Department of Electrical and Computer Engineering

ENEE4403 – Power Systems Lecture Notes

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Transmission Lines Parameters

» Introduction to transmission lines (T.L)
» Types of Overhead line Conductors.
» Resistance Calculation.
» Inductance Calculation.
» Capacitance Calculation.

☑ Overhead transmission System

1. Although underground AC transmission would present a solution to some of environmental and aesthetic (جمالي) problems in overhead transmission lines, there are technical and economic reasons that make the use of underground ac transmission not preferable.

2. The overhead transmission System is mostly used at high voltage level mainly because it is much cheaper Compared to underground system.

3. The selection of an economical voltage level for the T.L is based on the amount of power and the distance of transmission.

   ⟹ The economical voltage between Lines in 3ϕ is given by :-

$$V = 5.5\sqrt{0.62\,L + \frac{P}{100}} \quad , \text{ where}$$

   V = line voltage in kV.
   L = Length of T.L in km.
   P = Peak real power in kW.

4. Standard transmission voltages are established
   - → HV (30 - 230) kV
   - → EHV (230 - 765) kV
   - → UHV (765 - 1500) kV

Types of overhead line conductors based on → Conducting material, → the strength

1. The material to be chosen for conduction of power should be such that it has the lowest resistance. This would reduce the transmission losses.

(a) *

1) Silver resistivity 1.6 $\mu\Omega$cm
2) Copper resistivity 1.7 $\mu\Omega$cm
3) gold resistivity 2.35 $\mu\Omega$cm
4) aluminium resistivity 2.65 $\mu\Omega$cm

Problems: cost, theft, supply is quit limitted

(b) * The weight of material (density)

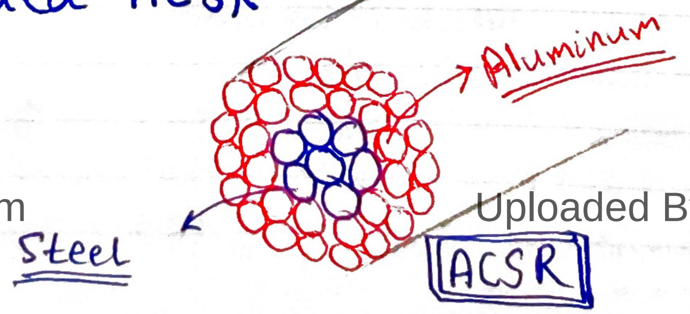
1) aluminium
2) copper
3) silver
4) gold

note: The weight the aluminium condu... having the same resis... as that of copper is roughly 60% less th... of copper.

2. In the early days of the transmission of electric power, conductors where usually copper, but aluminum conductors have completly replaced copper for overhead lines because of the much lower cost and lighter weight of an aluminum conductor compared with a copper conductor of the same resistance.

3. The most commonly used conductors for high voltage transmission lines are:-

* AAC All-Aluminum Conductors
* AAAC All-Aluminum-Alloy Conductors (أسلاك الألمنيوم)
* ACSR Aluminum Conductor, Steel-Reinforced (معزز، مقوى)
* ACAR Aluminum Conductor, Alloy-Reinforced.
* Expanded ACSR



» Aluminum-alloy conductors have higher tensile (قوة الشد) strength than the ordinary aluminum.

» ACSR consists of a central core of steel strands surrounded by layers of aluminum strands.

» ACAR has a central core of higher-strength aluminum surrounded by layers of aluminum.

» Expanded ACSR has a filler such as (paper, fiber) separating the inner steel strands from the outer aluminum strands. The filler gives a larger diameter (and hence, lower corona) for a given conductivity and tensile strength. Expanded ACSR is used for some extra-high voltage lines.

Stranded Conductors

» To increase the area stranded conductors are used. This increase the flexibility and the ability of the wire or cable to be bent.

» Generally the circular conductors of the same size are used for spiralling.

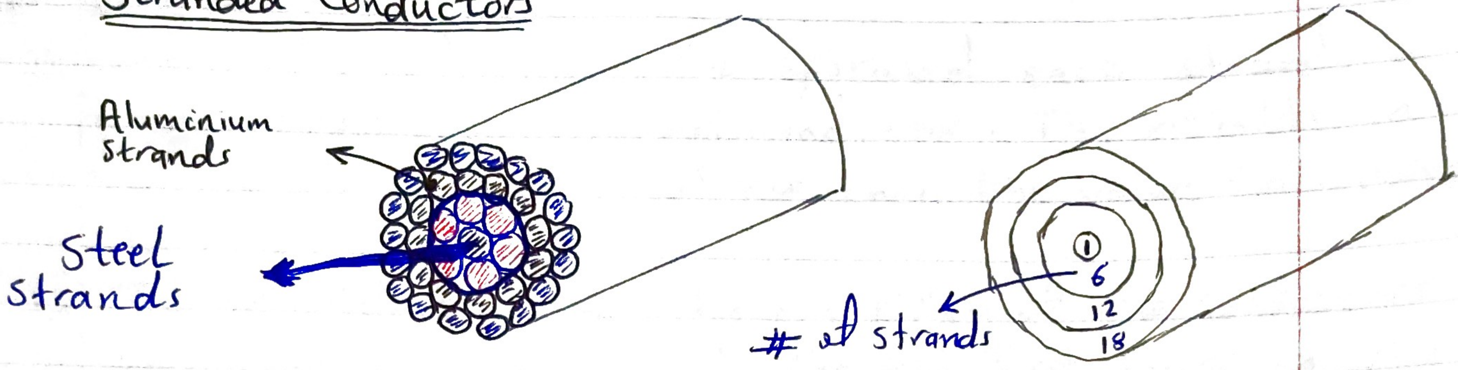
» Each layer of strands is spiraled in the opposite direction of its adjacent layer. This spiraling holds th strands in place (can't open up easily)

Stranded Conductors

ⓐ easier manufacturing (larger sizes)

ⓑ better mech. strength as well as better handling; much more flexible.

Stranded Conductors



Total # of strands → 1, 7, 19, 37, 61, 91

Line Resistance :-

- R_{ac} ~
- R_{dc} –

» The dc resistance of a solid round conductor at a specified temperature is given by :-

$$\boxed{R_{dc} = \frac{\rho^{T} l}{A} \ \Omega} \qquad (*)$$

where

- $\rho \equiv$ conductor resistivity at temp T (°C)
- $l \equiv$ conductor length (m)
- $A \equiv$ Conductor cross-sectional area (m²)

» Conductor resistance depends on the following factors :-
1. Temperature 2. Spiraling 3. Frequency

1. Temperature

Resistivity of conductor metals varies linearly over normal operating temperatures according to

$$\rho^{T_2} = \rho^{T_1}\left(\frac{T_2 + T}{T_1 + T}\right)$$

⇒ The conductor resistance increase as temp increases.

$$R_2 = R_1\left(\frac{T_2 + T}{T_1 + T}\right)$$

R_2 or R^{T_2}, R_1 or R^{T_1}

$T \triangleq$ temp. constant that depends on the conductor material.

⊙ For Aluminum
$T \cong 228$

2 Spiraling

» Since a stranded conductor is spiraled, each strand is longer than the finished conductor. This results in a slightly higher resistance than the value calculated using equation (*).

» The spiralling increase the resistivity of the conductors to an extent about 2% for the first layer on the centre conductor, about 4% for the second layer, and so on.

3 Frequency "skin effect"

» When ac flows in a conductor, the current distribution is not uniform over the conductor cross-sectional area and the current density is greatest at the surface of the conductor. This causes the ac resistance to be somewhat higher than the dc resistance. This behavior is known as skin effect.

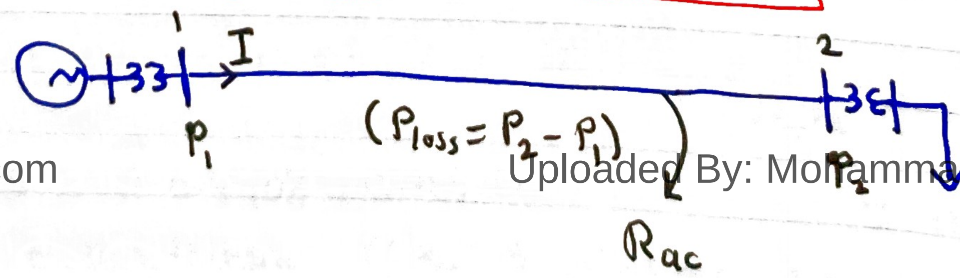
» This uneven distribution does not assume large proportion at 50 Hz up to a thickness of about 10 mm.

» At (50-60) Hz, the ac resistance is about 2 percent higher than the dc resistance.

Note:-

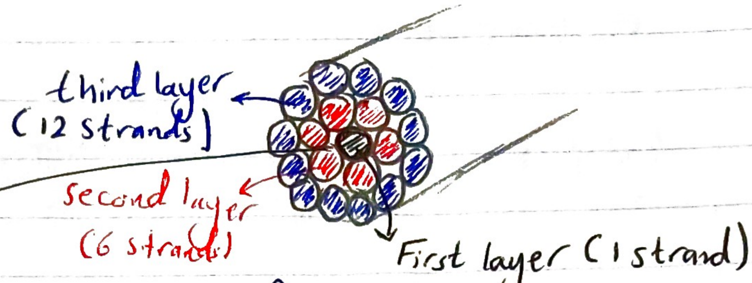
The ac resistance or effective resistance of a conductor is

$$\boxed{R_{ac} = \frac{P_{loss}}{I^2} \ \Omega}$$



Example A copper cable of 19 strands, each strand 2.032 mm in diameter is laid over a length of 1km. The temperature rise was found to be 40. Find the value of total R for this cable.

Solution



total # of strands = 19

For 1 strand 

$$A_{1s} = \frac{\pi d^2}{4} = \frac{\pi (0.2032)^2}{4}$$

$$= 0.03243 \text{ cm}^2$$

at 20°C

$$R_{1s} = \frac{\rho l}{A} = \frac{1.7 * 10^{-6} * 100000}{0.03243}$$

$$= 5.24\ \Omega$$

?!

$$R_{total} = \frac{5.24}{19} = 0.2758\ \Omega$$

1 Spiraling effect

First layer $R_{1con} = 5.24$

second layer $R_{6con} = \frac{5.24}{6} = 0.8733\ \Omega \xrightarrow{\text{Spir. eff}} R_{6con} = 0.8733 * 1.02$

$$= 0.8908\ \Omega$$

third layer $R_{12con} = \frac{5.24}{12} = 0.4367\ \Omega \xrightarrow{\text{Spir. eff}} R_{12con} = 0.4367 * 1.04$

$$= 0.4541\ \Omega$$

$R_{cable} = 5.24 \| 0.8908 \| 0.4541$

$R_{total} = 0.2844\ \Omega$ «3.1 % higher when we consider spiraling effect»

[2] Temperature effect

$$R_2 = R_1 \left(\frac{T + T_2}{T + T_1} \right) = 0.2844 \left(\frac{234.5 + 60}{234.5 + 20} \right)$$

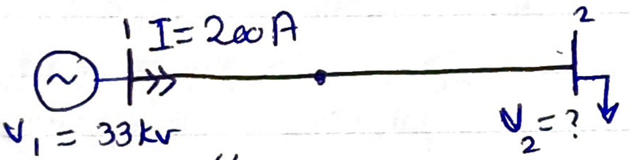
for copper

$$= 0.329 \ \Omega$$

the resistance at new temp.

$R = 0.2758 \ \Omega$ compared with (19.3 %)

<u>note</u>: If the cable was carrying a current 200A, the drop from one end to the other end would be about 65.8 volts due to resistance.



$((V_1 - V_2 = \text{voltage drop}))$

[3] frequency effect

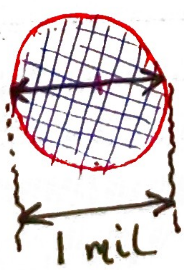
At freq 50 Hz the skin depth in a copper is of the order of 10 mm and hence would not have any significant effect as far as this problem is concerned.

<u>Note</u>:

» In english units, conductor cross-sectional area is expressed in circular mils (cmil)

» A circular mil (cmil) is a unit of area, equal to the area of a circle with a diameter of one mil (one thousandth of an inch)

* one inch = 1000 mils
mil = 0.001 inch
= 0.0254 mm

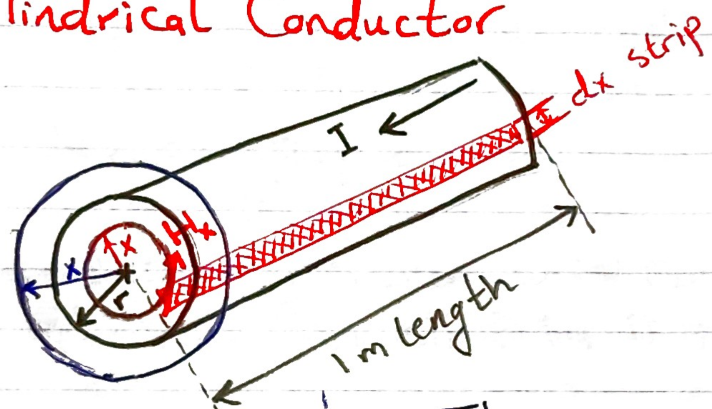


Inductance

» For Calculating Inductance we need to go to four steps :-

1. Magnetic Field Intensity H, from Ampere's Law
2. Magnetic Flux Density B, ($B = \mu H$)
3. Flux Linkages, (λ)
4. Inductance From Flux Linkages per ampere. ($L = \lambda / I$)

Solid Cylindrical Conductor



[A] Internal Flux Linkage   [B] External Flux Linkage

» The magnetic field intensity H_x, around a circle of radius x, is constant and tangent to the circle. The Ampere's Law relating H_x to the current I_x is given by:

$$\oint H_{tan}\, dl = I_{enclosed}$$

(محيط الدائرة)

$$\int_0^{2\pi x} H_x \cdot dl = I_x$$

$$\boxed{H_x = \frac{I_x}{2\pi x}} \quad \dots (1)$$

I_x is the Current enclosed at radius x.

[A] Internal Inductance

» A simple expression can be obtained for the internal flux linkage by neglecting the skin effect and assuming uniform current density throughout the conductor cross section, i.e.

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \Rightarrow I_x = \left(\frac{x}{r}\right)^2 I$$

• uniform current density

from (1) $\quad H_x = \dfrac{I_x}{2\pi x}$

$$H_x = \frac{I}{2\pi r^2}\, x$$

» For a nonmagnetic conductor with constant permeability μ_o, the magnetic flux density is given by:

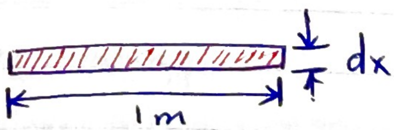
$$B_x = \mu_o H_x$$

$$B_x = \mu_o \left[\frac{I}{2\pi r^2}\, x\right]$$

$\mu_o \equiv$ permeability free space $\equiv 4\pi * 10^{-7}$ H/m

» The differential flux $d\phi$ for a small region of thickness dx and one meter length of the conductor is

$$d\phi_x = B_x \underbrace{dx \cdot 1}_{\text{area of strip}}$$



⊙ The flux $d\phi_x$ links only the fraction of the conductor from the center to radius x.
Thus, on the assumption of uniform current density, only the fraction $\frac{\pi x^2}{\pi r^2}$ of the total current is linked by the flux, i.e.,

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\phi_x$$

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\Phi_x$$

$$= \left(\frac{x^2}{r^2}\right)\left[B_x \, dx\right]$$

$$= \frac{x^2}{r^2}\left[\frac{\mu_0 I x}{2\pi r^2}\right] dx$$

$$d\lambda_x = \frac{\mu_0 I x^3}{2\pi r^4} dx$$

- $B_x = \mu_0 \left[\frac{I x}{2\pi r^2}\right]$
- $d\Phi_x = B_x \, dx$

» The total flux linkage

$$\lambda_{int} = \int_0^r d\lambda = \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3 \, dx$$

$$= \frac{\mu_0 I}{8\pi} \text{ Wb/m}$$

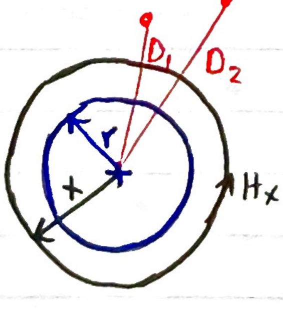
By def^n, for nonmagnetic material, the inductance L is the ratio of its total magnetic flux linkage to the current I, given by $L = \lambda / I$.

The Inductance due to the internal flux linkage is

$$L_{int} = \frac{\mu_0}{8\pi} = \frac{1}{2} * 10^{-7} \text{ H/m}$$

Note that L_{int} is independent of the conductor radius r.

$\boxed{B}$ Inductance due to external flux linkage



$$\oint H_{tan} \, dl = I_{enclosed}$$

$$\int_0^{2\pi x} H_x \, dl = I$$

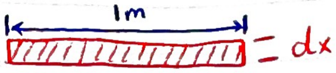
» $H_x (2\pi x) = I$

$H_x = \frac{I}{2\pi x}$ A/m $\quad x > r$

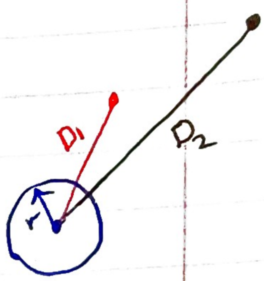
$$\gg B_x = \mu_0 H_x = 4\pi * 10^{-7}\left[\frac{I}{2\pi x}\right]$$

$$= 2*10^{-7}\,\frac{I}{x}$$

$$d\phi = B_x \cdot dx \cdot 1 = 2*10^{-7}\,\frac{I}{x}\,dx$$



» Total Flux Linkages between any two points



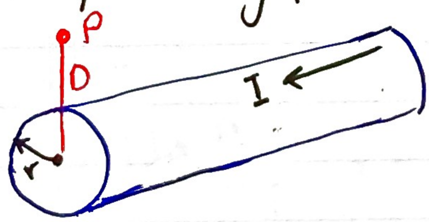
$$\lambda_{12} = \int_{D_1}^{D_2} d\lambda = 2*10^{-7}\, I \int_{D_1}^{D_2} \frac{1}{x}\,dx .$$

$$\lambda_{12} = \lambda_{ext} = 2*10^{-7}\, I \ln\frac{D_2}{D_1}$$

» The inductance between two points external to a Conductor is

$$\boxed{L_{ext} = 2*10^{-7}\ln\frac{D_2}{D_1} \quad H/m}$$

Total Flux Linkage up to any point P for this Conductor Carrying current I.



$$\lambda_p = \underbrace{\frac{1}{2}*10^{-7}\, I}_{\text{internal F.L.}} + \underbrace{2*10^{-7}\, I \ln\frac{D}{r}}_{\text{external F.L.}}$$

note:-
$\ln(a*b) = \ln(a) + \ln(b)$

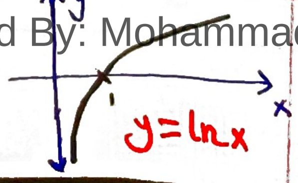
using $\frac{1}{2} = 2\ln e^{\frac{1}{4}}$

$$\lambda_p = 2*10^{-7}\, I\left(\ln e^{\frac{1}{4}} + \ln\frac{D}{r}\right) = 2*10^{-7}\, I \ln\frac{D}{e^{-\frac{1}{4}}\, r}$$

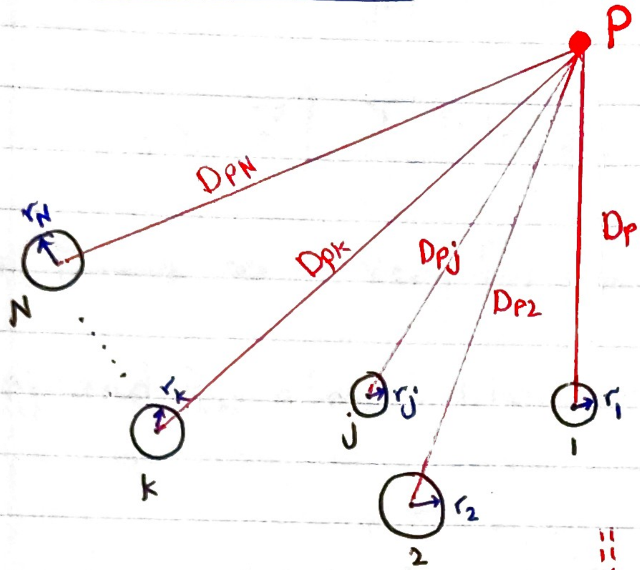
$$= 2*10^{-7}\, I \ln\frac{D}{r'}$$

where $r' = e^{-\frac{1}{4}}\, r = 0.7788r \triangleq$ effective radius due to internal flux

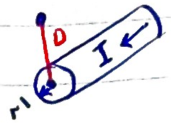
$$\boxed{L_p = \frac{\lambda_p}{I} = 2*10^{-7}\ln\left(\frac{D}{r'}\right) \quad H/m}$$



Composite Conductor :-



note :- $\lambda_P = 2*10^{-7} I \ln \frac{D}{r'}$



$$I_1 + I_2 + I_3 + \dots + I_N = 0$$

$$\sum_{j=1}^{N} I_j = 0$$

$$\lambda_{kPk} = 2*10^{-7} I_k \ln \frac{D_{Pk}}{r'_k}$$

$$\lambda_{kP1} = 2*10^{-7} I_1 \ln \frac{D_{P1}}{D_{k1}}$$

$$\lambda_{kPk} = 2*10^{-7} I_k \ln \frac{D_{Pk}}{D_{kk}} \text{ ; where } D_{kk} = r'_k$$

↳ Flux Linkages for the conductor k up to a point p due to the current flowing in conductor k.

λ_{kP} → Flux linkages for the conductor k up to a point p due to the currents flowing in conductors 1, 2, ... N.

$$\lambda_{kP} = \lambda_{kP1} + \lambda_{kP2} + \dots + \lambda_{kPN}$$

$$= 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{D_{Pj}}{D_{kj}} \text{ , where } D_{kk} = r'_k$$

$$= 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + 2*10^{-7} \sum_{j=1}^{N} I_j \ln D_{Pj}$$

j=1 → j=N-1 j=N

$$= 2*10^{-7} \left[\sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + \underbrace{\sum_{j=1}^{N-1} I_j \ln D_{Pj}}_{j=1 \to j=N-1} + \underbrace{I_N \ln D_{PN}}_{j=N} \right]$$

where

$$I_N = -(I_1 + I_2 + \dots + I_{N-1}) = -\sum_{j=1}^{N-1} I_j$$

$$\lambda_{kp} = 2*10^{-7}\left[\sum_{j=1}^{N} I_j \ln\frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln D_{pj} - \Big(\overbrace{\sum_{j=1}^{N-1} I_j}^{I_N}\Big)\ln D_{pN}\right]$$

$$= 2*10^{-7}\left[\sum_{j=1}^{N} I_j \ln\frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln\frac{D_{pj}}{D_{pN}}\right]$$

As $P \longrightarrow \infty$ very far away

D_{pj} and D_{pN} almost the same $(D_{pj} = D_{pN}) \Longrightarrow \left[\ln\frac{D_{pj}}{D_{pN}} = \ln 1 = 0\right]$

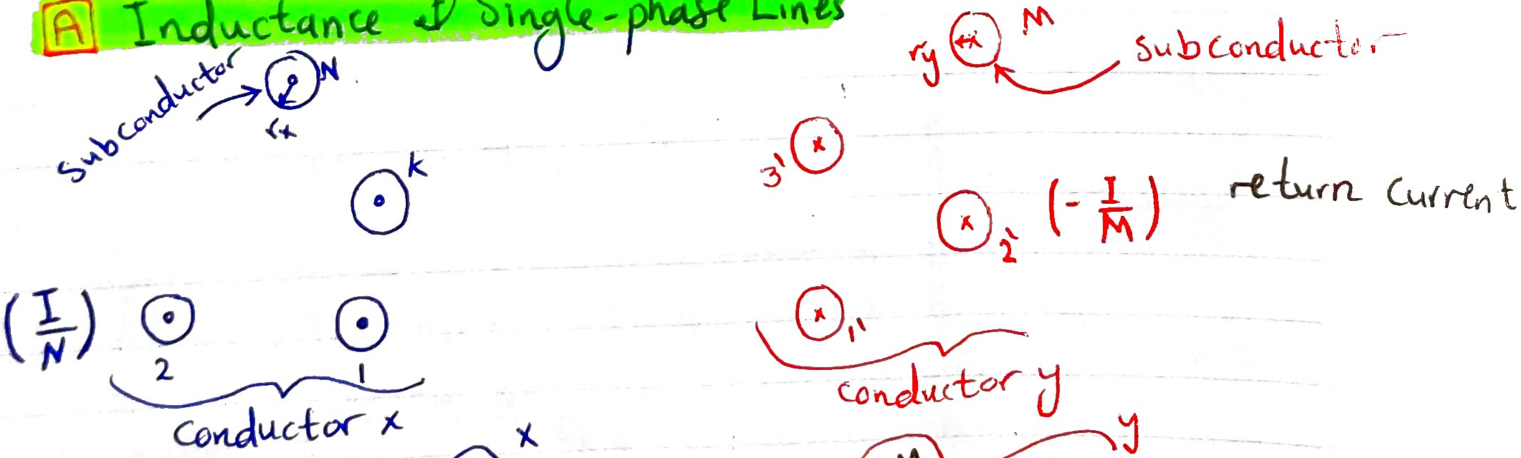
$$\boxed{\lambda_k = 2*10^{-7}\sum_{j=1}^{N} I_j \ln\frac{1}{D_{kj}}} \quad **$$

→ Total Flux Linkages for the Conductor k.

