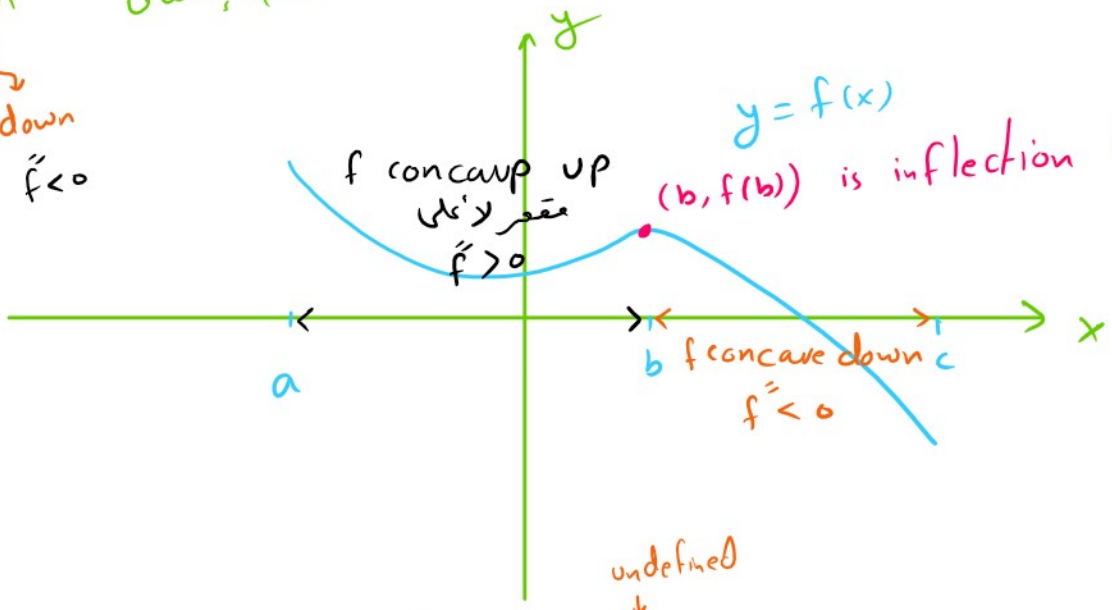
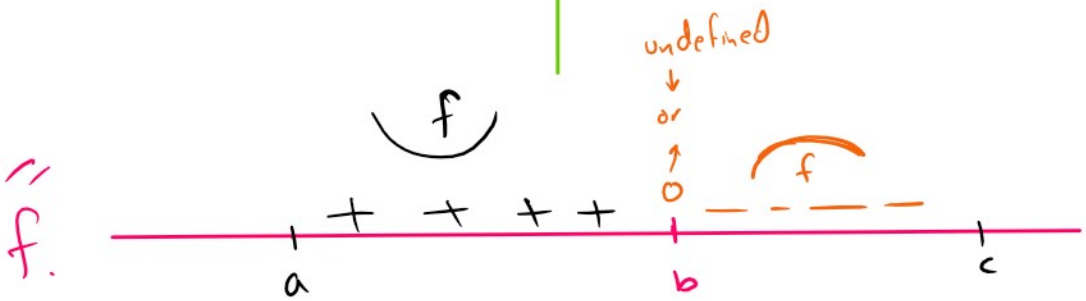
10.2 Concavity and Points of Inflections

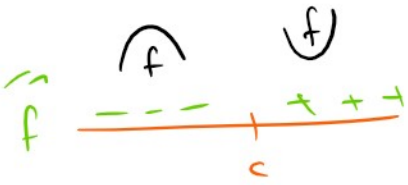
Saturday, May 28, 2022 6:34 PM

التقعر — نقاط الإنعطاف

up: $f'' > 0$ — down: $f'' < 0$







Def: f, f', f'' cont.

