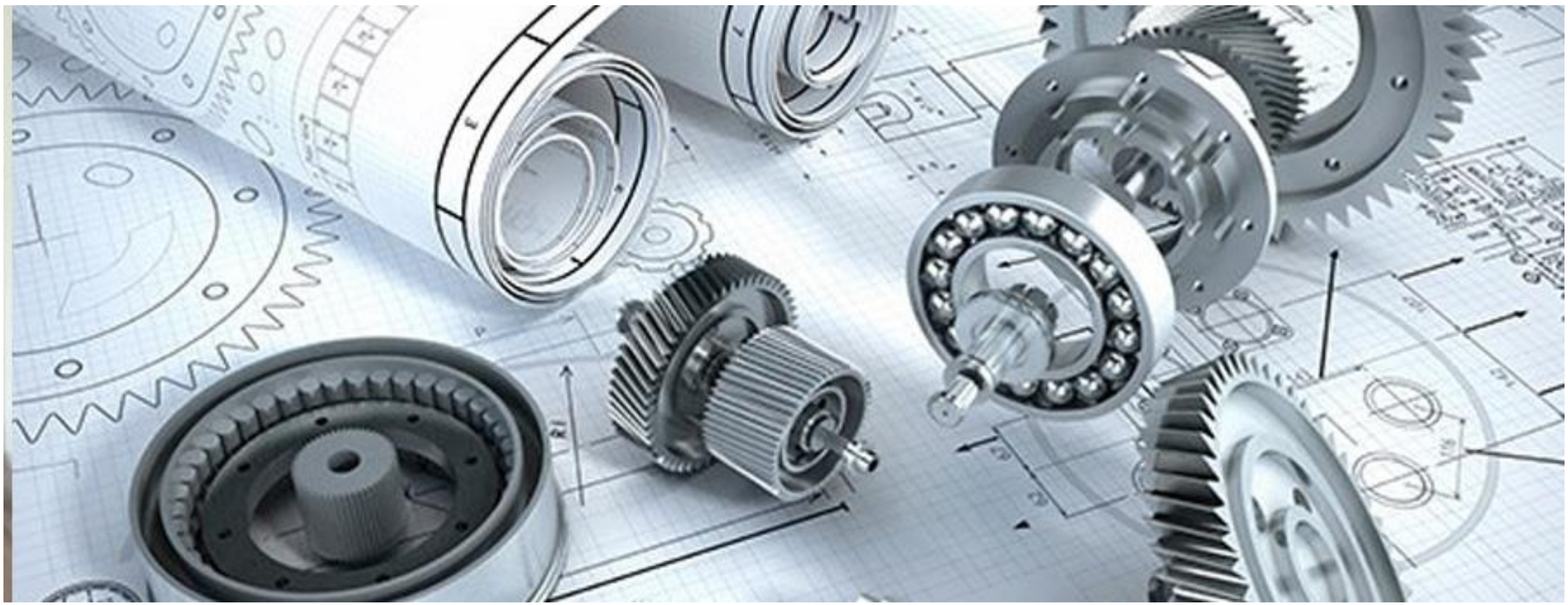
Machine Design 2
ENME 436

Department of Mechanical and Mechatronics Engineering

Dr. Rashad Mustafa



13.1 Types of Gears

Figure 13–1

Spur gears are used to transmit rotary motion between parallel shafts.

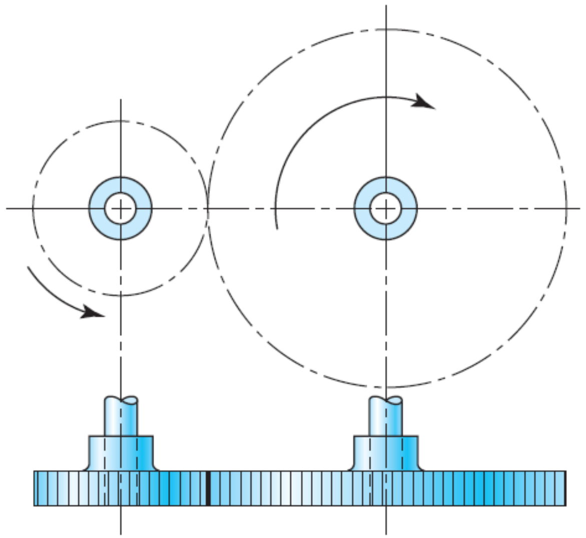
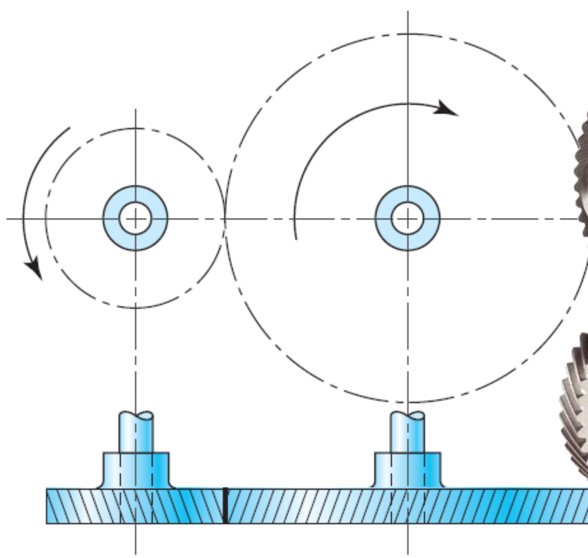


Figure 13–2

Helical gears are used to transmit motion between parallel or nonparallel shafts.



13.1 Types of Gears

Figure 13–3

Bevel gears are used to transmit rotary motion between intersecting shafts.

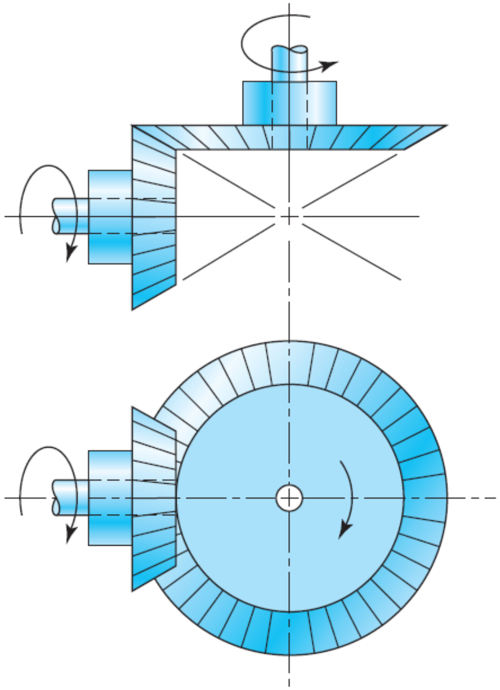
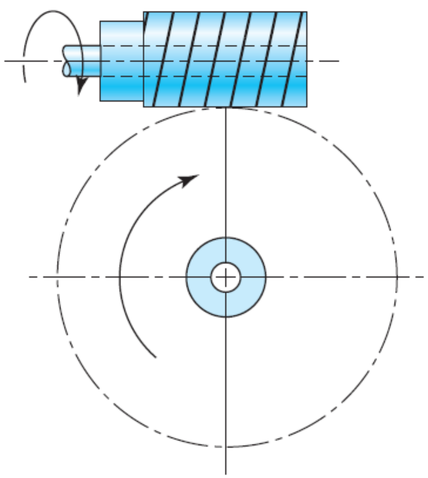
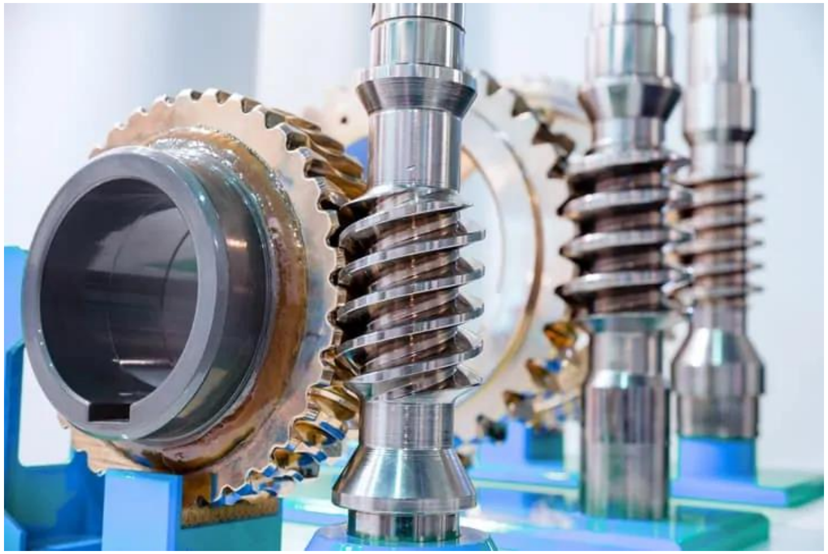


Figure 13–4

Worm gearsets are used to transmit rotary motion between nonparallel and nonintersecting shafts.





13.2 Nomenclature

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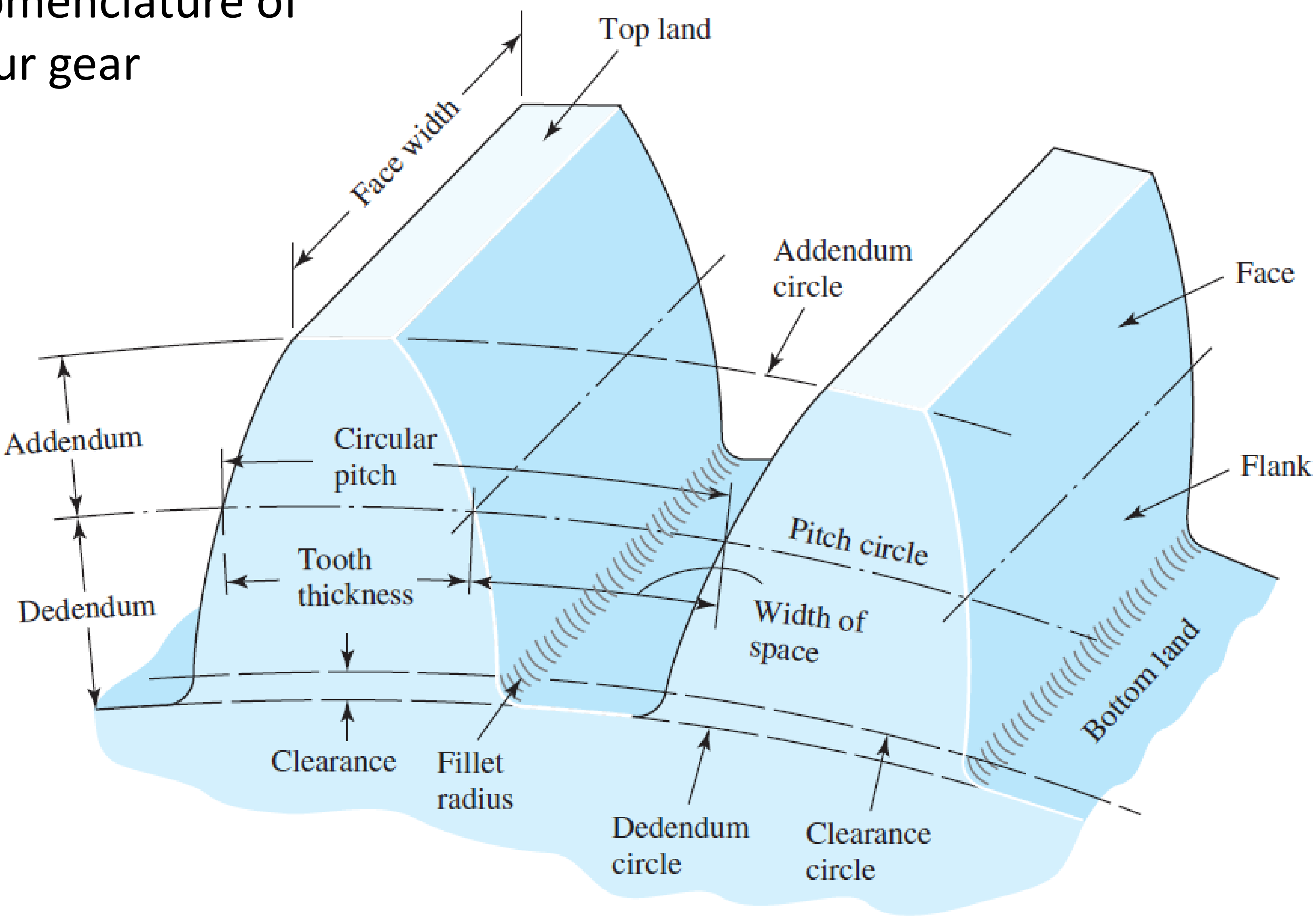
Nomenclature of spur gear



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13.2 Nomenclature

$$P = \frac{N}{d}$$

$$m = \frac{d}{N}$$

$$p = \frac{\pi d}{N} = \pi m$$

$$pP = \pi$$

where P = diametral pitch, teeth per inch
N = number of teeth
d = pitch diameter, in or mm
m = module, mm
p = circular pitch, in or mm

Cutters are generally available for the sizes shown below:

Table 13–2

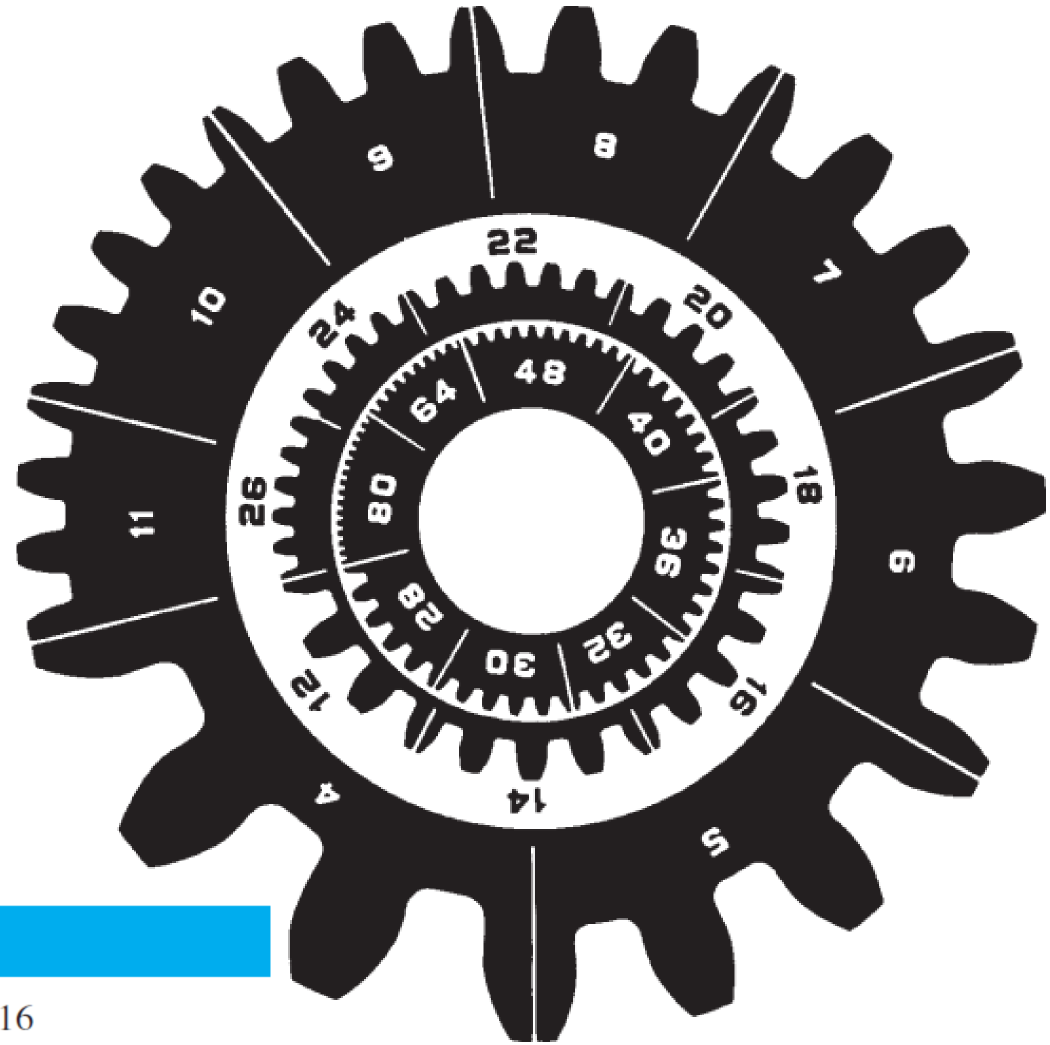
Tooth Sizes in General Uses

Diametral Pitch P(teeth/in)	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Module m(mm/tooth)	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

