# Circuit Analysis

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## Chapter 7

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# Chapter 7
# Natural and Step Response of RL and RC Circuits

In this chapter our goal is to find V and I that shows when energy is either released or acquired on the conductor (L) and the capacitor (C) in response to the sudden change in a DC voltage or current source

## Important Notes :

$t = 0^+$ —> After the sudden change
$t = 0$ —> At the sudden change
$t = 0^-$ —> Before the sudden change
$t(0^-) = t(0^+)$ (VERY IMPORTANT)

## What are we going to work on ?

### 1 – Natural Response :

Find I and V when we don't have a current or voltage source in the circuit , so the is released to a resistance network

### 2 – Step Response :

Find I and V when we have a current or voltage source in the circuit so the energy is being acquired by L and C

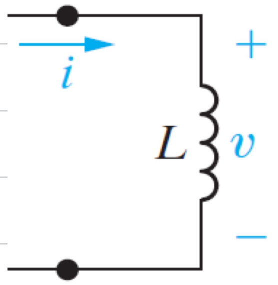
### 3 – General Case :

We use it to solve both cases above and get the complete response

* هاي الطريقة أسهل وأشمل من يلي فوق , لكن يلي فوق لازم نشتق القانون في كل سؤال , بالتالي رح استخدم هاي الطريقة في حل جميع الأسئلة على القانون العام .

The Rule : $X(t) = X(\infty) + [X(0^+) - X(\infty)]\, e^{-t/\tau}$

## First – The Inductor ( L ) :




$$\boxed{V_L = L\frac{di}{dt}}$$ ⟹ the value of L * I prime

$$I_L(t) = I_L(0) + \frac{1}{L}\int_0^t V_L(t)\,dt \quad , \; t \geq 0$$

1 – There is no voltage across the inductor if the current through it is not charging with time , so we convert it into short circuit

2 – a finite amount of energy can be stored in the. Inductor even if the voltage across it is zero , according to the following law

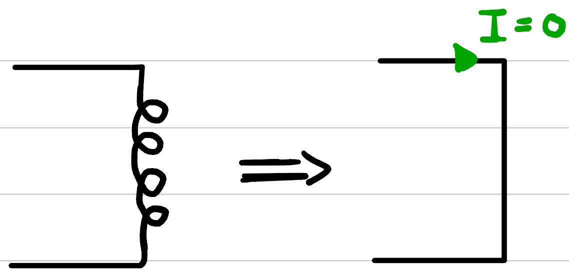
Energy of the inductor : $$\boxed{W_L(t) = \frac{1}{2} L\, I_L^2(t)}$$

3 – the inductor never dissipate energy , it only stores energy

$t = 0^+$

* At $t = 0^+$ the inductor can be considered as a current source.

$t = \infty$ AND $t = 0^-$

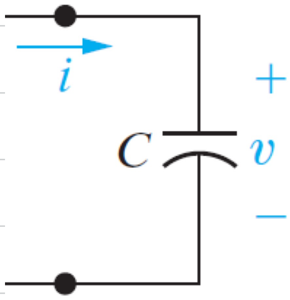



* At $t = \infty$ And $t = 0^-$, the inductor can be considered as a short circuit

* The inductor stores current

## Second – The Capacitor :

$\boxed{I_c = C\frac{dV}{dt}} \Rightarrow C$ value $*$ $V$ prime




$$V_c(t) = V_c(0) + \frac{1}{C}\int_0^t I_c(t)\,dt \quad , \quad t \geq 0$$

1 – There is no current across the capacitor if the voltage across it is not charging with time , so we convert it into open circuit

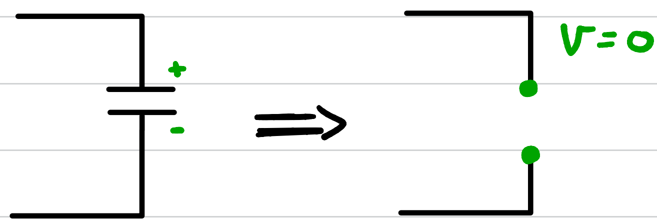
2 – a finite amount of energy can be stored in the capacitor even if the current across it is zero , according to the following law

Energy of the Capacitor : $\boxed{W_c(t) = \frac{1}{2} C V_c^2(t)}$

3 – the capacitor never dissipate energy , it only stores energy

$t = 0^+$

$t = \infty$ AND $t = 0^-$




* At $t = 0^+$ the Capacitor Can be Considered as a Voltage Source.