- f is concave up on (a,b) if $f''(x) > 0$ for all $x \in (a,b)$
- f " " down on (b,c) if

- f : : down on (b, c) if

$f''(x) < 0$ for all $x \in (b, c)$

Def The point $(b, f(b))$ is inflection Point if f changes concavity around $x = b$

- If $(b, f(b))$ is inflection Point (or Point of diminishing return) Then

$f''(b) = 0$ or $f''(b)$ undefined

Exp $f(x) = x^3 - 4x^2 + 3$

① Find inflection Points

$f' = 3x^2 - 8x$

$f'' = 6x - 8$

$f'' = 0$

$6x - 8 = 0$

$6x = 8$

$x = \frac{8}{6} = \frac{4}{3}$

f'' undefined

not possible لا يوجد مقدار في المقام

$\emptyset$

$f(\frac{4}{3}) = (\frac{4}{3})^3 - 4(\frac{4}{3})^2 + 3$

$= \frac{64}{27} - 4(\frac{16}{9}) + 3$

$= \frac{64}{27} - \frac{4 \times 3 \times 16}{27} + \frac{27}{27}$

$x = -\frac{6}{6} = -3$

inflection Point $\left(\frac{4}{3}, f\left(\frac{4}{3}\right)\right) = \left(\frac{4}{3}, \right)$

$= \frac{64 - 4\times3\times16 + 27\times3}{27}$

② Is f concave up at $x = 1$?

$f''(x) = 6x - 8$

$f''(1) = 6(1) - 8$

$= 6 - 8$

$= -2 < 0 \Rightarrow$ No $\Rightarrow$ f is concave down at $x = 1$

③ Is f concave down at $(2, -5)$?