13.2 Nomenclature

Actual Gear Tooth Sizes for Various Diametral Pitches

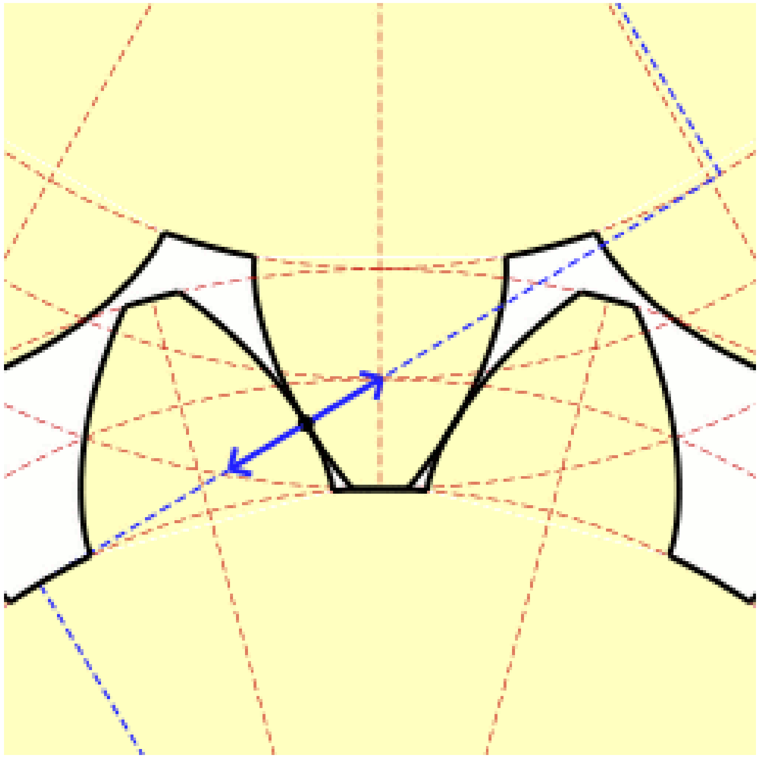


Diametral Pitch P(teeth/in)	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

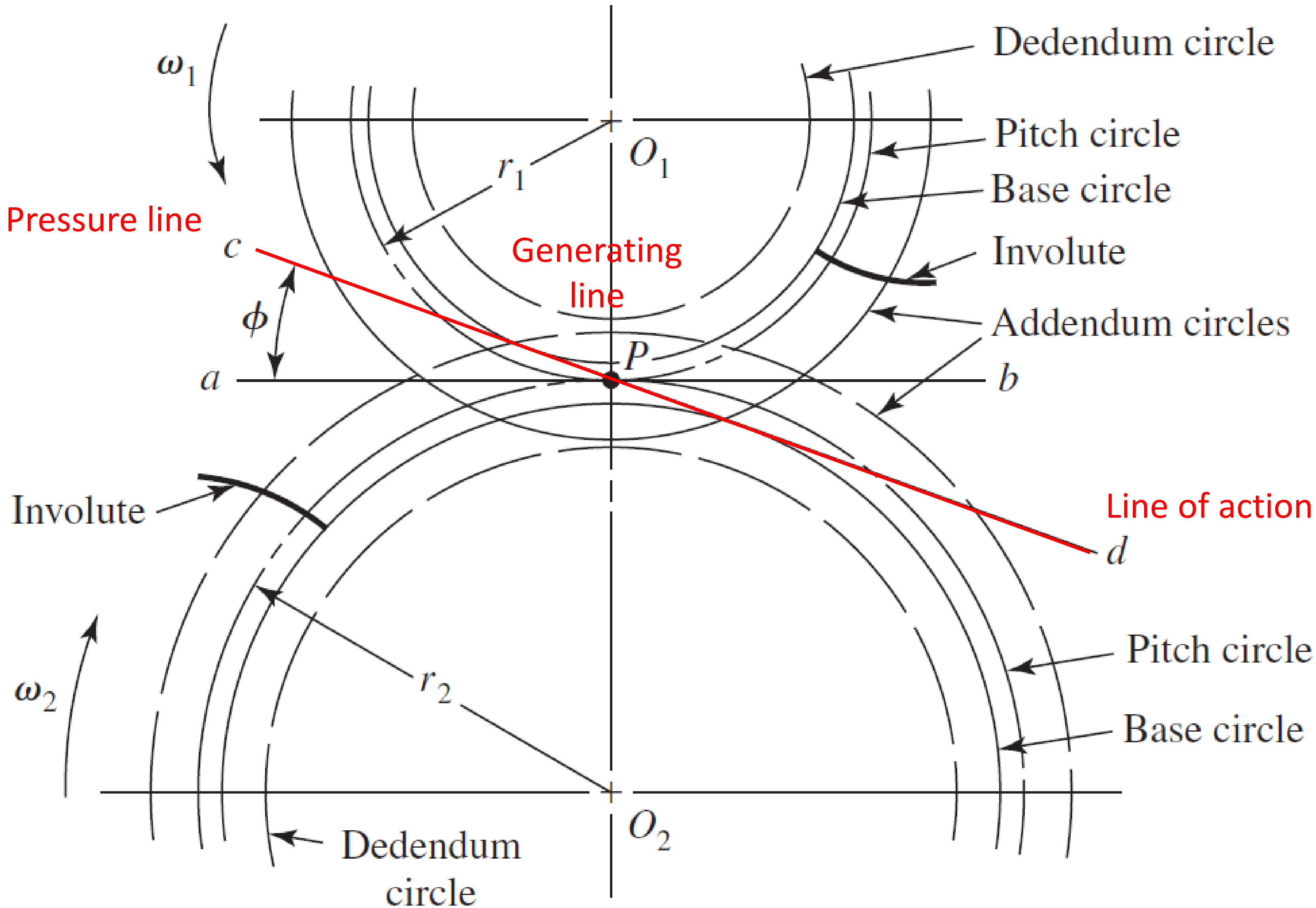
13.3 Conjugate Action & 13.4 Involute Properties

Conjugate action: When the tooth profiles, are designed so as to produce a **constant angular-velocity ratio** during meshing.

It is possible arbitrarily to select any profile for one tooth and then to find a profile for the meshing tooth that will give conjugate action → One of these solutions is the ***Involute Profile***.



13.4 Fundamentals

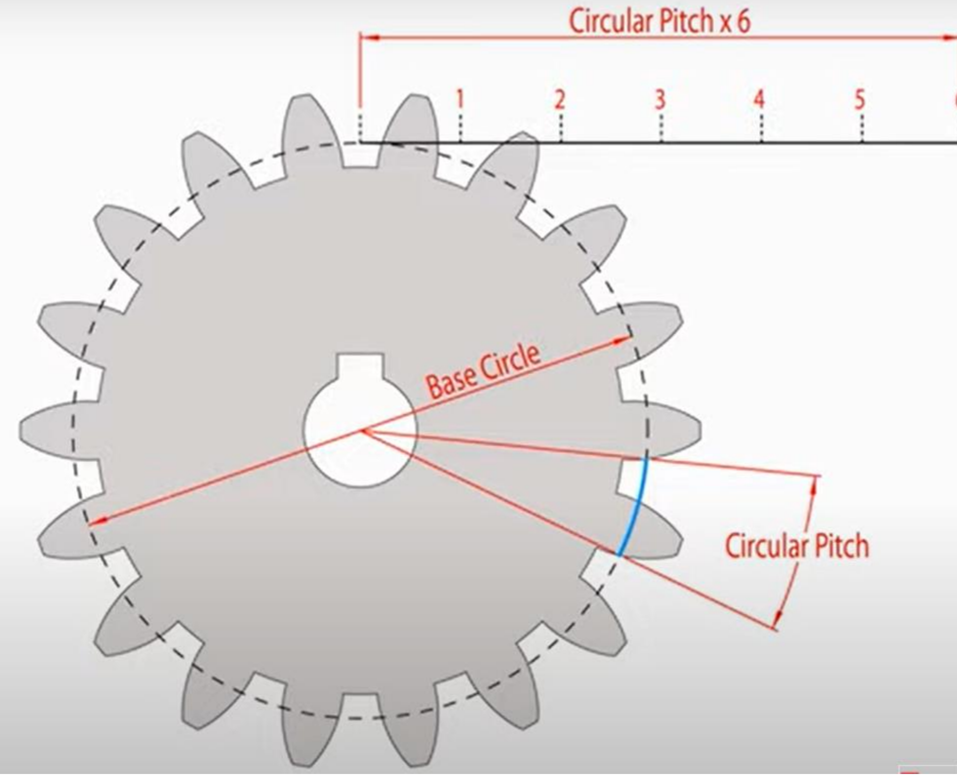


13.4 Involute Properties

Involute Gear Profile

Involute Profile

∴ *Measure 6 x circular pitch lengths*



13.4 Involute Properties

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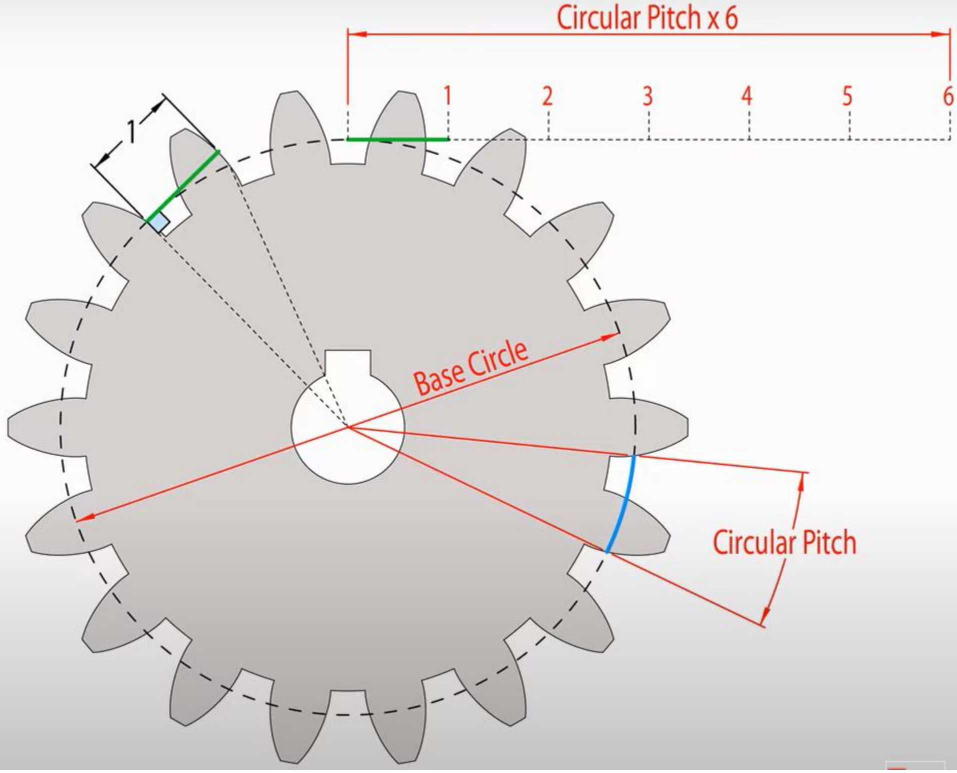
Involute Gear Profile

Involute Profile

- ∴ *Measure 6 x circular pitch lengths*
- ∴ *Mark a line to start the tooth arc*
- ∴ *Mark a line one circular pitch length around the base circle.*
- ∴ *Draw a straight line one circular pitch length at 90° from new mark*