Inductance

Inductance of Single-Phase Lines [A] — Inductance of 3 ϕ T.L [B]

[A] Inductance of Single-phase Lines



$$\phi_k = 2*10^{-7}\left[\overbrace{\frac{I}{N}\sum_{m=1}^{N}\ln\frac{1}{D_{km}}}^{x} - \overbrace{\frac{I}{M}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}}^{y}\right] \quad \text{using } **$$

→ The total flux for any subconductor k in conductor x.

$$\phi_k = 2*10^{-7}\left[\frac{I}{N}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{I}{M}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

Since only the fraction $\frac{1}{N}$ of the total conductor current I is linked by this flux, the flux linkage (λ_k) of sub conductor k is

$$\lambda_k = \frac{\phi_k}{N} = 2*10^{-7}\, I\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

The total flux linkage of conductor x is :-

$$\lambda_x = \sum_{k=1}^{N}\lambda_k$$

$$= 2*10^{-7}\, I\sum_{k=1}^{N}\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

$$= 2*10^{-7}\, I\ln\prod_{k=1}^{N}\frac{\left(\prod_{m=1'}^{M}D_{km}\right)^{\frac{1}{NM}}}{\left(\prod_{m=1}^{N}D_{km}\right)^{\frac{1}{N^2}}}$$

$$\ggg L_x = 2*10^{-7}\ln\frac{D_{xy}}{D_{xx}} \quad H/m/\text{Conductor}$$

$$\ggg L_y = 2*10^{-7}\ln\frac{D_{xy}}{D_{yy}} \quad H/m/\text{conductor}$$

where :

Geometric Mean Distance between x and y

$$D_{xy} = GMD_{x,y} = \sqrt[MN]{\prod_{k=1}^{N}\prod_{m=1'}^{M}D_{km}}$$

$$= \sqrt[MN]{(D_{11'}D_{12'}D_{13'}\cdots D_{1M})\cdots\cdots(D_{N1'}D_{N2'}\cdots\cdots D_{NM})}$$

$$D_{xx} = GMR_x = \sqrt[N^2]{\prod_{k=1}^{N}\prod_{m=1}^{N}D_{km}}$$

$$= \sqrt[N^2]{(D_{11}D_{12}D_{13}\cdots D_{1n})\cdots\cdots(D_{N1}D_{N2}\cdots D_{NN})}$$

Geometric Mean Radius of Conductor x

$$D_{yy} = GMR_y = \sqrt[M^2]{\prod_{k=1'}^{M}\prod_{m=1'}^{M}D_{km}}$$

$$= \sqrt[M^2]{(D_{1'1'}D_{1'2'}\cdots D_{1'M})\cdots(D_{M1'}D_{M2'}\cdots D_{MM})}$$

Geometric Mean Radius of Conductor y.

Note :

- $\frac{1}{N^2}\left(\ln\frac{1}{a}+\ln\frac{1}{b}+\ln\frac{1}{c}\right) - \frac{1}{NM}\left(\ln\frac{1}{x}+\ln\frac{1}{y}+\ln\frac{1}{z}\right)$

$= \frac{1}{N^2}\left[\ln\frac{1}{abc}\right] - \frac{1}{NM}\left(\ln\frac{1}{xyz}\right)$

$= \ln\frac{1}{(abc)^{\frac{1}{N^2}}} - \ln\frac{1}{(xyz)^{\frac{1}{NM}}}$

$= \ln\frac{\frac{1}{(abc)^{\frac{1}{N^2}}}}{\frac{1}{(xyz)^{\frac{1}{NM}}}}$

$= \ln\frac{(xyz)^{\frac{1}{NM}}}{(abc)^{\frac{1}{N^2}}}$

- $\ln A^{\alpha} = \alpha\ln A$

- $\sum\ln A_k = \ln\prod A_k$

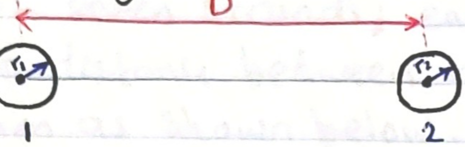
note that :-

$D_{11} = D_{22} = D_{33} = \cdots = D_{NN} = r'_x$

$D_{1'1'} = D_{2'2'} = D \cdots = \cdots = D_{MM} = r'_y$

$$\ggg L = L_x + L_y \quad H/m/\text{circuit}$$

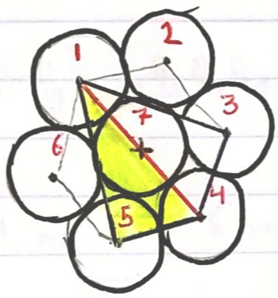
» if we have Single-phase two-wire Line



$$L_1 = 2 * 10^{-7} \ln \frac{D}{r_1'} \ \text{H/m} \qquad L_2 = 2 * 10^{-7} \ln \frac{D}{r_2'} \ \text{H/m}$$

$$r_1' = 0.7788\, r_1 \qquad r_2' = 0.7788\, r_2'$$

Example A stranded conductor consists of seven identical Strands each strand having a radius r as shown in Figure below, determine the GMR of the conductor interms of r.



$D_{12} = D_{16} = D_{17} = 2r$

$D_{14} = 4r$

$D_{13} = D_{15} = \sqrt{D_{14}^2 - D_{45}^2}$

$= \sqrt{16r^2 - 4r^2}$

$= \sqrt{12 r^2}$

$= 2\sqrt{3}\, r$

$$GMR = \sqrt[N^2]{(D_{11} D_{12} D_{13} D_{14} D_{15} D_{16} D_{17})(D_{21} D_{22} D_{23} \cdots D_{27}) \cdots\cdots (D_{71} \cdots)}$$

$$= \sqrt[49]{\underbrace{(r' \cdot 2r \cdot 2\sqrt{3}\, r \cdot 4r \cdot 2\sqrt{3}\, r \cdot 2r \cdot 2r)^6}_{1,2,3,4,5,6} \ \underbrace{(r')(2r)^6}_{7}}$$

$= 2.1767\, r$

» With Large number of strands the calculation of GMR can become very tedious. (مضجر، مُمل)

» Usually these are available in the manufacturer's data. (Tables)

» The design of a power Line requires the value of resistance and reactance to find out the active and reactive power, and the voltage drop in the process of power transfer over the transmission Line.

» Power losses should be limited to around (5-10)% of the total power transfered.

TABLE A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America)—ACSR

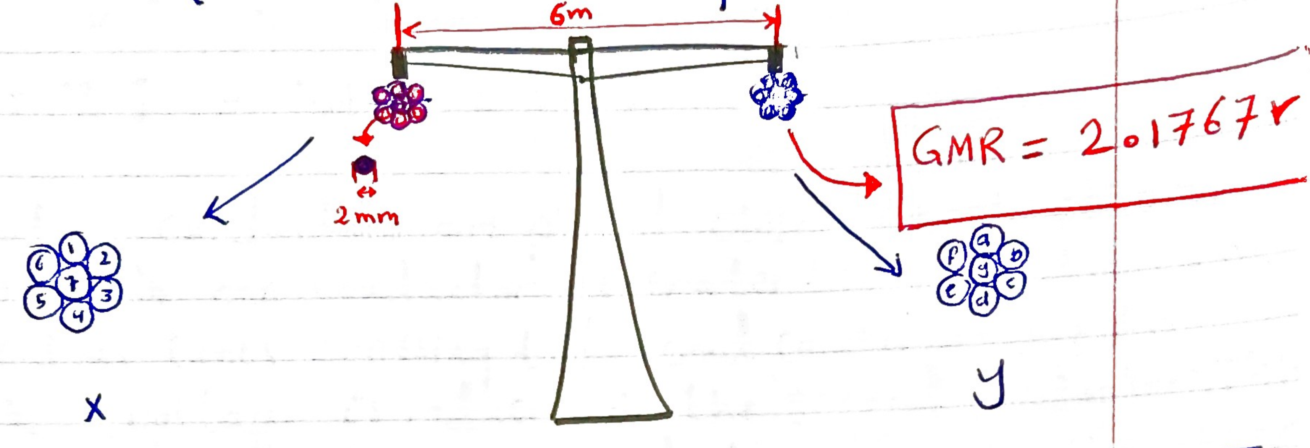
Code Word	Circular Mils Aluminum	Aluminum		Aluminum Strand Diameter (inches)	Steel	Steel Strand Diameter (inches)	Outside Diameter (inches)	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength (pounds)	Weight (pounds per mile)	Geometric Mean Radius at 60 Hz (feet)	Approx. Current Carrying Capacity† (amps)	r_a Resistance (Ohms per Conductor per Mile): 25°C (77°F) Small Currents, dc	25°C, 25 Hz	25°C, 50 Hz	25°C, 60 Hz	50°C (122°F) Current Approx. 75% Capacity‡, dc	50°C, 25 Hz	50°C, 50 Hz	50°C, 60 Hz	x_a Inductive Reactance (ohms per conductor per mile at 1 ft spacing all currents), 60 Hz	x'_a Shunt Capacitive Reactance (megohms per conductor per mile at 1 ft spacing), 60 Hz
Joree	2 515 000	76		0.1819	19	0.0849	1.880		61 700		0.0621									0.0450	0.337	0.0755
Thrasher	2 312 000	76		0.1744	19	0.0814	1.802		57 300		0.0595									0.0482	0.342	0.0767
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735		49 800		0.0570									0.0511	0.348	0.0778
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762		60 300		0.0588									0.0505	0.344	0.0774
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602		51 000		0.0534									0.0598	0.355	0.0802
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.359	0.0814
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.362	0.0821
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.365	0.0830
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.369	0.0838
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.372	0.0847
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.376	0.0857
Finch	1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.380	0.0867
Curlew	1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.385	0.0878
Cardinal	954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.390	0.0890
Canary	900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.393	0.0898
Crane	874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.395	0.0903
Condor	795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.401	0.0917
Drake	795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.399	0.0912
Mallard	795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.393	0.0904
Crow	715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.407	0.0932
Starling	715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.405	0.0928
Redwing	715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.399	0.0920
Flamingo	666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.412	0.0943
Rook	636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.414	0.0950
Grosbeak	636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.412	0.0946
Egret	636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.406	0.0937
Peacock	605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.417	0.0957
Squab	605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.415	0.0953
Dove	556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.420	0.0965
Eagle	556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.415	0.0957
Hawk	477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.430	0.0988
Hen	477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.424	0.0980
Ibis	397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.441	0.1015
Lark	397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235		Same as dc		0.259		Same as dc		0.435	0.1006
Linnet	336 400	26	2	0.1138	7	0.0855	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.451	0.1039
Oriole	336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.445	0.1032
Ostrich	300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.458	0.1057
Piper	300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.462	0.1049
Partridge	266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.465	0.1074

*Based on copper 97%, aluminum 61% conductivity.

†For conductor at 75°C, air at 25°C, wind 1.4 miles per hour (2 ft/sec), frequency = 60 Hz.

‡"Current Approx. 75% Capacity" is 75% of the "Approx. Current Carrying Capacity in Amps" and is approximately the current which will produce 50°C conductor temp. (25°C rise) with 25°C air temp., wind 1.4 miles per hour.

Example Power is transmitted over the live stranded conductor with seven strands; each strand 2 mm in diameter. The distance between the live and neutral wires is 6 mm as shown below. Calculate the inductance and reactance of the line in mH per km.



$$GMD_{xy} = \sqrt[49]{(D_{1a} D_{1b} D_{1c} D_{1d} D_{1e} D_{1f} D_{1g})(D_{2a} D_{2b} D_{2c} D_{2d} D_{2e} D_{2f} D_{2g}) \cdots (D_{7a} D_{7b} D_{7c} D_{7d} D_{7e} D_{7f} D_{7g})}$$

$$= 5.99999971 \text{ m} \cong 6 \text{ m}$$

$$GMR_x = GMR_y = 2.1767 r = (2.1767)(0.001)$$
$$= 0.0021767$$

$$L_x = 2 * 10^{-7} \ln \frac{D_{xy}}{D_{xx}} = 2 * 10^{-7} \ln \frac{6}{0.002177} \text{ H/m}$$

$$= 1.584 * 10^{-6} \text{ H/m per conductor}$$

$$L = L_x + L_y = 3.168 * 10^{-6} \text{ H/m}$$

$X_L = wL = 2\pi f L \triangleq$ Reactance per meter length of conductor

$$= 2\pi (50)(L)$$

$$= 9.954 * 10^{-4} \ \Omega/m$$

$$= 0.9954 \ \Omega/km$$

Notes

» The flux Linkage $\lambda = L \cdot I$

» The voltage drop due to this Flux Linkage is

$$V = Z I = j\omega L I = j\omega\lambda$$

» When two conductors are placed close to each other, current in one conductor generates the magnetic flux These flux Lines crossing the second conductor due to which a voltage is induced in the second conductor. This process of current in one conductor affecting the other conductor is the mutual inductance.

» If we define the two conductors as 1 and 2, then

$$M_{12} = \frac{\lambda_{12}}{I_2}$$

where
- M_{12} is the mutual inductance between conductors 1 and 2.
- λ_{12} is the flux Linkage between Conductors 1 and 2.
- I_2 is the current in conductor 2.

This in turn introduces the voltage drop in the first conductor which is defined by:

$$\boxed{V_1 = j\omega M_{12} I}$$

Inductance

Inductance of Single ϕ [A] | Inductance of 3ϕ T.L. [B]

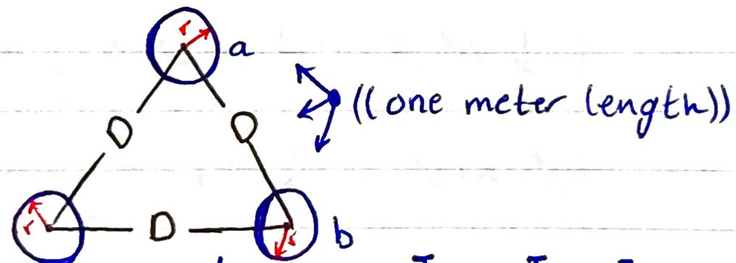
[B] Inductance of 3ϕ T.L.

a) Symmetrical Spacing (Equilateral Spacing).
b) Asymmetrical Spacing.
c) Transposition.
d) Bundled Conductor.

Composite Conductor :-

$$\lambda_k = 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}}$$

a) Three phase line with equilateral spacing.



Assuming Balanced 3ϕ currents :- $I_a + I_b + I_c = 0$

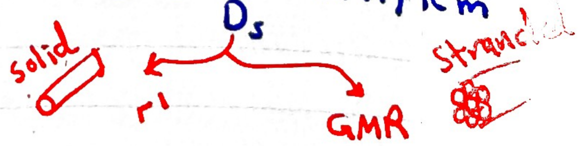
$\Rightarrow$ The total flux linkage of phase a conductor is :-

$$\lambda_a = 2*10^{-7}\left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D}\right)$$

$$= 2*10^{-7}\left(I_a \ln \frac{1}{r'} + (I_b + I_c) \ln \frac{1}{D}\right)$$

$$= 2*10^{-7}\left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D}\right) = 2*10^{-7} I_a \ln \frac{D}{r'}$$

$$L_a = \frac{\lambda_a}{I_a} = 2*10^{-7} \ln \frac{D}{r'} \ H/m = 0.2 \ln \frac{D}{D_s} \ mH/km$$



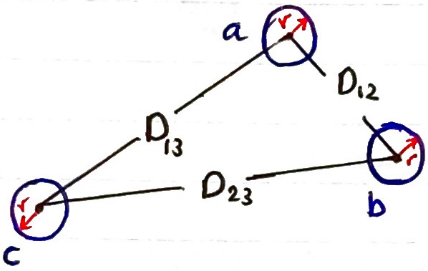
$$\lambda_a = \lambda_b = \lambda_c \Rightarrow L_a = L_b = L_c$$

** This means that the inductance per phase for 3ϕ circuit with equilateral spacing is the same as for one conductor of single phase circuit.

b)) Asymmetrical Spacing:-

» Practical transmission lines cannot maintain symmetrical spacing of conductors because of construction considerations.

» With asymmetrical spacing, even with balanced currents, the voltage drop due to line inductance will be unbalanced.



$$\lambda_a = 2*10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{13}}\right)$$

$$\lambda_b = 2*10^{-7}\left(I_a \ln\frac{1}{D_{12}} + I_b \ln\frac{1}{r'} + I_c \ln\frac{1}{D_{23}}\right)$$

$$\lambda_c = 2*10^{-7}\left(I_a \ln\frac{1}{D_{13}} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{r'}\right)$$

Or in matrix form $\boldsymbol{\lambda} = \mathbf{L}\,\mathbf{I}$

where the symmetrical inductance matrix $\mathbf{L}$ is given by:

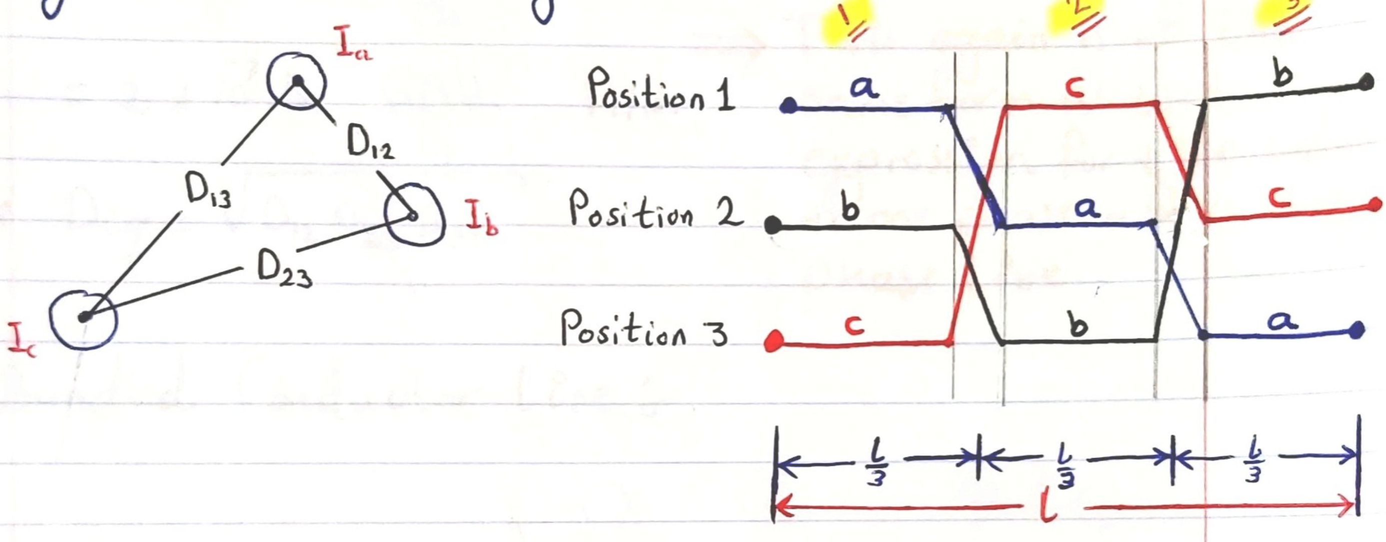
$$\mathbf{L} = 2*10^{-7}\begin{bmatrix} \ln\frac{1}{r'} & \ln\frac{1}{D_{12}} & \ln\frac{1}{D_{13}} \\ \ln\frac{1}{D_{12}} & \ln\frac{1}{r'} & \ln\frac{1}{D_{23}} \\ \ln\frac{1}{D_{13}} & \ln\frac{1}{D_{23}} & \ln\frac{1}{r'} \end{bmatrix}$$

⇒ The phase inductances are not equal

c)) Three phase transposed Line:

» One way to regain symmetry and obtain per-phase model is Consider transposition.

» The transposition consists of interchanging the phase configuration every one-third the length.



$$\lambda_{a_1} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right]$$

r' GMR

$$\lambda_{a_2} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right]$$

$$\lambda_{a_3} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{31}} + I_c \ln\frac{1}{D_{23}}\right]$$

$$\lambda_a = \frac{\lambda_{a1}\left(\frac{l}{3}\right) + \lambda_{a2}\left(\frac{l}{3}\right) + \lambda_{a3}\left(\frac{l}{3}\right)}{l} = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

$$= \frac{2*10^{-7}}{3}\left[3 I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}D_{23}D_{31}} + I_c \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

$$= \frac{2*10^{-7}}{3}\left[3 I_a \ln\frac{1}{D_s} - I_a \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

$$\lambda_a = 2*10^{-7} I_a \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s}$$

$$L_a = \frac{\lambda_a}{I_a} = 2*10^{-7} \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s} \quad \text{H/m per phase}$$

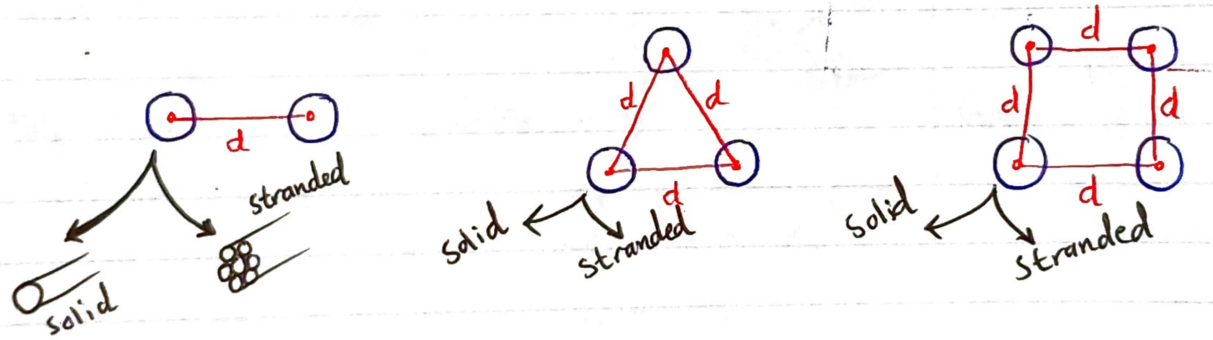
$$L_a = 2*10^{-7} \ln \frac{D_{eq}}{D_s} \quad \text{H/m}$$

$$= 2*10^{-7} \ln \frac{GMD}{D_s} \quad \text{H/m}$$

where $D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$

⇒ This again is of the same form as the expression for the induc of one phase of a single phase line.

d)) Bundled Conductor line :



>> Extra-high voltage transmission lines are usually constructed with bundled conductors. Bundling reduces the line reactance, which improves the line performance and increase the power capability of the line. Bundling also reduces the voltage surface gradient, which in turn reduces corona loss, radio interference, and surge impedance. ($\sqrt{\frac{L}{C}}$)

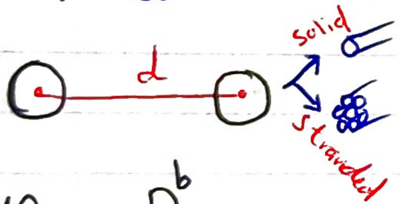
>> Typically, bundled conductors consists of two, three, or four subconducters symmetrically arranged in configuration as shown in Figure above.

» The subconductors within a bundle are separated at frequent intervals by spacer-dampers. Spacer-dampers prevent clashing, provide damping, and connect the subconductors in parallel.

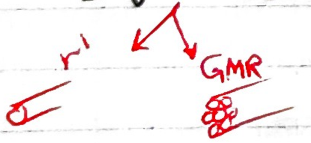
Bundling

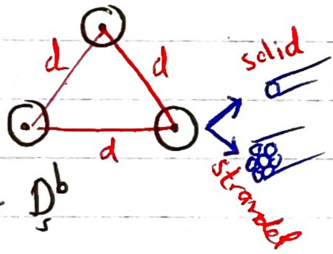
Reduces Electric Field Strength on Conductor Surface → **Reduces Corona**

Increases Effective Radius (GMR) → **Reduces Inductance**

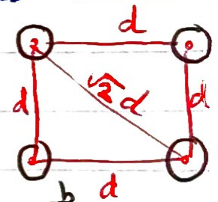


$$GMR_b = D_s^b = \sqrt[4]{(r' \cdot d)^2} = \sqrt[2]{r' \cdot d}$$





$$GMR_b = D_s^b = \sqrt[9]{(r' \cdot d \cdot d)^3} = \sqrt[3]{r'\ d^2}$$



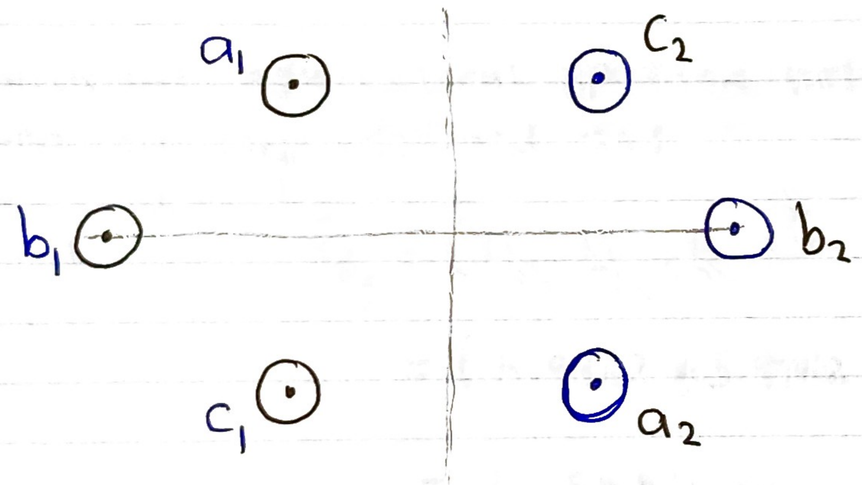
$$GMR_b = D_s^b = \sqrt[16]{(r' \cdot d \cdot d \cdot d\sqrt{2})^4} = 1.091\sqrt[4]{r'\ d^3}$$

$$\boxed{L_a = 2 * 10^{-7} \ln \frac{D_{eq}}{D_s^b} \quad H/m}$$

>> Three-phase Lines - Parallel Circuits.

>> Three-phase Double-Circuit Lines.

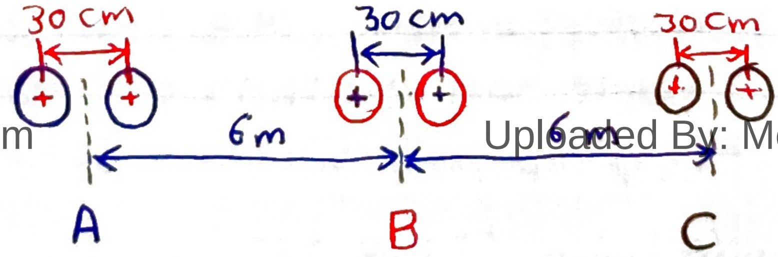
A three-phase double-circuit line consists of two identical 3ϕ circuits. The circuits are operated with abc, cba in parallel. Because of geometrical differences between conductors, voltage drop due to line inductance will be unbalanced. To achieve balance, each phase conductor must be transposed within its group and with respect to parallel 3ϕ line.



example The conductor configuration of a completely transposed 3-ϕ overhead transmission line with bundled conductor is shown below. All the conductors have a radius of 0.74 cm with a 30 cm bundle spacing.

a)) Determine the inductance per-phase in mH/km and in mH/m.

b)) Find the inductive line reactance per phase in Ω/m at $f = 50$ Hz.



$$D_{ab} = \sqrt[4]{d_{13}\, d_{14}\, d_{23}\, d_{24}}$$

$$= (6 * 6.3 * 5.7 * 6)^{1/4} = 5.9962 \text{ m}$$

Similarly,

$$D_{bc} = 5.9962 \text{ m}$$

$$D_{ca} = \sqrt[4]{d_{15}\, d_{16}\, d_{25}\, d_{26}}$$

$$= (12 * 12.3 * 11.7 * 12)^{\frac{1}{4}} = 11.9981 \text{ m}$$

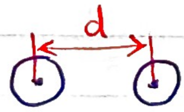
The equivalent equilateral spacing between the phases is given by Deq defined as:-

$$D_{eq} = (D_{ab} \cdot D_{bc} \cdot D_{ca})^{\frac{1}{3}}$$

$$= (5.9962 * 5.9962 * 11.9981)^{\frac{1}{3}}$$

$$= 7.5559 \text{ m}$$

$$D_s^b = \sqrt[2]{r' d}$$



$$= (0.7788 * r * 30)^{\frac{1}{2}} = 4.1580 \text{ cm}$$

a)) Inductance per phase for the given system is:-

$$L = 2 * 10^{-7} \ln \frac{D_{eq}}{D_s^b} \quad \text{H/m/phase}$$

$$= 1.04049 * 10^{-6} \quad \text{H/m/phase}$$

$$= 1.04049 * 10^{-3} \quad \text{mH/m/phase} = 1.04049 \text{ mH/km/phase}$$

b)) The inductive line reactance per phase

$$X_L = 2\pi f L = 2\pi(50)(1.04049) * 10^{-6} \ \Omega\text{/m/phase}$$

$$= 3.270 * 10^{-4} \ \Omega\text{/m/phase}$$

Transmission Lines Parameters

T.L Resistance — T.L Inductance — T.L Capacitance

Transmission Line Capactance:

» Capacitance of transmission line is the result of the potential difference between the conductors, it causes them to be charged in the same manner as the plates of a capacitor, when there is a potential difference between them the capacitance between conductors is the charge per unit of the potential difference.