$f''(x) = 6x - 8$

$f''(2) = 6(2) - 8$

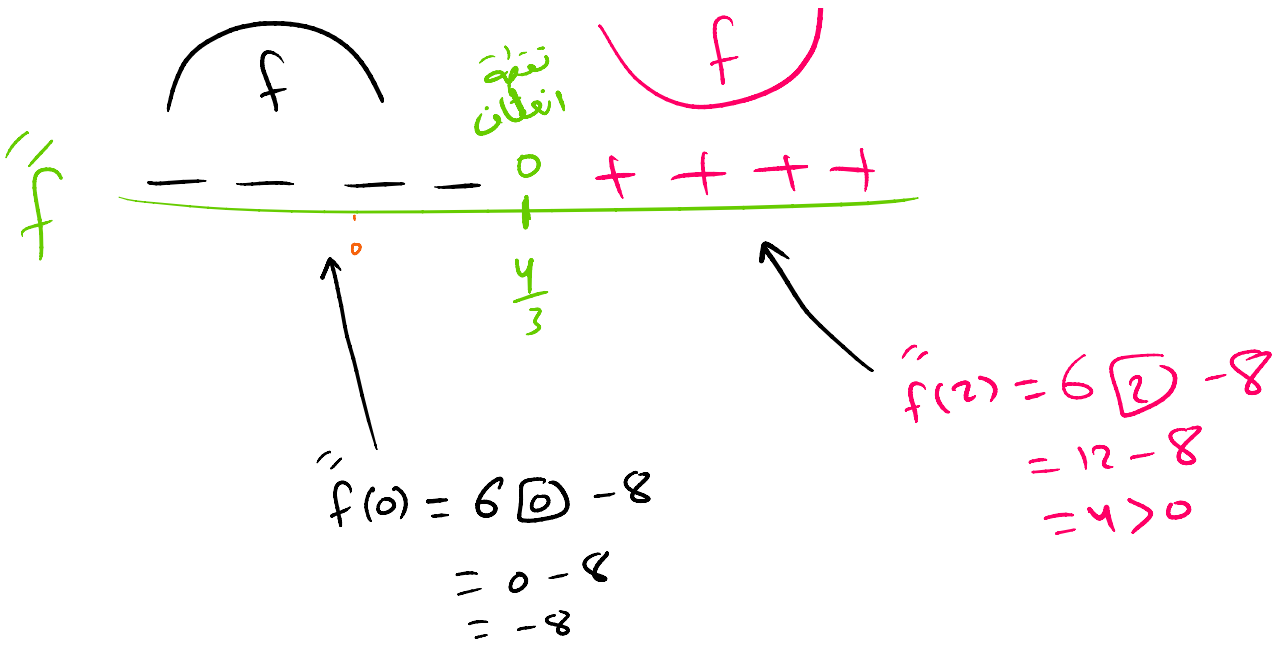
$= 12 - 8$

$= 4 > 0 \Rightarrow$ NO

$\Rightarrow$ f is concave up at $x = 2$

④ Find intervals where f is concave up and concave down

$f''(x) = 6x - 8$



f is concave up on $(\frac{4}{3}, \infty)$

f is concave down on $(-\infty, \frac{4}{3})$

⑤ Sketch f(x)

