Introduction to Machinery Principles
Suggested Problems

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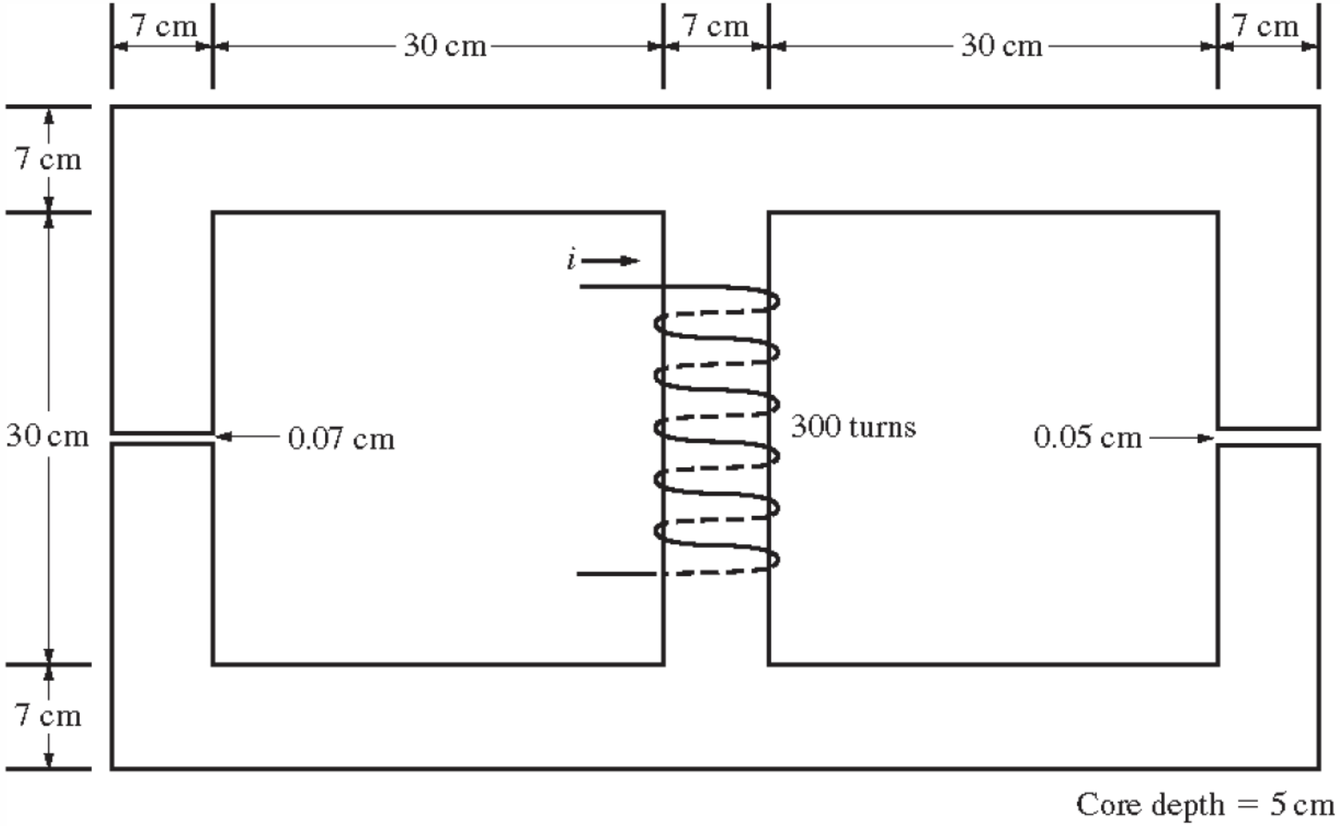
BZU-2020

1-6. A ferromagnetic core with a relative permeability of 1500 is shown in Figure P1-3. The dimensions are as shown in the diagram, and the depth of the core is 5 cm. The air gaps on the left and right sides of the core are 0.050 and 0.070 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are 300 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?



SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{TOT} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11\ \text{m}}{(1500)(4\pi \times 10^{-7}\ \text{H/m})(0.07\ \text{m})(0.05\ \text{m})} = 168\ \text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0007\ \text{m}}{(4\pi \times 10^{-7}\ \text{H/m})(0.07\ \text{m})(0.05\ \text{m})(1.05)} = 152\ \text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11\ \text{m}}{(1500)(4\pi \times 10^{-7}\ \text{H/m})(0.07\ \text{m})(0.05\ \text{m})} = 168\ \text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0005\ \text{m}}{(4\pi \times 10^{-7}\ \text{H/m})(0.07\ \text{m})(0.05\ \text{m})(1.05)} = 108\ \text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37 \text{ m}}{(1500)\left(4\pi \times 10^{-7} \text{ H/m}\right)(0.07 \text{ m})(0.05 \text{ m})} = 56.1 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 56.1 + \frac{(168+152)(168+108)}{168+152+168+108} = 204 \text{ kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(300 \text{ t})(1.0 \text{ A})}{204 \text{ kA} \cdot \text{t/Wb}} = 0.00147 \text{ Wb}$$

