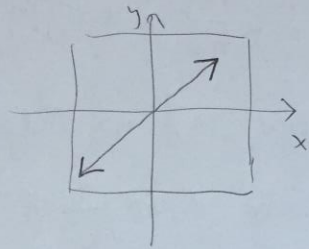


Ch6: Polarization of Light:

- $\uparrow \vec{E}_0$ Direction of $\vec{E}$ field
Polarization

HW: 4.5, 4.9

- If $\vec{E}_0$ oscillates in a plane:
Linearly polarized.



- If $\vec{E}_0$ oscillates (spirals around a plane)
while wave propagates:
Circular or elliptical
polarization.

6.1 Linear, Circular, Elliptical Polarization

(2)

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

If direction of $\vec{k}$ is $\hat{z}$ direction

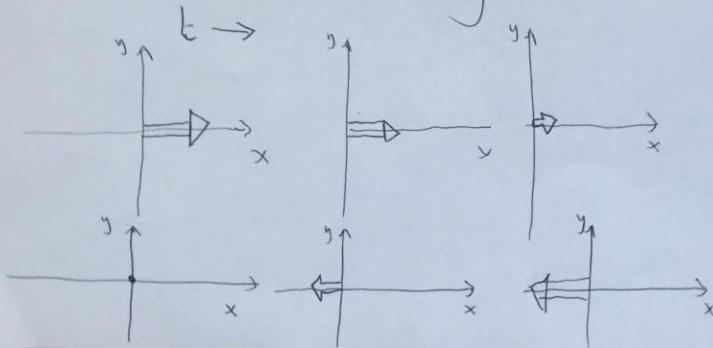
$$\vec{E}(z, t) = (E_x \hat{x} + E_y \hat{y}) e^{i(kz - \omega t)}$$

We are interested in Real part.

E_x, E_y can be complex.

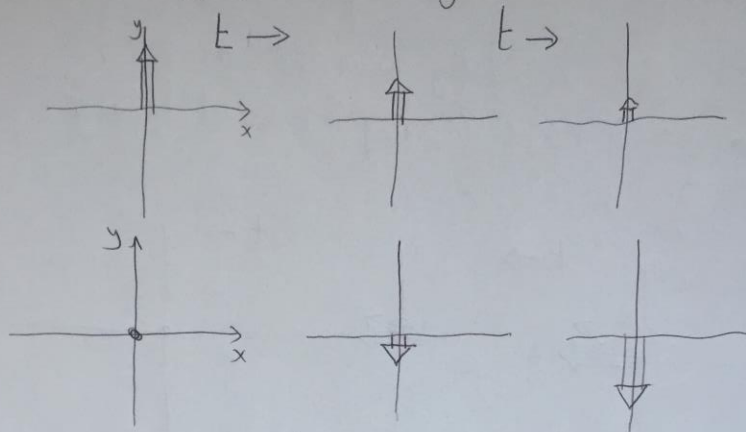
The relation between E_x, E_y describes the polarization.

Case 1: $E_y = 0$: Linearly polarized light.
Along $\hat{x}$ -direction.

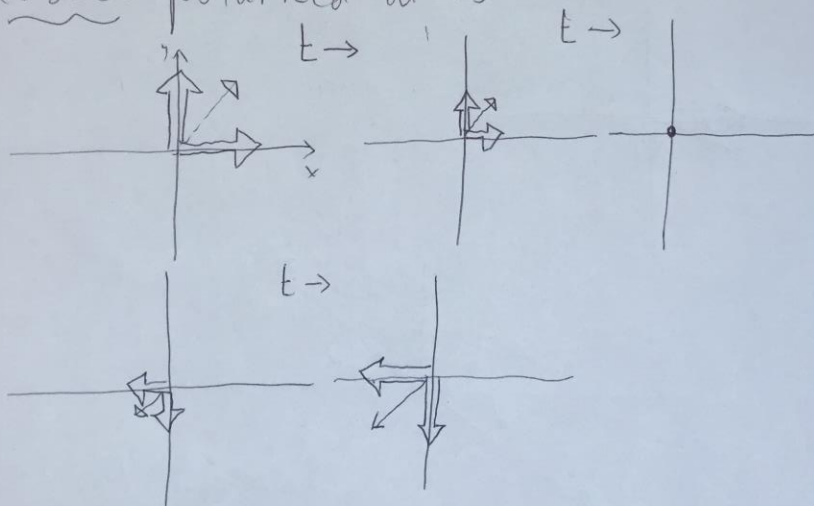


Case 2: $\hat{y}$ polarized light.