$f(x) = x^3 - 4x^2 + 3$

$f(0) = 3$

$f' = 3x^2 - 8x = 0$

$x(3x^2 - 8) = 0$

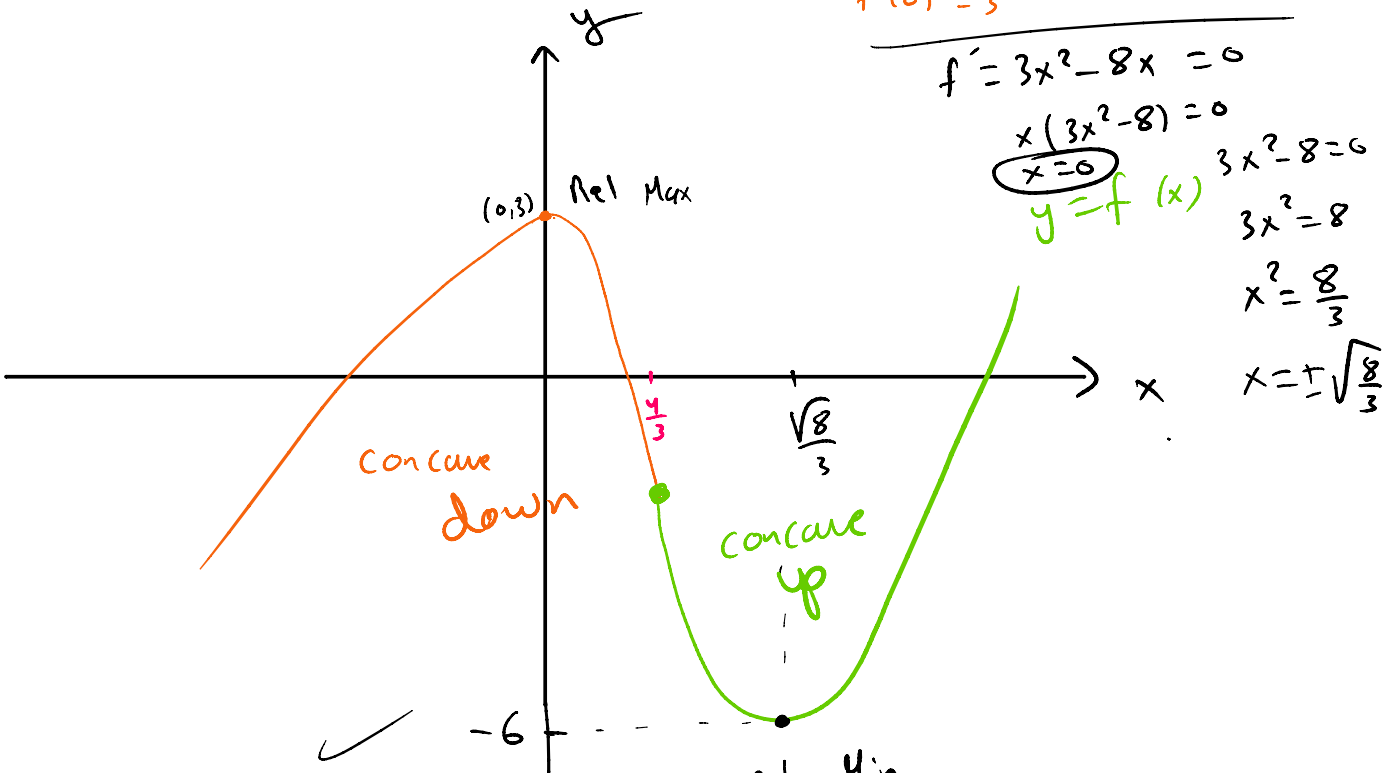
$x = 0$

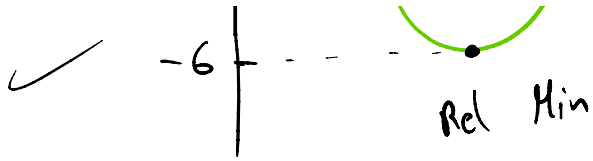
$3x^2 - 8 = 0$

$3x^2 = 8$

$x^2 = \frac{8}{3}$

$x = \pm\sqrt{\frac{8}{3}}$





Exp $f(x) = \frac{x^4}{2} - x^3 + 5$

(1) Find Inflection Points (2) Find intervals where f is concave up/down

(3) Sketch f(x)

$f' = 2x^3 - 3x^2$

$f'' = 6x^2 - 6x$

(1) check $\begin{cases} f'' = 0 \Rightarrow 6x^2 - 6x = 0 \\ 6x(x-1) = 0 \\ x=0 \;,\; x=1 \\ f'' \text{ undefined} \end{cases}$