* At $t = \infty$ And $t = 0^-$, the Capacitor Can be Considered as a open Circuit

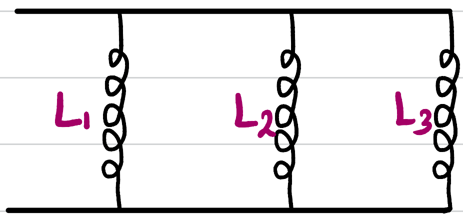
* The Capacitor Stores Voltage

# Series parallel combination of inductors and capacitors

## 1.1 – series inductors :

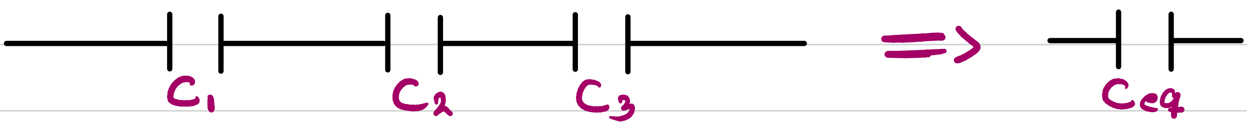

$$L_{eq} = L_1 + L_2 + L_3$$

## 1.2 – parallel inductors :



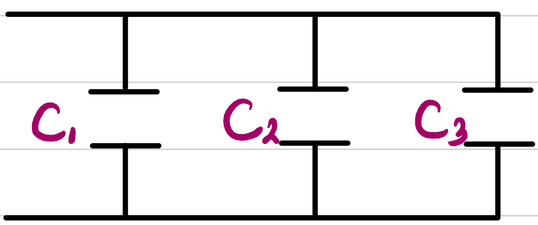

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

## 2.1 – series capacitors :




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C_{eq}$$

## 2.1 – parallel capacitors :




$$C_{eq} = C_1 + C_2 + C_3$$

# First Order Circuits (RL and RC)

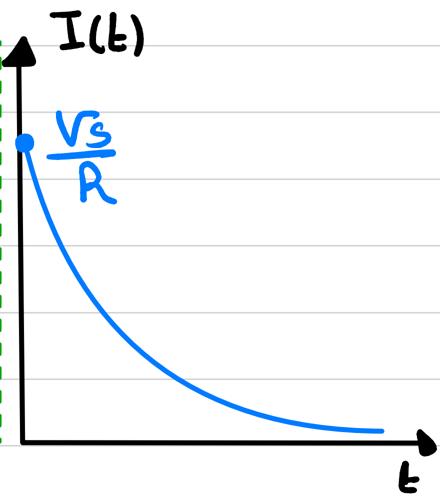
- The circuits that include inductor or capacitor BUT not both
- First order circuit = First order differential equation

## 1 – Natural Response (discharging) :

It happens when we switch from the battery to a resistive network
And it can be RL or RC circuit

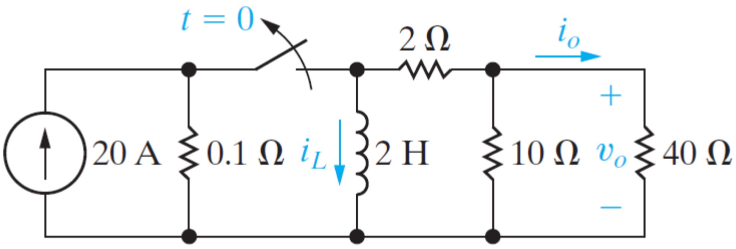
### ① RL circuit

- $I_L(0^+) = I_L(0^-) = \dfrac{V_s}{R_{eq}} = I_{\text{Current source}}$
- time constant : $\tau = L / R$
- $i(t) = i(\infty) + [i(0^+) - i(\infty)]\, e^{-t/\tau}$