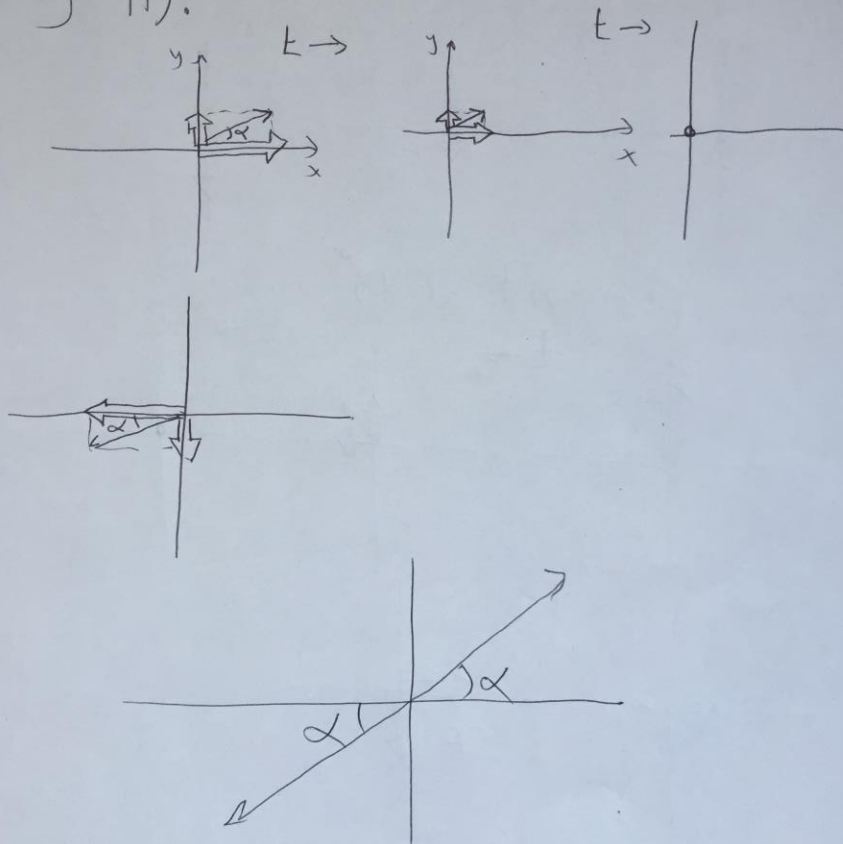
(3)



Case 3: polarized at 45°



In general: Linearly polarized light if E_x & E_y have the same complex phase (or phases differing by π). (4)



Circular Polarized light:

(5)

Suppose $E_y = iE_x = (e^{i\frac{\pi}{2}})E_x$; (delay $\frac{\pi}{2}$)

let E_x be real.

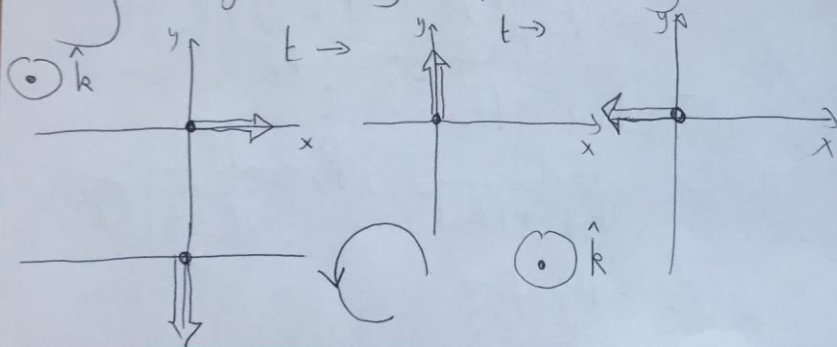
$$\vec{E}(z,t) = \text{Re}[E_x e^{i(kz - \omega t)}] \hat{x} + \text{Re}[e^{i\pi/2} E_x e^{i(kz - \omega t)}] \hat{y}$$

$$= E_x \cos(kz - \omega t) \hat{x}$$

$$+ E_x \cos(kz - \omega t + \frac{\pi}{2}) \hat{y}$$

$$= E_x [\cos(kz - \omega t) \hat{x} - \sin(kz - \omega t) \hat{y}]$$

$\hat{y}$ lags $\hat{x}$ by a quarter cycle.