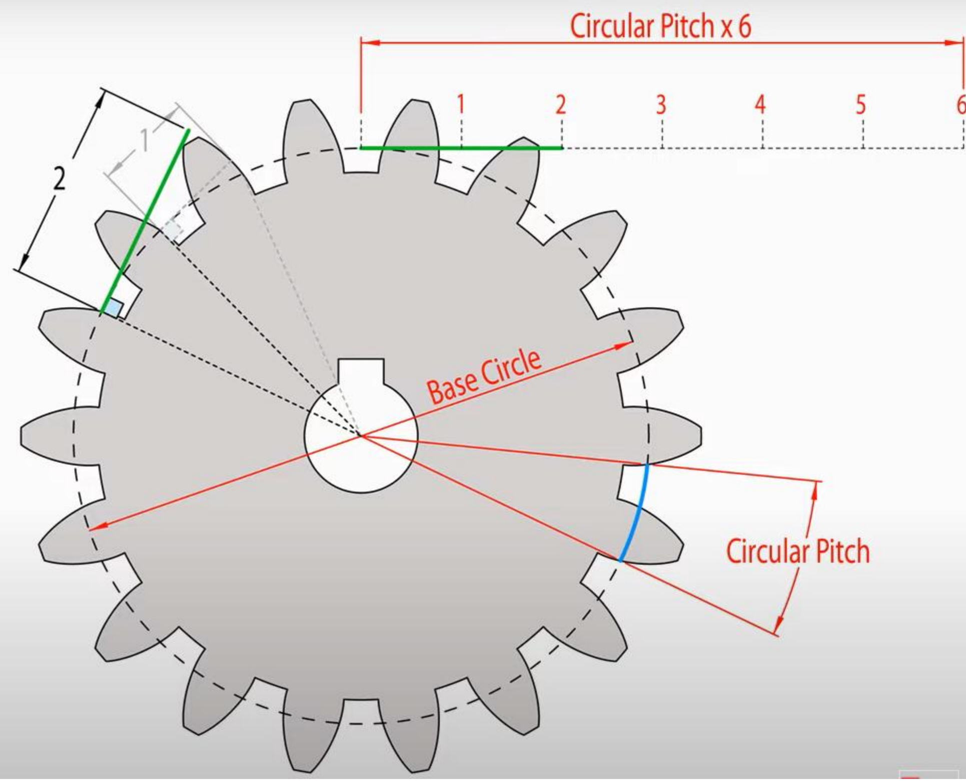


13.4 Involute Properties

Involute Gear Profile

Involute Profile

∴ *Mark a new line two circular pitch lengths around the base circle*



13.4 Involute Properties

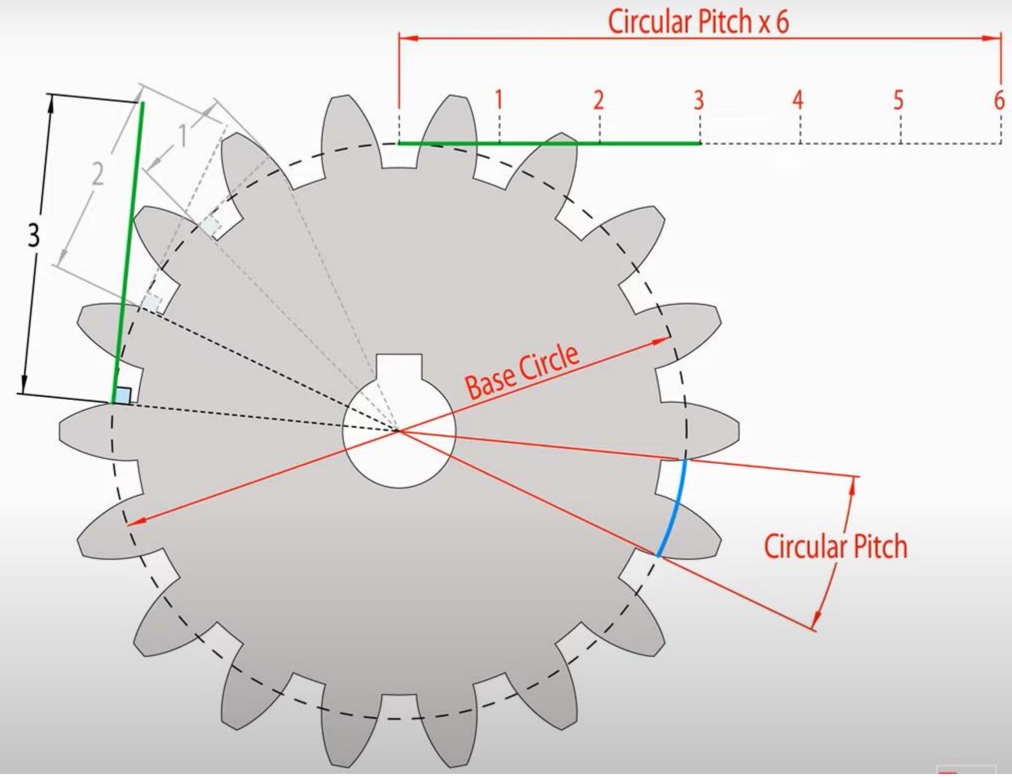
Involute Gear Profile

Involute Profile

∴ *Mark a new line two circular pitch lengths around the base circle*

∴ *Draw a straight line two circular pitch length at 90° from new mark*

∴ *Repeat for length 3*



13.4 Involute Properties

Involute Gear Profile

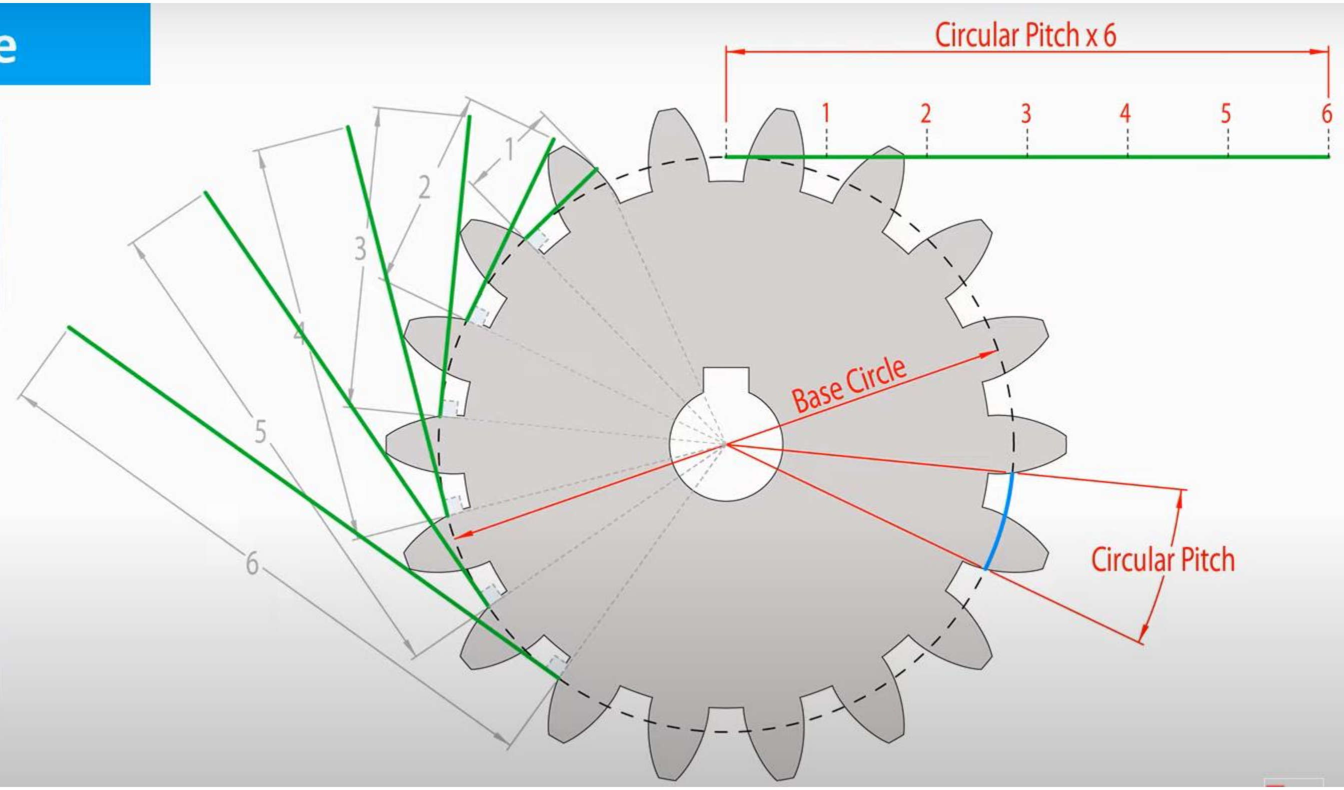
Involute Profile

∴ *Mark a new line two circular pitch lengths around the base circle*

∴ *Draw a straight line two circular pitch length at 90° from new mark*

∴ *Repeat for length 3*

∴ *Repeat for lengths 4, 5 & 6*



13.4 Involute Properties

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Involute Gear Profile

Involute Profile

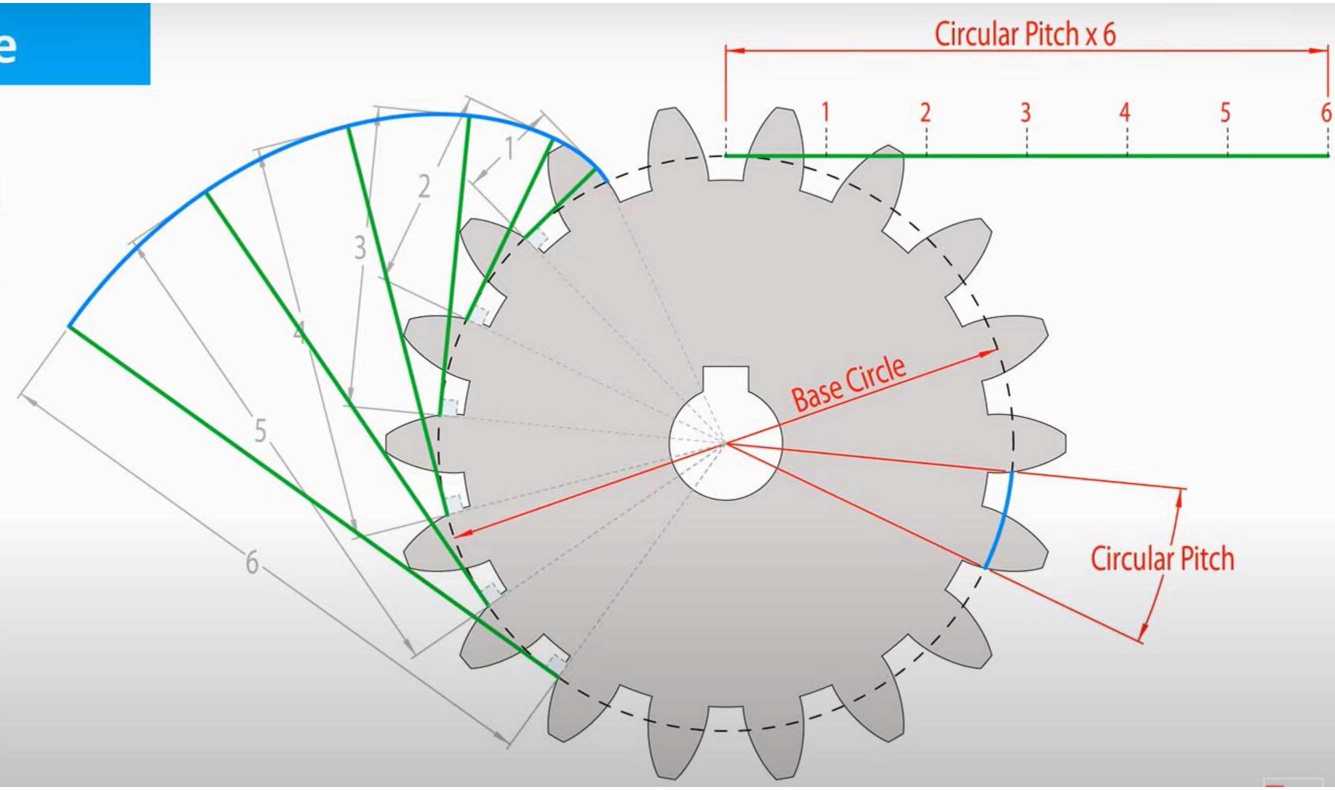
∴ *Mark a new line two circular pitch lengths around the base circle*

∴ *Draw a straight line two circular pitch length at 90° from new mark*

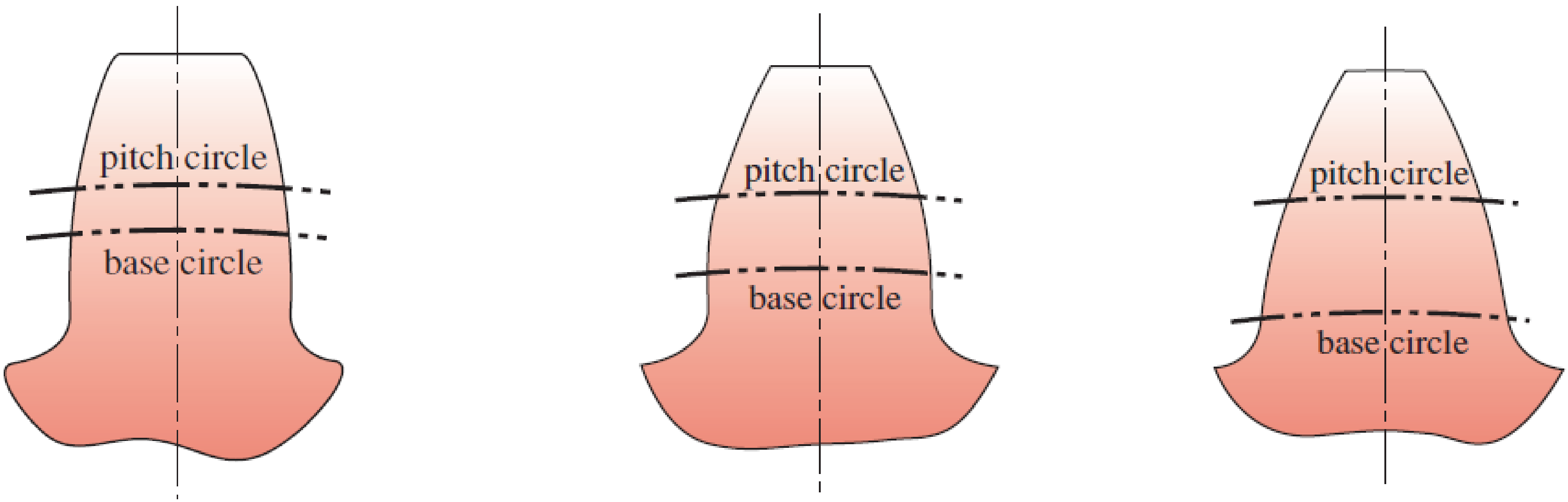
∴ *Repeat for length 3*

∴ *Repeat for lengths 4, 5 & 6*

∴ Draw the resultant arc



13.4 Fundamentals

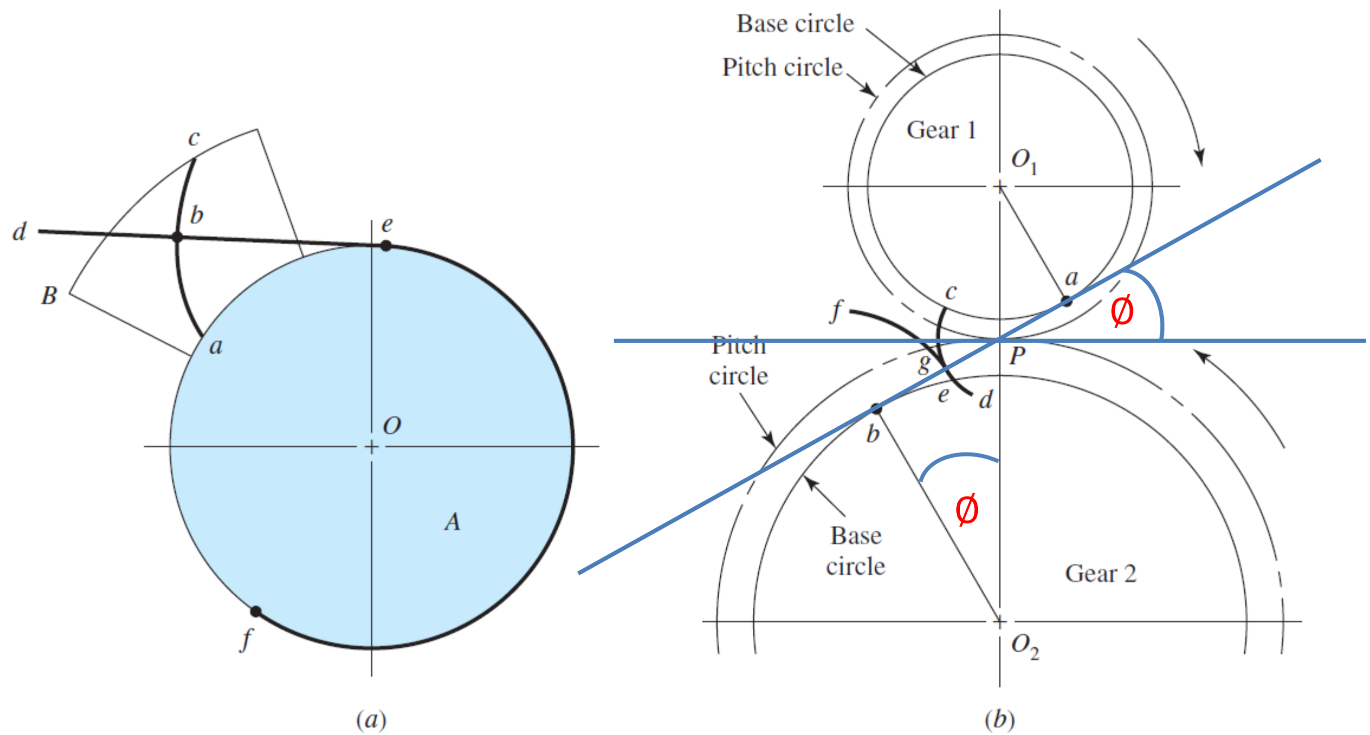


(a) $\phi = 14.5°$ (b) $\phi = 20°$ (c) $\phi = 25°$

13.5 Fundamentals

Figure 13–7

(*a*) Generation of an involute;
(*b*) involute action.



Ø: *Pressure angle*

13.5 Fundamentals

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Table 13–4

Standard Tooth Proportions for Helical Gears

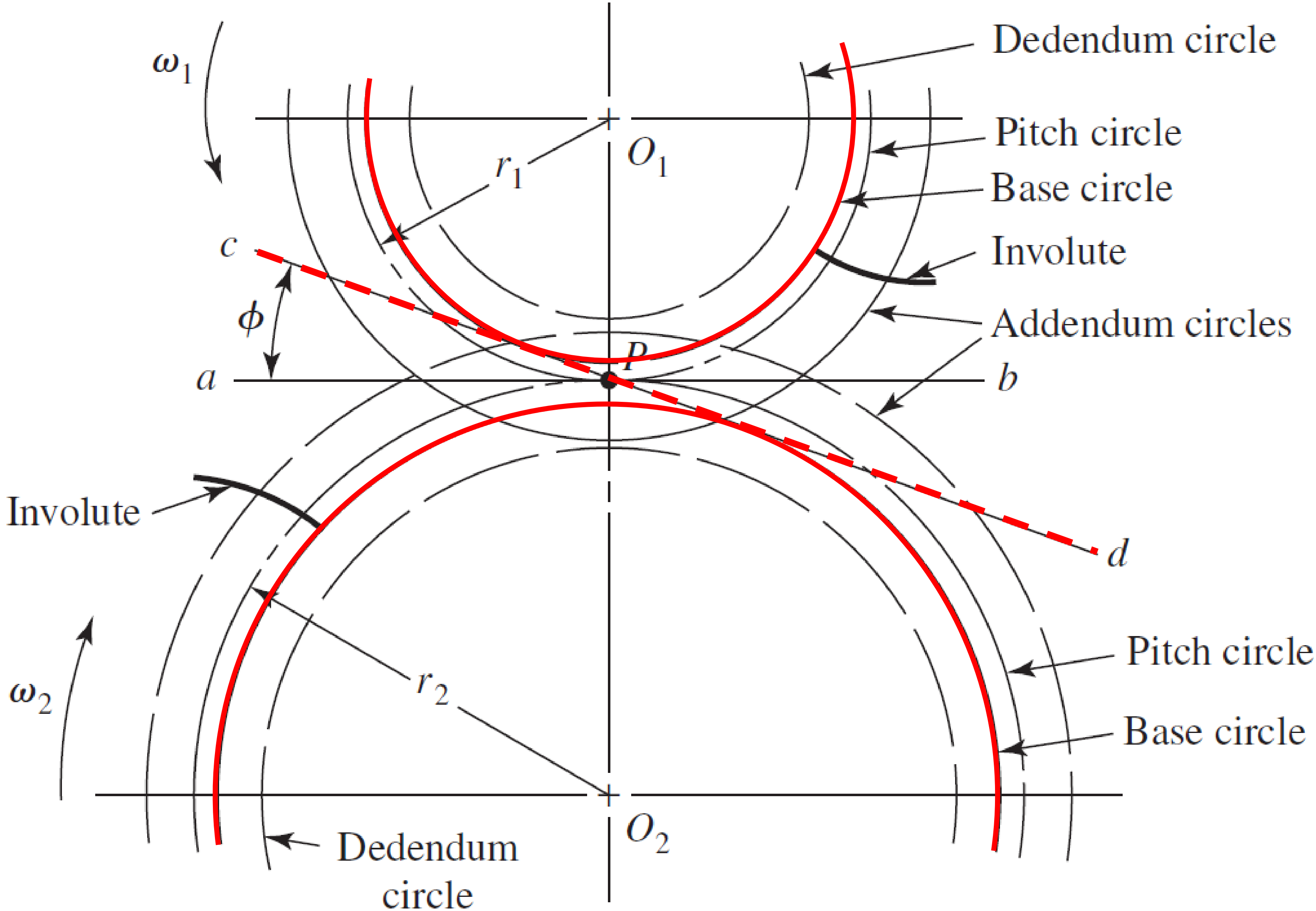
Quantity*	Formula	Quantity*	Formula
Addendum	$\frac{1.00}{P_n}$	External gears:	
Dedendum	$\frac{1.25}{P_n}$	Standard center distance	$\frac{D+d}{2}$
Pinion pitch diameter	$\frac{N_P}{P_n \cos \psi}$	Gear outside diameter	$D + 2a$
Gear pitch diameter	$\frac{N_G}{P_n \cos \psi}$	Pinion outside diameter	$d + 2a$
Normal arc tooth thickness†	$\frac{\pi}{P_n} - \frac{B_n}{2}$	Gear root diameter	$D - 2b$
Pinion base diameter	$d \cos \phi_t$	Pinion root diameter	$d - 2b$
		Internal gears:	
Gear base diameter	$D \cos \phi_t$	Center distance	$\frac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan \psi \cos \phi_t)$	Inside diameter	$D - 2a$
		Root diameter	$D + 2b$

*All dimensions are in inches, and angles are in degrees.