**Example 1**

after switching it out find
1- $I_L(t)$ for $t \geq 0$

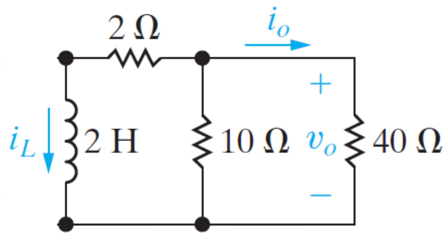



**Step 1 :-**

Find $I(0^-)$, from the circuit directly or on the law
since we have a current source $\Rightarrow I_L(0^-) = 20\,A$

**Step 2 :-**

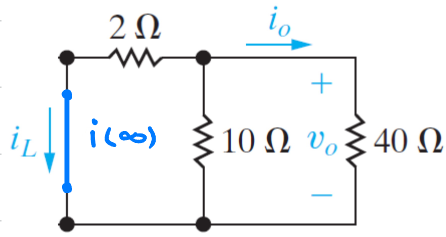
After switching we get a natural response circuit, then we find :




$\boxed{1 \cdot \tau}$ $\tau = L / R$ , $R : 10 // 40 + 2 \Rightarrow R = 10\,\Omega$

$\tau = 2/10 \Rightarrow \tau = 0.2\,s$

$\boxed{2 \cdot i(\infty)}$ You have to find $i$ when the Capacitor became open Circuit or the inductor became short circuit :




we don't have any Sources $\Rightarrow i(\infty) = 0$

$$i(t) = i(\infty) + (i(0^+) - i(\infty))\, e^{-t/\tau}$$

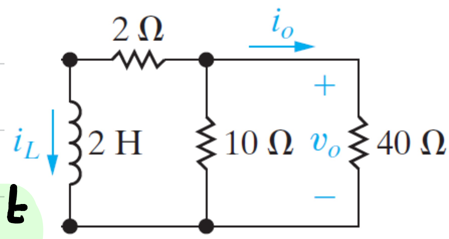
$$i(t) = 0 + (20 - 0)\, e^{-5t}$$

$$i(t) = 20\, e^{-5t}$$

**Note :** Usually $i(\infty)$ in the natural response is equal to zero because we don't have ay source to supply the circuit

## 2- $I_o(t)$ for $t \geq 0^+$

we use Current divider rule




$$\frac{10}{10 + 40} * (-I_L(t))$$

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عكس الاتجاه

$$= 0.2 * -20\, e^{-5t} \Rightarrow \boxed{I_o(t) = -4\, e^{-5t}}$$

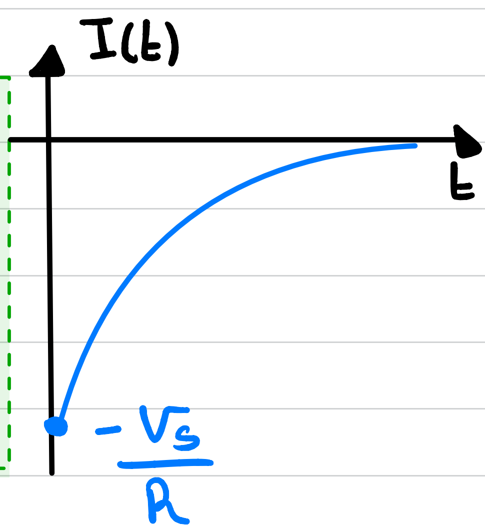
## 3- $V_o(t)$ for $t \geq 0^+$

$$V_o = I_o * R_{40} \Rightarrow V_o = -4\, e^{-5t} * 40$$

$$\Rightarrow \boxed{V_o = -160\, e^{-5t}}$$

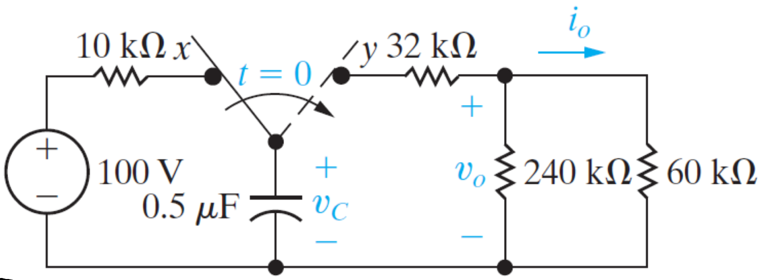
## ② RC circuit

- $V_c(0^-) = V_c(0^+) = V_s$
- time constant : $\tau = RC$
- $V(t) = V(\infty) + [V(0^+) - V(\infty)]\, e^{-t/\tau}$