1)) Electric Field and Voltage Calculation
2)) Transmission Line Capacitance for:-
  A Single-phase Line.
  B 3ϕ Lines with equal spacing.
  C 3ϕ Lines, bundled conductor, and unequal spacing.

1)) Gauss's Law → Electric Field Strength (E)
→ Voltage between Conductors
→ Capacitance $C = q/V$

Gauss's Law:- Total electric flux leaving a closed surface = Total charge within the vollume enclosed by the closed surface.

⇓ Leads to

Normal Electric Flux density integrated over the closed surface = charge enclosed by this closed surface.

Surface integral over closed surface $\leftarrow \oiint D_\perp \, ds = \oiint \varepsilon E_\perp \, ds = Q_{enclosed}$

Where,

$\varepsilon \triangleq$ permittivity of the medium $= \varepsilon_r \varepsilon_0$

$\varepsilon_0 = 8.854 * 10^{-12}$ F/m

$D_\perp \triangleq$ normal component of electric flux density.

$E_\perp \triangleq$ normal component of electric field strength.

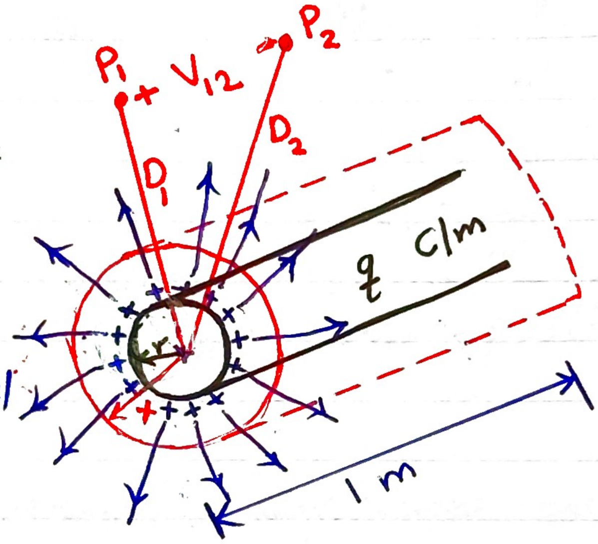
ds = the differential surface area.

Note :-

Inside the perfect conductor, Ohm's law give $E_{int} = 0$

$\Downarrow$

That is, the internal electric field $E_{int} = 0$



$$\oiint \varepsilon E_\perp \, ds = Q_{enclosed}$$

$$\varepsilon E_x (2\pi x)(1) = q \, (1) \quad \leftarrow \text{1 m length}$$

$$E_x = \frac{q}{2\pi\varepsilon x} \quad \text{V/m}$$

$$V_{12} = \int_{D_1}^{D_2} E_x \, dx = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon x} dx$$

$$V_{12} = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \quad \text{Volts}$$

$D_2 \rightarrow P_2$ −, $D_1 \rightarrow P_1$ +

where,

$\varepsilon = \varepsilon_r \varepsilon_0$

$\varepsilon_0 = 8.854 * 10^{-12}$ F/m

note $\Downarrow$

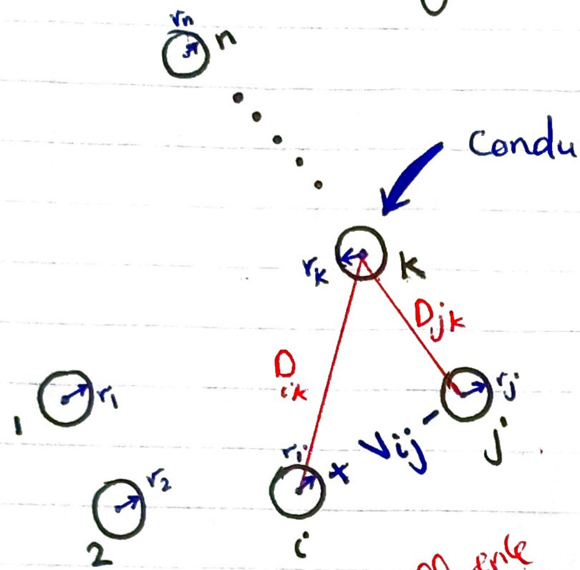
− P_2

V_{12}

+ P_1

$$\boxed{V_{12} = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \text{ Volts}}$$

Multi-Conductor System :



conductor k has radius r_k and charge q_k ((per meter length of the conductor))

$$V_{ijk} = \frac{q_k}{2\pi\varepsilon} \ln \frac{D_{jk}}{D_{ik}} \quad \text{volts}$$

$$\boxed{V_{ij} = \sum_{k=1}^{n} \frac{q_k}{2\pi\varepsilon} \ln \frac{D_{jk}}{D_{ik}} \quad \text{volts}}$$

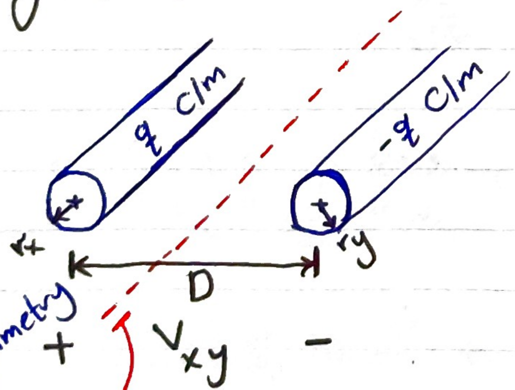
voltage difference due to charges in all conductors

Super-position Theorem

Transmission Line Capacitance

Single-Phase Line [A] — Three-Phase Lines [B]

[A] Single-Phase Line



due to symmetry

→ zero-voltage
→ zero-potential
→ potential of neutral

$$V_{xy} = \frac{1}{2\pi\varepsilon}\left[q \ln \frac{D_{yx}}{D_{xx}} - q \ln \frac{D_{yy}}{D_{xy}} \right]$$

$$= \frac{q}{2\pi\varepsilon} \ln \frac{D_{yx} D_{xy}}{D_{xx} D_{yy}}$$

$$= \frac{q}{\pi\varepsilon} \ln \frac{D}{\sqrt{r_x r_y}} \quad \text{Volts}$$

$$C_{xy} = \frac{q}{V_{xy}} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)} \quad F/m$$

ooo Notes ooo

» $V_{12}(q_1) = \frac{q_1}{2\pi\varepsilon} \ln \frac{D}{r}$

» $V_{12}(q_2) = \frac{q_2}{2\pi\varepsilon} \ln \frac{r}{D}$

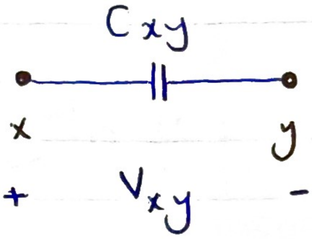
» $V_{21}(q_2) = \frac{q_2}{2\pi\varepsilon} \ln \frac{D}{r} = -V_{12}$

» $V_{12} = V_{12}(q_1) + V_{12}(q_2)$

$q_2 = -q_1$

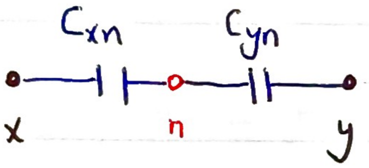
$$C_{xy} = \frac{\pi \varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)} \qquad \text{if } r_x = r_y$$

$$C_{xy} = \frac{\pi \varepsilon}{\ln\left(\frac{D}{r}\right)}$$

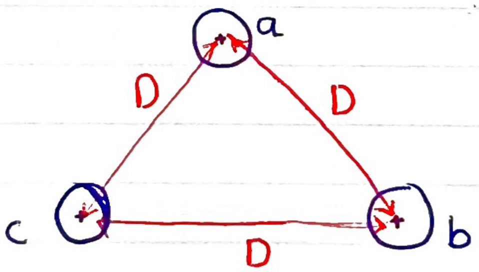


$$V_{xn} = V_{yn} = \frac{V_{xy}}{2}$$

$$C_n = C_{xn} = C_{yn} = \frac{q}{V_{xn}} = 2\,C_{xy} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \quad F/m$$



$\boxed{B}$ Three-Phase Line with Equilateral Spacing:



$$q_a + q_b + q_c = 0$$

$$\Rightarrow V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ba}}{D_{aa}} + q_b \ln\frac{D_{bb}}{D_{ab}} + q_c \ln\frac{D_{bc}}{D_{ac}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + \cancel{q_c \ln\frac{D}{D}}^{\,0}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right] \quad \text{Volts}$$

$$\Rightarrow V_{ac} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ca}}{D_{aa}} + q_b \ln\frac{D_{cb}}{D_{ab}} + q_c \ln\frac{D_{cc}}{D_{ac}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{D}{D} + q_c \ln\frac{r}{D}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_c \ln\frac{r}{D}\right]$$

$V_{ab} + V_{ac}$

$$V_{ab} + V_{ac} = \left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln\frac{D}{r} + \underbrace{(q_b + q_c)}_{-q_a} \ln\frac{r}{D}\right]$$

$$\boxed{V_{an} = \frac{1}{3}(V_{ab} + V_{ac})}$$

$$= \frac{1}{3}\left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln\frac{D}{r} + q_a \ln\frac{D}{r}\right]$$

$$= \frac{q_a}{2\pi\varepsilon} \ln\frac{D}{r}$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} \quad F/m \quad \text{Line to neutral}$$

Notes :

$$V_{ab} = \sqrt{3}\, V_{an} \angle{+30} = \sqrt{3}\, V_{an}\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right]$$

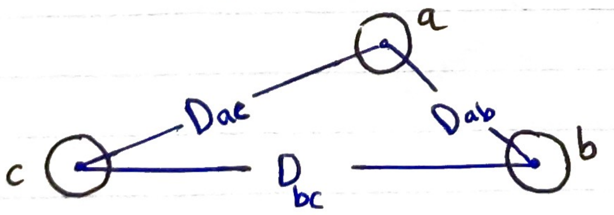
$$V_{ac} = -V_{ca} = \sqrt{3}\, V_{an} \angle{-30} = \sqrt{3}\, V_{an}\left[\frac{\sqrt{3}}{2} - j\frac{1}{2}\right]$$

+

$$V_{ab} + V_{ac} = 3 V_{an}$$

$$\boxed{V_{an} = \frac{1}{3}(V_{ab} + V_{ac})}$$

C) 3ϕ with asymmetrical Spacing

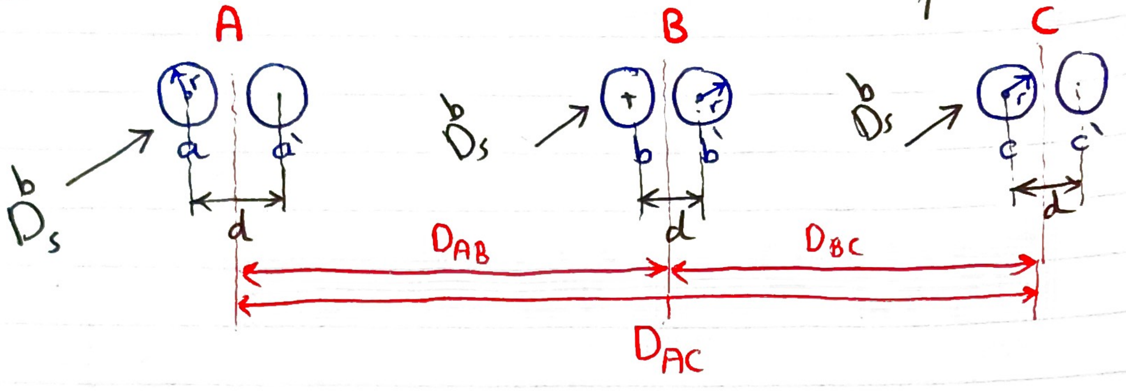


$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}\right)} , \quad D_{eq} = \sqrt[3]{D_{ab}\, D_{ac}\, D_{bc}}$$

(r) solid

$\left(\frac{\text{outside diameter}}{2}\right)$ stranded

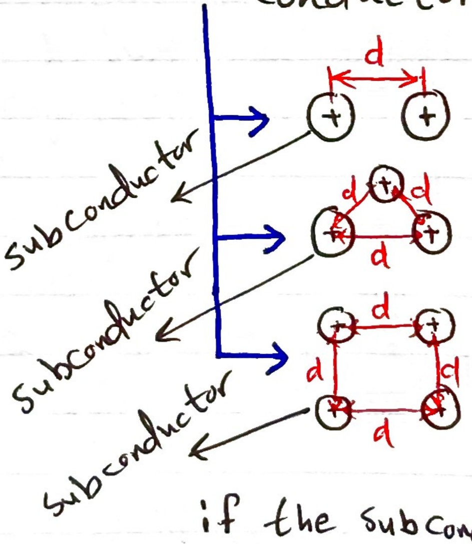
D 3ϕ Bundled Conductor with unequal spacing



$D_{AB} = GMD_{A,B}$, $D_{BC} = GMD_{B,C}$, $D_{AC} = GMD_{A,C}$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{D_s^b}\right)}$$

$D_{eq} = \sqrt[3]{D_{AB} \, D_{BC} \, D_{AC}}$, $D_s^b \triangleq$ GMR for the bundled conductor



$D_s^b = \sqrt[2]{r d}$

$D_s^b = \sqrt[3]{r \, d^2}$

$D_s^b = 1.091 \sqrt[4]{r d^3}$

if the subconductor is stranded

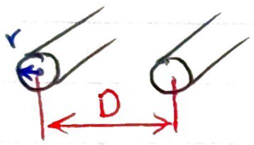
$r \longrightarrow \left[\frac{\text{outside diameter}}{2}\right]$

from manufacture's data (Tables)

$$C$$

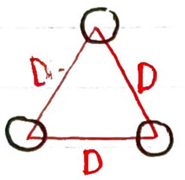
1ϕ

$$C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D}{r}}$$

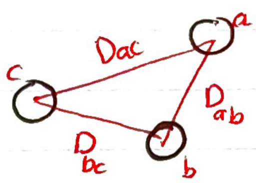


3ϕ

$$C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D}{r}}$$

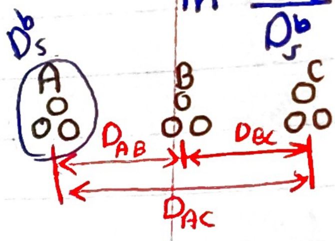


$$C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{r}}$$



3ϕ Bundel

$$C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{D_s^b}}$$



[if stranded conductor exist instead of solid
$r \longrightarrow \frac{\text{outside diameter}}{2}$]

example

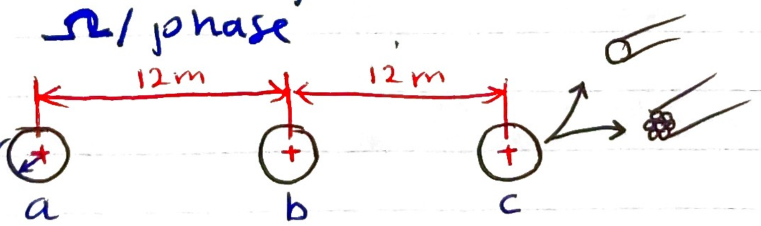
A three-phase, 400 kV, 50 Hz, 350 km overhead T.L. has flat horizontal spacing with three identical conductors. The conductors have an outside diameter of 3.28 cm with 12 m between adjacent conductors.

>> Determine the capacitive reactance-to-neutral in Ω/m/phase

>> Determine the capacitive reactance for the Line in Ω/phase

Solution

outside diameter = 3.28 cm



$$D_{eq} = \sqrt[3]{D_{ab} D_{ac} D_{bc}} = \sqrt[3]{(12)(24)(12)} = 15.119 \text{ m}$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}\right)} = 8.163 * 10^{-6}\ \mu F/m$$

note: $Z_c = \frac{1}{j\omega C}$

$Y_c = \omega C$

$$Y_n = 2\pi * 50 * C_n = 2.565 * 10^{-9}\ \mho/m/phase$$

Length = 350 km

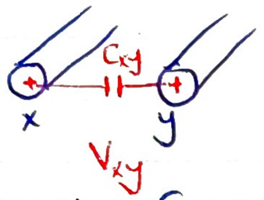
$$Y_n = 8.978 * 10^{-4}\ \mho/phase$$

$$\text{Reactance} = X_n = \frac{1}{Y_n} = 1.1138 * 10^{-3}\ \Omega/phase$$

Line charging current:-

The current supplied to the transmission line capacitance is called charging current.

- For a single-phase circuit operating at line-to-line voltage $V_{xy} = V_{xy}\angle 0^\circ$.



» The charging current is

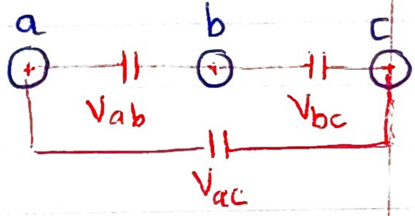
$$I_{chg} = Y_{xy} V_{xy} = j\omega C_{xy} V_{xy} \quad \text{Amp}$$

» The capacitor delivers reactive power, the reactive power delivered by this line-to-line capacitance is

$$Q_c = \frac{V_{xy}^2}{X_c} = Y_{xy} V_{xy}^2$$

$$= \omega C_{xy} V_{xy}^2 \quad \text{var}$$

- For a completely transposed 3ϕ line that has $V_{an} = V_{LN}\angle 0^\circ$



» The phase a charging current i

$$I_{chg} = Y_{an} V_{an} = j\omega C_{an} V_{LN}$$

» The reactive power delivered by phase a is

$$Q_{C1\phi} = Y_{an} V_{an}^2 = \omega C_{an} V_{LN}^2 \quad \text{Var}$$

» The total reactive power supplied by the 3ϕ line is

$$Q_{C3\phi} = 3Q_{C1\phi} = 3\omega C_{an} V_{LN}^2$$

$$= \sqrt{3}\sqrt{3}\,\omega C_{an} V_{LN} V_{LN}$$

$$Q_{C3\phi} = \omega C_{an} V_{LL}^2 \quad \text{Var}$$

Transmission Line Modeling

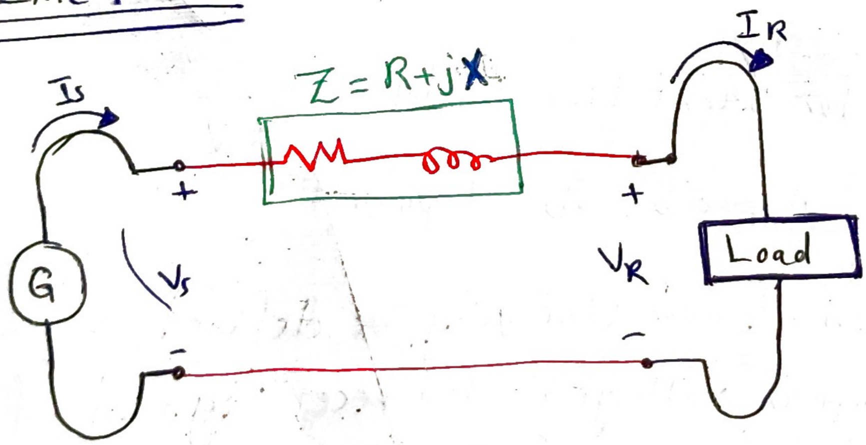
- Short Line Model (Less than 80 km)
- Medium Line Model (80km < L < 250 km)
- Long Line Model (L ≥ 250 km)

» Lumped parameter system.

» Distributed parameter system.

- we use Lumped parameters which give good accuracy for short lines and for lines of medium length.
- If an overhead line is classified as short, shunt capacitance is so small that it can be omitted entirely with little loss of accuracy, and we need to consider only the series resistance R and the series inductance L for the total length of the Line.

[1] Short Line Model :-



» line length < 80 km

» Generally MV/ LV Lin

» Capacitance cab be neglected

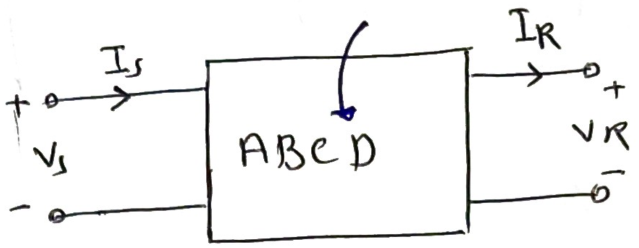
$$Z = (r + j\omega L)\,l$$
$$= R + jX$$

where r and L are the per-phase resistance and inductance per unit length, respectively, and l is the line length.

The phase voltage at the sending end is

$$V_S = V_R + Z I_R \quad \text{——— } (1)$$

$$I_S = I_R$$



Two-port representation of a T.L

$$V_S = A V_R + B I_R$$
$$I_S = C V_R + D I_R$$
$$\Rightarrow \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Since we are dealing with a linear passive, bilateral (ثنائي) two-port network, the det of the transmission matrix is unity :-

$$AD - BC = 1$$

$$\Rightarrow \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

- According to (1) for short Line model

$A = 1$ per unit, $B = Z\ \Omega$, $C = 0\ S$, $D = 1$ per unit

- Voltage regulation of the line may be defined as the percentage change in voltage at the receiving end of the line (expressed as percent of full load voltage) in going from no-load to full load.

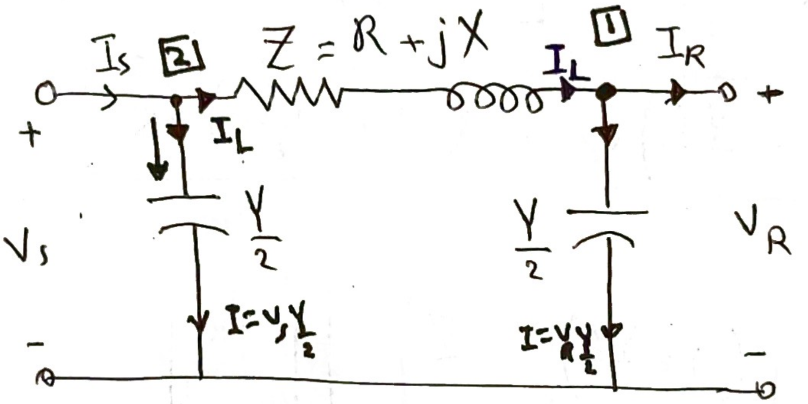
$$\boxed{\text{Percent } VR = \frac{|V_{R(NL)}| - |V_{R(FL)}|}{|V_{R(FL)}|} * 100}$$

Voltage regulation is a measure of line voltage drop.

At no Load $I_R = 0 \Rightarrow V_{R(NL)} = \frac{V_S}{A}$ ← $A=1$ for Short Line.

Medium Line Model

- 80km < length < 250km.
- As the length of line increases, the line charging current becomes appreciable and the shunt capacitance must be considered.
- For medium length lines, half of the shunt capacitance may be considered to be lumped at each end of the line. This is referred to as the nominal π model and is shown in Figure below :-



$$\boxed{I_L = I_R + V_R \frac{Y}{2}}$$

in terms of

$V_s(V_R, I_R)$

$I_s(V_R, I_R)$

$Z \equiv$ total series impedance of the Line.

$Y \equiv$ total shunt admittance of the Line.

$$Y = (g + j\omega C)\, l$$

Under normal conditions, the shunt conductance per unit length, which represents the leakage current over the insulators and due to corona, is negligible and g is assumed to be zero. C is the line to neutral capacitance per km, and l is the line length.

1. $V_s = V_R + Z I_L$

$$= V_R + Z\left(\overbrace{I_R + V_R \cdot \frac{Y}{2}}^{I_L}\right)$$

$$\boxed{V_s = \left(1 + \frac{YZ}{2}\right) V_R + Z I_R}$$

$$\boxed{\begin{aligned} V_s &= A V_R + B I_R \\ I_s &= C V_R + D I_R \end{aligned}}$$

2. $I_s = I_L + V_s \cdot \frac{Y}{2}$

$$= \left(\overbrace{I_R + V_R \cdot \frac{Y}{2}}^{I_L}\right) + \frac{V_s Y}{2}$$

$$= I_R + \boxed{\frac{V_R Y}{2}} + \left[\left(1 + \frac{YZ}{2}\right) V_R + Z I_R\right] \frac{Y}{2}$$

$$\boxed{I_s = Y\left(1 + \frac{YZ}{4}\right) V_R + \left(1 + \frac{YZ}{2}\right) I_R}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

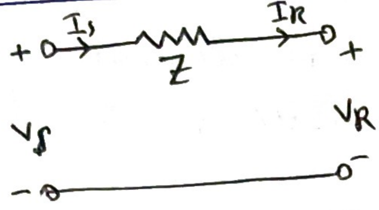
$A = D = 1 + \frac{YZ}{2}$ per unit

since the π model is a symmetrical two-port network $(A = D)$

$B = Z\ \Omega$

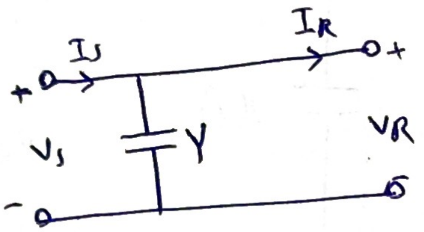
$C = Y\left(1 + \frac{YZ}{4}\right) S$

ABCD Matrix



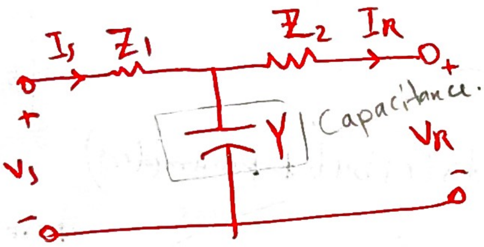
$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

Short line



$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Compensat for Reactive power

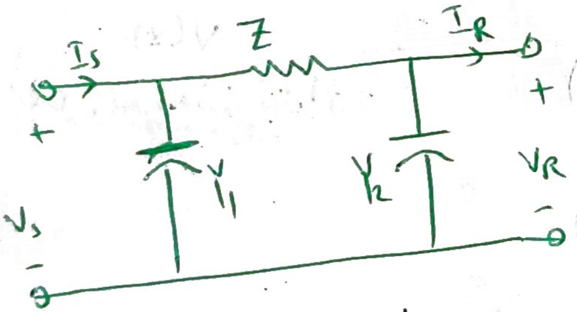


T- circuit

$$\begin{bmatrix} (1+YZ_1) & (Z_1+Z_2+YZ_1Z_2) \\ Y & (1+YZ_2) \end{bmatrix}$$

$\boxed{AD-BC=1}$

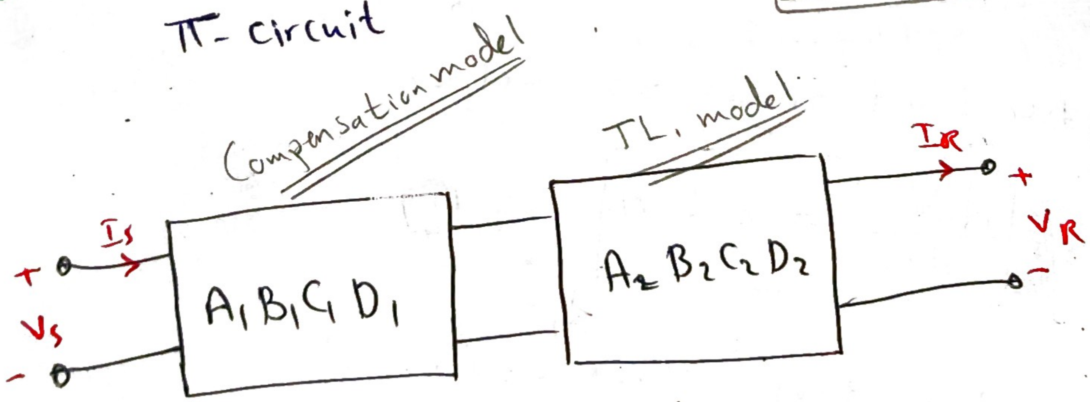
we can ...