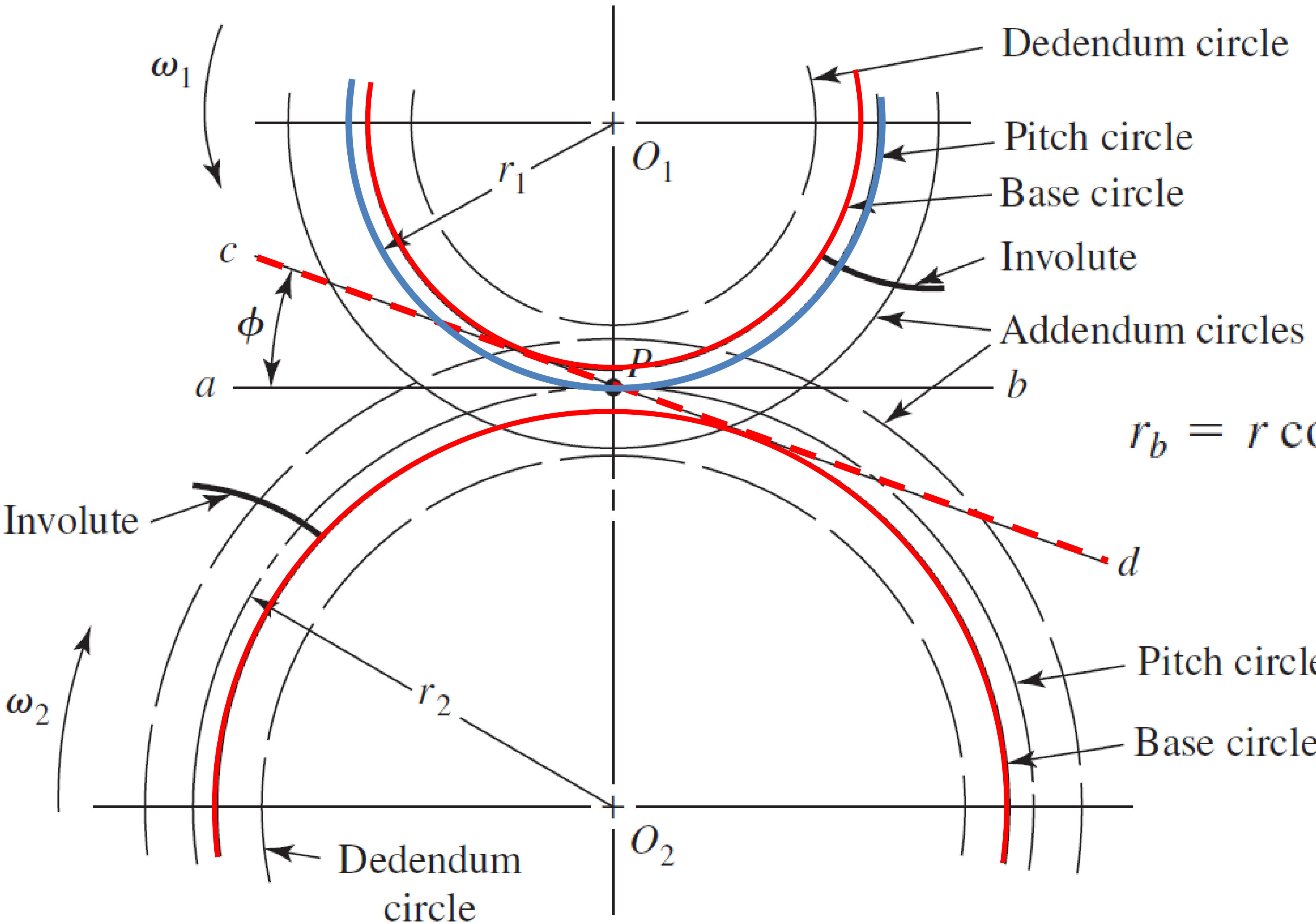
†B_n is the normal backlash.

13.5 Fundamentals

Figure 13-9: Circles of a gear layout



13.5 Fundamentals



$$r_b = r\cos\phi$$

13.4 Fundamentals

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Table 13–1

Standard and Commonly Used Tooth Systems for Spur Gears

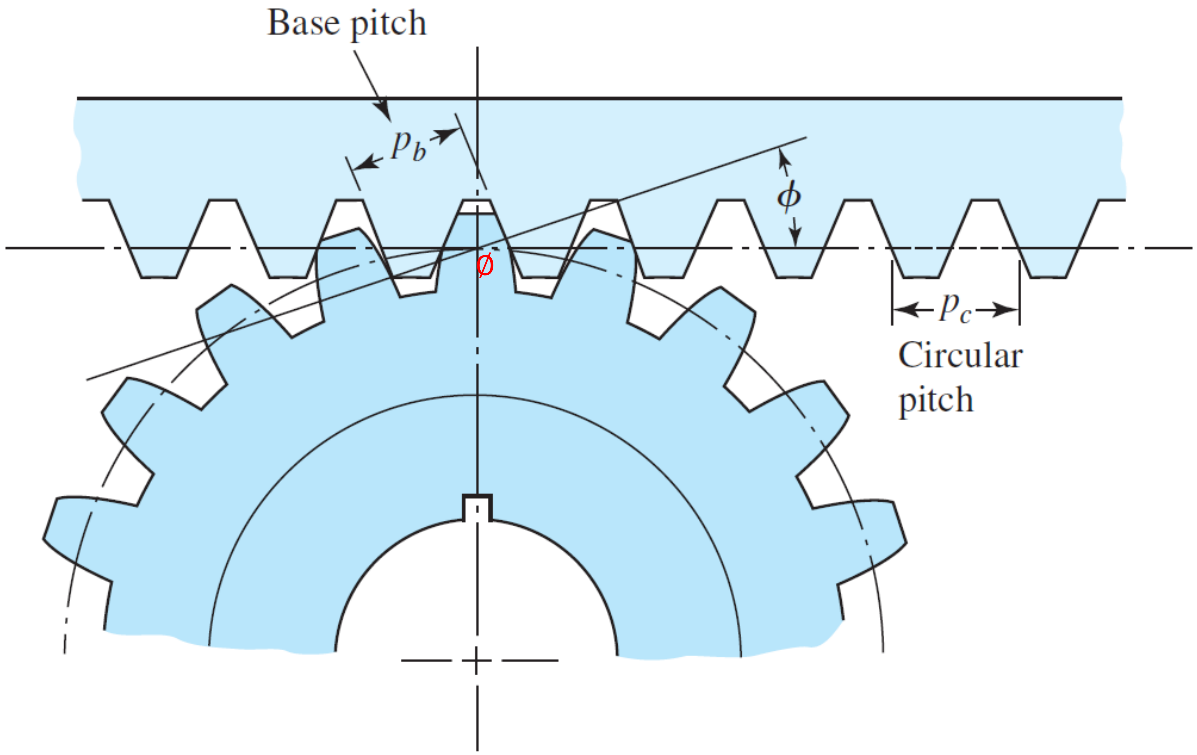
Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
	$22\frac{1}{2}$	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
	25	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
Stub	20	$0.8/P$ or $0.8m$	$1/P$ or m

The table contains the standards most used for spur gears. A 14.5° pressure angle was once used for these but is now obsolete; the resulting gears had to be comparatively larger to avoid interference problems.

13.4 Fundamentals

Figure 13–13

Involute-toothed pinion and rack.

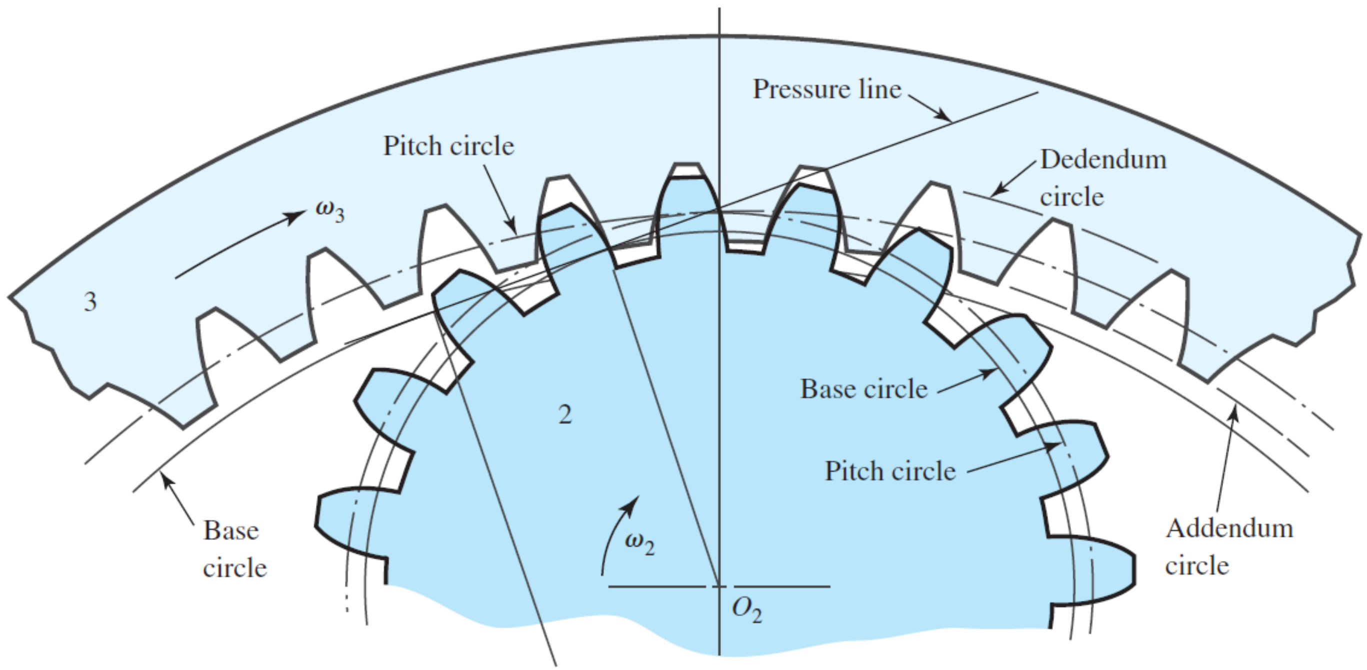


$$p_b = p_c \cos \phi \qquad (13\text{–}7)$$

13.4 Fundamentals

Figure 13–14

Internal gear and pinion.



13.4 Fundamentals

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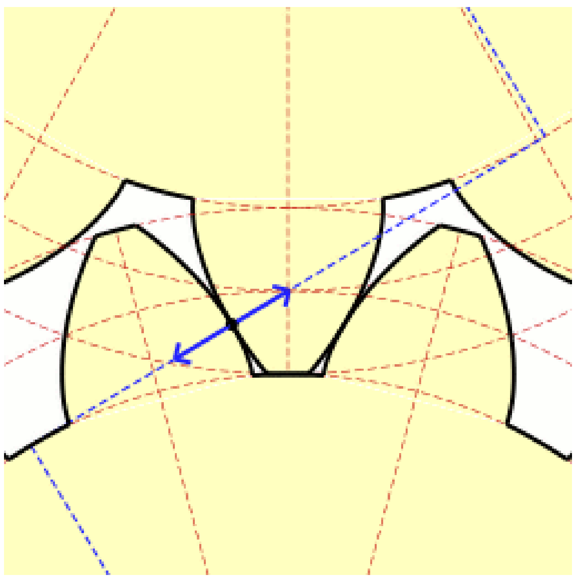
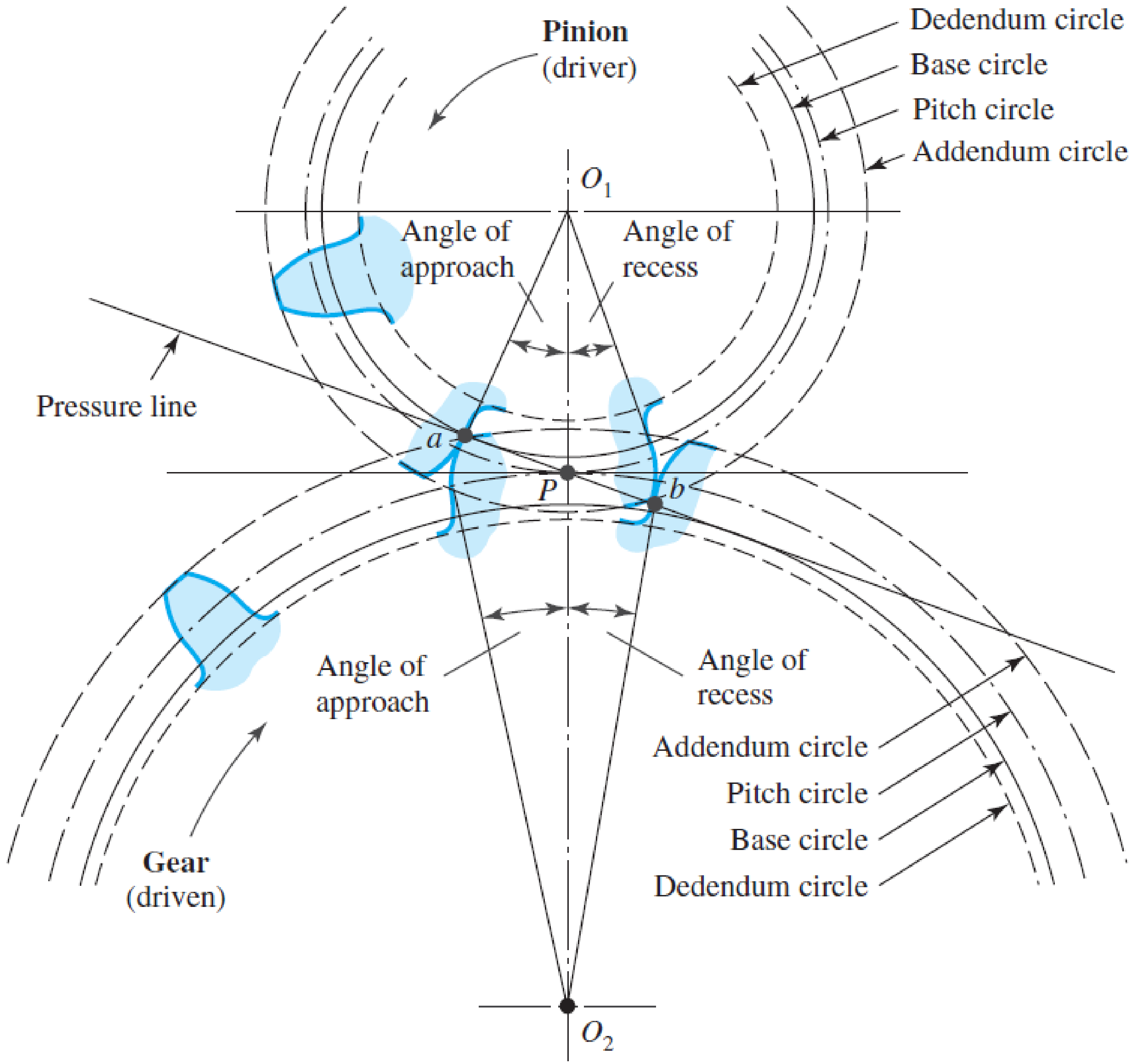
Figure 13–12

Tooth action.

