Discussion 7.3

26 Find $\frac{dy}{dx}$ if $\ln(xy) = e^{x+y}$

$$\ln x + \ln y = e^{x+y}$$

$$\frac{1}{x} + \frac{y'}{y} = e^{x+y}(1+y')$$

$$y'\left(\frac{1}{y} - e^{x+y}\right) = e^{x+y} - \frac{1}{x}$$

$$y'\left(\frac{1 - y\,e^{x+y}}{y}\right) = \frac{x\,e^{x+y} - 1}{x}$$

$$y' = \frac{y}{x}\left(\frac{x\,e^{x+y} - 1}{1 - y\,e^{x+y}}\right)$$

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42 Evaluate $\int \frac{e^{-1/x^2}}{x^3}\,dx$

$u = -\frac{1}{x^2} = -x^{-2}$

$du = 2x^{-3}\,dx$

$\frac{du}{2} = \frac{dx}{x^3}$

$$= \int e^u \frac{du}{2}$$

$$= \frac{1}{2} e^u + c$$

$$= \frac{1}{2} e^{-x^{-2}} + c$$

$$= \frac{1}{2} e^{-\frac{1}{x^2}} + c$$

57 Find $\frac{dy}{ds}$ if $y = 5^{\sqrt{s}}$

$$\frac{dy}{ds} = y' = 5^{\sqrt{s}} \frac{1}{2\sqrt{s}} \ln 5$$

$$= \frac{\ln 5}{2} \frac{5^{\sqrt{s}}}{\sqrt{s}}$$

92 Evaluate $\int \frac{x\, 2^{x^2}}{1 + 2^{x^2}}\, dx$

$u = 1 + 2^{x^2}$

$du = 2^{x^2} (2x) \ln 2\, dx$

$\frac{du}{2 \ln 2} = x\, 2^{x^2} dx$

$$= \int \frac{\frac{du}{2\ln 2}}{u}$$

$$= \frac{1}{2\ln 2} \int \frac{du}{u}$$

$$= \frac{1}{\ln 4} \ln |u| + c$$

$$= \frac{1}{\ln 4} \ln (1 + 2^{x^2}) + c$$ since $1 + 2^{x^2} > 0$