لا يوجد لها قيمة

$f(1) = \frac{1}{2} - 1 + 5$

$= \frac{1}{2} - 1 + 5$

$= \frac{1}{2} + 4$

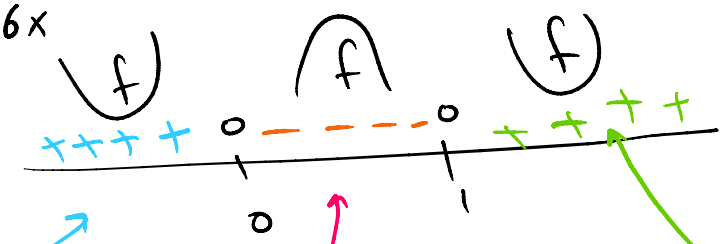
$= \frac{1}{2} + \frac{8}{2}$

$= \frac{9}{2}$

Inflections Points ✓ $(0, f(0)) = (0, 5)$

✓ $(1, f(1)) = (1, \frac{9}{2})$

$f''(x) = 6x^2 - 6x$



$f''(-1) = 6(-1)^2 - 6(-1)$

$= 6 + 6$

$= 12 > 0$

$f''(\frac{1}{2}) = 6(\frac{1}{2})^2 - 6(\frac{1}{2})$

$= 6(\frac{1}{4}) - 3$

$= 1.5 - 3 < 0$

$f''(2) = 6(2)^2 - 6(2)$

$= 6(4) - 12$

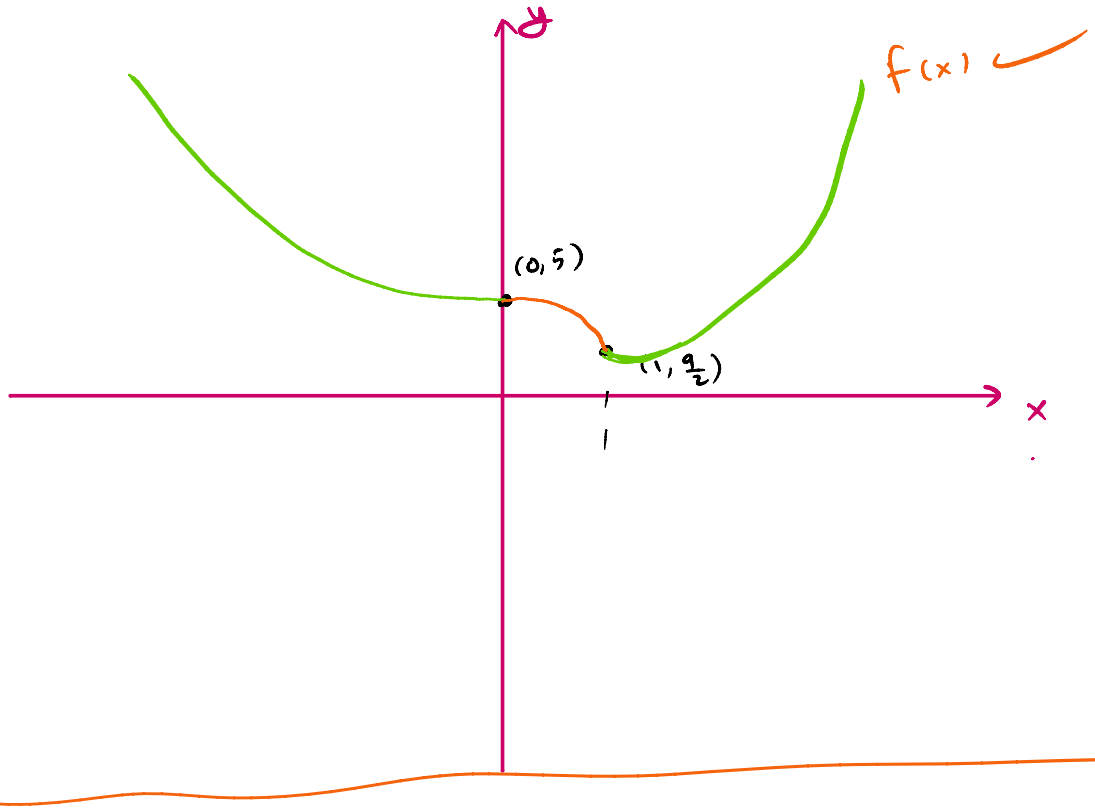
$= 24 - 12$

$= 12 > 0$

(2) f concave up on $(-\infty, 0) \cup (1, \infty)$

f concave down on $(0, 1)$

(3)



Exp If $f''(c) = 0$. Does this mean that f has inflection Point at $x = c$?