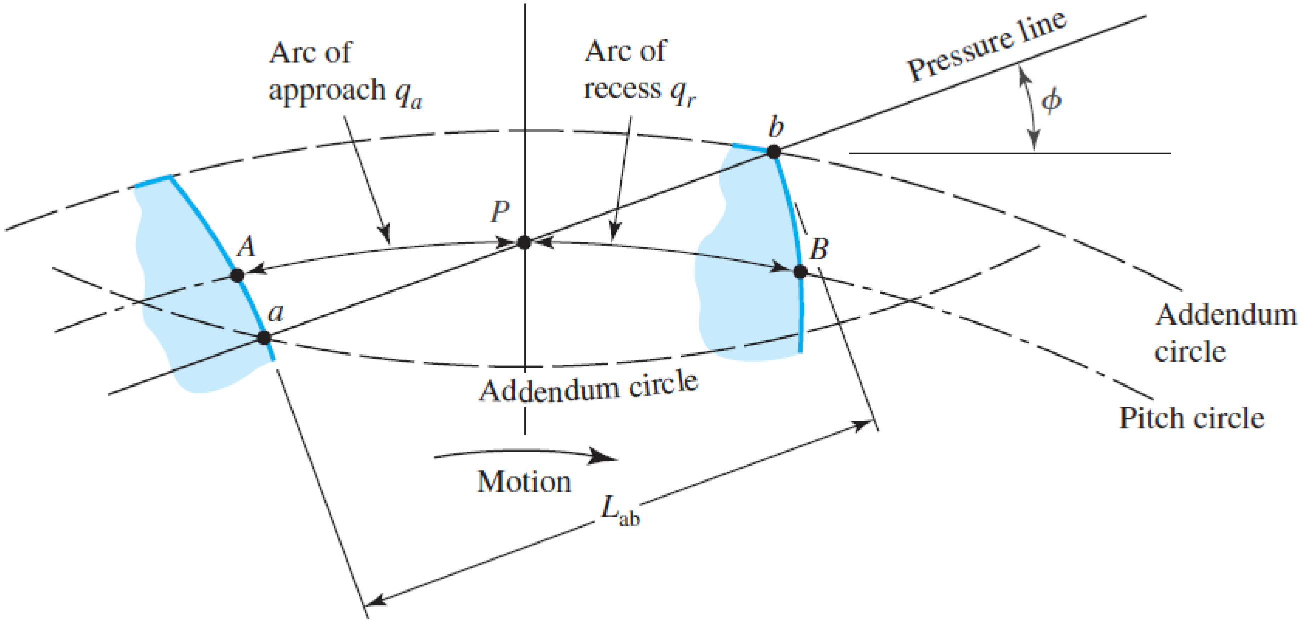




13.6 Contact Ratio

Figure 13–15

Definition of contact ratio.

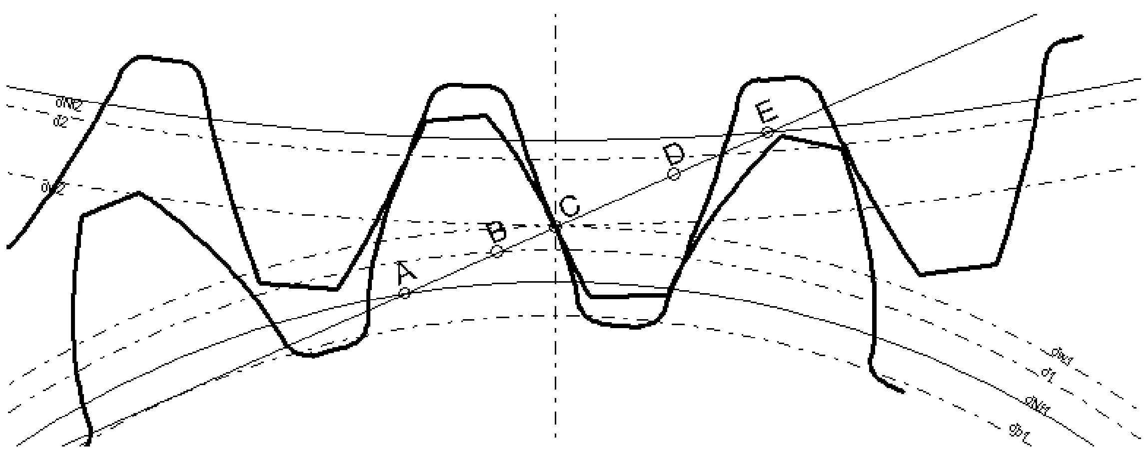


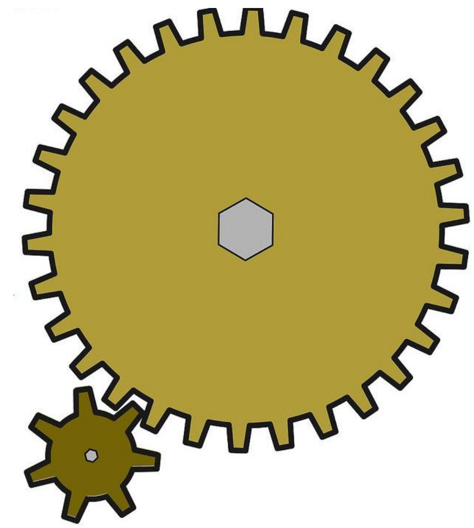
$$m_c = \frac{q_t}{p} \tag{13–8}$$

$$m_c = \frac{L_{ab}}{p \cos \phi} \tag{13–9}$$

13.6 Contact Ratio

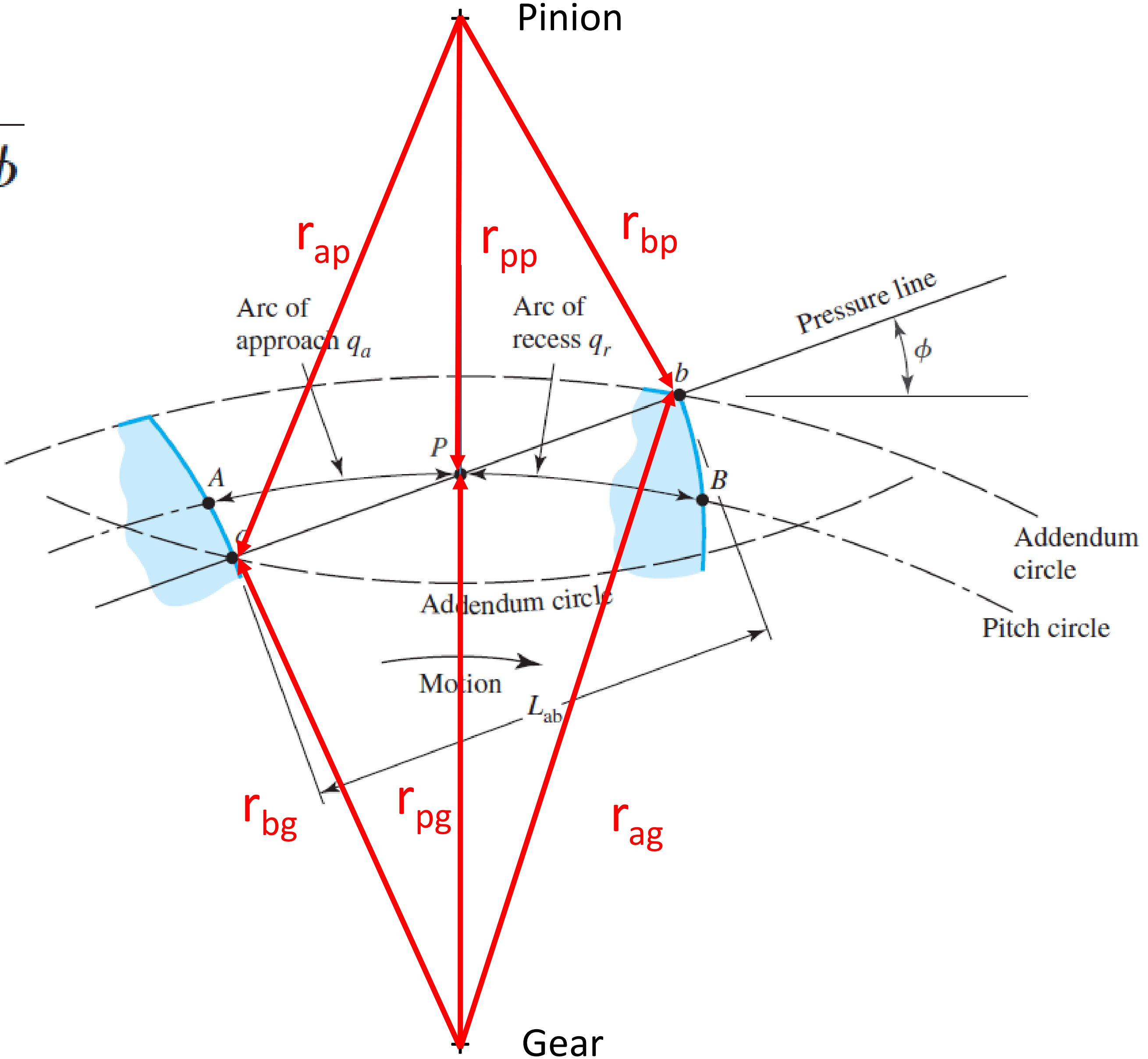
Gears should not generally be designed having contact ratios less than about **1.20**, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level.