π- circuit

$$\begin{bmatrix} (1+Y_2Z) & Z \\ (Y_1+Y_2+Y_1Y_2Z) & (1+Y_1Z) \end{bmatrix}$$

$\boxed{AD-BC=1}$



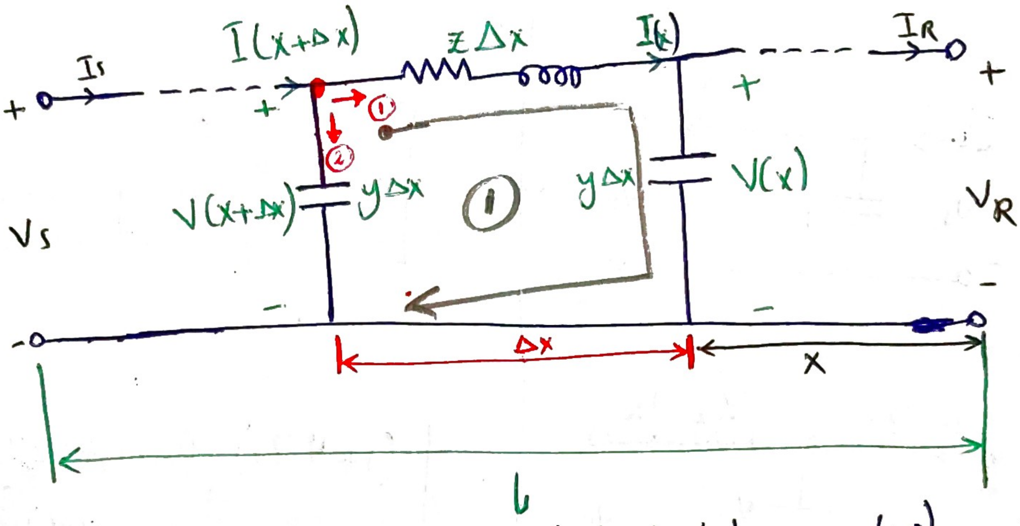
Cascaded networks

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} (A_1A_2+B_1C_2) & (A_1B_2+B_1D_2) \\ (C_1A_2+D_1C_2) & (C_1B_2+D_1D_2) \end{bmatrix}$$

3 Long Line Model :-

* For the short and medium length lines ~~reasonably~~ app. accurate models were obtained by assuming the line parameters to be lumped.
For lines 250 km and longer and for a more accurate solution the exact effect of the distributed parameters must be considered.

$V\big|_{x=l} = V_s$ $\qquad\qquad$ $V_R = \left(V\big|_{x=0}\right)$



(Long-line with distributed parameters)

- $z = r + j\omega L$
- $y = g + j\omega C$

target → $V(x)$, $I(x)$

بدلالة par المعروضة بالرأس

① KVL

$\boxed{1}$ $V(x+\Delta x) = z\,\Delta x\, I(x) + V(x)$

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = z\, I(x)$$

Taking the limit as $\Delta x \to 0$, we have

$$\boxed{\frac{dV(x)}{dx} = z\, I(x)} \quad \text{—— ①}$$

KCL

$\boxed{2}$ $I(x+\Delta x) = I(x) + y\,\Delta x\, V(x+\Delta x)$

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y\, V(x+\Delta x)$$

$\Delta x \to 0$

$$\frac{dI(x)}{dx} = y\, V(x)$$

and from ① ⟹ نرجع للمعادلة رقم ①

$$\frac{dV(x)}{dx} = z\,I(x) \quad \text{— (1)}$$ from (1) return to 1

$$\boxed{\frac{d^2V(x)}{dx^2} = z\,\frac{dI(x)}{dx}}$$ substituting

$$\Rightarrow \frac{dI(x)}{dx} = y\,V(x) \quad \text{(2)}$$ from (2) return to 1

$$\frac{d^2V(x)}{dx^2} = z\,\underbrace{\boxed{\frac{dI(x)}{dx}}}_{yV_x}$$

$$\frac{d^2V(x)}{dx^2} = \boxed{zy}\,V(x)$$

$$zy = \gamma^2 ,$$

$$\boxed{\frac{d^2V(x)}{dx^2} - \gamma^2 V(x) = 0}$$

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

where $\gamma \equiv$ propagation constant $= \sqrt{zy} = \alpha + j\beta$

attenuation constant (α), phase constant (β).

$$= \sqrt{\underbrace{(r + j\omega L)}_{z}\underbrace{(g + j\omega C)}_{y}}$$

$$\therefore V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$\boxed{V_x \quad I_x}$ → $V(x)$ ✓, $I(x)$?

we want to find I_x

$$I(x) = \frac{1}{z}\frac{dV(x)}{dx} \equiv \text{from — (1)}$$

$$= \frac{\gamma}{z}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$$= \sqrt{\frac{y}{z}}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$$= \frac{1}{Z_c}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$Z_c \equiv$ characteristic impedance

$$Z_c = \sqrt{z/y}$$

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$$I(x) = \frac{1}{Z_c}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$A_1 = ?!$, $A_2 = ?!$

Two boundary conditions:-

at $x=0$

$x=0$

① $V(x) = V_R$

② $I(x) = I_R$

$$\Longrightarrow \quad V_R = A_1 + A_2$$

$$I_R = \frac{A_1 - A_2}{Z_c}$$

$$\Longrightarrow \quad A_1 = \frac{V_R + Z_c I_R}{2}$$

$$A_2 = \frac{V_R - Z_c I_R}{2}$$

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I(x) = \frac{\frac{V_R}{Z_c} + I_R}{2} e^{\gamma x} - \frac{\frac{V_R}{Z_c} - I_R}{2} e^{-\gamma x}$$

$V(x) = \square V_R + \square I_R$

$I(x) = \square V_R + \square I_R$

Rearranged

$V(x) = \square V_R + \square I_R$

$I(x) = \square V_R + \square I_R$

$\cosh \gamma x$ $\quad$ $\sinh \gamma x$

$$V(x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} V_R + Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2} I_R$$

$$I(x) = \frac{1}{Z_c}\frac{e^{\gamma x} - e^{-\gamma x}}{2} V_R + \frac{e^{\gamma x} + e^{-\gamma x}}{2} I_R$$

$\sinh \gamma x$ $\quad$ $\cosh \gamma x$

$$V(x) = \cosh \gamma x \, V_R + Z_c \sinh \gamma x \, I_R$$

$$\Longrightarrow \quad I(x) = \frac{1}{Z_c} \sinh \gamma x \, V_R + \cosh \gamma x \, I_R$$

We are particularly interested in the relation between the sending end and the receiving end of the line.

Setting $x = l$

$V(l) = V_s$

$I(l) = I_s$

$$\Rightarrow \boxed{\begin{aligned} V_s &= \cosh \gamma l \, V_R + Z_c \sinh \gamma l \, I_R \\ I_s &= \frac{1}{Z_c} \sinh \gamma l \, V_R + \cosh \gamma l \, I_R \end{aligned}} \quad \text{—— } (1)$$

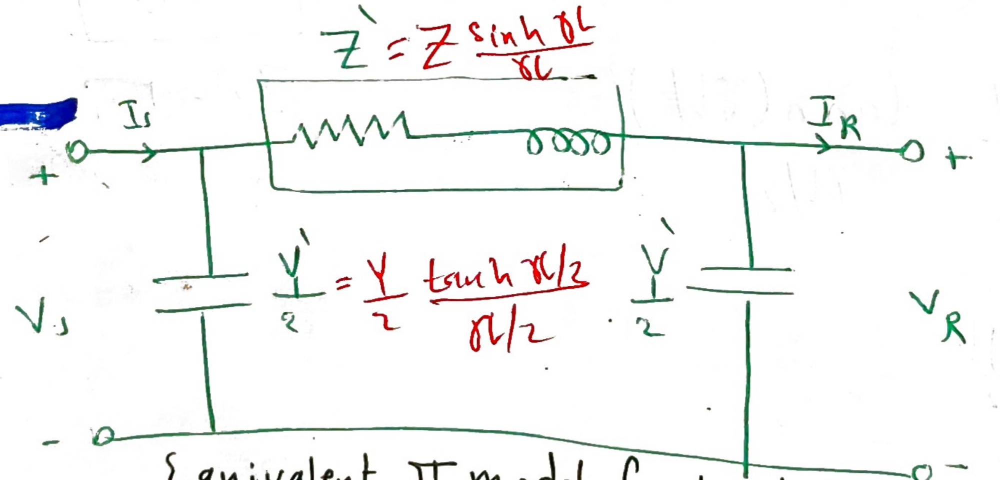
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_c} \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

(ABCD matrix)

before

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} (1 + \frac{YZ}{2}) & Z \\ Y(1 + \frac{YZ}{4}) & (1 + \frac{YZ}{2}) \end{bmatrix}$$

note that, as before, $A = D$ and $AD - BC = 1$.



Equivalent π model for long length Line.

$$\boxed{\begin{aligned} V_s &= \left(1 + \frac{Z'Y'}{2}\right) V_R + Z' I_R \\ I_s &= Y'\left(1 + \frac{Z'Y'}{4}\right) V_R + \left(1 + \frac{Z'Y'}{2}\right) I_R \end{aligned}} \quad \text{—— } (2)$$

$Z_c \sinh \gamma l$

$\cosh \gamma l$

Comparing (1) with (2)

(1) $$Z' = Z_c \sinh \gamma l$$

$$= \sqrt{\frac{z}{y}} \sinh \gamma l$$

$$= z l \frac{\sinh \gamma l}{\sqrt{zy}\, l} = Z \frac{\sinh \gamma l}{\gamma l}$$

(2) $$\cosh \gamma l = 1 + \frac{Z' Y'}{2}$$

$$\cosh \gamma l = 1 + \frac{(Z_c \sinh \gamma l \, Y')}{2} = 1 + Z_c \sinh \gamma l \cdot \frac{Y'}{2} = \cosh \gamma l$$

$$\frac{Y'}{2} = \frac{1}{Z_c} \cdot \boxed{\frac{\cosh \gamma l - 1}{\sinh \gamma l}} \leftarrow \tanh \frac{\gamma l}{2}$$

$$= \frac{1}{Z_c} \tanh \frac{\gamma l}{2}$$

$$\boxed{Y = yl}$$

$$Z_c = \sqrt{z/y}$$

$$= \frac{Y}{2} \frac{\tanh(\gamma l/2)}{\gamma l/2}$$

$$= \frac{yl}{2} \frac{\tanh(\gamma l/2)}{\frac{\sqrt{zy}\, l}{2}}$$

Note:-

$$\cosh(\gamma l) = \cosh(\alpha l) \cdot \cos(\beta l) + j \sinh(\alpha l) \cdot \sin(\beta l)$$

$$\sinh(\gamma l) = \sinh(\alpha l) \cdot \cos(\beta l) + j \cosh(\alpha l) \cdot \sin(\beta l)$$

Loss Less Line :-

→ A, B, C, D par.
→ Z', $\frac{Y'}{2}$ (model)

loss-less
صعب cosh
بدون approx. idea
good idea in power flow analysis.

$Z = j\omega L \quad \Omega/m \qquad (r = 0)$

$y = j\omega C \quad S/m \qquad (g = 0)$

$$Z_s = \sqrt{\frac{Z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \equiv \text{Surge Impedance}$$

purely resistive.

lossless Line

- A, B, C, D — par
- Z', $\frac{Y'}{2}$ — model
- wave length λ
- SIL

$$\gamma = \sqrt{zy} = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta \ m^{-1}$$

purely imag.

γ: real α attenuation constant; Imag β phase constant

$\beta = \omega\sqrt{LC}$ = phase constant ; $\alpha = 0$ since there is no loss in the Line.

ABCD Parameters (Lossless Line) :-

$$* \ A(x) = D(x) = \cosh(\gamma x) = \cosh(\underline{j\beta}\, x) = \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x) \quad \text{per unit}$$

not hyp. Function

note

$$\sinh(\gamma x) = \sinh(j\beta x) = j\,\frac{e^{j\beta x} - e^{-j\beta x}}{2j} = j\sin(\beta x) \quad \text{per unit}$$

(not) hyp function

$$* \ B(x) = Z_c \sinh(\gamma x) = j Z_c \sin(\beta x)$$
$$= j\sqrt{\frac{L}{C}} \cdot \sin(\beta x) \ \Omega$$

$$* \ C(x) = \frac{\sinh(\gamma x)}{Z_c} = \frac{j\sin(\beta x)}{\sqrt{\frac{L}{C}}} \ S$$

π-model for loss-less line →

model :-

$\odot \; Z' = Z_c \sinh \gamma l \quad \leftarrow j\sin(\beta l)$

$= j \underbrace{Z_c \sin(\beta l)}$

$= j X'$

$\gamma = j\beta$

$$\odot \; \frac{Y'}{2} = \frac{Y}{2} \frac{\tanh \frac{\gamma l}{2}}{\gamma l/2} = \frac{Y}{2} \frac{\tanh(j\beta l/2)}{j\beta l/2}$$

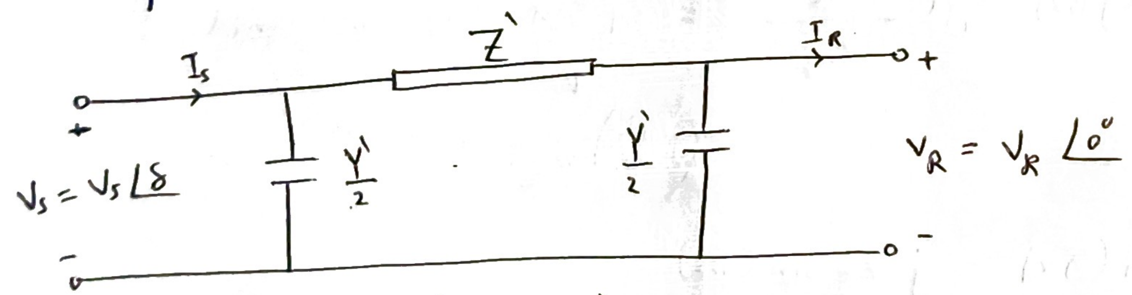
$$= \frac{Y}{2} \frac{\sinh(j\beta l/2)}{(j\frac{\beta l}{2})\cosh(\frac{j\beta l}{2})}$$

$$= \left(\frac{j\omega C l}{2}\right) \frac{\cancel{j}\sin(\beta l/2)}{(\cancel{j}\frac{\beta l}{2})\cos(\beta l/2)}$$

$$= \frac{j\omega C l}{2} \frac{\tan(\beta l/2)}{\beta l/2}$$

$$= \frac{j\omega C' l}{2}$$

π- Equivalent Circuit ((Loss Less Line)) :-



$$Z' = (j\omega L l)\left(\frac{\sin\beta l}{\beta l}\right) = jX' \;\Omega$$

$$\frac{Y'}{2} = \left(\frac{j\omega C l}{2}\right)\frac{\tan(\beta l/2)}{(\beta l/2)} = \frac{j\omega C' l}{2} \;S$$

For Loss less line :-

$V(x) = A(x) V_R + B(x) I_R$ $\quad || \quad$ $I(x) = C(x) V_R + D(x) I_R$

$= \cos(\beta x) V_R + j Z_c \sin(\beta x) I_R$ $\quad || \quad$ $= \frac{j\sin(\beta x)}{Z_c} V_R + \cos(\beta x) I_R$

Wave Length (Loss Less Line) :- A wavelength is the distance required to change the phase of the voltage or current by 2π radians or 360°.

$$v = \frac{w}{\beta} = \frac{2\pi f}{\beta}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{w\sqrt{LC}} = \frac{1}{f\sqrt{LC}} \quad m$$

* The expression for the inductance per unit length L and capacitance per unit length C of a transmission line were derived in previous chapter. When the internal flux linkage of a conductor is neglected $GMR_L = GMR_C$

$$r' = r$$

$$\lambda \cong \frac{1}{f\sqrt{\mu_o \varepsilon_o}}$$

$\mu_o = 4\pi * 10^{-7}$ $\Longrightarrow$ $\lambda = 6000\ km$, for $50\ Hz$

$\varepsilon_o = 8.85 * 10^{-12}$

$$\Longrightarrow f\lambda = v = \frac{1}{\sqrt{LC}} \cong \frac{1}{\sqrt{\mu_o \varepsilon_o}} \cong 3 * 10^{8}\ m/sec.$$

$\equiv$ Velocity of propagation of voltage and current waves on loss-less Line

Surge Impedance Loading :- (SIL) is the power delivered by a lossless line to a load resistance equal to the surge impedance Z_c.

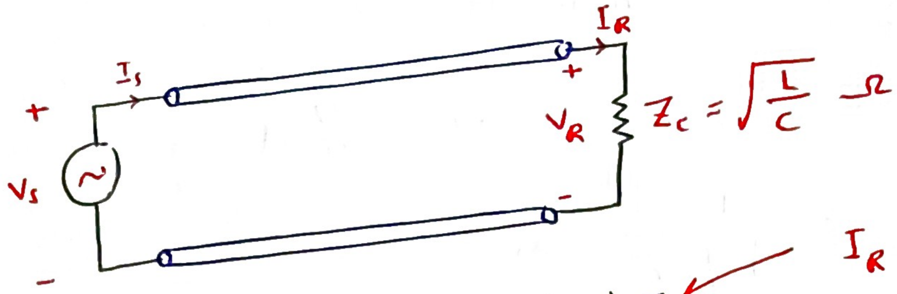
$$\boxed{Z_c = \sqrt{L/C}}$$

* $V(x) = A(x)\, V_R + B(x)\, I_R$

$$= \cos(\beta x)\, V_R + j Z_c \sin(\beta x)\, I_R$$

* $I(x) = C(x)\, V_R + D(x)\, I_R$

$$= j\,\frac{\sin(\beta x)}{Z_c}\, V_R + \cos(\beta x)\, I_R$$



$I_R = \frac{V_R}{Z_c}$

① $V(x) = \cos(\beta x)\, V_R + j\, Z_c \sin(\beta x)\, I_R$

$= \cos(\beta x)\, V_R + j\, Z_c \sin(\beta x) \left(\frac{V_R}{Z_c}\right)$

$= \underbrace{\left[\cos(\beta x) + j \sin(\beta x)\right]}_{}\, V_R$

$= e^{j\beta x}\, V_R$ volts.

$|V(x)| = |V_R|$ volts ; Voltage is constant along the line.

② $I(x) = j\,\frac{\sin(\beta x)}{Z_c}\, V_R + \cos(\beta x)\, \frac{V_R}{Z_c}$

$= \left[\cos\beta x + j \sin\beta x\right] \frac{V_R}{Z_c}$

$= \left[e^{j\beta x}\right] \frac{V_R}{Z_c}$ A.

$S(x) = P(x) + j\,Q(x) = V(x)\, I^*(x)$

$= \left[e^{j\beta x}\, V_R\right]\left[\frac{e^{j\beta x}\, V_R}{Z_c}\right]^*$

$= \frac{|V_R|^2}{Z_c}$; Real power along the line is constant and reactive power flow is zero.

⊙ at rated line voltage

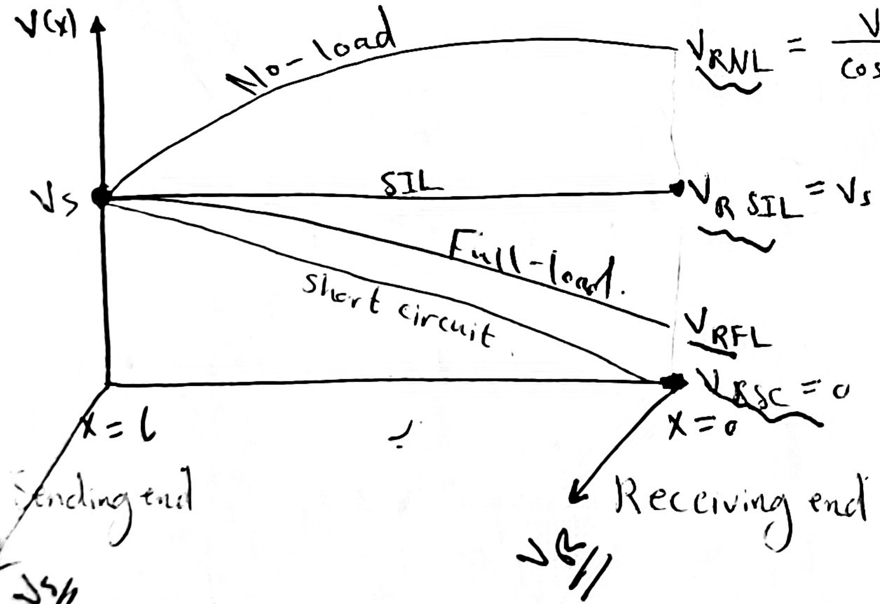
$SIL = \frac{V^2_{rated}}{Z_c}$ $\boxed{L-L}$

V_{rated} (kV)	$Z_c = \sqrt{L/C}$	$SIL = V_{rated}^2 / Z_c$ (MW)
230	380	140
345	285	420
500	250	1000
765	257	2280

Voltage Profiles:-

$V_{NL}(x) = [\cos(\beta x)]\, V_{RNL}$

$V_{SC}(x) = Z_c \sin\beta x \; I_{RSC}$



$I_{RNL} = 0$

$V_{NL}(x) = (\cos\beta x)\, V_{RNL}$

Voltage profiles of an uncompensated lossless line with fixed sending end voltage.

example 5.5
160

$$\boxed{V(x) = \cos(\beta x)\, V_R + j Z_c \sin(\beta x)\, I_R}$$

① no-load, $I_{RNL} = 0$

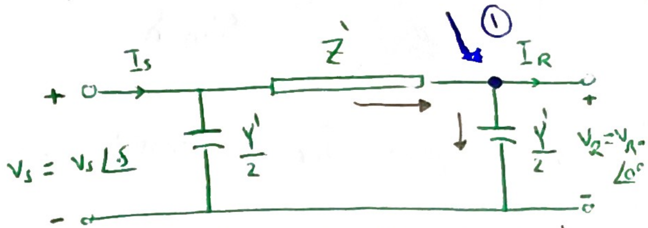
$V_{NL}(x) = \cos(\beta x)\, V_{RNL}$

$V_{RNL} = \frac{V_s}{\cos(\beta l)}$

② for short circuit at the load V_{RSC}

$V_{SC}(x) = (Z_c \sin\beta x)\, I_{RSC}$

Steady-State Stability Limit :



$Z' = jX'$ (loss-less line)

$\frac{Y'}{2} = j\frac{\omega c' l}{2}$

KCL at node ①

$$I_R = \frac{V_s - V_R}{Z'} - \frac{Y'}{2} V_R$$

$$= \frac{V_s e^{j\delta} - V_R}{jX'} - j\frac{\omega c' l}{2} V_R$$

complex power at the receiving end

$$S_R = V_R I_R^* = V_R \left(\frac{V_s e^{j\delta} - V_R}{jX} \right)^* + j\frac{\omega c' l}{2} V_R^2$$

$$= V_R \frac{j}{j} \left(\frac{V_s e^{-j\delta} - V_R}{-jX'} \right) + j\frac{\omega c' l}{2} V_R^2$$

$$\boxed{A e^{j\theta} = A\cos\theta + jA\sin\theta}$$

$$= \frac{j V_R V_s \cos\delta + V_R V_s \sin\delta - j V_R^2}{X'} + j\frac{\omega c' l}{2} V_R^2$$

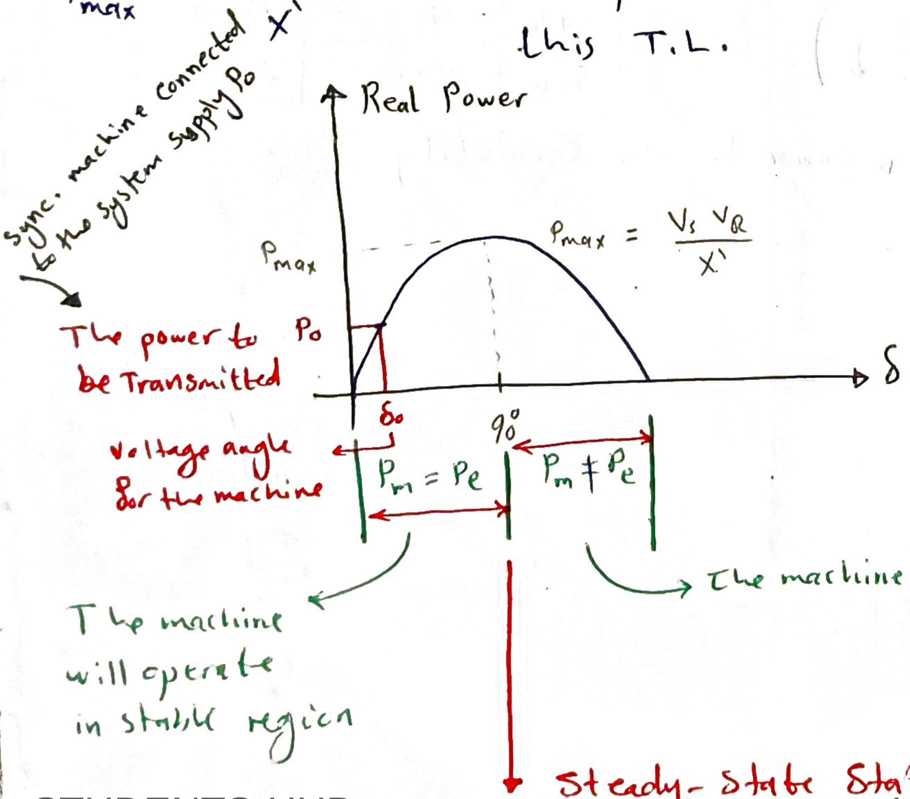
real power

$$P = \underbrace{P_s = P_R}_{\text{loss-less line}} = Re(S_R) = \frac{V_R V_s}{X'} \underbrace{\sin\delta}_{} \ W$$

real part

max when $\delta = 90°$

$P_{max} = \frac{V_R V_s}{X'}$ W , max power that can be transmitted over this T.L.



if an attempt were made to exceed this limit, then the machine would loss synchronism

notes :-

① $P_{max} = \frac{V_s V_R}{X'}$ ← ثابت ← عكسي

$V_s \cong V_R \cong 1$ per unit

bundel

GMR ↑ , L ↓ , X ↓ , P_{max} ↑

allow you to Transmitt more power on the T.L.