# Example 2

After switching $x \to y$ find




1- $V_c(t)$ for $t \geq 0$

like the steps above

$V_c(0^-) = V_s \Rightarrow V_c(0^-) = 100$

$\tau = RC$ , $R: (240 // 60) + 32 \Rightarrow R = 80\,k\Omega$

$\tau = 0.5 * 80 \Rightarrow \tau = 40\,ms$

$V(\infty) = 0$

$V(t) = 0(100-0)e^{-25t} \Rightarrow V(t) = 100e^{-25t}$

2- $V_o(t)$ for $t \geq 0^+$

by voltage divider rule , $\frac{48}{48+32} * 100e^{-25t}$

$V_o(t) = 60e^{-25t}$ for $t \geq 0^+$

3- $I_o(t)$ for $t \geq 0^+$

$I_o(t) = \frac{60e^{-25t}}{60} \Rightarrow I_o(t) = e^{-25t}\,mA$

4- Total energy dissipated by $60\,k\Omega$

step 1

find the power at $60\,k\Omega \Rightarrow P = I_o^2 * R$

$P = (e^{-25})^2 * 60k \Rightarrow P = 60e^{-50}\,mW$

Step 2

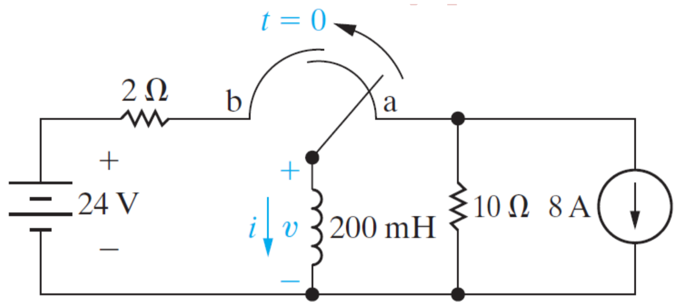
do integration from ZERO to $\infty$

$w = \int_0^\infty 60e^{-50} = 0 - (-60/50) \Rightarrow w = 1.2\,mJ$

## Example 3 for Step response

1- find $i(t)$

we do source transformation to get $V_s = IR \Rightarrow V_s = 80$




$$i(0^-) = \frac{V_s}{R} \Rightarrow i(0^-) = -8A$$

$$i(\infty) = \frac{V}{R} = \frac{24}{2} \Rightarrow i(\infty) = 12A$$

$$\tau = L/R \Rightarrow \tau = (200 * 10^{-3})/2 \Rightarrow \tau = 0.1$$

$$i(t) = 12 + (-8-12)\, e^{\frac{-t}{0.1}} \Rightarrow i(t) = 12 - 20\, e^{-10t}$$

2- V at inductor after switching to b

$$V = L\frac{di}{dt} \Rightarrow V = 0.2 * 200\, e^{-10t} \Rightarrow V = 40\, e^{-10t}$$

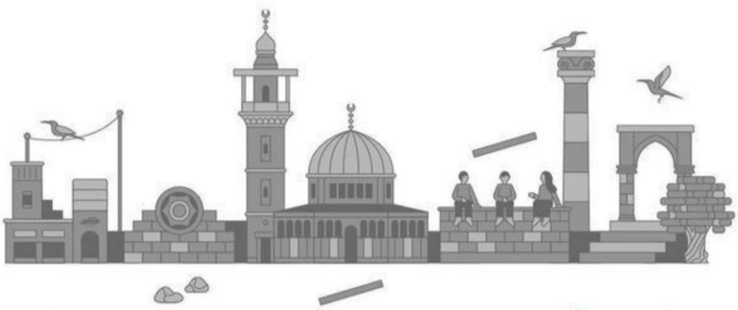
3- t when the Voltage of the inductor = 25 V