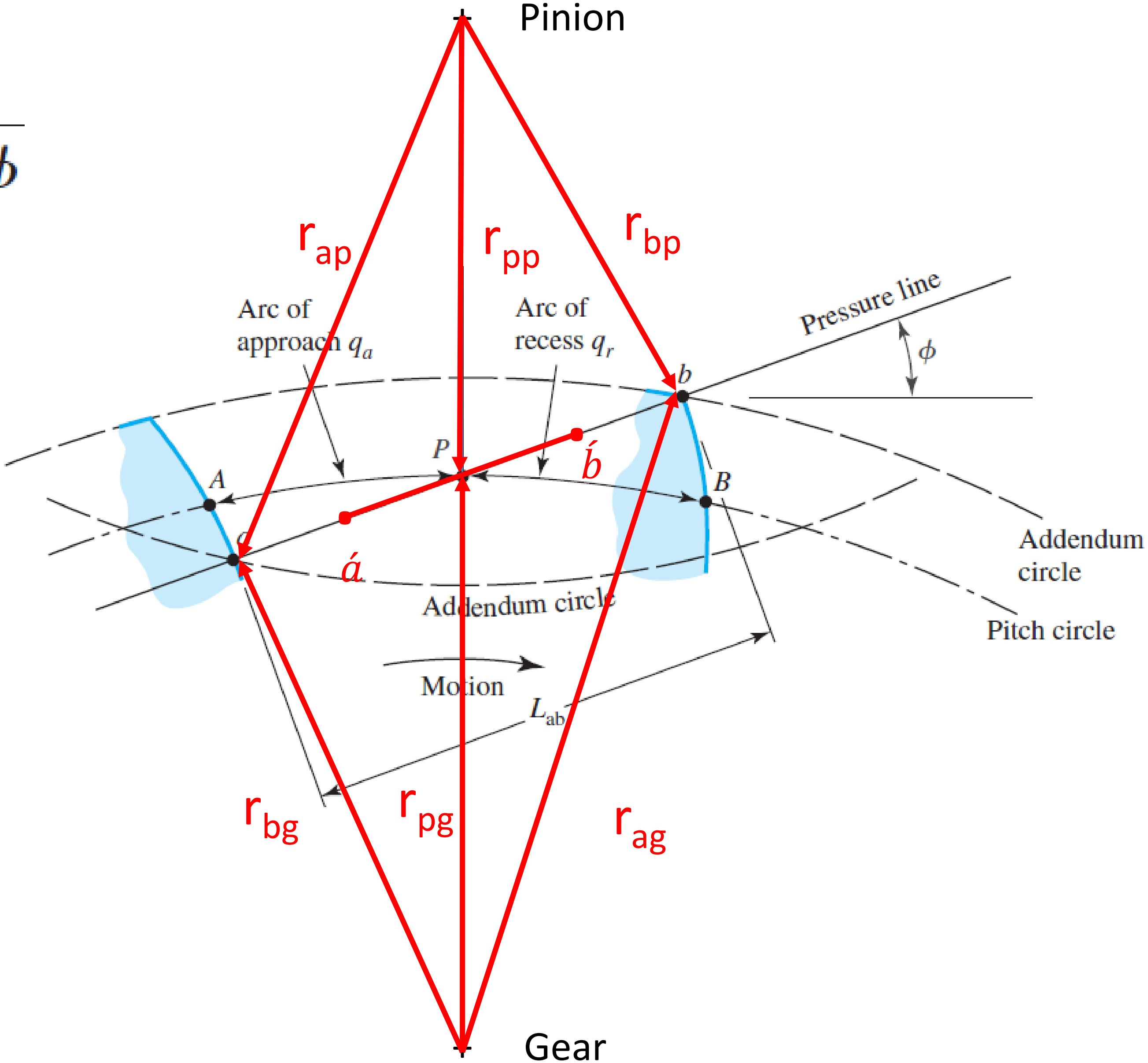
13.6 Contact Ratio

$$m_c = \frac{L_{ab}}{p \cos \phi}$$



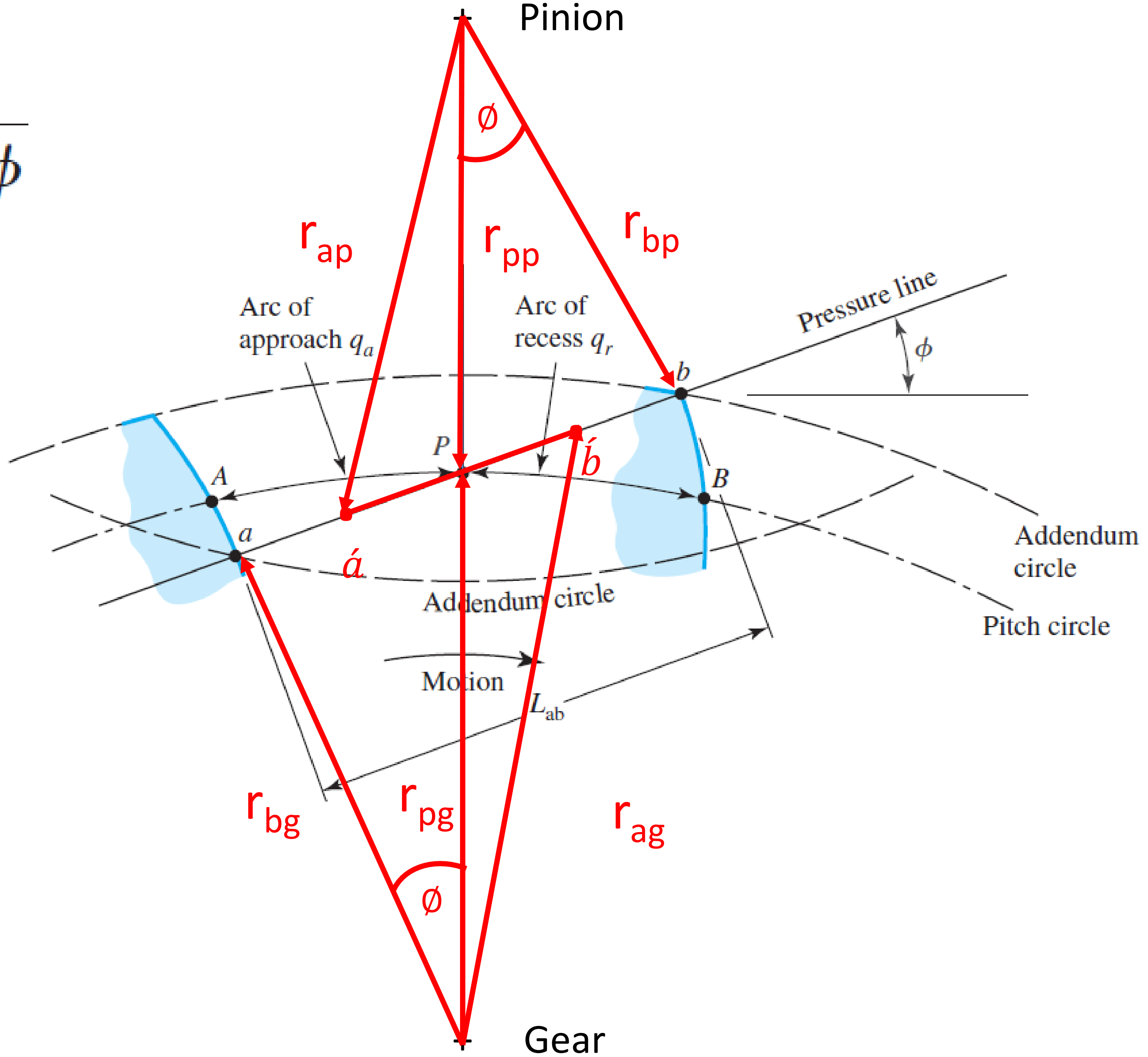
13.6 Contact Ratio

$$m_c = \frac{L_{ab}}{p \cos \phi}$$



13.6 Contact Ratio

$$m_c = \frac{L_{ab}}{p \cos \phi}$$



13.6 Contact Ratio

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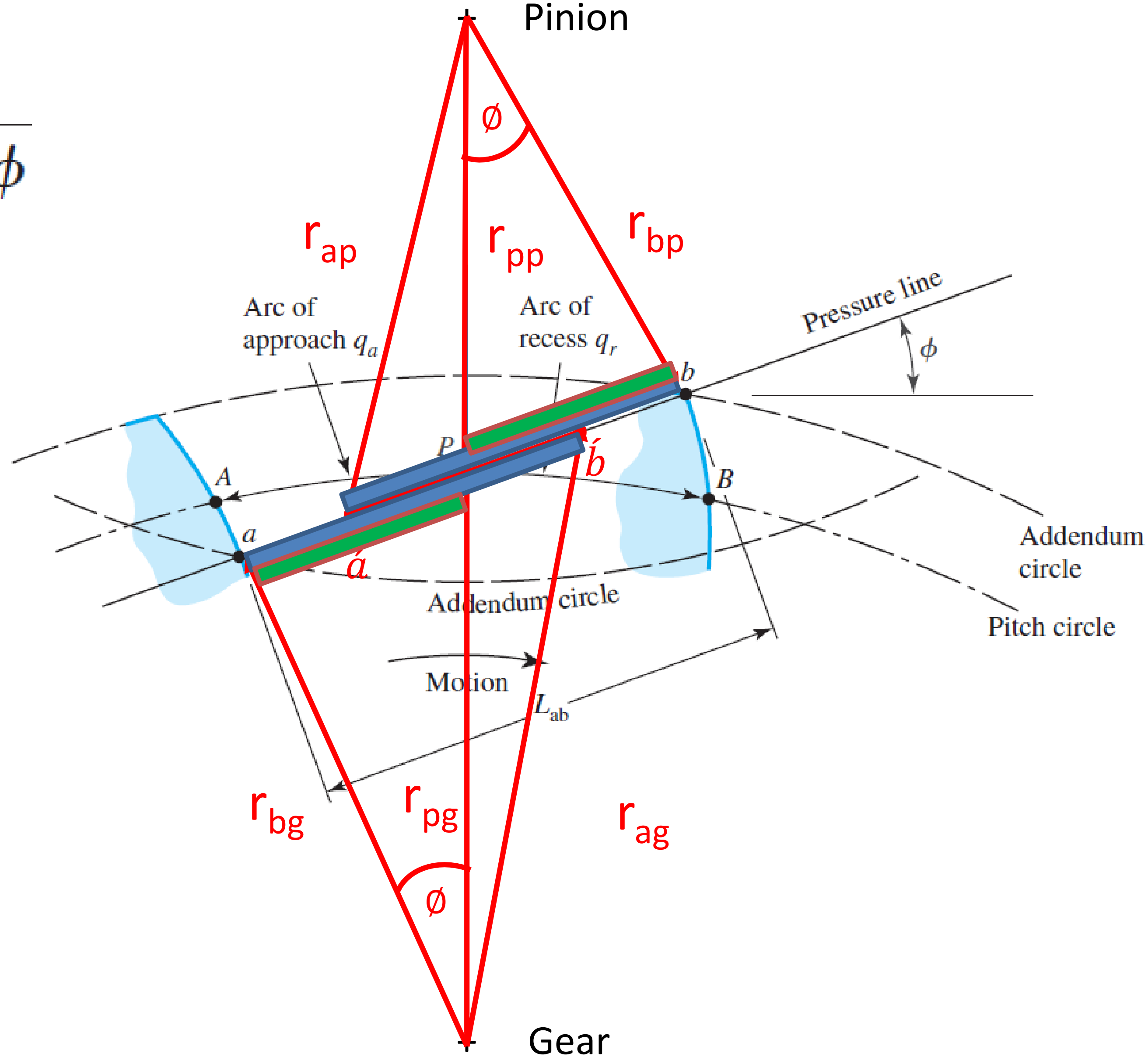
$$m_c = \frac{L_{ab}}{p \cos \phi}$$



13.7 Interference

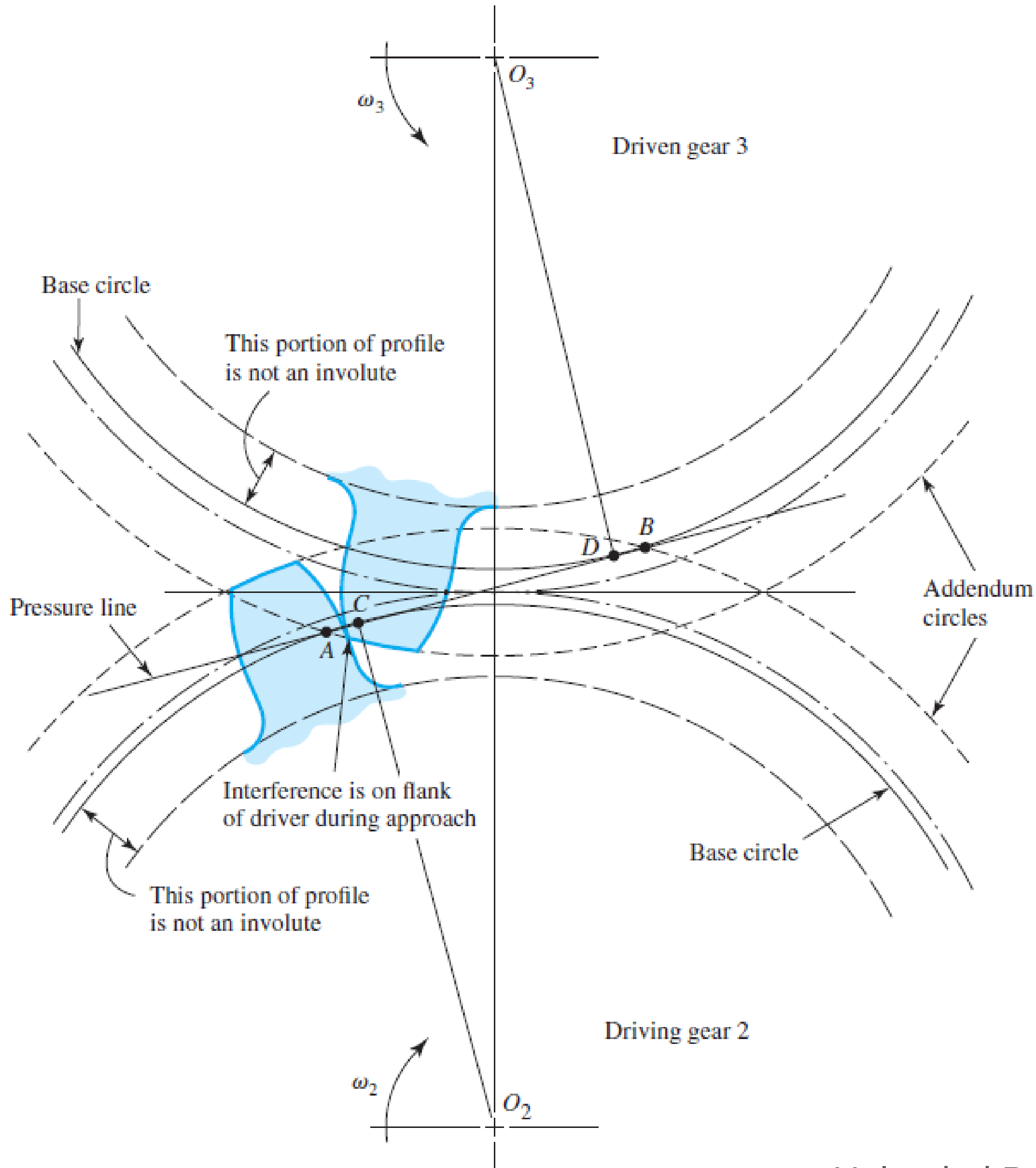


Figure 13–16

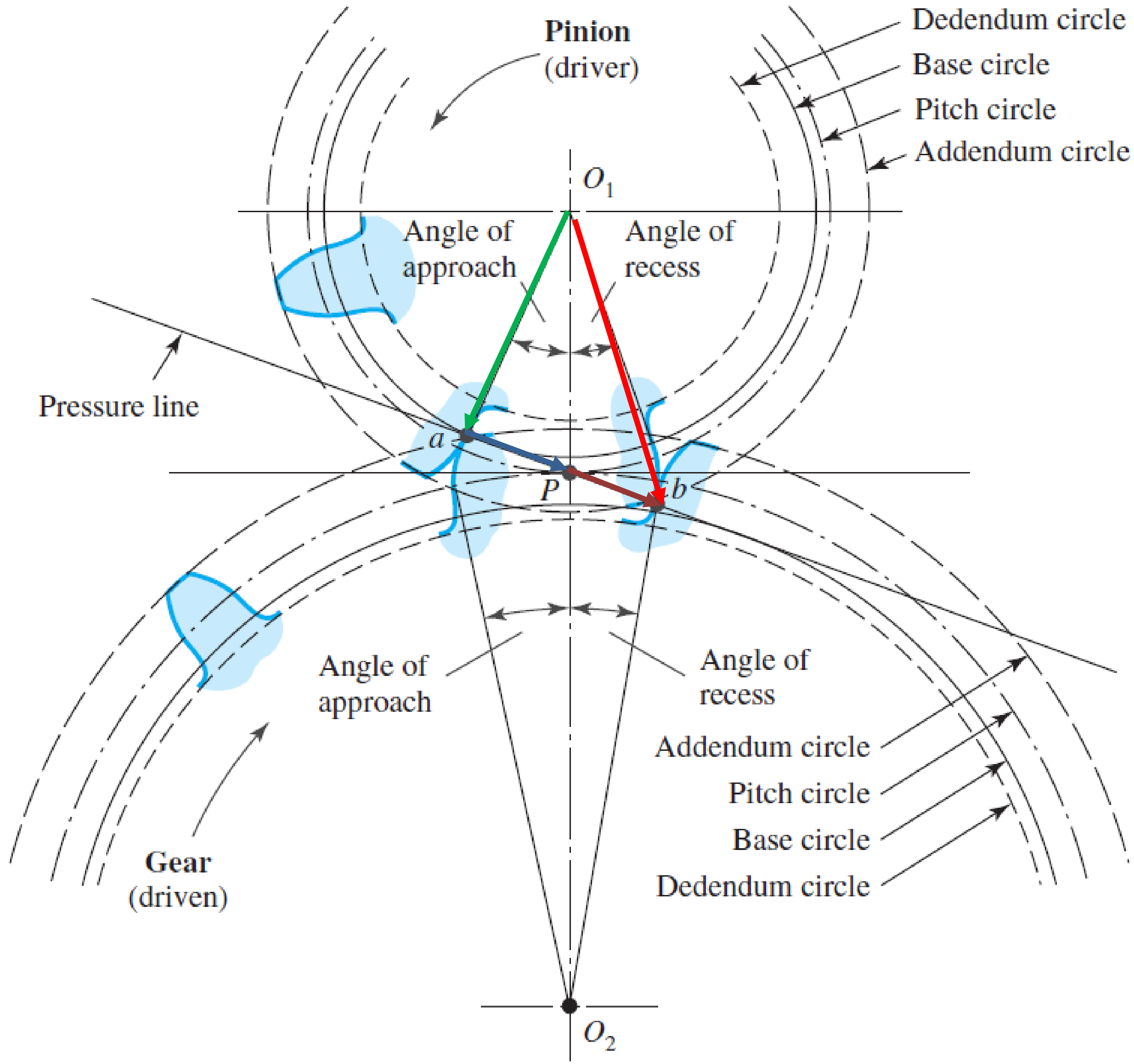
Interference in the action of gear teeth.

13.7 Interference

Figure 13–12

Tooth action.

r_{amax}



13.7 Interference

Case 1: One-To-One Gear Teeth Ratio

The smallest number of teeth on a spur pinion and gear without Interference:

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \qquad (13\text{–}10)$$

where $k = 1$ for full-depth teeth, 0.8 for stub teeth and ϕ = pressure angle.
For a 20° pressure angle, with $k = 1$,

$$N_P = \frac{2(1)}{3\sin^2 20^\circ}\left(1 + \sqrt{1 + 3\sin^2 20^\circ}\right) = 12.3 = 13 \text{ teeth}$$

13.7 Interference

Case 2: Mating Gear has more teeth than the pinion

The smallest number of teeth on a spur pinion without Interference:

$$N_P = \frac{2k}{(1 + 2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1 + 2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

For example, if $m = 4$, $\phi = 20°$,

$$N_P = \frac{2(1)}{[1 + 2(4)]\sin^2 20°}\left[4 + \sqrt{4^2 + [1 + 2(4)]\sin^2 20°}\right] = 15.4 = 16 \text{ teeth}$$

13.7 Interference

Case 2: Mating Gear has more teeth than the pinion

The smallest number of teeth on a spur gear without Interference:

The largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi} \qquad (13\text{–}12)$$

For example, for a 13-tooth pinion with a pressure angle ϕ of 20°,

$$N_G = \frac{13^2 \sin^2 20^\circ - 4(1)^2}{4(1) - 2(13) \sin^2 20^\circ} = 16.45 = 16 \text{ teeth}$$

13.7 Interference

Case 3: Mating Rack and Pinion:

The smallest number of teeth on a spur pinion without Interference:

$$N_P = \frac{2(k)}{\sin^2 \phi} \qquad (13\text{–}13)$$

For a 20° pressure angle full-depth tooth the smallest number of pinion teeth to mesh with a rack is

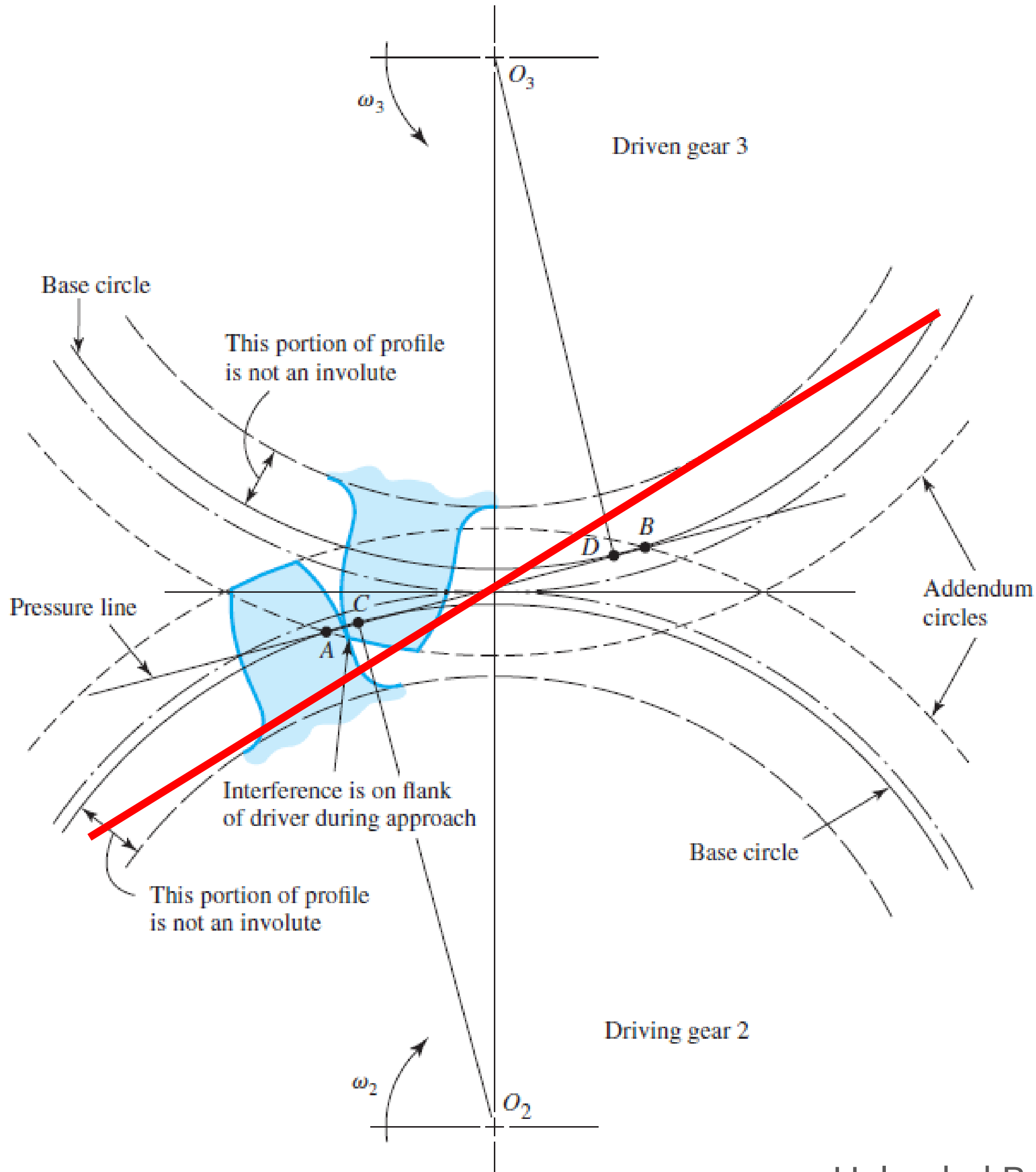
$$N_P = \frac{2(1)}{\sin^2 20°} = 17.1 = 18 \text{ teeth}$$

13.7 Interference

Figure 13–16

Interference in the action of gear teeth.