② mechanical input → [ ] → electrical output

input ↑ , δ ↑ , Po ↑

In tearms of SIL

$$P = \frac{V_R \, V_s}{X'} \sin\delta$$

→ real power for loss-less line.

- $Z' = Z_c \sinh \gamma l$
  $= j \underbrace{Z_c \sin(\beta l)}$
  $= j X'$

- $\lambda = \frac{2\pi}{\beta} \Rightarrow \beta = \frac{2\pi}{\lambda}$

$$= \frac{V_s \, V_R \, \sin\delta}{Z_c \sin \beta l}$$

$$= \left(\frac{V_s . V_R}{Z_c}\right) \cdot \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

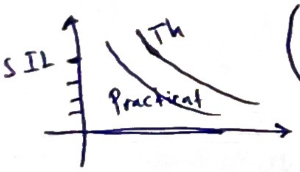
$$= \left(\frac{V_s}{V_{rated}}\right)\left(\frac{V_R}{V_{rated}}\right) \cdot \left(\frac{V^2_{rated}}{Z_c}\right) \cdot \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

$$= \left(V_{s.p.u}\right)\left(V_{R.p.u}\right)\left(SIL\right) . \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)} \quad W$$

$$P_{max} = \frac{V_{s.p.u} \, V_{R.p.u} \, SIL}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

of t… Line.

Voltage kV	SIL (MW)	Typical Thermal Rating (MW)
230	150	400
345	400	1200
500	900	2600



$(l \uparrow \; , \; P_{max} \downarrow)$

→ power transfer capability

Maximum Power Flow (Lossy Line):-

real
$$A = \cosh(\gamma l) = A \angle \theta_A$$
img

real
$$B = Z' = Z' \angle \theta_Z$$
img

$$I_R = \frac{V_s - A V_R}{B} = \frac{V_s e^{j\delta} - A V_R e^{j\theta_A}}{Z' e^{j\theta_z}}$$

$$S_R = P_R + j Q_R = V_R^* I_R^* = V_R \left[\frac{V_s e^{j(\delta - \theta_z)} - A V_R e^{j(\theta_A - \theta_Z)}}{Z'} \right]^*$$

$$= \frac{V_R V_s}{Z'} e^{j(\theta_Z - \delta)} - \frac{A V_R^2}{Z'} e^{j(\theta_Z - \theta_A)}$$

$$P_R = Re(S_R) = \underbrace{\frac{V_R V_s}{Z'} \cos(\theta_Z - \delta)} - \underbrace{\frac{A V_R^2}{Z'} \cos(\theta_Z - \theta_A)}$$

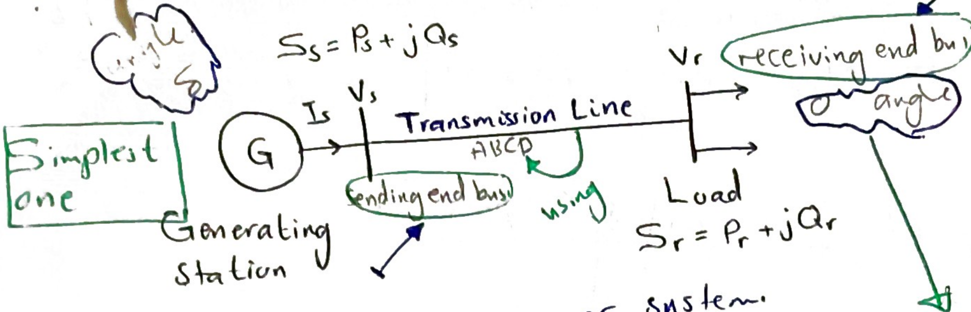
Two component

same as previous — راح نقللها هنا اكثر

$$P_{max} \Big|_{\theta_Z = \delta}$$

Transmission Line Steady State Operation :-

[when we talk about the S.S.O on T.L what we really mean is how the Line is perform when we want to Transmit ~~[illegible]~~ certain amount of power throug it.]



A two bus power system.

Power flow on transmission Lines :-

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

① $V_s = AV_R + BI_R$

$\frac{1}{B} V_s = \frac{A}{B} V_R + I_r$

$I_r = \frac{1}{B} V_s - \frac{A}{B} V_r$

② $I_s = CV_R + DI_r$

$I_s = CV_R + \frac{D}{B} V_s - \frac{DA}{B} V_r$

$= \frac{D}{B} V_s + \left(\frac{CB}{B} - \frac{DA}{B}\right) V_R$

$= \frac{D}{B} V_s - \frac{1}{B} V_R$

rec end current: $I_r = \frac{1}{B} V_s - \frac{A}{B} V_r$ ------ ①

sending end current: $I_s = \frac{D}{B} V_s - \frac{1}{B} V_r = \frac{A}{B} V_s - \frac{1}{B} V_r$ --- ②

we know that $\boxed{A = D}$

Let $V_r = |V_r| \angle 0$ (as a refrense phaser)

$V_s = |V_s| \angle \delta$, ↑ V_s leads V_r by δ

[δ : the angle ~~by~~ which the V_s leads V_r.]

complex number ← $D = A = |A| \angle \alpha$

complex number ← $B = |B| \angle \beta$

Then, from ① and ②

$$I_r = \frac{|V_s|}{|B|} \angle(\delta - \beta) - \frac{|A||V_r|}{|B|} \angle(\alpha - \beta)$$

$$I_s = \frac{|A||V_s|}{|B|} \angle(\alpha + \delta - \beta) - \frac{|V_r|}{|B|} \angle -\beta$$

The Conjugates of I_r and I_s are:-

$$I_r^* = \frac{|V_s|}{|B|} \angle(\beta - \delta) - \frac{|A||V_r|}{|B|} \angle(\beta - \alpha)$$

$$I_s^* = \frac{|A||V_s|}{|B|} \angle(\beta - \alpha - \delta) - \frac{|V_r|}{|B|} \angle \beta$$

Complex Power

(a) $S_r = P_r + jQ_r = V_r I_r^*$

$$= |V_r| \angle 0 \left[\frac{|V_s|}{|B|} \angle(\beta - \delta) - \frac{|A||V_r|}{|B|} \angle(\beta - \alpha) \right]$$

$$= \frac{|V_s||V_r|}{|B|} \angle(\beta - \delta) - \frac{|A||V_r|^2}{|B|} \angle(\beta - \alpha)$$

(b) $S_s = P_s + jQ_s = V_s I_s^*$

$$= |V_s| \angle \delta \left[\frac{|A||V_s|}{|B|} \angle(\beta - \alpha - \delta) - \frac{|V_r|}{|B|} \angle \beta \right]$$

$$= \frac{|A||V_s|^2}{|B|} \angle(\beta - \alpha) - \frac{|V_r||V_s|}{|B|} \angle(\beta + \delta)$$

real and reactive power

(a)

real power: $P_r = \frac{|V_s||V_r|}{|B|} \cos(\beta - \delta) - \frac{|A||V_r|^2}{|B|} \cos(\beta - \alpha)$

δ: power angle or voltage angle at the sending end / voltage phase

reactive power: $Q_r = \frac{|V_s||V_r|}{|B|} \sin(\beta - \delta) - \frac{|A||V_r|^2}{|B|} \sin(\beta - \alpha)$

δ: 1 paramter variable δ note ① ②

(b)

real power: $P_s = \frac{|A||V_s|^2}{|B|} \cos(\beta - \alpha) - \frac{|V_r||V_s|}{|B|} \cos(\beta + \delta)$

$Q_s = \frac{|A||V_s|^2}{|B|} \sin(\beta - \alpha) - \frac{|V_r||V_s|}{|B|} \sin(\beta + \delta)$

notes

① For agiven system voltage level V_s and V_r will be very near to the system voltage and they don't not chang much. (33kV, ...)

② β, α T.L paramters and they already there so, they are fixed.

$(\beta = \delta)$ $P_{r\,max} = \frac{|V_s||V_r|}{|B|} - \frac{|A||V_r|^2}{|B|} \cos(\beta - \alpha)$

max power which can be transmitted or recieved.

$Q_r = -\frac{|A||V_r|^2}{|B|} \sin(\beta - \alpha)$

For Short Line :- $\Longrightarrow$ $A = D = 1\angle 0^\circ$

$B = Z\angle\theta$

(حتى نفهم ال Conspet أحسن for power flow

Simpler

$$P_r = \frac{|V_s||V_r|}{|Z|}\cos(\theta - \delta) - \frac{|V_r|^2}{|Z|}\cos\theta$$

$$Q_r = \frac{|V_s||V_r|}{|Z|}\sin(\theta - \delta) - \frac{|V_r|^2}{|Z|}\sin\theta$$

$$P_s = \frac{|V_s|^2}{|Z|}\cos\theta - \frac{|V_s||V_r|}{|Z|}\cos(\theta + \delta)$$

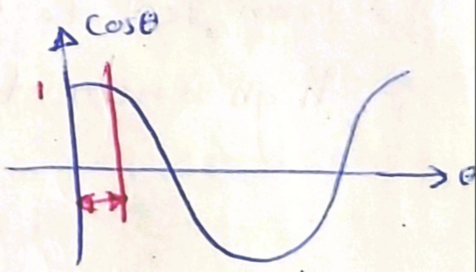
$$Q_s = \frac{|V_s|^2}{|Z|}\sin\theta - \frac{|V_s||V_r|}{|Z|}\sin(\theta + \delta)$$

$(Z = R + jX)$

As $R \ll X$, $|Z| \approx X$ and $\theta = 90^\circ$, substituting these values in the above equations

$$\boxed{P_r = \frac{|V_s||V_r|}{X}\sin\delta}$$

$$Q_r = \frac{|V_s||V_r|}{X}\cos\delta - \frac{|V_r|^2}{X}$$



As δ is normally small ; $\cos\delta \cong 1$

$\cos 15 = 0.966$

$\cos 20 = 0.94.$

$$Q_r = \frac{|V_s||V_r|}{X} - \frac{|V_r|^2}{X}$$

$$\boxed{Q_r = \frac{|V_r|}{X}\left(|V_s| - |V_r|\right)}$$

From these relationships we Can Conclude the following points

1. For fixed values of V_s, V_r and X the real power depends on angle δ the phase angle by which V_s leads V_r. This angle δ is called power angle. When $\delta = 90°$ P is maximum. For system stability considerations δ has to be kept well below $90°$.

لأنها هي اللي تحدد كمية البور اللي سأنقلها من خلال T.L

range $(20°-30°)$ ↑↑ disturbances will → $90°$

2. Power can be transferred over line even when $|V_s| \lesssim |V_r|$.

The phase difference δ between V_r and V_s causes the flow of power in the line. Power systems are operated with almost the same voltage magnitudes (i.e., 1pu) at important busses by using methods of Voltage Control.

because this provides a much better operating conditions for the system

δ: + V_s leads V_r $\left[V_s \xrightarrow{\text{power flow}} V_r \right.$

δ: − V_s lags V_r $\left. V_r \xrightarrow{\text{power flow}} V_s \right]$

direction of power flow (δ) not $|V|$

3. The maximum real power transferred over a line increases with increase in V_s and V_r.

An increase of 100% in V_r and V_s increases the power transfer to 400%. This is the reason for adopting high and extra high transmission voltages.

100% double the voltage

تبنّي

(تتخذ أسلوب أو وجهة نظر معينة)

4. The maximum real power depends on the reactance X which is directly proportional to line inductance. A decrease in inductance increases the line Capacity. The line inductance can be decreased by using bundled Conductors.

>> Another method of reducing line inductance is by inserting capacitance in series with the line. This method is known as series compensation. The series capacitors are usually installed at the middle of the line.

(Positive Reactance L + negative Reactance C) → effective Reactance will be ↓
series

5. The reactive power transferred over a line is directly proportional to $(|V_s| - |V_r|)$ i.e., voltage drop along the line, and is independent of power angle. This means the voltage drop on the line is due to the transfer of reactive power over the line. To maintain a good voltage profile, reactive power control is necessary.

Voltage Control

Reactive Power compensation equipment has the following effect:-

1. Reduction in current. $S = P + jQ$, $Q\downarrow$, $S\downarrow$, $I\downarrow$, V = constant = nominal value
   $S = VI^*$

   V_s, V_r ± 5% (nominal value) → (10% loss in line)

2. Maintain ~~xy~~ voltage profile within limits. يحافظ
   $Q_r = \frac{|V_r|}{X}(|V_s| - |V_r|)$
3. Reduction of losses in the system. $(I^2R)\downarrow$ since $I\downarrow$
   d
4. Reduction in investment (الأموال, استثمار) in the system per kW of load supplied. $(Q\downarrow, \downarrow I, r\downarrow)$
5. Decrease in KVA loading of generators and lines. This decrease in KVA loading relieves (يريح, يخفف) overload condition or releases (يبعث, يطلق) capacity for additional load growth.
6. Improvement in power factor of generators.

→ Reactive compensation of T.L. ←

1. Static Var Compensation.
2. Rotating Compensators (synchronous compensator)
3. Using Transformer. (Tap transformer)
4. Using Power Electronics (STAT COM)

Static Compensation

The performance of transmission lines, especially those of medium length and longer, can be improved by reactive compensation of a series or parallel type.

1. Series compensation consists of a capacitor bank placed in series with each phase conductor of the line. Series compensation reduces the series impedance of the line, which is the principal (رئيسي) cause of voltage drop and the most important factor in determining the maximum power which the line can transmit.

2. Shunt compensation refers to:

   a. The placement of inductors from each line to neutral to reduce partially or completely the shunt susceptance of a high-voltage line, which is particularly important at light loads when the voltage at the receiving end may otherwise become very high. (Shunt Reactors)

   b. Shunt Capacitors are used for lagging Power factor circuits created by heavy loads. The effect is to supply the requisite reactive power to maintain the receiving end voltage at satisfactory level.

example: A 50 Hz, 138 kV, 3-phase transmission Line is 200 km long. The distributed line parameters are

$R = 0.1 \ \Omega/km$

$L = 1.2 \ mH/km$

$C = 0.01 \ \mu F/km$

$G = 0$

The transmission line delivers 40 MW (P_r, 3φ power) at 132 kV (V_r) with 0.95 power factor lagging. Find the sending end voltage and current, and also the transmission line efficiency.

Solution: For the given values of R, L and C, we have for $w = 2\pi(50)$,

$z = 0.1 + j\underset{(wL)}{0.377} = 0.39\angle 75.14^\circ \ \Omega/km.$

$y = j\underset{(jwC)}{3.14 * 10^{-6}} = 3.14 * 10^{-6}\angle 90 \ \mho/km.$

$$\boxed{\begin{aligned} V_1 &= V_2 \cosh\gamma l + Z_c I_2 \sinh\gamma l \\ I_1 &= I_2 \cosh\gamma l + \left(\frac{V_2}{Z_c}\right)\sinh\gamma l \end{aligned}}$$

From the above values

$Z_c = \sqrt{(z/y)} = 352.42\angle -7.43^\circ \ \Omega$

$\gamma l = 200\sqrt{zy} = 0.2213\angle 82.57^\circ = 0.0286 + j\,0.2194$

⇨ $\sinh\gamma l = \dfrac{e^{\gamma l} - e^{-\gamma l}}{2} = 0.2195\angle 82.67^\circ$

⇒ $\cosh\gamma l = \dfrac{e^{\gamma l} + e^{-\gamma l}}{2} = 0.975\angle 0.37^\circ$

The values of power and voltage specified in the problem refers to 3-phase and line-to-line quantities.

$\boxed{V_{LN}}$ $|V_2| = 132/\sqrt{3} = 76.2 \ kV$

also, using V_2 as reference; $\angle V_2 = 0^\circ$, we get

$V_2 = 76.2\angle 0^\circ \ kV$

Note:

$$\boxed{\begin{aligned} \cosh(\gamma l) &= \cosh(\alpha l) * \cos(\beta l) + j\sinh(\alpha l) * \sin(\beta l) \\ \sinh(\gamma l) &= \sinh(\alpha l) * \cos(\beta l) + j\cosh(\alpha l) * \sin(\beta l) \end{aligned}}$$

Now, per phase power supplied to the load.

$$P_{load} = \frac{40}{3} = 13.33 \text{ MW}.$$

Given the value of power factor = 0.95, we can find I_2

$$P_{load} = 0.95 \, |V_2| \cdot |I_2|$$

Thus, $|I_2| = 184.1$

Also, since I_2 lags V_2 by $\cos^{-1} 0.95 = 18.195°$,

$$I_2 = 184.1 \angle -18.195°$$

Finally, we have:-

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$

$$V_1 = 82.96 \angle 8.6 \text{ kV}$$

Sending end voltage.

⇒ L-to-N voltage at the sending end.

Similarly,

$$I_1 = I_2 \cosh \gamma l + \left(\frac{V_2}{Z_c}\right) \sinh \gamma l$$

$$= 179.46 \angle 17.79$$

Sending end current.

* Power P_s

We now calculate the efficiency of transmission.

Perphase input power, $P_{in} = Re(V_1 I_1^*)$

$$= 14.69 \text{ MW}$$

Hence, $\eta = \dfrac{13.33}{14.69} = 0.907.$

Thus, the efficiency of transmission is 90.7%

example

A 3 phase 132 kV overhead line delivers 60 MVA (S_r) at 132 kV (V_r, V_2) and power factor (PF_r) 0.8 lagging at its receiving end. The constants of the line are $A = 0.98 \angle 3^\circ$ and $B = 100 \angle 75^\circ$ ohms per phase. Find

(a) sending end voltage and power angle.

(b) sending end active and reactive power.

(c) line losses and vars absorbed by the line.

(d) and (e)

Solution :-

((a))

V_{LN} (phase voltage)

$$V_r = \frac{132000}{\sqrt{3}} = 76210 \angle 0^\circ$$ ← refrence voltage.

$$\frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$I_r = \left[\frac{60 \times 10^6}{3}\right] \Big/ \left[\frac{132000}{\sqrt{3}}\right]$$

$$S_r = V_r I_r^*$$

$$\boxed{I_r = 262.43 \angle -36.87^\circ}$$ ← $-\cos^{-1}$ p.F

$$V_s = A \cdot V_r + B \cdot I_r$$

$$= (0.98 \angle 3^\circ)(76210 \angle 0) + (100 \angle 75^\circ)(262.43 \angle -36.87^\circ)$$

$$= 97.33 \times 10^3 \angle 11.92^\circ \ \text{V}$$

L-L

* Sending end line voltage $= (\sqrt{3})(97.33)$ kV

$= 168.58$

* Power angle (δ) $= 11.92^\circ$

(d) capacity of static compensation equipment at the receiving end to reduce the sending end voltage to 145 kV for the same load conditions. part (a) $V_s \downarrow$ (we need to reduce)

$= 145$ kV → 132 kV

(e) The unity power factor load which can be supplied at the receiving end with 132 kV as the line voltage at both the ends.

Load → Power → voltage → P.f.

132 kV — 132 k } Purely resistive load.

We have 3 phase power given as

$$S_s = \frac{|A||V_s|^2}{|B|} \angle(\beta-\alpha) - \frac{|V_r||V_s|}{|B|} \angle(\beta+\delta)$$

L-L

$$= \frac{(0.98)*(168.58)^2}{(100)} \angle(75-3^\circ) - \frac{(132)(168.58)}{(100)} \angle(75^\circ+11.92^\circ)$$

L-L

$$= 278.49 \angle 72^\circ - 222.53 \angle(86.92^\circ)$$

notes :-

3-ϕ power
$\Rightarrow$ Vs and Vr
L-L voltages

$\Rightarrow$ Sending end active power

$$P_s = 278.49 \cos(72^\circ) - 222.53 \cos(86.92^\circ)$$

$$= 86.06 - 11.96 = 74.10 \text{ MW}$$

1-ϕ power
$\Rightarrow$ Vs and Vr
L-N voltage

$\Rightarrow$ Sending end reactive power

$$Q_s = 278.49 \sin 72^\circ - 222.53 \sin 86.92^\circ$$

$$= 264.89 - 222.21$$

$$= 42.65 \text{ MVar lagging}$$

((c)) * Line Losses $= P_s - P_r$

$$= 74.10 - \underbrace{60 * 0.8}_{48 \text{ MW}}$$

$$= 26.10 \text{ MW}$$

* MVar absorbed by line $= Q_s - Q_r$

$\sin\theta$, $\theta = \cos^{-1} pf$

$$= 42.65 - \underbrace{60 * 0.6}_{36 \text{ MVar}}$$

$$= 6.65 \text{ MVar.}$$

(d) $P_r = 60 * 0.8 = 48 \text{ MW}$

$|V_s| = 145 \text{ KV}$

$|V_r| = 132 \text{ KV}$

* $P_r = |V_s||V_r||B|^{-1}\cos(\beta-\delta) - |A||V_r|^2|B|^{-1}\cos(\beta-\alpha)$

$48 = \frac{(145)(132)}{100}\cos(\beta-\delta) - \frac{(0.98)(132)^2}{(100)}\cos(75-3^\circ)$

? for this particular operating conditions.

$48 = 191.4\cos(\beta-\delta) - 170.75\cos(72)$

$\cos(\beta-\delta) = 0.5275$

$\angle(\beta-\delta) = \cos^{-1}(0.5275) = 58.16^\circ$

β ✓

δ ? ✓

* $Q_r = |V_s||V_r||B|^{-1}\sin(\beta-\delta) - |A||V_r|^2|B|^{-1}\sin(\beta-\alpha)$

$= \frac{(145)(132)}{(100)}\sin(58.16) - \frac{(0.98)(132)^2}{(100)}\sin(72^\circ)$

$= 162.60 - 162.40$

$= 0.20 \text{ MVar}$

$Q_c = V_{rms} \cdot I_{rms}\sin(\theta_v-\theta_i)$ (-90)

$= V_{rms} I_{rms}(-1)$

$\boxed{Q_c = -V_{rm} I_{rms}}$ I_c

$= -V_{rms}[wC\, V_{rms}]$

$= -V_{rms}^2 * wC$

Thus for $V_s = 145$ kV, $V_r = 132$ kV and $P_r = 48$ MW, a lagging MVar of 0.2 will be supplied from the line along with the real power of 48 MW. Since the load requires 36 MVar lagging ($60 * \sin\theta$), the static compensation equipment must deliver 36 − 0.2, i.e, 35.8 MVar lagging (or must absorb 35.8 MVar leading). The capacity of static capacitor is, therefore, 35.8 MVar.

$\boxed{Q_c = -wC\, V_{rms}^2}$

$|V_s| = |V_r| = 132 \text{ kV}$, $Q_r = 0$

$$Q_r = |V_s||V_r||B|^{-1} \sin(\beta - \delta) - |A||V_r|^2 |B|^{-1} \sin(\beta - \alpha)$$

$$0 = \frac{(132)(132)}{(100)} \sin(\beta - \delta) - \frac{(0.98)(132)^2}{(100)} \sin(75^\circ - 3^\circ)$$

$$\underline{/(\beta - \delta)} = 68.75^\circ$$

e)

$$P_r = |V_s||V_r| |B|^{-1} \cos(\beta - \delta) - |A||V_r|^2 |B|^{-1} \cos(\beta - \alpha)$$

$$= \frac{(132)(132)}{(100)}[\cos(68.75)] - \frac{(0.98)(132)^2}{(100)} \cos(72^\circ)$$

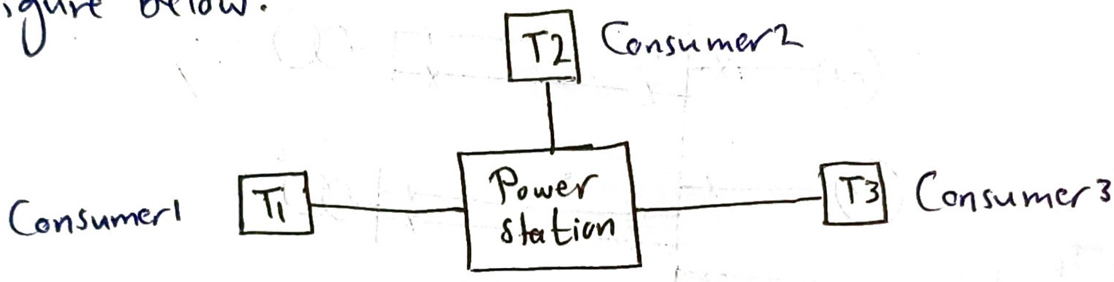
$$= 63.13 - 52.77$$

$$= 10.36 \text{ MW}$$

Power Flow Analysis

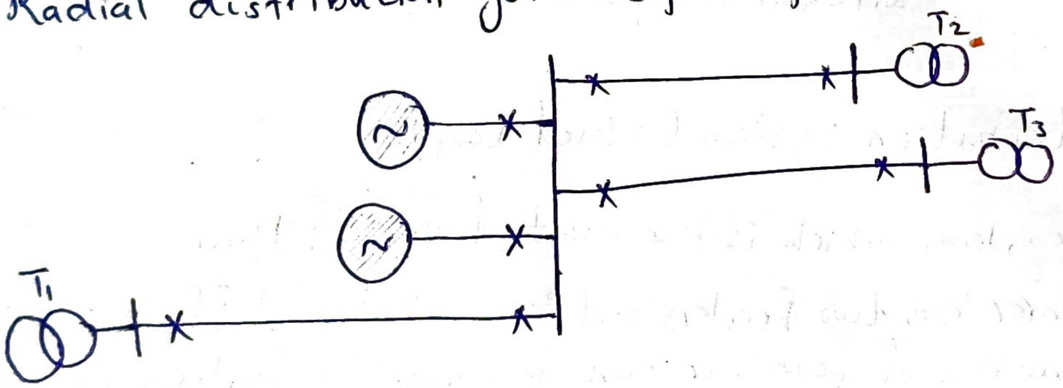
» The development of simple distribution system [open-loop / closed-loop]
Network arrangements

When a consumer requests electrical power from a supply authority, ideally all that is required is a cable and a transformer, shown physically as in Figure below.