from the low above $V = 40\, e^{-10t}$

$$25 = 40\, e^{-10t} \Rightarrow \frac{25}{40} = e^{-10t} \Rightarrow \ln\left(\frac{25}{40}\right) = \ln e^{-10t}$$

$$-0.47 = -10t \Rightarrow t = 0.047\ s$$

*** Note:** $i(0^+) = i(0^-)$ Just for the inductor
$V(0^+) = V(0^-)$ Just for the Capacitor

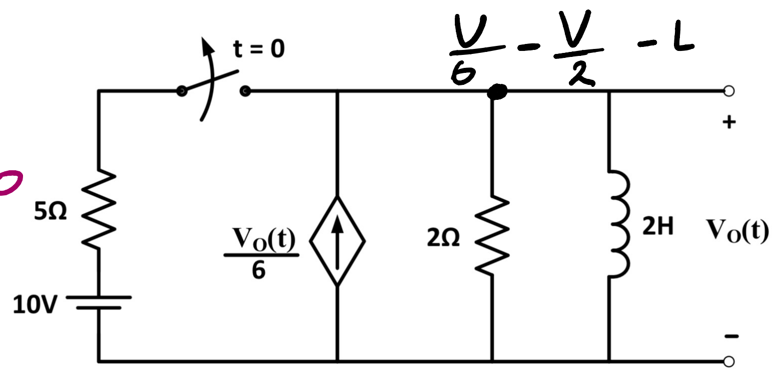


## Example 4 : Dependent Source

1- find $V(t)$

Step 1

Normally find $i$ at $t=0^-, \infty$

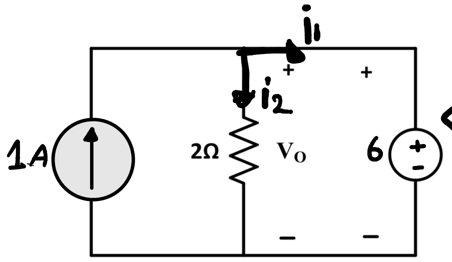


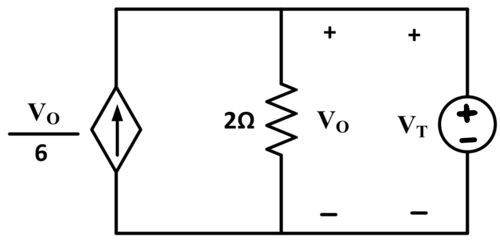

$$i(0^-) = 10/5 \Rightarrow i(0^-) = 2$$

$$V(\infty) = 0$$

Step 2

Since we have dependent source we find Thevinin equivalent circuit at $t = \infty$




let $V_T = 6 \Rightarrow$ then we make source transfer. and apply KVL

$$-6 + 2i_T + 2 = 0 \Rightarrow 2i_T = 4 \Rightarrow i_T = 2A$$

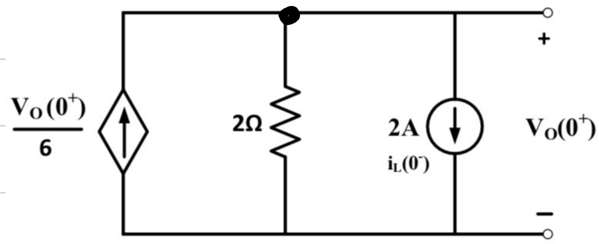
$$R_{th} = \frac{V_T}{i_T} \Rightarrow R_{th} = \frac{6}{2} \Rightarrow R_{th} = 3\,\Omega$$

$$\tau = L / R_{th} \Rightarrow \tau = \frac{2}{3}$$

Step 3

Find $V$ or $i$ at $t = 0^+$ Using KCL

- Capacitor $\rightarrow$ Voltage source
- Inductor $\rightarrow$ Current source




$$\frac{V_o}{6} - \frac{V_o}{2} - 2 = 0 \Rightarrow V_o = V(0^+) = -6V$$

$$V(t) = 0 + (-6-0)\,e^{-\frac{3t}{2}} \Rightarrow V(t) = -6\,e^{-\frac{3t}{2}}$$