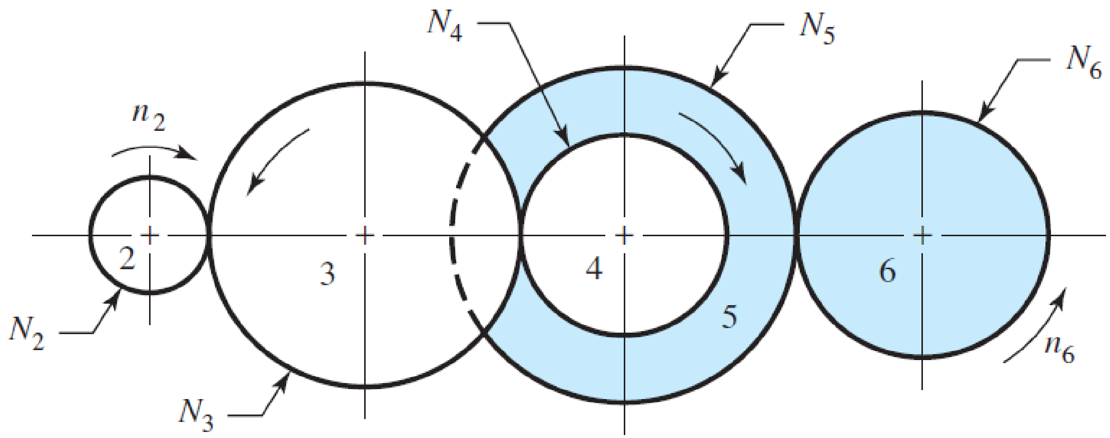
Effects of Increasing Pressure Angle - Involute Profile



13.13 Gear Trains – Simple Gear Train

Figure 13–27

A gear train.



$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$

Idler Gear: 3

Driver Gears: 2, 3, and 5

Driven Gears: 3, 4, and 6

13.13 Gear Trains – Compound Gear Train

Figure 13–28

A two-stage compound gear train.

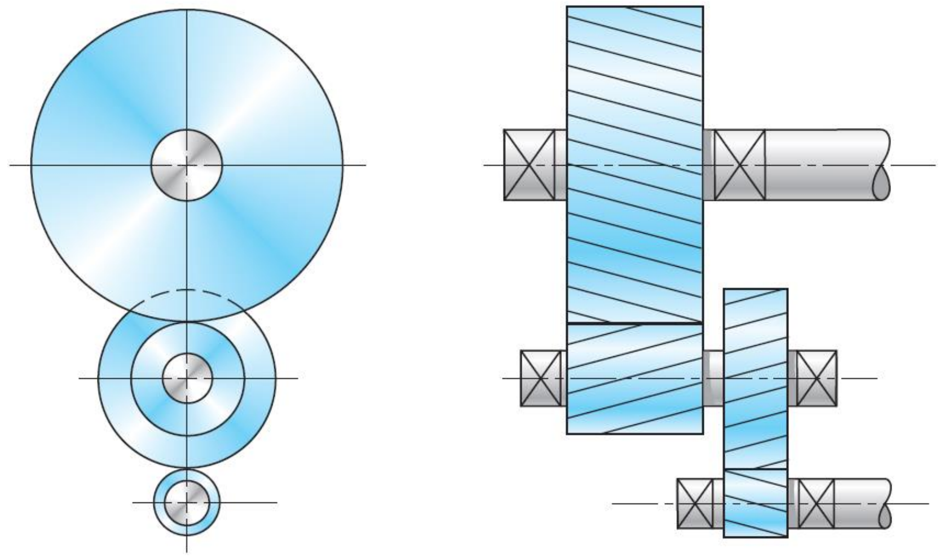
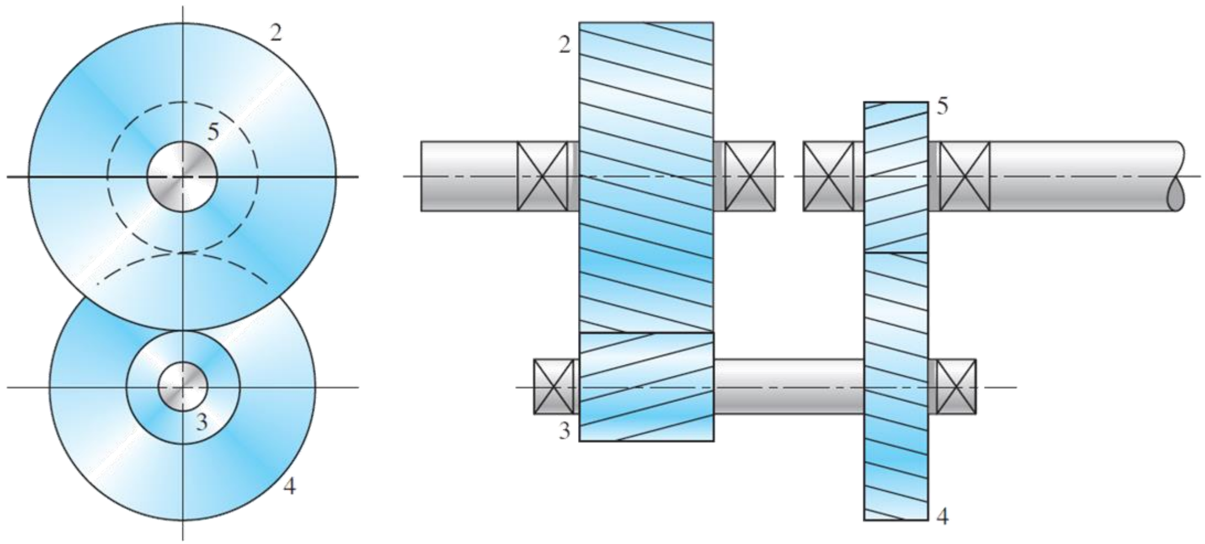


Figure 13–29

A compound reverted gear train.



13.13 Gear Trains – Planetry Gear Train

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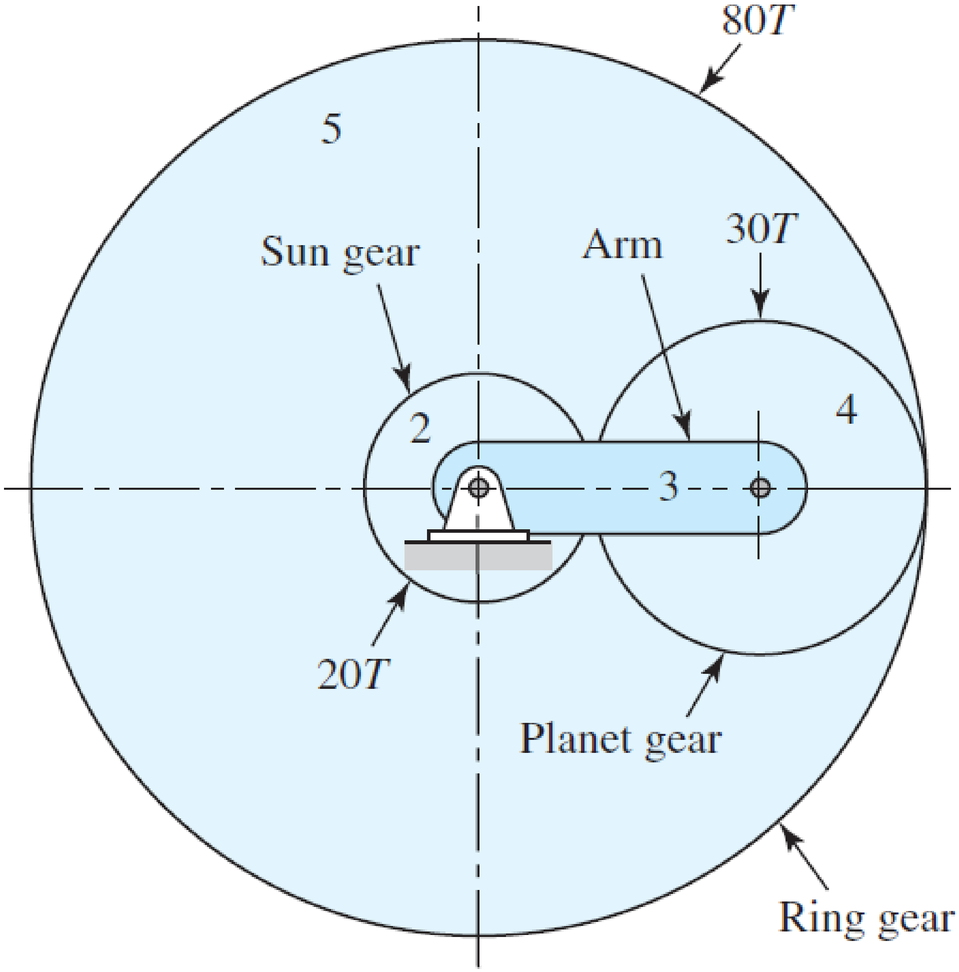
Figure 13–30

A planetary gear train.

$$e = \frac{n_5 - n_3}{n_2 - n_3}$$



13.13 Gear Trains – Planetry Gear Train

Problem 13-30

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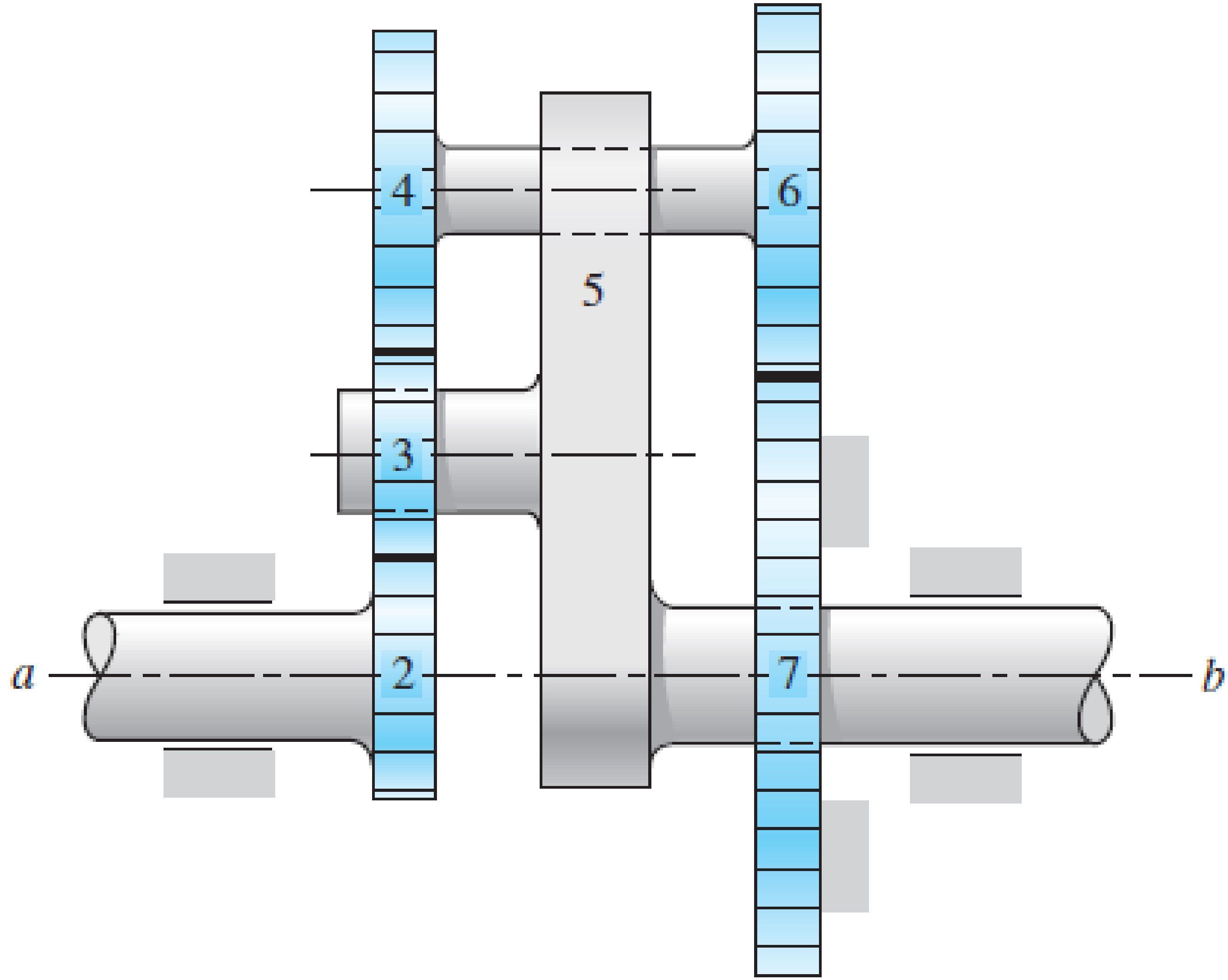
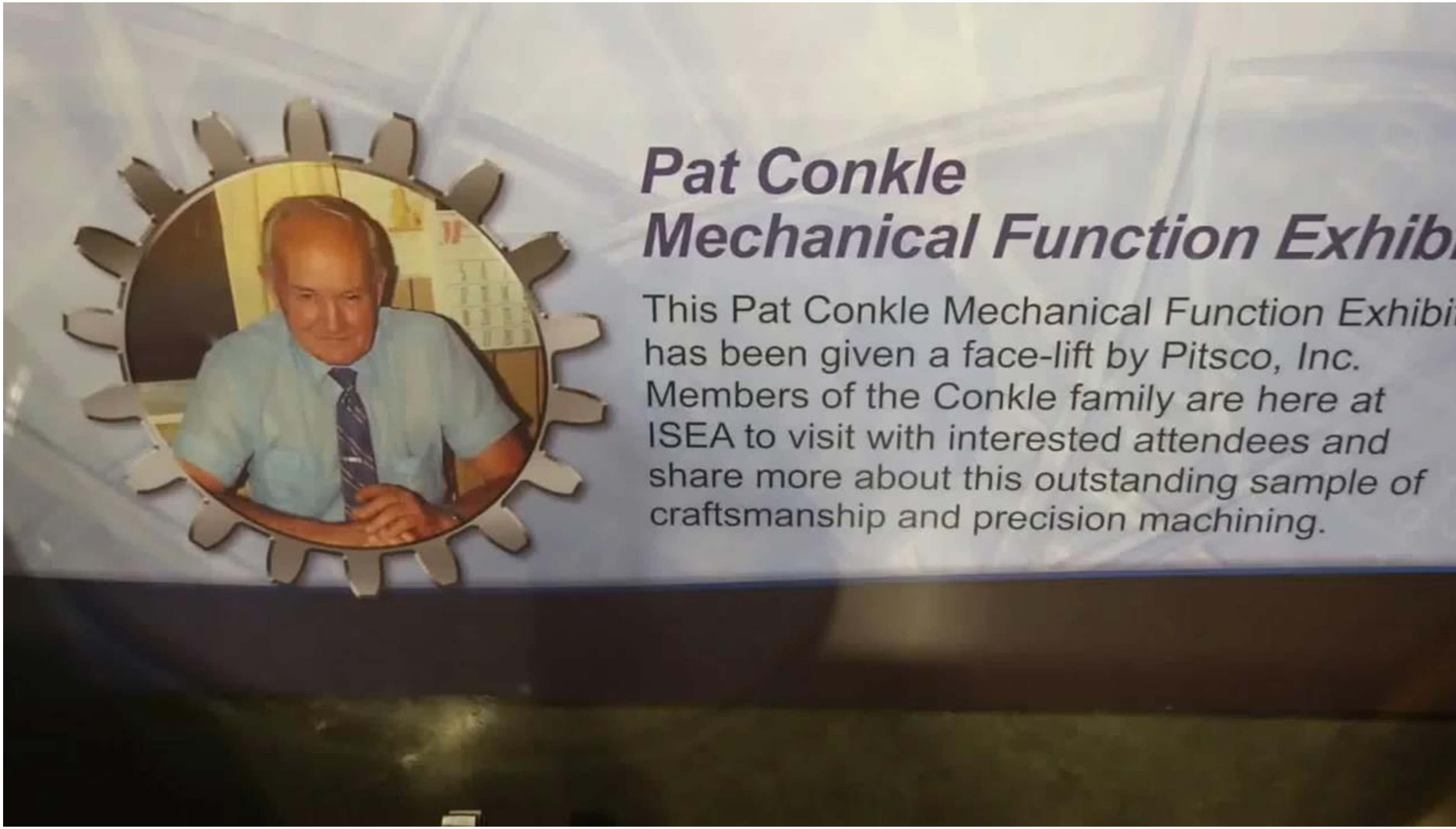
13–30 The tooth numbers for the gear train illustrated are $N_2 = 20$, $N_3 = 16$, $N_4 = 30$, $N_6 = 36$, and $N_7 = 46$. Gear 7 is fixed. If shaft *a* is turned through 10 revolutions, how many turns will shaft *b* make?