This is left handed circularly polarized light. (6)

[Rotates CW looking in $\hat{k}$ direction]

6.2 Jones Vectors

- Deals with light that has a definite Polarization, such as plane waves.
- Two-Dimensional Matrix algebra.

$$\vec{E}(z, t) = (|E_x| e^{i\phi_x} \hat{x} + |E_y| e^{i\phi_y} \hat{y}) e^{i(kz - \omega t)}$$

$$= E_{\text{eff}} (A \hat{x} + B e^{i\delta} \hat{y}) e^{i(kz - \omega t)}$$

$$E_{\text{eff}} = \sqrt{|E_x|^2 + |E_y|^2} e^{i\phi_x}$$

$$A \equiv \frac{|E_x|}{\sqrt{|E_x|^2 + |E_y|^2}} ; B \equiv \frac{|E_y|}{\sqrt{|E_x|^2 + |E_y|^2}}$$

$$\delta = \phi_y - \phi_x$$

$$A^2 + B^2 = 1$$

(7)

Case 1: $E_y = 0$; $B = 0$; everything well defined.

Case 2: $E_x = 0$, then $e^{i\phi}$ is indeterminate.

We let $E_{eff} = |E_y| e^{i\phi_y}$, $B = 1$, $\delta = 0$

Using 2.62: $\bar{I} = \frac{1}{2} n c \epsilon_0 \vec{E} \cdot \vec{E}^*$

we get $\bar{I} = \frac{1}{2} n c \epsilon_0 |E_{eff}|^2$??

Jones Vector: $A \hat{x} + B e^{i\delta} \hat{y}$

$$(A \hat{x} + B e^{i\delta} \hat{y}) \cdot (A \hat{x} + B e^{i\delta} \hat{y})^* = 1 \quad (\text{unit vector})$$

we use matrix notation

$$\begin{bmatrix} A \\ B e^{i\delta} \end{bmatrix}$$

This describes the polarization state of any plane wave field.

① Case 1: linear Polarization along $\hat{x}$.

⑧

$$E_y = 0 \quad A = 1 ; B = 0$$

(we don't care about δ).

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

② Linear Polarization along $\hat{y}$.

$$E_x = 0 ; B = 1 ; \delta = 0 ; A = 0$$

special case $\rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

③ Linearly polarized at angle α (measured from x -axis). e.g. $\alpha = 45^\circ$

$$|E_x| = |E_y| \quad \phi_x = \phi_y = 0$$

$$A = \frac{1}{\sqrt{2}} ; B = \frac{1}{\sqrt{2}} \quad \delta = 0 \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|E_x| = \cos \alpha ; |E_y| = \sin \alpha$$

$$A = \cos \alpha ; B = \sin \alpha$$

$$\rightarrow \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix}$$

Right circularly polarized.

(9)

$$|E_x| = |E_y| \quad \phi_x = \phi_y + \frac{\pi}{2}$$

$$A = \frac{1}{\sqrt{2}} ; B = \frac{1}{\sqrt{2}} ; \delta = \frac{\pi}{2}$$

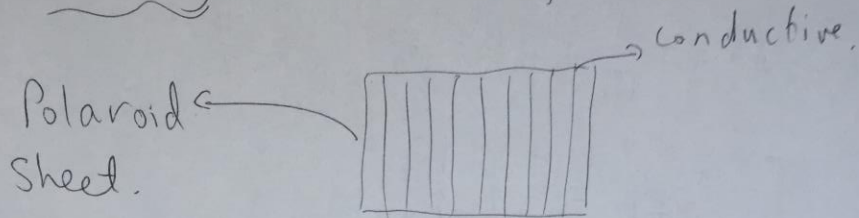
$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} e^{i\frac{\pi}{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

6.3 Elliptically Polarized Light

Elliptically Polarized: A state between
Linear and circular polarization.

6.4 Linear Polarizers & Jones Matrices (10)

- History: 1928 Land. ; Polarizer.



$\updownarrow \vec{E}$ gets absorbed.
 $\longleftrightarrow \vec{E}$ gets through.