A simple distribution system

① Radial distribution system (open loop)

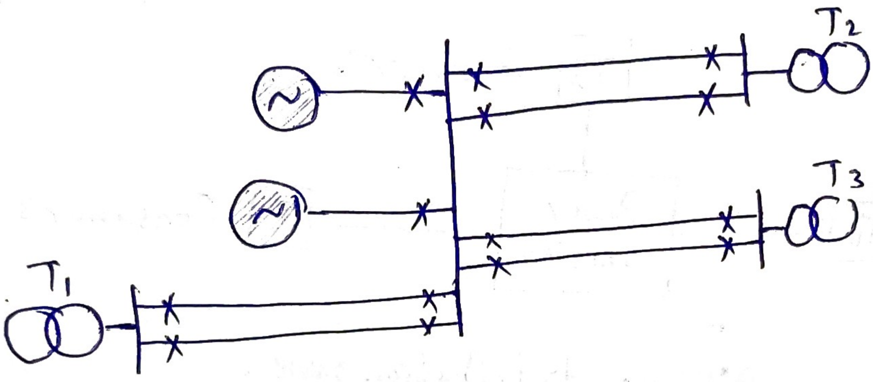


Advantages
If a fault occurs at T2 then only the protection on one leg connecting T2 is called into operation to isolate this leg. The other consumers are not affected.

Disadvantages
If the conductor to T2 fails, then supply to this particular consumer is lost completely and cannot be restored until the conductor is replaced / repaired.

② Radial distribution system with parallel feeders (open Loop)

This disadvantage (radial) can be overcome by introducing addi (parallel) feeders (as shown below) connecting each of the consumers radially. However, this requires more cabling and is not always economical.

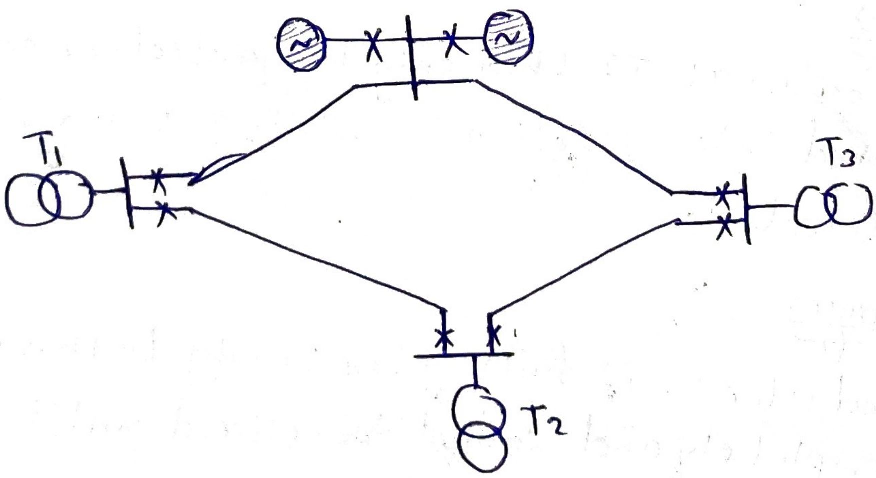


Radial distribution system with parallel feeders

Loop

③ Ring main distribution system (closed Loop)

The Ring main system, which is the most favored.
Here each consumer has two feeders but connected in different paths to ensure continuity of power, in case of conductor failure in any section.



<u>Advantages</u>:

.. Essentially, meets the requirements of two alternative feeds to give 100% continuity of supply, whilst saving in cabling compared to parallel feeds.

Disadvantages :-

For faults at T_1 fault current is fed into fault via two parallel paths effectively reducing the impedance from the source to the fault location, and hence the fault current is much higher compared to a radial path. The fault current in particular could vary depending on the exact location of the fault.

Protection must therefore be fast and discriminate correctly so that other consumers are not interrupted.

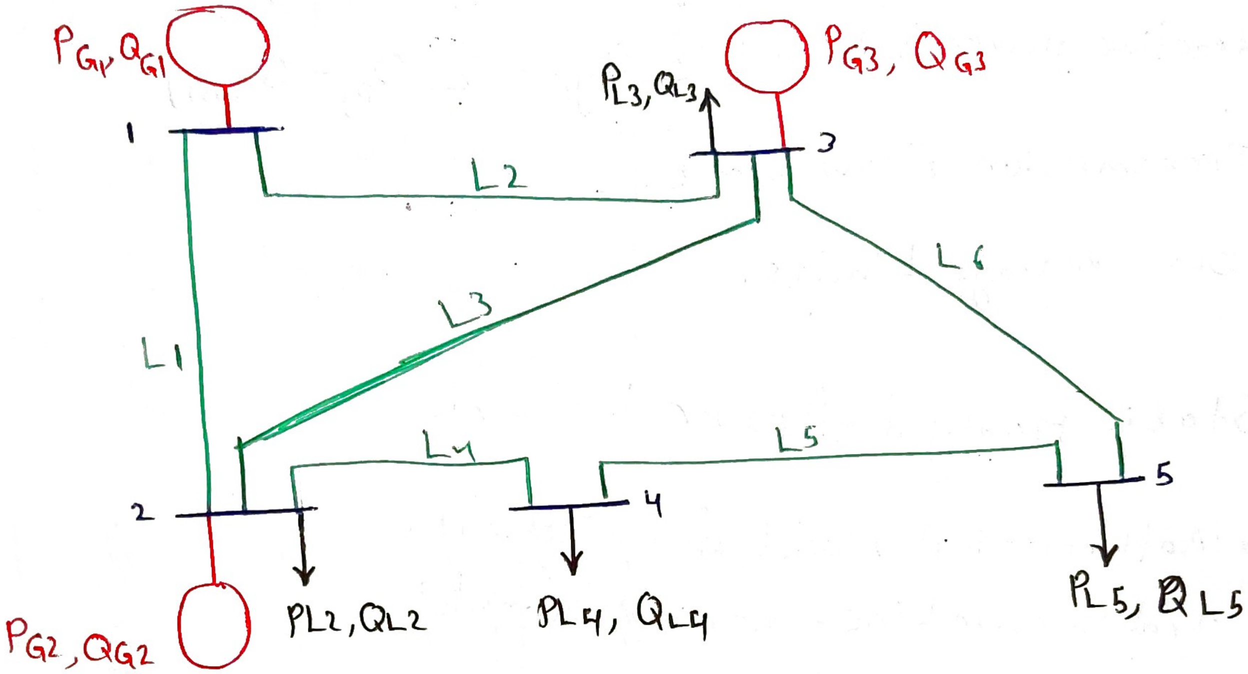
$V = Z\,I$

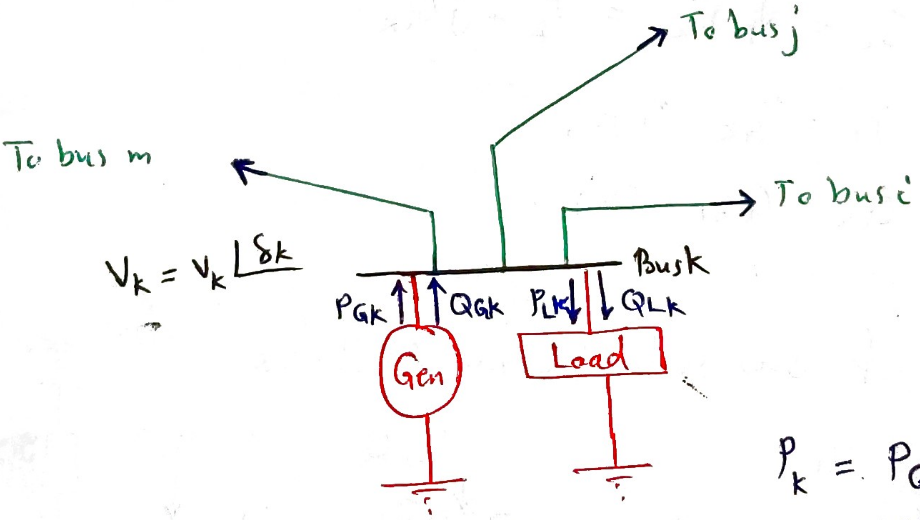
$I = \frac{V}{Z}$

④ Inter connected, Network system

Power Flow Analysis

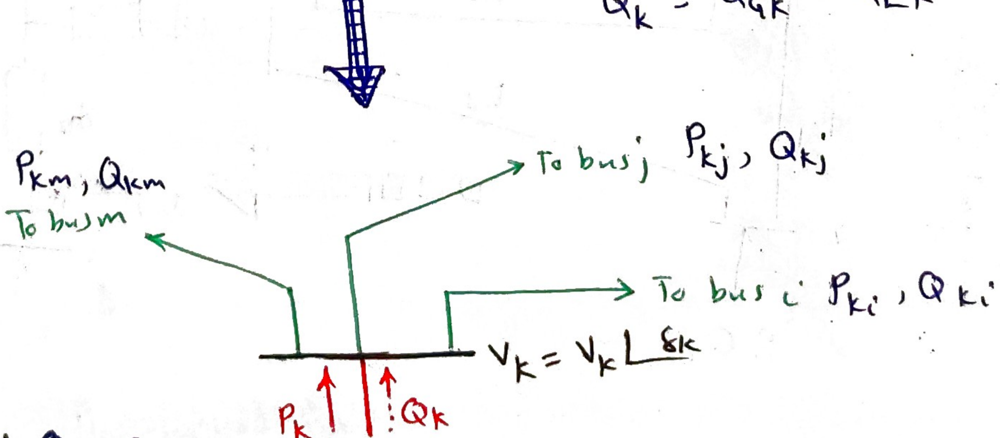
Load Flow Analysis





$$P_k = P_{Gk} - P_{Lk}$$

$$Q_k = Q_{Gk} - Q_{Lk}$$



$P_k = P_{Gk} - P_{Lk}$ | $Q_k = Q_{Gk} - Q_{Lk}$

$P_k = P_{ki} + P_{kj} + P_{km}$ | $Q_k = Q_{ki} + Q_{kj} + Q_{km}$

Power Flow Study :-

- Static Analysis of power Network
- Real power balance $\left(\sum P_{g_i} - \sum P_{D_j} - P_{loss}\right)$
- Reactive power balance $\left(\sum Q_{g_i} - \sum Q_{D_j} - Q_{loss}\right)$
- Transmission Flow Limit.
- Bus Voltage Limits.

» Static Analysis of power Network

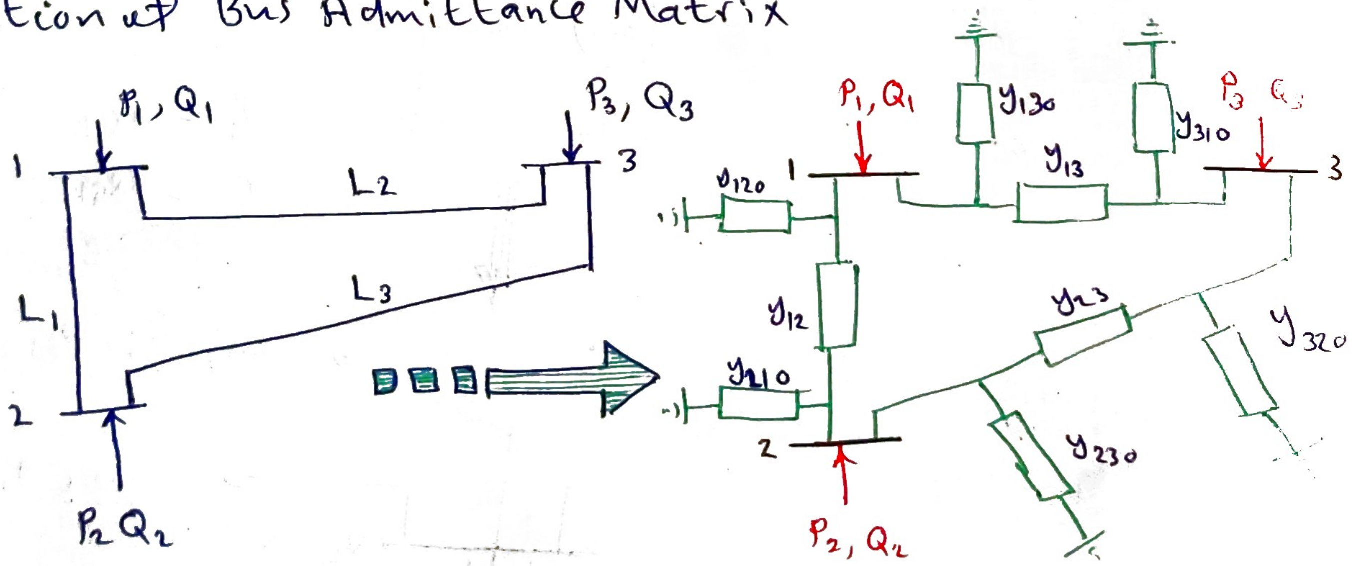
- Mathematical Model of the Network.
- Transmission Line – nominal π model.
- Bus power injections –

$$S_k = V_k I_k^* = P_k + jQ_k$$

$$P_k = P_{Gk} - P_{Lk}.$$

$$Q_k = Q_{Gk} - Q_{Lk}.$$

» Formation of Bus Admittance Matrix



$$I_1 = y_{120} V_1 + y_{12}(V_1 - V_2) + y_{130} V_1 + y_{13}(V_1 - V_3)$$

$$I_2 = y_{210} V_2 + y_{12}(V_2 - V_1) + y_{230} V_2 + y_{23}(V_2 - V_3)$$

$$I_3 = y_{310} V_3 + y_{13}(V_3 - V_1) + y_{320} V_3 + y_{23}(V_3 - V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} (y_{120} + y_{12} + y_{130} + y_{13}) & -y_{12} & -y_{13} \\ -y_{21} & (y_{210} + y_{12} + y_{230} + y_{23}) & -y_{23} \\ -y_{31} & -y_{32} & (y_{310} + y_{13} + y_{320} + y_{23}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$Y_{11} = y_{120} + y_{12} + y_{130} + y_{13}$$

$$Y_{22} = y_{210} + y_{12} + y_{230} + y_{23}$$

$$Y_{33} = y_{310} + y_{13} + y_{320} + y_{23}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$

- Y_{ii} is called Self-Admittance (Driving Point Admittance)

  Y_{ij} is called Transfer-Admittance (Mutual Admittance)

$$I_{Bus} = Y_{Bus} V_{Bus} \; ; \; V_{Bus} = Z_{Bus} I_{Bus}$$

Characteristics of Ybus Matrix:-

- Dimension of Y_{bus} is $(N \times N)$ $\longrightarrow$ $N \equiv$ Number of buses.
- Y_{bus} is symmetric matrix
- Y_{bus} is a sparse matrix (up to 90% to 95% sparse) متفرق متناثر
- Diagonal Elements Y_{ii} are obtained as Algebraic sum of all elements Incident to bus 'i'
- Off-diagonal Elements $Y_{ij} = Y_{ji}$ are obtained as negative of admittance Connecting bus i and j

Power Flow Equations:-

$$I_{BUS} = Y_{BUS} V_{BUS}$$

$$I_k = \sum_{n=1}^{N\ (\#\text{ of buses})} Y_{kn} V_n \Longrightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\underbrace{S_k}_{\text{bus } k} = P_k + jQ_k = V_k I_k^*$$

$$P_k + jQ_k = V_k \left[\sum_{n=1}^{N} Y_{kn} V_n \right]^* \qquad k = 1, 2, 3, \dots, N$$

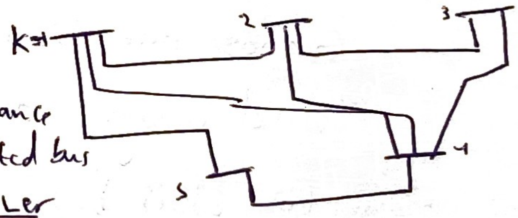
$$V_n = V_n e^{j\delta_n}$$

$$Y_{kn} = Y_{kn} e^{j\theta_{kn}} \quad \leftarrow \text{angle of the admittance.} \qquad k, n = 1, 2, 3, \dots, N$$

$$P_k + jQ_k = V_k \sum_{n=1}^{N} Y_{kn} V_n e^{j(\delta_k - \delta_n - \theta_{kn})}$$

$$\boxed{\begin{aligned} P_k &= V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}) \\ Q_k &= V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}) \end{aligned}}$$

The admittance that connected bus <u>k to all other buses</u>



Characteristics of Power Flow Equations:-

* Power Flow Equations are Algebraic ((There is no driffative or driffrentiance))
  - Static System. because we use
* Power Flow Equations are Non-linear (sin, cos) (and multiplication)
  - Iterative Solution
* Relate P, Q in terms of V, δ and Y_{BUS} Elements
  - P, Q → f(V, δ)

Consumers Controlled by (شغب) (إضراب) and the Power System has no control on them

Characterization of Variables :-

* Load $(P_L, Q_L) \Rightarrow$ Uncontrolled (Disturbance) Variable.
* Generation $(P_G, Q_G) \Rightarrow$ Control Variable. ((depends on the Load)) — Economic dispatch problem
* Voltage $(V, \delta) \Rightarrow$ State Variable.

For a Given Operating Condition $\rightarrow$ Loads and Generations at all buses are known (Specified)

$\Rightarrow$ Find the Voltage Magnitude and Angle (V, δ) at each bus.

Problem in Power Flow $\rightarrow$

All generation Variables (P_G, Q_G) can not be specified as (بما) Losses are not known a priori.

why $\rightarrow$

Solution $\rightarrow$

Choose one bus as reference where Voltage Magnitude and angle are specified. The losses are assigned to this bus. This bus is called "Slack Bus".

Classification of Busbars :-

مراجعة مهمة

Each bus k is classified into one of the following three bus types :-

$\boxed{1}$ Swing Bus - There is only one swing bus, which for convenience (وسيلة للراحة) is numbered bus 1.

The swing bus is a reference bus for which $V_1 \angle \delta_1$, typically $1.0\angle 0^\circ$ per unit, is specified (input data).

The power-flow program computes P_1 and Q_1.

[2] Load bus - P_k and Q_k are specified (input data).

The power flow program computes V_k and δ_k.

* [3] Voltage Controlled bus - P_k and V_k are input data.

The power flow program computes Q_k and δ_k.

Examples are buses which generators, switched shunt capacitors, or static var system are connected.

Maximum and minimum var limits $Q_{Gk,max}$, $Q_{Gk,min}$ that this equipment can supply are also input data. Another examples is a bus to which a tap changing transformer is connected;

6.4 POWER FLOW SOLUTION

Power flow studies, commonly known as *load flow*, form an important part of power system analysis. They are necessary for planning, economic scheduling, and control of an existing system as well as planning its future expansion. The problem consists of determining the magnitudes and phase angle of voltages at each bus and active and reactive power flow in each line.

In solving a power flow problem, the system is assumed to be operating under balanced conditions and a single-phase model is used. Four quantities are associated with each bus. These are voltage magnitude $|V|$, phase angle δ, real power P, and reactive power Q. The system buses are generally classified into three types.

Slack bus One bus, known as *slack* or *swing bus*, is taken as reference where the magnitude and phase angle of the voltage are specified. This bus makes up the difference between the scheduled loads and generated power that are caused by the losses in the network.

Load buses At these buses the active and reactive powers are specified. The magnitude and the phase angle of the bus voltages are unknown. These buses are called P-Q buses.

Regulated buses These buses are the *generator buses*. They are also known as *voltage-controlled buses*. At these buses, the real power and voltage magnitude are specified. The phase angles of the voltages and the reactive power are to be determined. The limits on the value of the reactive power are also specified. These buses are called P-V buses.

6.4.1 POWER FLOW EQUATION

Consider a typical bus of a power system network as shown in Figure 6.7. Transmission lines are represented by their equivalent π models where impedances have been converted to per unit admittances on a common MVA base.

Application of KCL to this bus results in

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n \end{aligned} \quad (6.23)$$

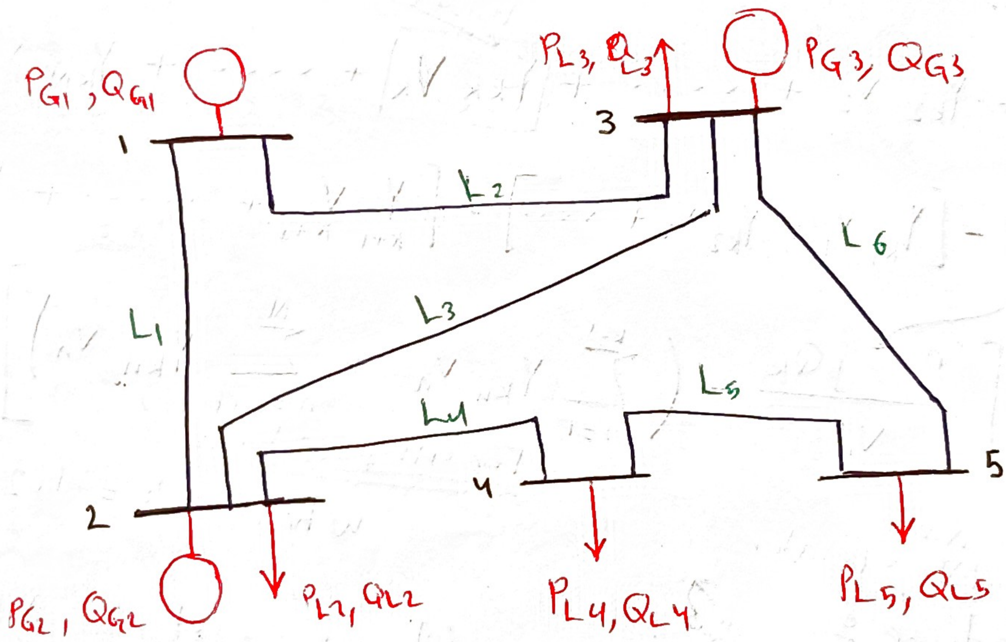
or

Classification of Busbars:-

1. Swing Bus (V_i, δ_i)
2. PV Bus (Voltage Control Bus)
3. PQ Bus (Load Bus)

With each bus i, 4 variables (P_i, Q_i, V_i, and δ_i) are associated. Depending on the type of bus two variables are specified (known) and two unknown variables are obtained from power flow solution.

5-Bus Power System



Bus Data

Bus	Type	V Per unit	δ deg	PG per unit	QG per unit	PL per unit	QL per unit	QGmax per unit	QGmin per unit
1	Swing	1.03	0	–	–				
2									
3									
4									
5									

Power Flow Solution by Gauss-Seidel Method

Non-linear, Algebraic ⟶ Iterative Solution

$$I_{BUS} = Y_{BUS} V_{BUS}$$

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n$$

$$S_k = P_k + jQ_k = V_k I_k^*$$

$$P_k + jQ_k = V_k \left[\sum_{n=1}^{N} Y_{kn} V_n \right]^* \quad , k = 1, 2, \ldots, N$$

$$(P_k + jQ_k)^* = V_k^* I_k$$

$$P_k - jQ_k = V_k^* I_k$$

$$I_k = \frac{P_k - jQ_k}{V_k^*}$$

$$I_k = \frac{P_k - jQ_k}{V_k^*} \quad , \text{ Also}$$

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n \quad , \text{or}$$

leave this

$$I_k = Y_{k1} V_1 + Y_{k2} V_2 + \cdots + \boxed{Y_{kk} V_k} + \cdots + Y_{kN} V_N$$

$$Y_{kk} V_k = I_k - [Y_{k1} V_1 + Y_{k2} V_2 + \cdots] - [Y_{k+1} V_{k+1} + \cdots + Y_{kN} V_N]$$

Find ?!

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n + \sum_{n=k+1}^{N} Y_{kn} V_n \right) \right]$$

For all P-Q buses

where $k = 1, 2, 3, \ldots N$

PQ, Slack bus (V, δ), ~~PV~~ (no) → later

Iterative Procedure :

1. Make an initial guess (تخمين) $|V_i|^{(0)}$ and $\delta_i^{(0)}$ (10-30)
   - Flat Start $|V_i|^{(0)} = 1.0$ and $\delta_i^{(0)} = 0.0$

2. Use this solution in PFE (Power Flow Eq) to obtain a better first solution.

3. First Solution is used to obtain a better second solution and so on.

$$V_k^{i+1} = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^{*}} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n^{i+1} + \sum_{n=k+1}^{N} Y_{kn} V_n^{i} \right) \right]$$

updating the buses values

$k = 1, 2, \ldots, N$, i is iteration count.

Continue iteration till $|V_k^{i+1} - V_k^{i}| \leq \varepsilon$

Algorithm Steps :-

1. With P_{gi}, Q_{gi}, P_{di}, and Q_{di} known Calculate bus injections P_i, Q_i
delivered

2. Form Y_{BUS} Matrix

3. Set initial voltage $V_i^{(0)}$, $\delta_i^{(0)}$

4. Iteratively solve equation

updating value

the value which is available

$$V_k^{i+1} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^*} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n^{i+1} + \sum_{n=k+1}^{N} Y_{kn} V_n^{i}\right)\right]$$

to obtain new values of bus voltages.

Algorithm Modification when PV Buses are also Present

$$Q_i = -I_m\left[\underbrace{V_i^*}\ \underbrace{\sum_{k=1}^{n} Y_{ik} V_k}_{I}\right]$$

$P_k + jQ_k = V_k I^*$

$P_k - jQ_k = V_k^* I_k$

$Q_k = -\,Im[V_k^* I_k]$

$$Q_i^{(r+1)} = -\,Im\left[(V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + (V_i^{(r)})^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)}\right]$$

The revised value of δ_i is obtained from immediately following step 1. Thus

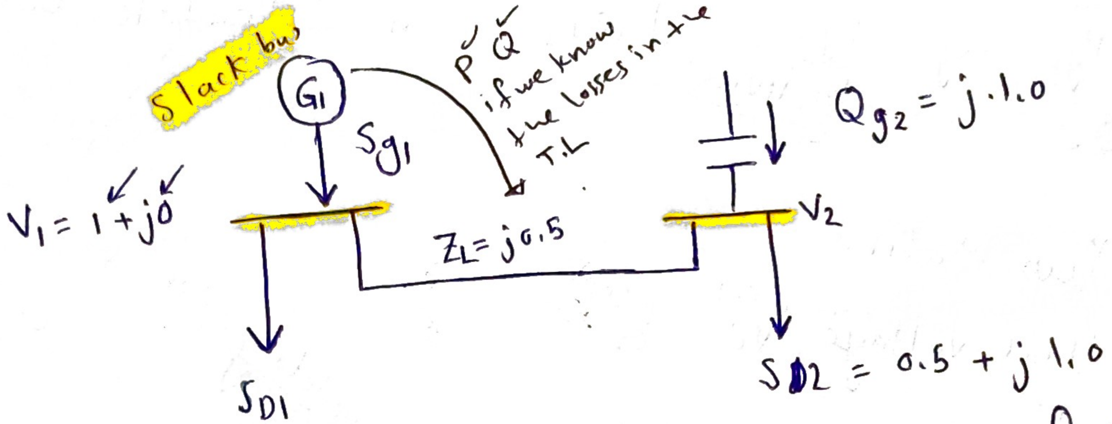
$$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$$

$$= \text{Angle of}\left[\frac{A_i^{(r+1)}}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)}\right]$$

Where $A_i^{(r+1)} = \dfrac{P_i - jQ_i^{(r+1)}}{Y_{ii}}$

The algorithm for PQ buses remains unchanged.