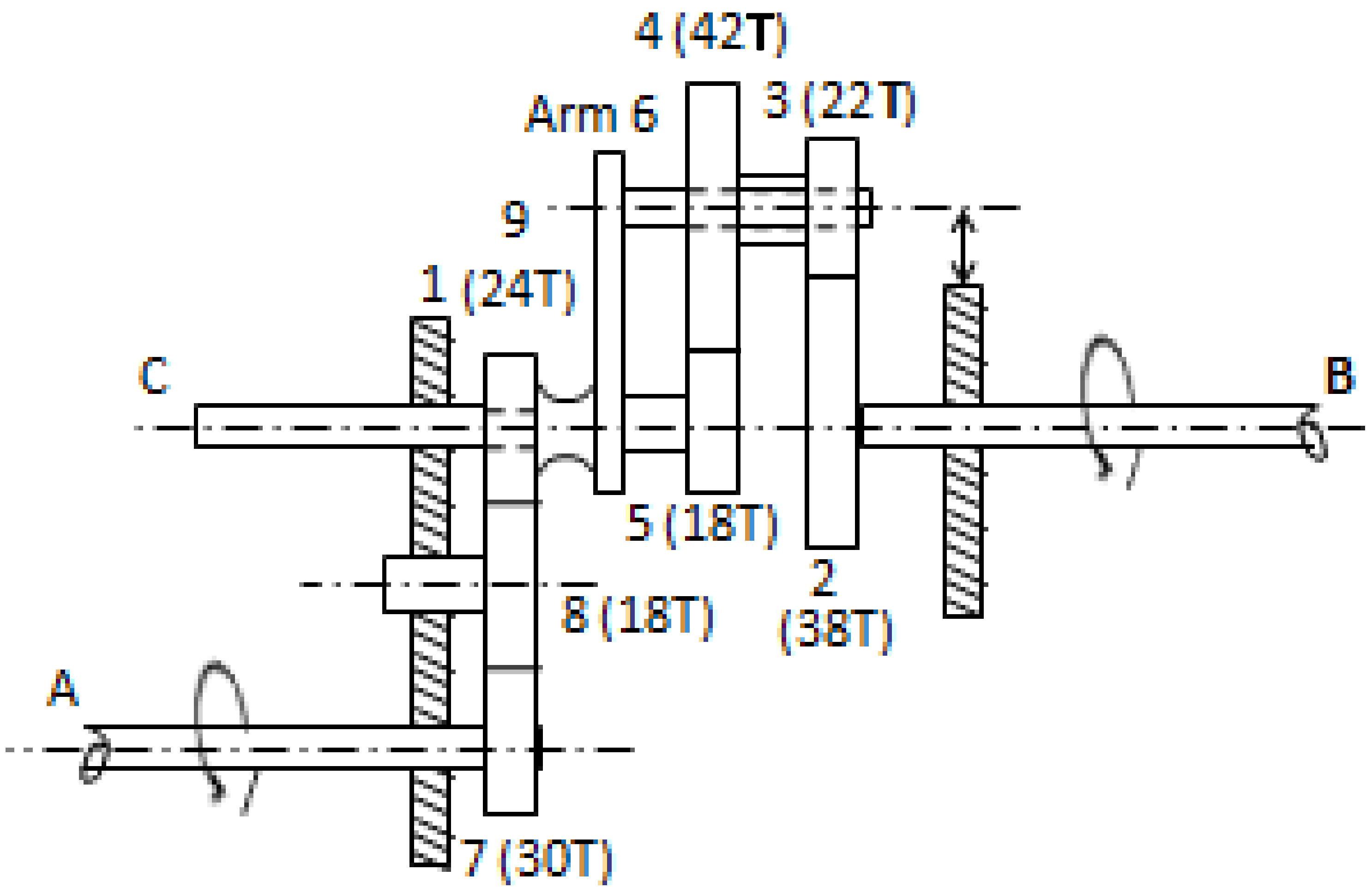


13.13 Gear Trains – Planetry Gear Train



13.13 Gear Trains – Planetry Gear Train - Example

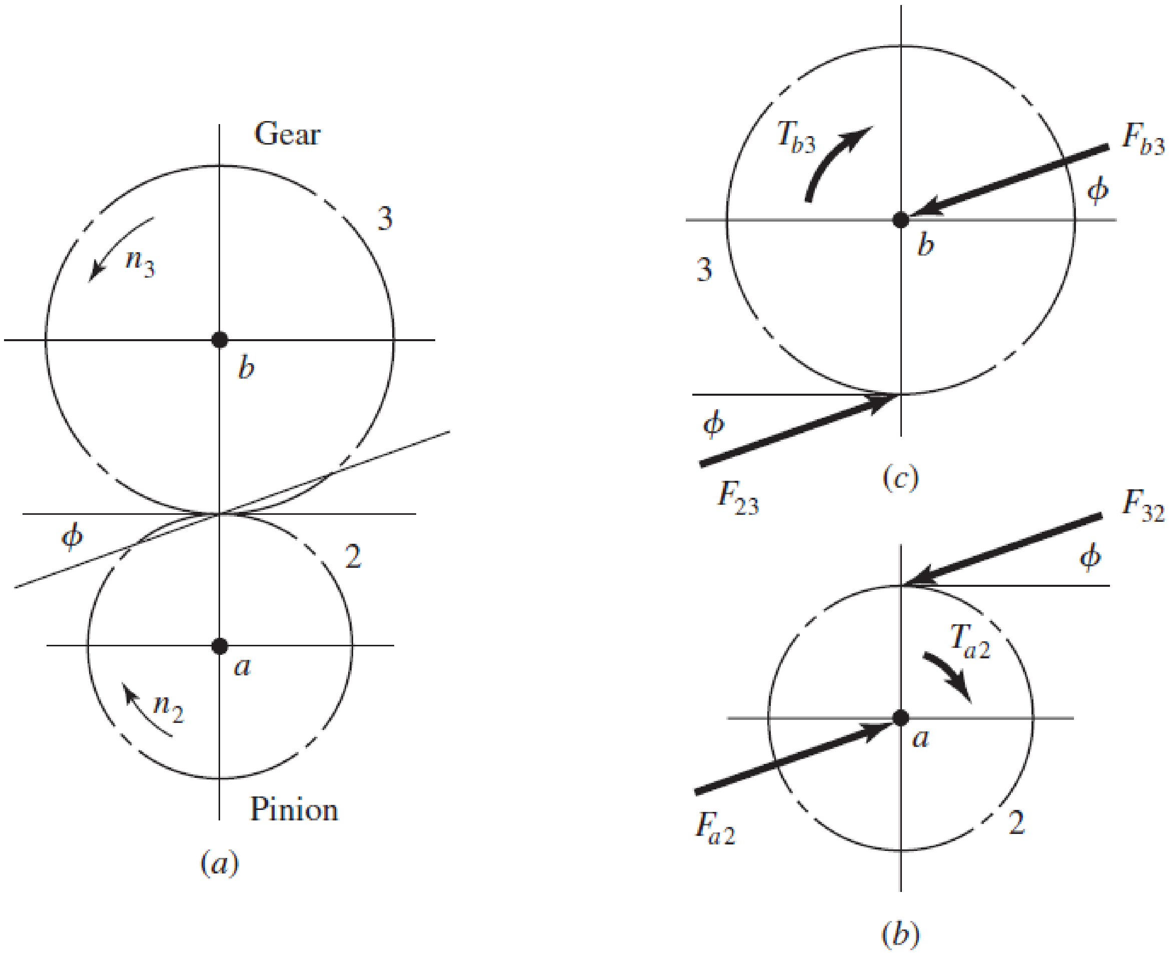
Shaft *A* rotates at 300 rpm and shaft *B* at 600 rpm in the directions shown. Determine the speed and direction of rotation of shaft *C*.



13.14 Force Analysis

Figure 13–32

Free-body diagrams of the forces and moments acting upon two gears of a simple gear train.



13.14 Force Analysis

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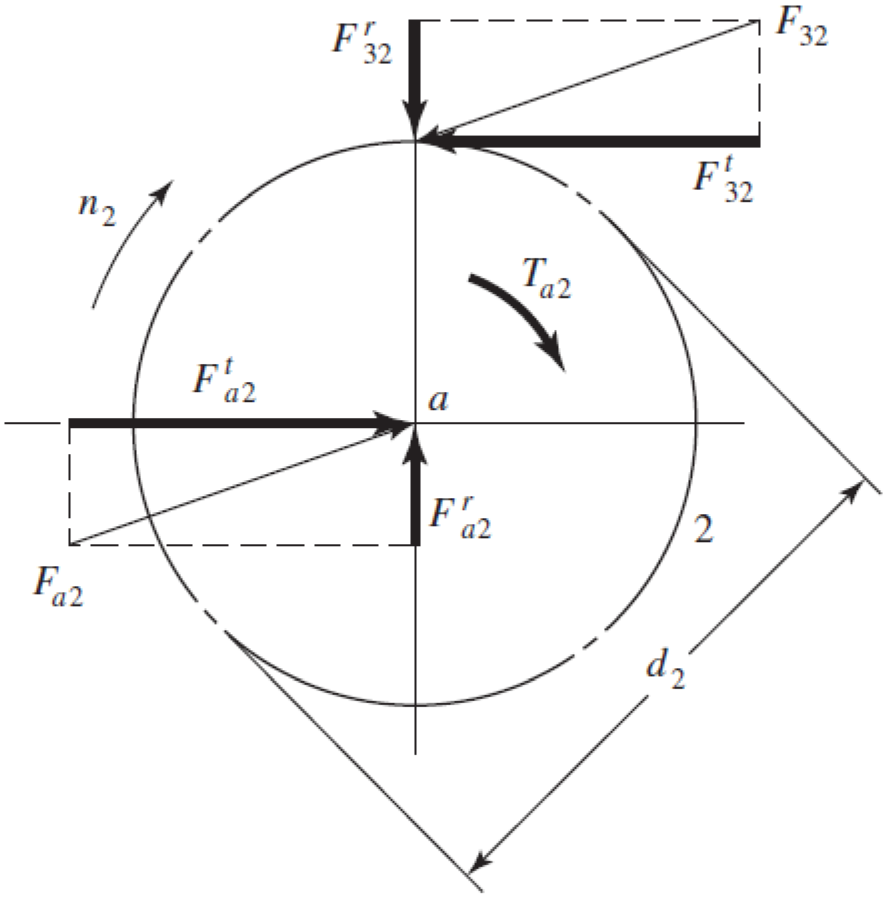
Figure 13–33

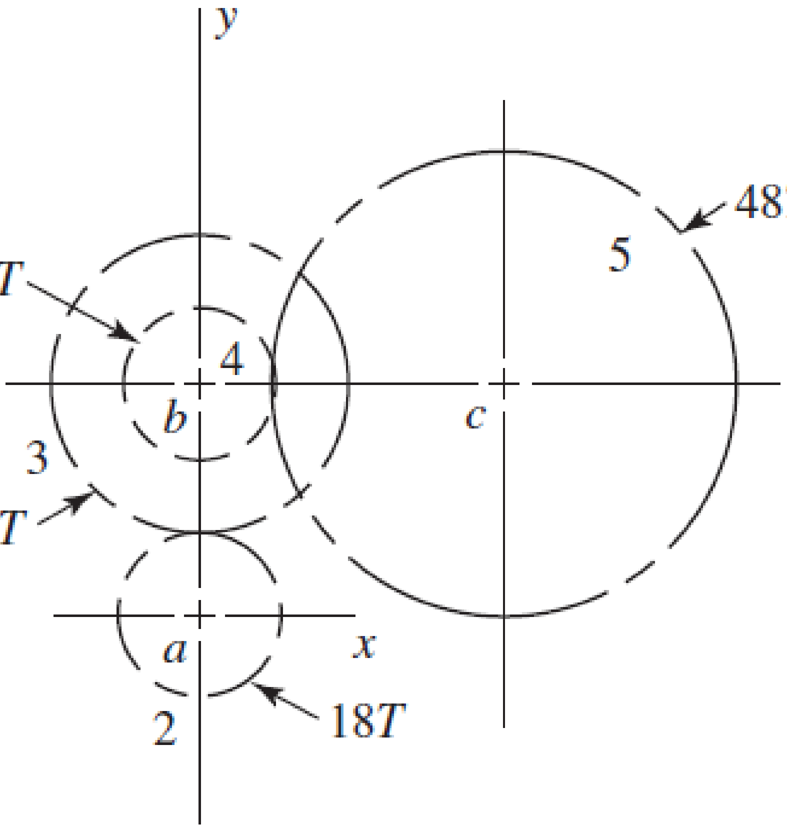
Resolution of gear forces.



13.14 Force Analysis – Problem 13-13

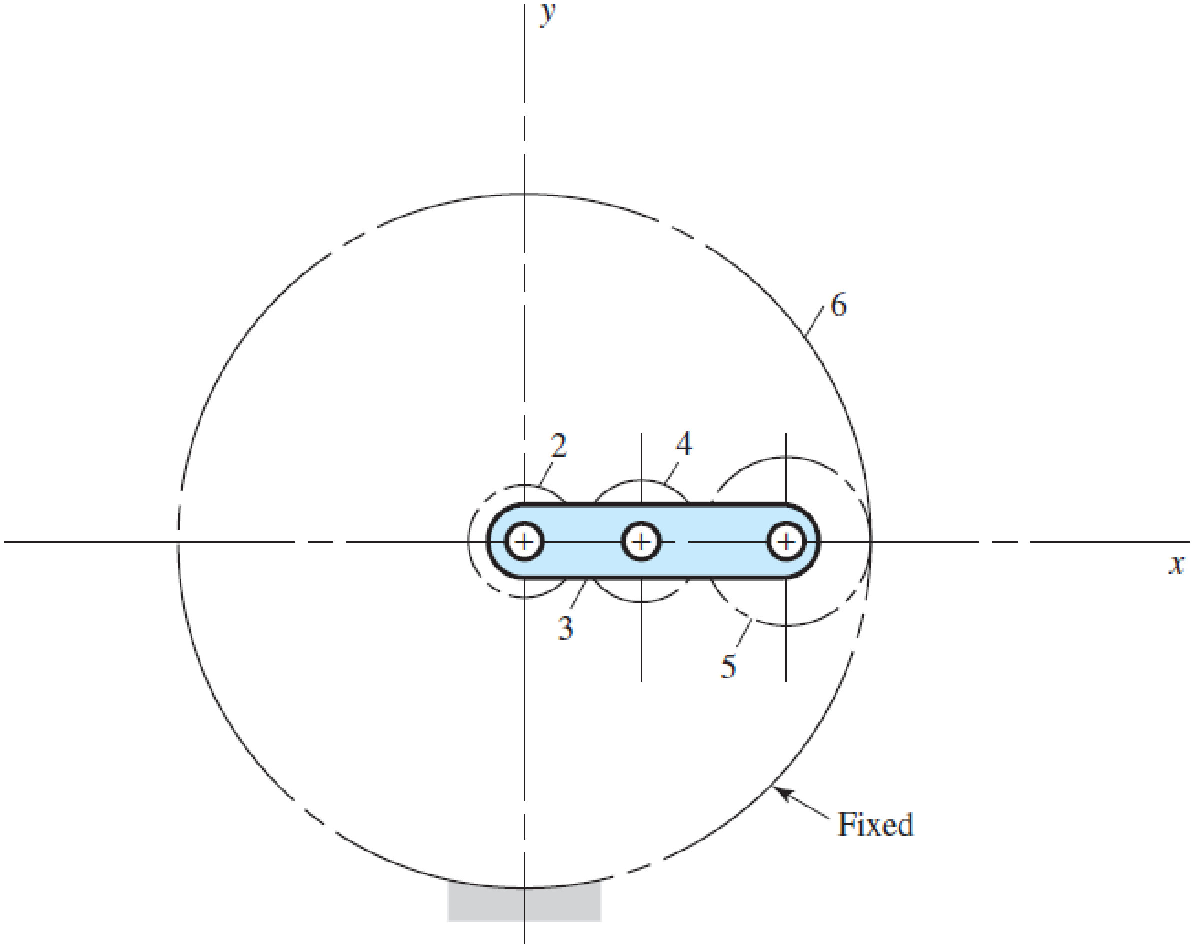
The gears shown in the figure have a module of 12 mm and a 20° pressure angle. The pinion rotates at 1800 rev/min clockwise and transmits 150 kW through the idler pair to gear 5 on shaft *c*. What forces do gears 3 and 4 transmit to the idler shaft?

Problem 13–33



13.14 Force Analysis – Problem 13-32

13–32 The 24T 6-pitch 20° pinion 2 shown in the figure rotates clockwise at 1000 rev/min and is driven at a power of 25 hp. Gears 4, 5, and 6 have 24, 36, and 144 teeth, respectively. What torque can arm 3 deliver to its output shaft? Draw free-body diagrams of the arm and of each gear and show all forces that act upon them.



Problem 13–32