No

$f(x) = x^4 \Rightarrow f' = 4x^3$

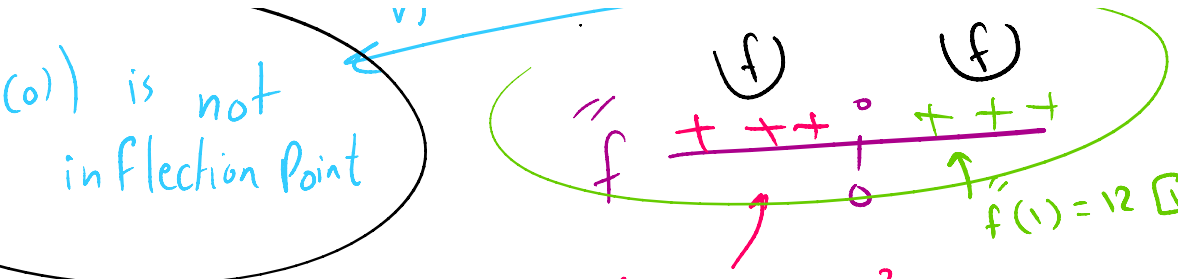
$f'' = 12x^2 = 0$

$x = 0$

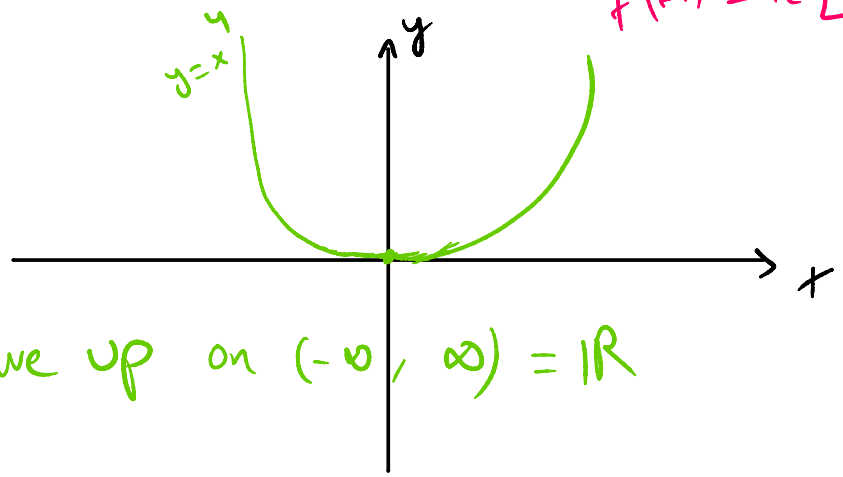
but $f''(0) = 0$ ✓

(0, f(0)) is not (0, f(0))

$(0, f(0))$ is not inflection Point



$f(x) = x^4$



f concave up on $(-\infty, \infty) = \mathbb{R}$

Exp $f(x) = \sqrt{x}$ Find inflection Points and intervals of concavity

$f(x) = x^{\frac{1}{2}}$

$f' = \frac{1}{2} x^{-\frac{1}{2}}$

$f'' = -\frac{1}{4} x^{-\frac{1}{2}-1} = -\frac{1}{4} x^{-\frac{3}{2}} = -\frac{1}{4} \frac{1}{x^{\frac{3}{2}}} = \frac{-1}{4\sqrt{x^3}}$

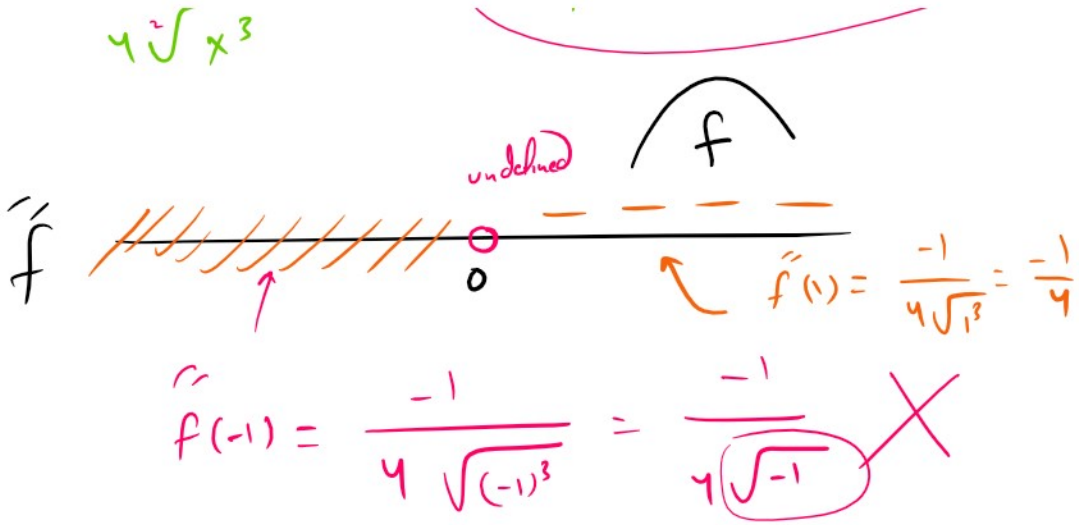
inflection Points $\begin{cases} f'' = 0 \quad \not\exists \\ \text{or} \\ f'' \text{ undefined at } x = 0 \end{cases}$

(غير قابلة للاشتقاق)