The fluxes in the left and right legs can be found by the “flux divider rule”, which is analogous to the current divider rule.

$$\phi_{\text{left}} = \frac{(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{\text{TOT}} = \frac{(168+108)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00068 \text{ Wb}$$

$$\phi_{\text{right}} = \frac{(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{\text{TOT}} = \frac{(168+152)}{168+152+168+108}(0.00147 \text{ Wb}) = 0.00079 \text{ Wb}$$

The flux density in the air gaps can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00068 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.185 \text{ T}$$

$$B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00079 \text{ Wb}}{(0.07 \text{ cm})(0.05 \text{ cm})(1.05)} = 0.215 \text{ T}$$

1-17. Figure P1-13 shows the core of a simple dc motor. The magnetization curve for the metal in this core is given by Figure 1-10*c* and *d*. Assume that the cross-sectional area of each air gap is 18 cm^2 and that the width of each air gap is 0.05 cm. The effective diameter of the rotor core is 5 cm.

(a) We wish to build a machine with as great a flux density as possible while avoiding excessive saturation in the core. What would be a reasonable maximum flux density for this core?

(b) What would be the total flux in the core at the flux density of part *(a)*?

(c) The maximum possible field current for this machine is 1 A. Select a reasonable number of turns of wire to provide the desired flux density while not exceeding the maximum available current.