Example:- For the system shown, $Z_L = j0.5$, $V_1 = 1\angle 0^\circ$, $S_{G2} = j1.0$ and $S_{D2} = 0.5 + j1.0$. Find V_2 using Gauss-Seidel iteration technique.



Solution: Firstly, we calculate the elements of the Y_{BUS}

For $Z_L = j0.5$, we have

$Y_{11} = -j2$

$Y_{12} = j2 = -y_{12}$

$Y_{21} = j2 = -y_{21}$

$Y_{22} = -j2$

We iterate on V_2 using the equation given

$$V_2^{n+1} = \frac{1}{Y_{22}}\left[S_2^* / (V_2^n)^* - Y_{21} * V_1\right] \quad \text{---- (1)}$$

Given $V_1 = 1\angle 0^\circ$

$S_2 = S_{G2} - S_{D2} = -0.5$

Putting the values of V_1, S_2, Y_{22} and Y_{21} in equation (1), we get

$$V_2^{n+1} = -j\left[0.25 / (V_2^n)^*\right] + 1.0 \quad \text{--- (2)}$$

Start with a guess, taking $V_2^0 = 1\angle 0^\circ$ and iterate using equation (2).

We have, $V_2^0 = 1 + j0$

Putting in equation (2), and iterating for V_2, we get

$$V_2^1 = -j[0.25/(1+j0)^*] + 1.0$$

$$= 1.0 - j0.25$$

$$V_2^1 = 1.030776\angle -14.036243^\circ$$

$$V_2^2 = -j[0.25/(1.0-j0.25)^*] + 1.0$$

$$= 1.0 - j0.25/(1.0+j0.25)^*]$$

$$= 1.0/(1.0+j0.25)$$

$$= 0.970143\angle -14.036249^\circ$$

Similarly, we can iterate it further. The results of the iteration are tabulated below

① V_1 ✓ V_2 ✓ Y ✓
② ⇓ I ✓
③ ⇓ loss ✓
④ P, Q ✓

	Iteration #	V_2
	0	$1\angle 0^\circ$
0.030776	1	$1.030776\angle -14.036243^\circ$
0.060633	2	$0.970143\angle -14.036249^\circ$
0.000118	3	$0.970261\angle -14.931409^\circ$
0.004026	4	$0.966235\angle -14.931416^\circ$
0.000001	5	$0.966236\angle -14.995078^\circ$
0.000756	6	$0.965948\angle -14.995072^\circ$

Since, the difference in the values for the voltage doesn't change much between 5th and 6th iteration, we can stop after the 6th. Hence, we can see that starting with the value $V_2 = 1\angle 0^\circ$, convergence is reached in six steps.

6

unit. The scheduled loads at buses 2 and 3 are as marked on the diagram. Line impedances are marked in per unit on a 100-MVA base and the line charging susceptances are neglected.

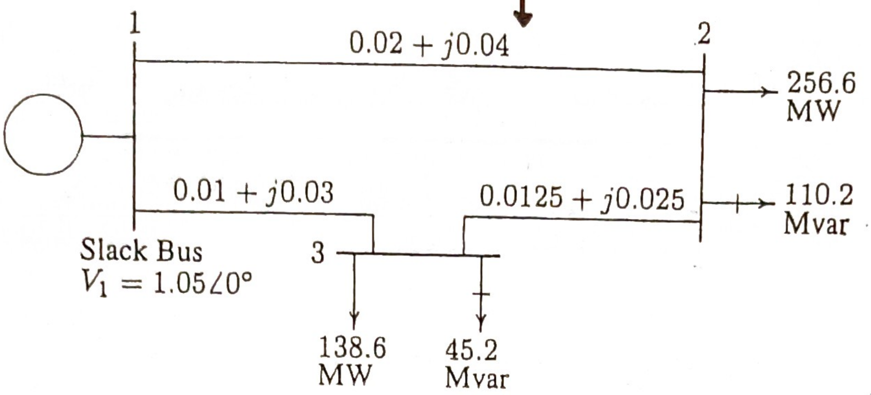


FIGURE 6.9
One-line diagram of Example 6.7 (impedances in pu on 100-MVA base).

(a) Using the Gauss-Seidel method, determine the phasor values of the voltage at the load buses 2 and 3 (P-Q buses) accurate to four decimal places.
(b) Find the slack bus real and reactive power.
(c) Determine the line flows and line losses. Construct a power flow diagram showing the direction of line flow.

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The admittances are marked on the network shown in Figure 6.10.
At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(256.6 + j110.2)}{100} = -2.566 - j1.102 \quad \text{pu}$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452 \quad \text{pu}$$

Since the actual admittances are readily available in Figure 6.10, for hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

$$\boxed{V_k^{i+1} = \frac{1}{Y_{KK}}\left[\frac{P_k - jQ_k}{V_k^{*(i)}} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n^{i+1} + \sum_{n=k+1}^{N} Y_{kn} V_n^{i}\right)\right]}$$

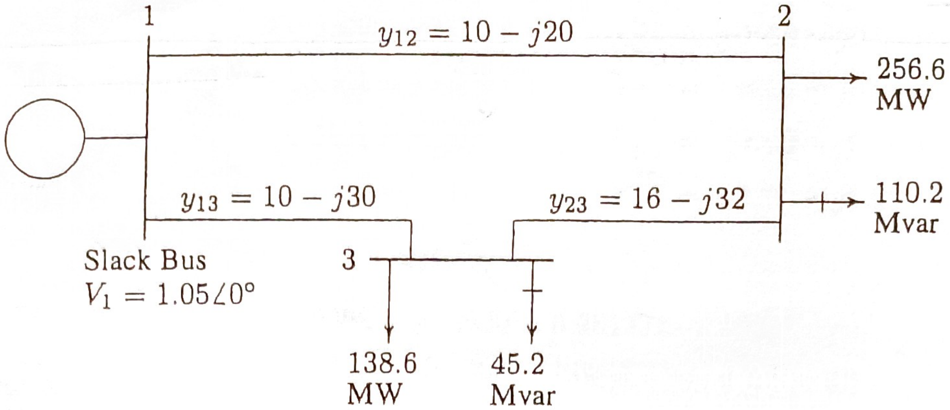


FIGURE 6.10
One-line diagram of Example 6.7 (admittances in pu on 100-MVA base).

$$V_2^{(1)} = \frac{\frac{-2.566+j1.102}{1.0-j0} + (10-j20)(1.05+j0) + (16-j32)(1.0+j0)}{(26-j52)}$$

$$= \underline{0.9825} - j\underline{0.0310}$$

and $\hookrightarrow$ To four decimal places.

$$V_3^{(1)} = \frac{\frac{P_3^{sch}-jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13}+y_{23}}$$

$$= \frac{\frac{-1.386+j0.452}{1-j0} + (10-j30)(1.05+j0) + (16-j32)(0.9825-j0.0310)}{(26-j62)}$$

$$= 1.0011 - j0.0353$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-2.566+j1.102}{0.9825+j0.0310} + (10-j20)(1.05+j0) + (16-j32)(1.0011-j0.0353)}{(26-j52)}$$

$$= 0.9816 - j0.0520$$

and

$$V_3^{(2)} = \frac{\frac{-1.386+j0.452}{1.0011+j0.0353} + (10-j30)(1.05+j0) + (16-j32)(0.9816-j0.052)}{(26-j62)}$$

$$= 1.0008 - j0.0459$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} per unit in seven iterations as given below.

$$V_2^{(3)} = 0.9808 - j0.0578 \qquad V_3^{(3)} = 1.0004 - j0.0488$$

$$V_2^{(4)} = 0.9803 - j0.0594 \qquad V_3^{(4)} = 1.0002 - j0.0497$$
$$V_2^{(5)} = 0.9801 - j0.0598 \qquad V_3^{(5)} = 1.0001 - j0.0499$$
$$V_2^{(6)} = 0.9801 - j0.0599 \qquad V_3^{(6)} = 1.0000 - j0.0500$$
$$V_2^{(7)} = 0.9800 - j0.0600 \qquad V_3^{(7)} = 1.0000 - j0.0500$$

The final solution is

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle{-3.5035^\circ} \quad \text{pu}$$
$$V_3 = 1.0000 - j0.0500 = 1.00125\angle{-2.8624^\circ} \quad \text{pu}$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.05[1.05(20 - j50) - (10 - j20)(0.98 - j.06) - \\ &\quad (10 - j30)(1.0 - j0.05)] \\ &= 4.095 - j1.890 \end{aligned}$$

or the slack bus real and reactive powers are $P_1 = 4.095$ pu $= 409.5$ MW and $Q_1 = 1.890$ pu $= 189$ Mvar.

(c) To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1.05 + j0) - (0.98 - j0.06)] = 1.9 - j0.8$$
$$I_{21} = -I_{12} = -1.9 + j0.8$$
$$I_{13} = y_{13}(V_1 - V_3) = (10 - j30)[(1.05 + j0) - (1.0 - j0.05)] = 2.0 - j1.0$$
$$I_{31} = -I_{13} = -2.0 + j1.0$$
$$I_{23} = y_{23}(V_2 - V_3) = (16 - j32)[(0.98 - j0.06) - (1 - j0.05)] = -.64 + j.48$$
$$I_{32} = -I_{23} = 0.64 - j0.48$$

The line flows are

$$\begin{aligned} S_{12} = V_1 I_{12}^* &= (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84 \ \text{pu} \\ &= 199.5 \ \text{MW} + j84.0 \ \text{Mvar} \\ S_{21} = V_2 I_{21}^* &= (0.98 - j0.06)(-1.9 - j0.8) = -1.91 - j0.67 \ \text{pu} \\ &= -191.0 \ \text{MW} - j67.0 \ \text{Mvar} \\ S_{13} = V_1 I_{13}^* &= (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05 \ \text{pu} \\ &= 210.0 \ \text{MW} + j105.0 \ \text{Mvar} \end{aligned}$$

$$S_{31} = V_3 I_{31}^* = (1.0 - j0.05)(-2.0 - j1.0) = -2.05 - j0.90 \text{ pu}$$
$$= -205.0 \text{ MW} - j90.0 \text{ Mvar}$$
$$S_{23} = V_2 I_{23}^* = (0.98 - j0.06)(-0.656 + j0.48) = -0.656 - j0.432 \text{ pu}$$
$$= -65.6 \text{ MW} - j43.2 \text{ Mvar}$$
$$S_{32} = V_3 I_{32}^* = (1.0 - j0.05)(0.64 + j0.48) = 0.664 + j0.448 \text{ pu}$$
$$= 66.4 \text{ MW} + j44.8 \text{ Mvar}$$

and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 8.5 \text{ MW} + j17.0 \text{ Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 5.0 \text{ MW} + j15.0 \text{ Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.8 \text{ MW} + j1.60 \text{ Mvar}$$

The power flow diagram is shown in Figure 6.11, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

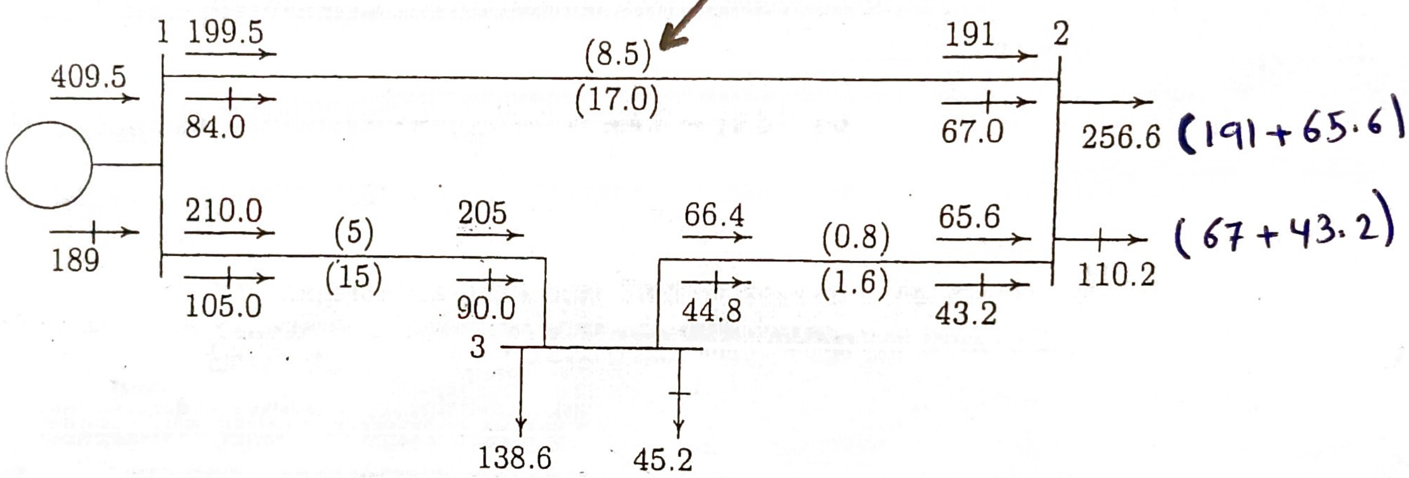


FIGURE 6.11
Power flow diagram of Example 6.7 (powers in MW and Mvar).

Example 6.8 (chp6ex8)

Figure 6.12 shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

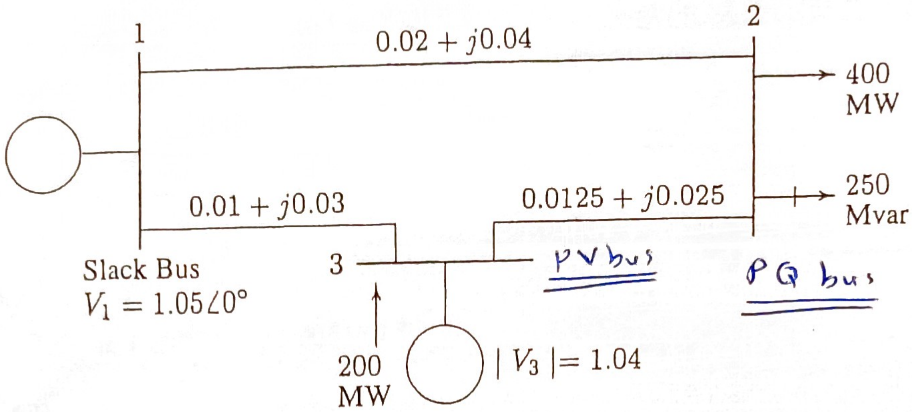


FIGURE 6.12
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The load and generation expressed in per units are

$$\text{(Load)}\quad S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$\text{(gen.)}\quad P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.04 + j0.0$, V_2 and V_3 are computed from (6.28).

$$\begin{aligned}
V_2^{(1)} &= \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\
&= \frac{\frac{-4.0 + j2.5}{1.0 - j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.04 + j0)}{(26 - j52)} \\
&= 0.97462 - j0.042307
\end{aligned}$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$\begin{aligned}
Q_3^{(1)} &= -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\
&= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - \\
&\quad (16 - j32)(0.97462 - j0.042307)]\} \\
&= 1.16
\end{aligned}$$

$$Q_i^{(r+1)} = -\mathrm{Im}\left[(V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + (V_i^{(r)})^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)} \right]$$

$$Q_i = -\mathrm{Im}\left[V_i^* \underbrace{\sum_{k=1}^{n} Y_{ik} V_k}_{I} \right]$$

The value of $Q_3^{(1)}$ is used as Q_3^{sch} for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{2.0 - j1.16}{1.04 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.97462 - j0.042307)}{(26 - j62)}$$

$$= 1.03783 - j0.005170$$ ثبت

غير

يحتفظ به

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = -0.005170$, and its real part is obtained from

$|V_3| = 1.04$

$| \ |^2 = (\text{real})^2 + (\text{Img})^2$

$\text{real} = \sqrt{| \ |^2 - (\text{Img})^2}$

$$\text{real part} = e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$$

Thus

$$V_3^{(1)} = 1.039987 - j0.005170$$

For the second iteration, we have

$$V_2^{(2)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}}$$

$$= \frac{\frac{-4.0 + j2.5}{.97462 + j.042307} + (10 - j20)(1.05) + (16 - j32)(1.039987 + j0.005170)}{(26 - j52)}$$

$$= 0.971057 - j0.043432$$

$$Q_3^{(2)} = -\Im\{V_3^{*(1)}[V_3^{(1)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$

$$= -\Im\{(1.039987 + j0.005170)[(1.039987 - j0.005170)(26 - j62) - (10 - j30)(1.05 + j0) - (16 - j32)(0.971057 - j0.043432)]\}$$

$$= 1.38796$$

$$V_{c3}^{(2)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{2.0 - j1.38796}{1.039987 + j0.00517} + (10 - j30)(1.05) + (16 - j32)(.971057 - j.043432)}{(26 - j62)}$$

$$= 1.03908 - j0.00730$$

تغير ثبت

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = -0.00730$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2} = 1.039974$$

or

$$V_3^{(2)} = 1.039974 - j0.00730$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} pu in seven iterations as given below.

$$
\begin{array}{lll}
V_2^{(3)} = 0.97073 - j0.04479 & Q_3^{(3)} = 1.42904 & V_3^{(3)} = 1.03996 - j0.00833 \\
V_2^{(4)} = 0.97065 - j0.04533 & Q_3^{(4)} = 1.44833 & V_3^{(4)} = 1.03996 - j0.00873 \\
V_2^{(5)} = 0.97062 - j0.04555 & Q_3^{(5)} = 1.45621 & V_3^{(5)} = 1.03996 - j0.00893 \\
V_2^{(6)} = 0.97061 - j0.04565 & Q_3^{(6)} = 1.45947 & V_3^{(6)} = 1.03996 - j0.00900 \\
V_2^{(7)} = 0.97061 - j0.04569 & Q_3^{(7)} = 1.46082 & V_3^{(7)} = 1.03996 - j0.00903
\end{array}
$$

The final solution is

$$V_2 = 0.97168\angle{-2.6948^\circ} \quad \text{pu}$$

$$S_3 = 2.0 + j1.4617 \quad \text{pu}$$
$$V_3 = 1.04\angle{-.498^\circ} \quad \text{pu}$$
$$S_1 = 2.1842 + j1.4085 \quad \text{pu}$$

Line flows and line losses are computed as in Example 6.7, and the results expressed in MW and Mvar are

$$
\begin{array}{lll}
S_{12} = 179.36 + j118.734 & S_{21} = -170.97 - j101.947 & S_{L\,12} = 8.39 + j16.79 \\
S_{13} = 39.06 + j22.118 & S_{31} = -38.88 - j\,21.569 & S_{L\,13} = 0.18 + j0.548 \\
S_{23} = -229.03 - j148.05 & S_{32} = 238.88 + j167.746 & S_{L\,23} = 9.85 + j19.69
\end{array}
$$

The power flow diagram is shown in Figure 6.13, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

* for slack bus.

$$P_1 - jQ_1 = V_1^* \left[V_1 (y_{12} + y_{13}) - (y_{12} V_2 + y_{13} V_3) \right]$$

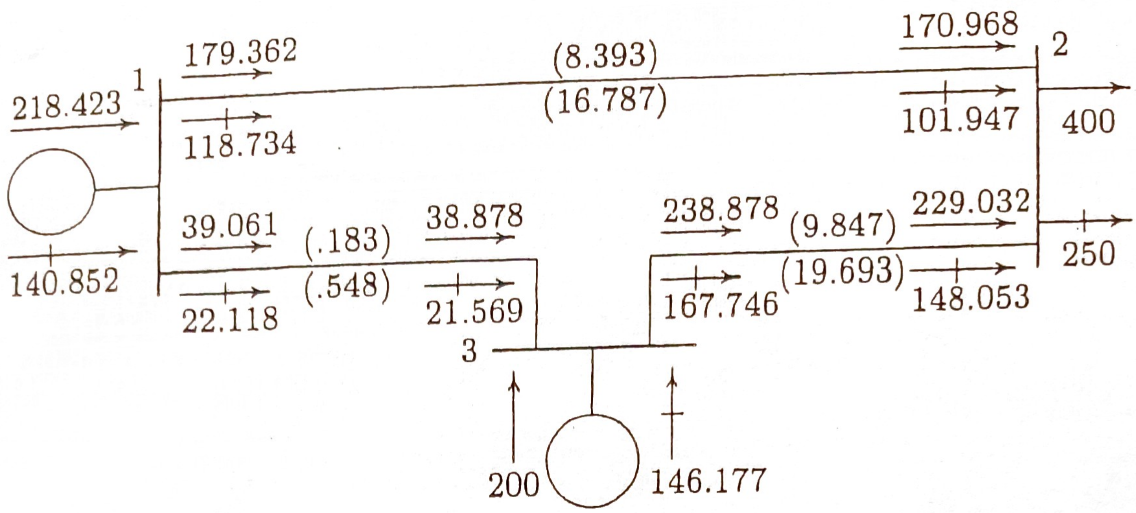


FIGURE 6.13
Power flow diagram of Example 6.8 (powers in MW and Mvar).

6.7 TAP CHANGING TRANSFORMERS

In Section 2.6 it was shown that the flow of real power along a transmission line is determined by the angle difference of the terminal voltages, and the flow of reactive power is determined mainly by the magnitude difference of terminal voltages. Real and reactive powers can be controlled by use of tap changing transformers and regulating transformers.

In a tap changing transformer, when the ratio is at the nominal value, the transformer is represented by a series admittance y_t in per unit. With off-nominal ratio, the per unit admittance is different from both sides of the transformer, and the admittance must be modified to include the effect of the off-nominal ratio. Consider a transformer with admittance y_t in series with an ideal transformer representing the off-nominal tap ratio 1:a as shown in Figure 6.14. y_t is the admittance in per unit based on the nominal turn ratio and a is the per unit off-nominal tap position allowing for small adjustment in voltage of usually ± 10 percent. In the case of phase shifting transformers, a is a complex number. Consider a fictitious bus x between the turn ratio and admittance of the transformer. Since the complex power on either side of the ideal transformer is the same, it follows that if the voltage goes through a positive phase angle shift, the current will go through a negative phase angle shift. Thus, for the assumed direction of currents, we have

$$V_x = \frac{1}{a}V_j \tag{6.43}$$

$$I_i = -a^* I_j \tag{6.44}$$

The current I_i is given by

$$I_i = y_t(V_i - V_x)$$

Balanced and Unbalanced Faults

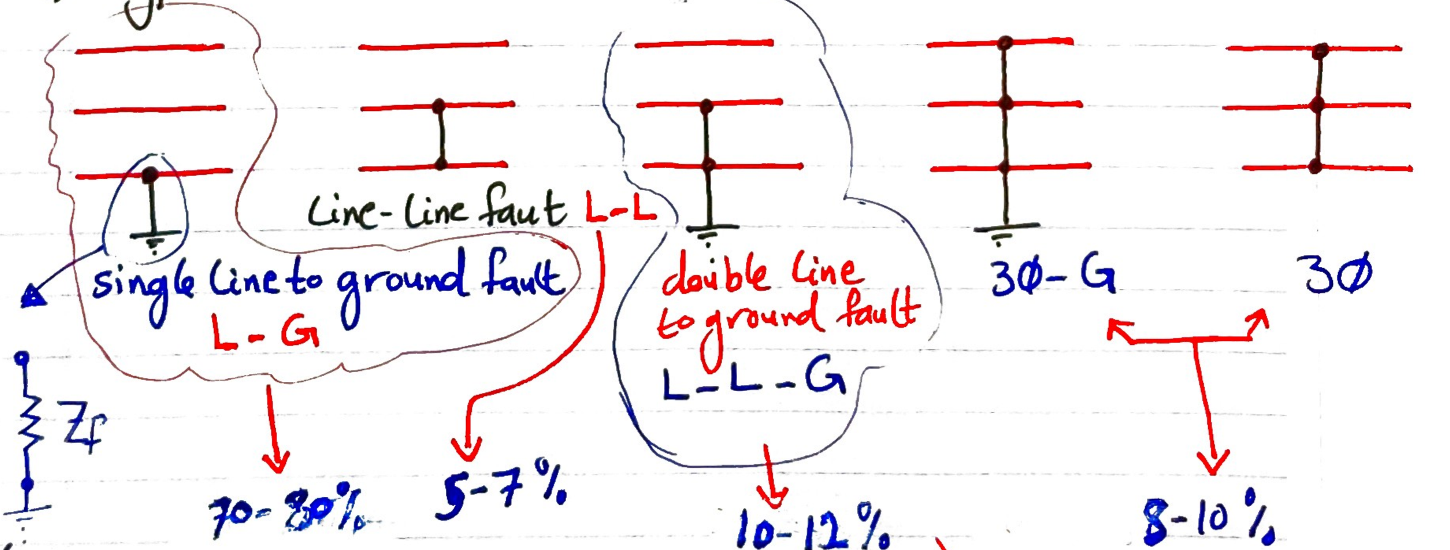
Fault Analysis

» An essential part of a power network is the calculation of the currents which flow in the components when faults of various types occur.

» In a fault survey, faults are applied at various points in the network and the resulting currents obtained by hand calculation, or, most likely now on large networks, by computer softwares

» The magnitude of the fault currents give the engineer the current settings for the protection to be used and the ratings of the circuit breakers

» Types of Short Circuit:



Asymmetrical Faults
Unbalanced Faults

Symmetrical Faults
Balanced Faults

if $Z_f = 0 \Rightarrow$ Solid Fault, Bolted Fault

» The most common of these faults is the short circuit of a single phase to ground fault.

» Often the path to ground contains resistance in the form of an arc as shown in the previous figure.

» Although the single line to ground fault is the most common, calculations are frequently performed to 3ϕ faults.

» 3ϕ faults (Balanced faults) are the most severe fault and easy to calculate.

» The problem consists of determining bus voltages and line currents during various types of faults.

9.2 BALANCED THREE-PHASE FAULT

1) This type of fault is defined as the simultaneous short circuit across all three phases. It occurs infrequently, but it is the most severe type of fault encountered. Because the network is balanced, it is solved on a per-phase basis. The other two phases carry identical currents except for the phase shift.

3) . the reactance of the synchronous generator under short-circuit conditions is a time-varying quantity, and for network analysis three reactances were defined. a The subtransient reactance X_d'', for the first few cycles of the short circuit current, b transient reactance X_d', for the next (say) 30 cycles, and c the synchronous reactance X_d, thereafter. Since the duration of the short circuit current depends on the time of operation of the protective system, it is not always easy to decide which reactance to use. Generally, the subtransient reactance is used for determining the interrupting capacity of the circuit breakers. In fault studies required for relay setting and coordination, transient reactance is used. Also, in typical transient stability studies, transient reactance is used.