$f'' = \frac{-1}{4\sqrt{x^3}}$

$f''(0)$ undefined

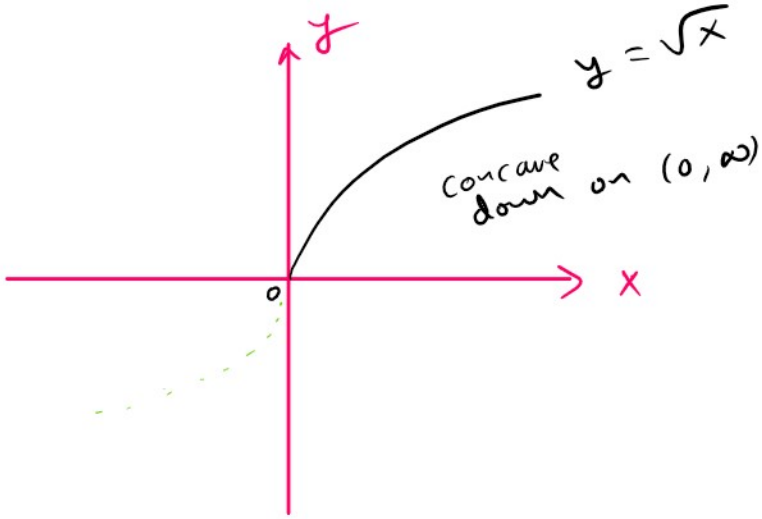
$\sqrt[4]{x^3}$



f is concave down on $(0, \infty)$

No inflection Points

$y = \sqrt{x}$



Exp Suppose the annual profit for a store (in thousands of dollars) is given by

$$P(x) = -0.2x^3 + 3x^2 + 6$$

where x is the # of years past 2006.

[redacted] diminishing return for the

Find the point of diminishing return for the profit.

check

$p' = 0$ | p' undefined

$p'(x) = -0.6x^2 + 6x$

$p''(x) = -1.2x + 6$

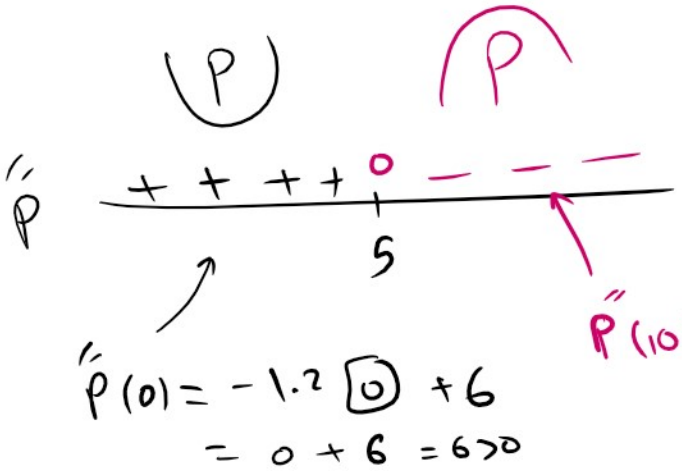
$-1.2x + 6 = 0$

$1.2x = 6$

$x = \frac{6}{1.2} = \frac{1}{0.2} = \frac{1}{\frac{2}{10}}$

0.2

$= \frac{10}{2} = 5$



$p''(0) = -1.2(0) + 6$

$= 0 + 6 = 6 > 0$

$p''(10) = -1.2(10) + 6$

$= -12 + 6$

$= -6 < 0$

$\Rightarrow$ $(5, 56)$ is inflection point

$(5, p(5))$

$p(x) = -0.2x^3 + 3x^2 + 6$

$p(5) = -0.2(5)^3 + 3(5)^2 + 6$

$= -0.2(125) + 3(25) + 6$

$= -25 + 75 + 6$

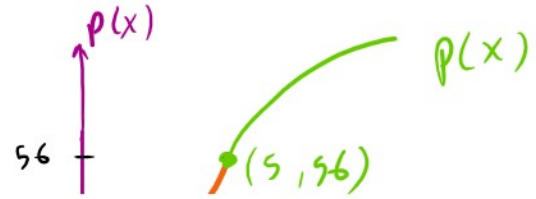
$= 50 + 6$

$= 56$

$p(0) = 6$

$p(x)$ is concave up on $(0, 5)$ and

concave down on $(5, \infty)$



$(5, \infty)$