2) A fault represents a structural network change equivalent with that caused by the addition of an impedance at the place of fault. If the fault impedance is zero, the fault is referred to as the *bolted fault* or the *solid fault*. The faulted network can be solved conveniently by the Thévenin's method. The procedure is demonstrated in the following example.

Example 9.1 (chp9ex1)

The one-line diagram of a simple three-bus power system is shown in Figure 9.1. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common 100 MVA base, and for simplicity, resistances are neglected. The following assumptions are made.

(*i*) Shunt capacitances are neglected and the system is considered on no-load.

(*ii*) All generators are running at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current, the bus voltages, and the line currents during the fault when a balanced three-phase fault with a fault impedance $Z_f = 0.16$ per unit occurs on

(a) Bus 3.
(b) Bus 2.
(c) Bus 1.

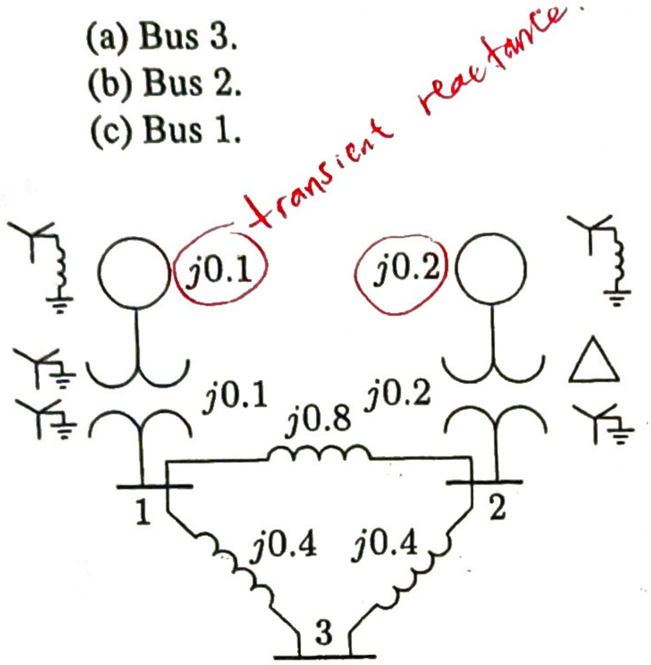


FIGURE 9.1
The impedance diagram of a simple power system.

The fault is simulated by switching on an impedance Z_f at bus 3 as shown in Figure 9.2(a). Thévenin's theorem states that the changes in the network voltage caused by the added branch (the fault impedance) shown in Figure 9.2(a) is equivalent to those caused by the added voltage $V_3(0)$ with all other sources short-circuited as shown in Figure 9.2(b).

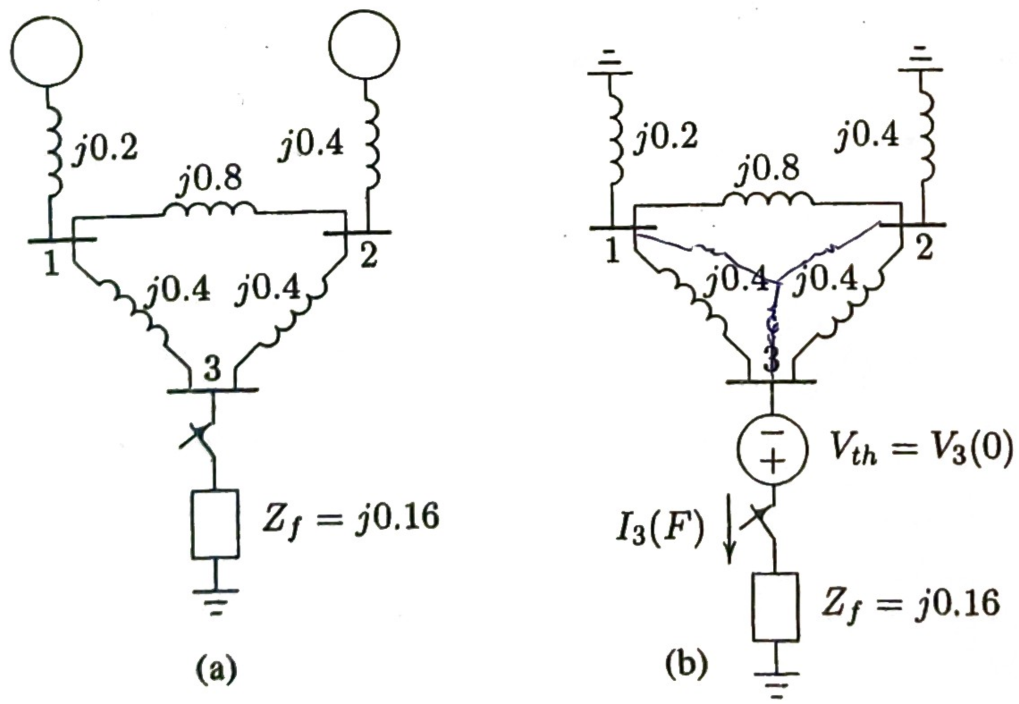


FIGURE 9.2
(a) The impedance network for fault at bus 3. (b) Thévenin's equivalent network.

(a) From 9.2(b), the fault current at bus 3 is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f}$$

where $V_3(0)$ is the Thévenin's voltage or the prefault bus voltage. The prefault bus voltage can be obtained from the results of the power flow solution. In this example, since the loads are neglected and generator's emfs are assumed equal to the rated value, all the prefault bus voltages are equal to 1.0 per unit, i.e.,

$$V_1(0) = V_2(0) = V_3(0) = 1.0 \quad \text{pu}$$

Z_{33} is the Thévenin's impedance viewed from the faulted bus.

To find the Thévenin's impedance, we convert the Δ formed by buses 123 to an equivalent Y as shown in Figure 9.3(a).

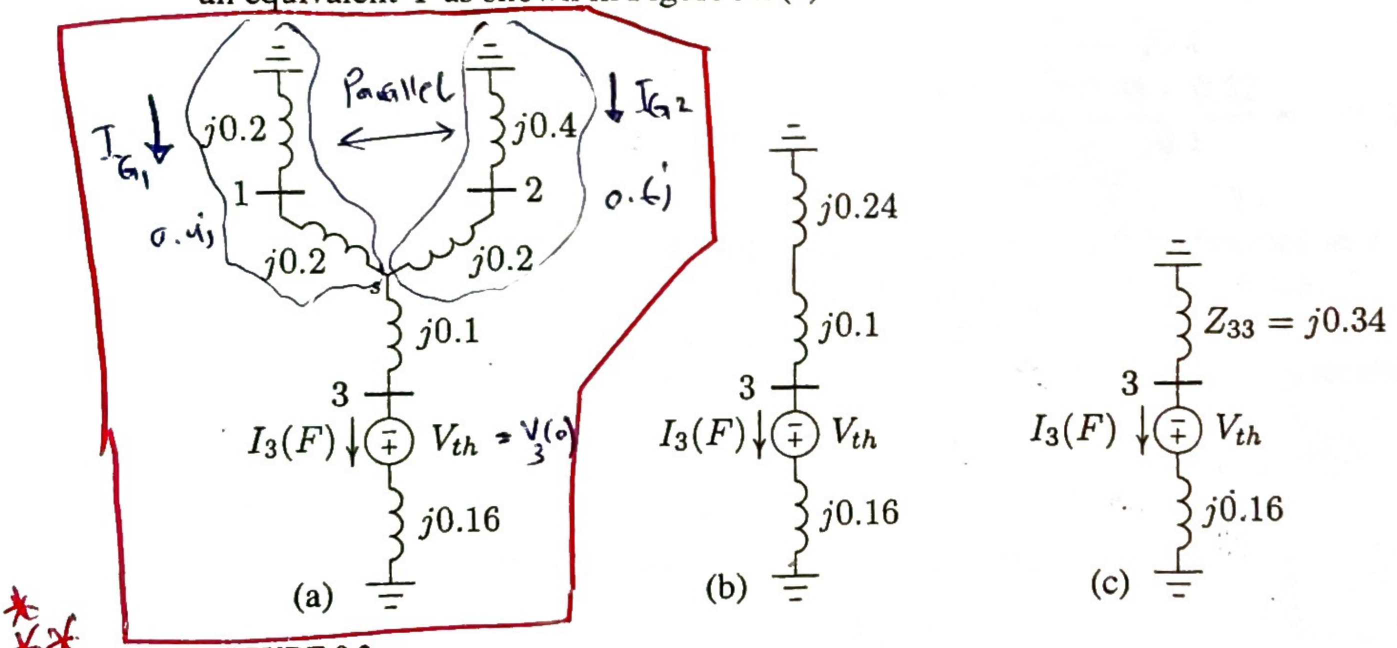


FIGURE 9.3
Reduction of Thévenin's equivalent network.

$$Z_{1s} = Z_{2s} = \frac{(j0.4)(j0.8)}{j1.6} = j0.2 \qquad Z_{3s} = \frac{(j0.4)(j0.4)}{j1.6} = j0.1$$

Combining the parallel branches, Thévenin's impedance is

$$\begin{aligned} Z_{33} &= \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1 \\ &= j0.24 + j0.1 = j0.34 \end{aligned}$$

From Figure 9.3(c), the fault current is

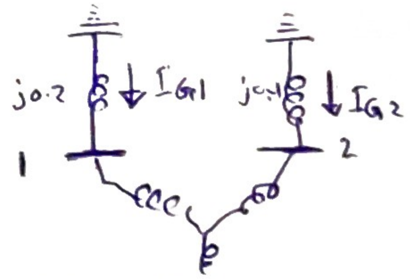
1) $$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

With reference to Figure 9.3(a), the current divisions between the two generators are

2)

$$I_{G1} = \frac{j0.6}{j0.4 + j0.6} I_3(F) = -j1.2 \quad \text{pu}$$
$$I_{G2} = \frac{j0.4}{j0.4 + j0.6} I_3(F) = -j0.8 \quad \text{pu}$$

For the bus voltage changes from Figure 9.3(b), we get



3)

$$\Delta V_1 = 0 - (j0.2)(-j1.2) \quad = -0.24 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j0.8) \quad = -0.32 \quad \text{pu}$$
$$\Delta V_3 = (j0.16)(-j2) - 1.0 = -0.68 \quad \text{pu}$$

4) * The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.2(b), i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.24 = 0.76 \quad \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.32 = 0.68 \quad \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.68 = 0.32 \quad \text{pu}$$

5) * The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$

(b) The fault with impedance Z_f at bus 2 is depicted in Figure 9.4(a), and its Thévenin's equivalent circuit is shown in Figure 9.4(b). To find the Thévenin's impedance, we combine the parallel branches in Figure 9.4(b). Also, combining parallel branches from ground to bus 2 in Figure 9.5(a), results in

$$Z_{22} = \frac{(j0.6)(j0.4)}{j0.6 + j0.4} = j0.24$$

From Figure 9.5(b), the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.24 + j0.16} = -j2.5 \quad \text{pu}$$

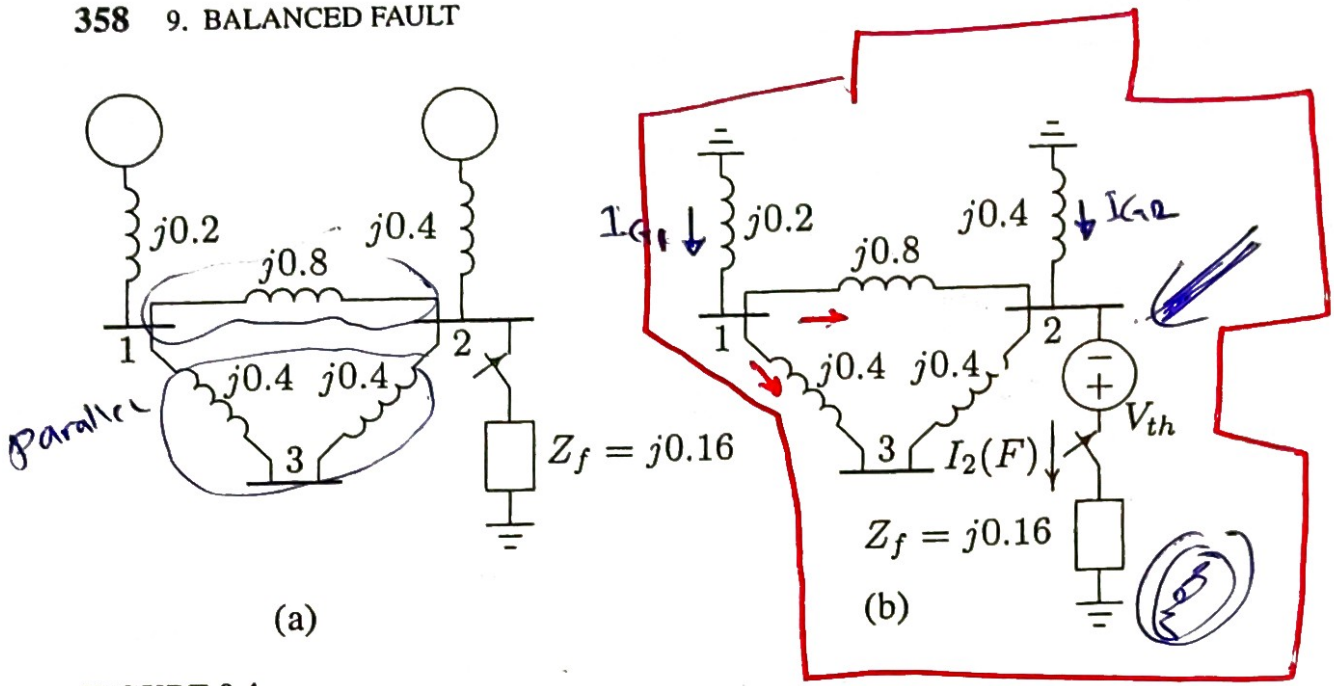


FIGURE 9.4
(a) The impedance network for fault at bus 2. (b) Thévenin's equivalent network.

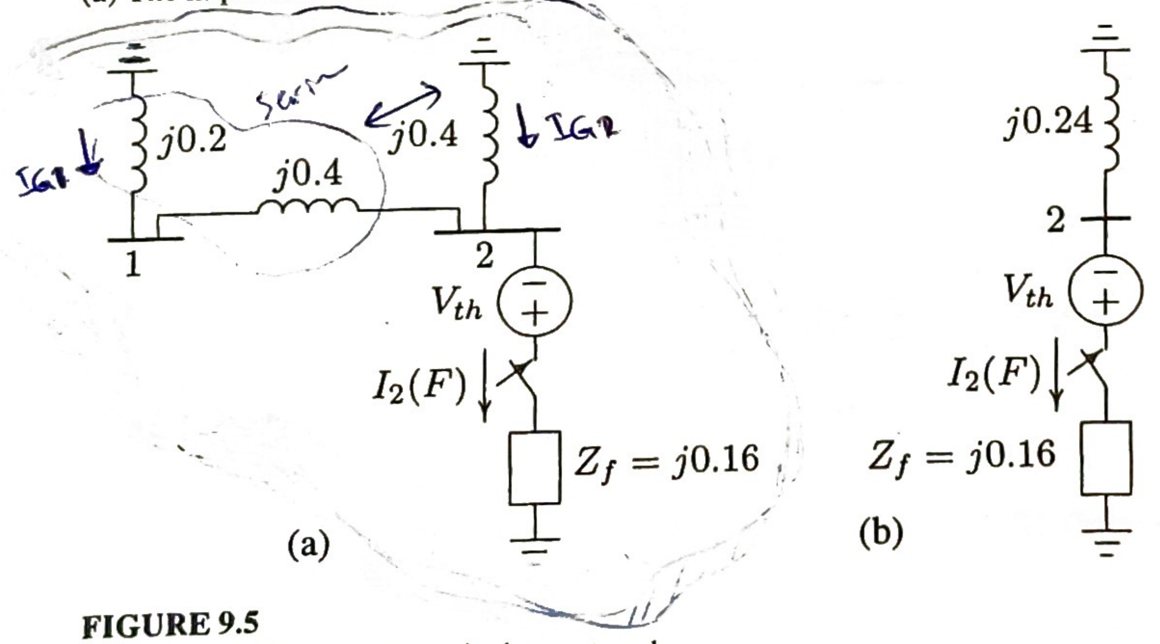


FIGURE 9.5
Reduction of Thévenin's equivalent network.

With reference to Figure 9.5(a), the current divisions between the generators are

$$I_{G1} = \frac{j0.4}{j0.4 + j0.6} I_2(F) = -j1.0 \quad \text{pu}$$

$$I_{G2} = \frac{j0.6}{j0.4 + j0.6} I_2(F) = -j1.5 \quad \text{pu}$$

For the bus voltage changes from Figure 9.4(a), we get

$$\Delta V_1 = 0 - (j0.2)(-j1.0) = -0.2 \quad \text{pu}$$

$$\Delta V_2 = 0 - (j0.4)(-j1.5) = -0.6 \quad \text{pu}$$

$$\Delta V_3 = -0.2 - (j0.4)\left(\frac{-j1.0}{2}\right) = -0.4 \quad \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.4(b), i.e.,

$$
\begin{aligned}
V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.2 = 0.8 \quad \text{pu} \\
V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.6 = 0.4 \quad \text{pu} \\
V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.4 = 0.6 \quad \text{pu}
\end{aligned}
$$

The short circuit-currents in the lines are

$$
\begin{aligned}
I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.8 - 0.4}{j0.8} = -j0.5 \quad \text{pu} \\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.8 - 0.6}{j0.4} = -j0.5 \quad \text{pu} \\
I_{32}(F) &= \frac{V_3(F) - V_3(F)}{z_{32}} = \frac{0.6 - 0.4}{j0.4} = -j0.5 \quad \text{pu}
\end{aligned}
$$

(c) The fault with impedance Z_f at bus 1 is depicted in Figure 9.6(a), and its Thévenin's equivalent circuit is shown in Figure 9.6(b).

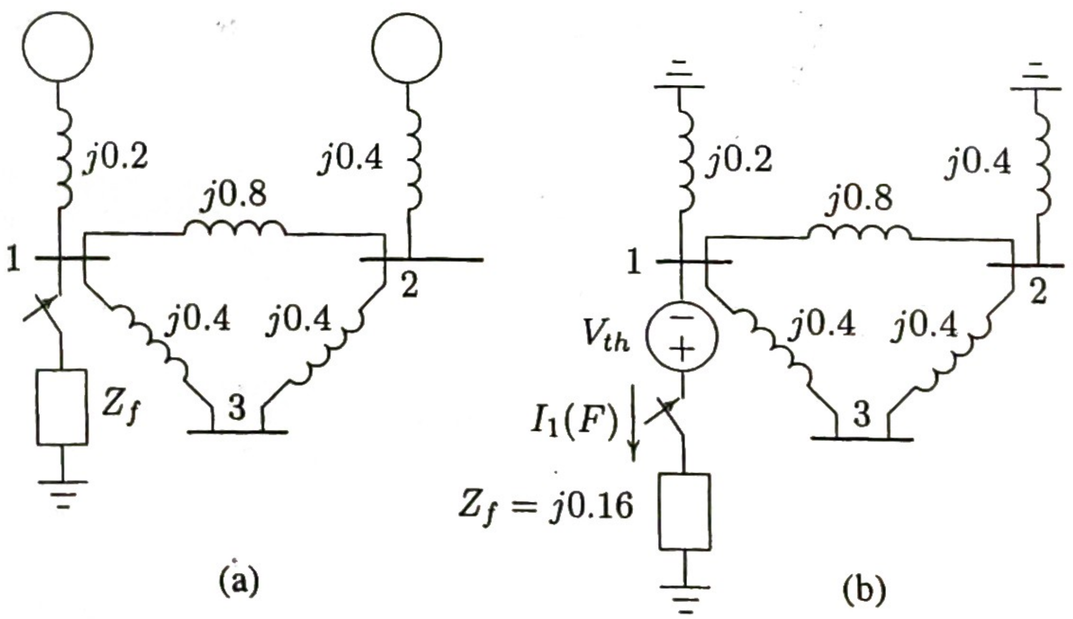


FIGURE 9.6
(a) The impedance network for fault at bus 1. (b) Thévenin's equivalent network.

To find the Thévenin's impedance, we combine the parallel branches in Figure 9.6(b). Also, combining parallel branches from ground to bus 1 in Figure 9.7(a),

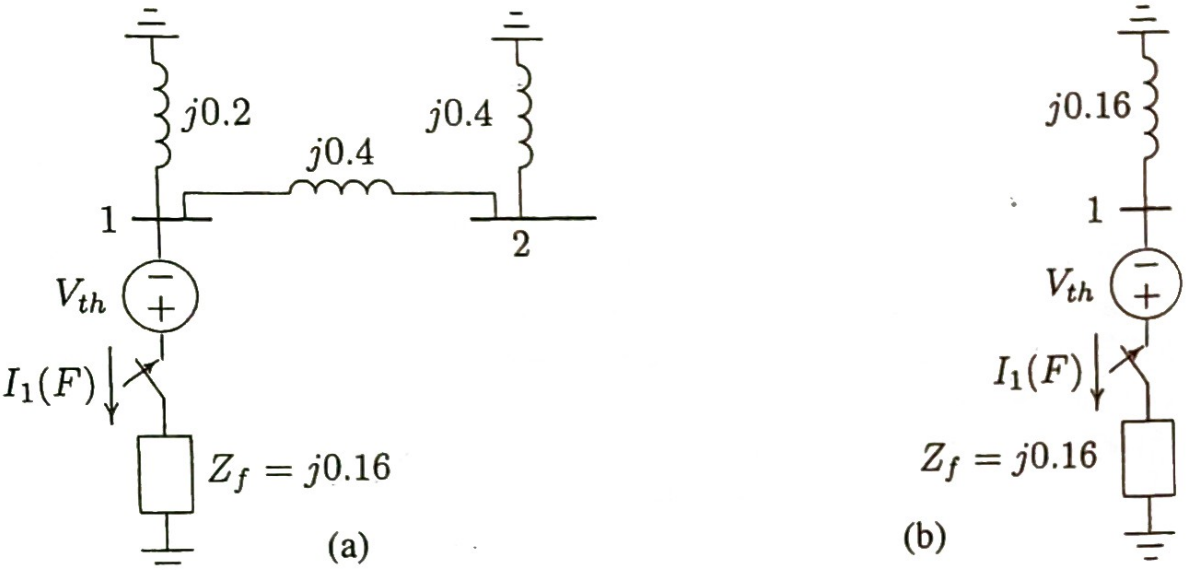


FIGURE 9.7
Reduction of Thévenin's equivalent network.

results in

$$Z_{11} = \frac{(j0.2)(j0.8)}{j0.2 + j0.8} = j0.16$$

From Figure 9.7(b), the fault current is

$$I_1(F) = \frac{V_1(0)}{Z_{11} + Z_f} = \frac{1.0}{j0.16 + j0.16} = -j3.125 \quad \text{pu}$$

With reference to Figure 9.7(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.8}{j0.2 + j0.8} I_2(F) = -j2.50 \quad \text{pu}$$
$$I_{G2} = \frac{j0.2}{j0.2 + j0.8} I_2(F) = -j0.625 \quad \text{pu}$$

For the bus voltage changes from Figure 9.6(b), we get

$$\Delta V_1 = 0 - (j0.2)(-j2.5) = -0.50 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j0.625) = -0.25 \quad \text{pu}$$
$$\Delta V_3 = -0.5 + (j0.4)\left(\frac{-j0.625}{2}\right) = -0.375 \quad \text{pu}$$

Bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.6(b), i.e.,

$$\begin{aligned} V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.50 = 0.50 \quad \text{pu} \\ V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.25 = 0.75 \quad \text{pu} \\ V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.375 = 0.625 \quad \text{pu} \end{aligned}$$

The short-circuit currents in the lines are

$$\begin{aligned} I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{21}} = \frac{0.75 - 0.5}{j0.8} = -j0.3125 \quad \text{pu} \\ I_{31}(F) &= \frac{V_3(F) - V_1(F)}{z_{31}} = \frac{0.625 - 0.5}{j0.4} = -j0.3125 \quad \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.75 - 0.625}{j0.4} = -j0.3125 \quad \text{pu} \end{aligned}$$

Notes :-

1) In the above example the load currents were neglected and all prefault bus voltages were assumed to be equal to 1.0 per unit. For more accurate calculation, the prefault bus voltages can be obtained from the power flow solution. ~~As we have seen in Chapter 6, in a power system, loads are specified and the load currents are unknown.~~ One way to include the effects of load currents in the fault analysis is to express the loads by a constant impedance evaluated at the prefault bus voltages. ~~This is a very good approximation which results in linear nodal equations.~~ The procedure is summarized in the following steps.

- The prefault bus voltages are obtained from the results of the power flow solution.
- In order to preserve the linearity feature of the network, loads are converted to constant admittances using the prefault bus voltages.
- The faulted network is reduced into a Thévenin's equivalent circuit as viewed from the faulted bus. Applying Thévenin's theorem, changes in the bus voltages are obtained.
- Bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages computed in the previous step.
- The currents during the fault in all branches of the network are then obtained.

~~Short-~~ Short-Circuit Capacity (SCC):-

The SCC at a bus is a common measure of the strength of a bus.

The SCC or the short-circuit MVA at bus k is defined as the product of the magnitude of the rated bus voltage and the fault current.

$$SCC = \sqrt{3}\, V_{LK}\, I_K(F) * 10^{-3}\ \text{MVA}$$

$I_K(F)$ → expressed in amp.

V_{LK} → the line-to-line voltage expressed in kV

But

$$I_K(F) = I_K(F)_{pu} * I_B$$

$$= \boxed{I_K(F)_{pu}} * \frac{S_B * 10^3}{V_B\sqrt{3}}$$

S_B → base MVA

V_B → ~~base~~ [L-L base voltage in kV]

$I_K(F)_{pu}$ → Symmetrical 3-φ fault current in per unit.

$$= \frac{V_K(0)}{X_{KK}} * \frac{S_B * 10^3}{V_B\sqrt{3}}$$

$$\boxed{I_K(F) = \frac{V_K(0)}{Z_{KK}+Z_f}}$$

$V_K(0)$: per unit prefault bus voltage

X_{KK}: is the per unit reactance to the point of fault.

$$\boxed{R \ll X\ ,\ Z\downarrow\ ,\ I(F)\uparrow}$$ ← good approx.

R → system resistance is neglected

$$SCC = \frac{V_K(0)\, S_B \cancel{* 10^3}}{X_{KK}\, V_B \cancel{\sqrt{3}}} * \cancel{\sqrt{3}}\, V_{LK} \cancel{* 10^{-3}}\ \text{MVA}$$

$$= \frac{V_K(0)\, S_B\, V_{LK}}{X_{KK}\, V_B}$$

if $V_B = V_{LK}$, $V_K(0) = 1\,p.u$ $\Longrightarrow$ $\boxed{SCC \cong \frac{S_B}{X_{KK}}\ \text{MVA}}$