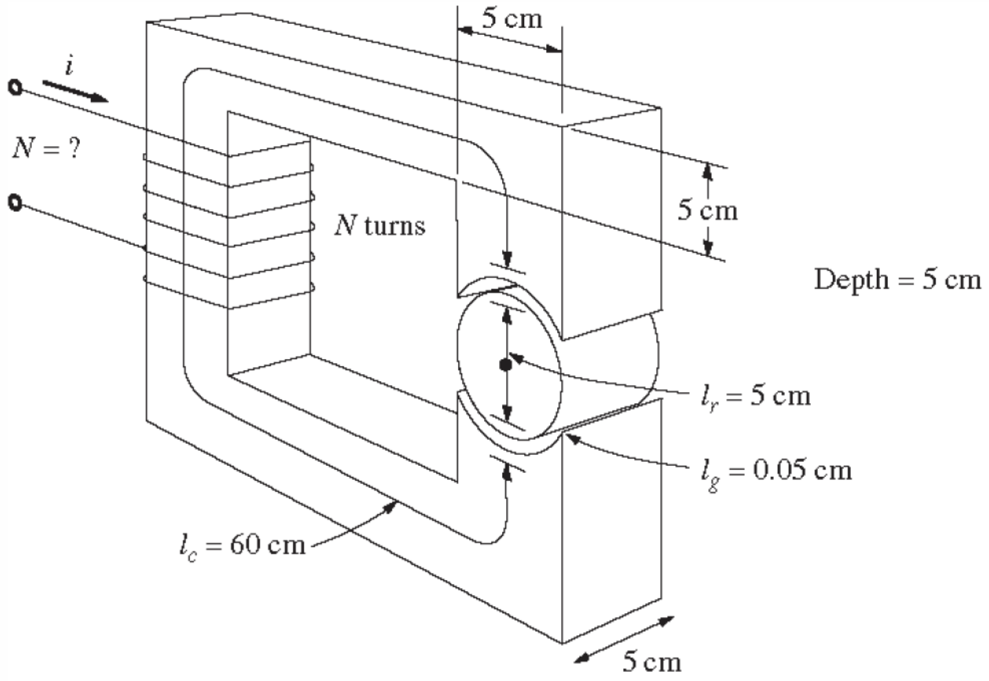
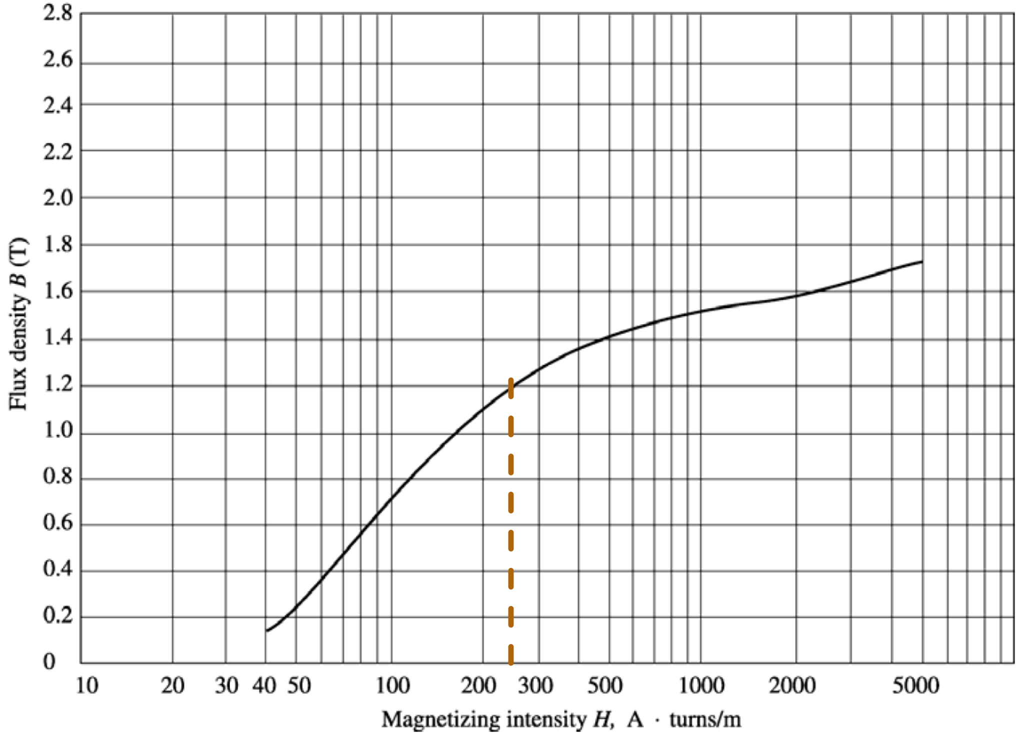
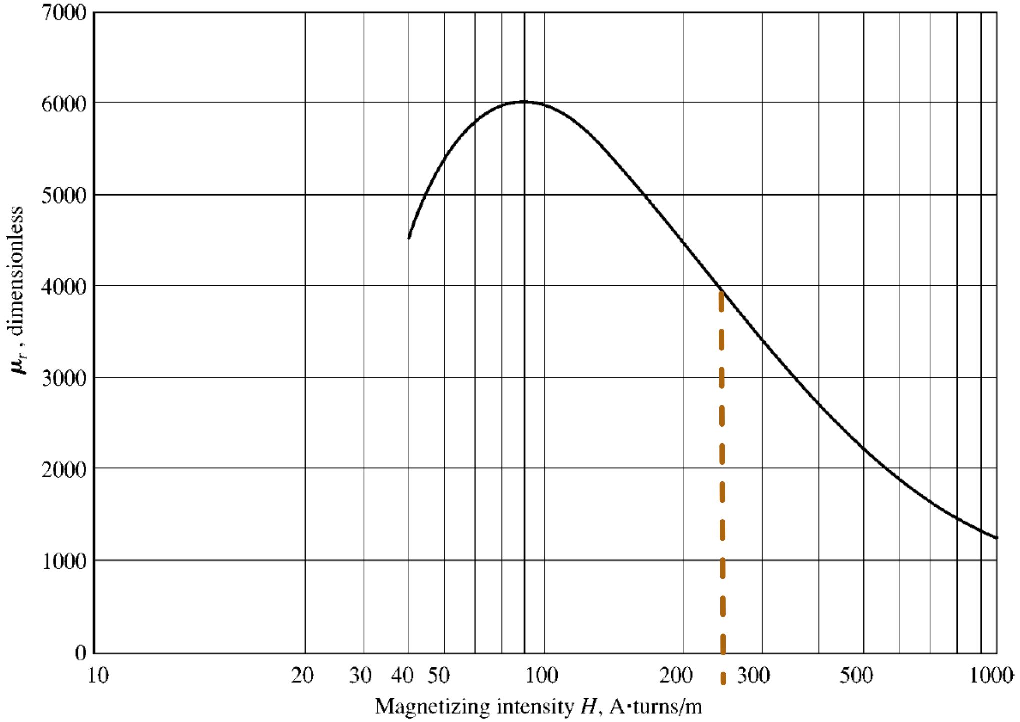


FIGURE P1–13

SOLUTION The magnetization curve for this core is shown below:



The relative permeability of this core is shown below:



(a) From Figure 1-10*c*, a reasonable maximum flux density would be about 1.2 T. Notice that the saturation effects become significant for higher flux densities.

(b) At a flux density of 1.2 T, the total flux in the core would be

$$\phi = BA = (1.2\ \text{T})(0.05\ \text{m})(0.05\ \text{m}) = 0.0030\ \text{Wb}$$

(c) The total reluctance of the core is:

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_{\text{stator}} + \mathcal{R}_{\text{air gap 1}} + \mathcal{R}_{\text{rotor}} + \mathcal{R}_{\text{air gap 2}}$$

At a flux density of 1.2 T, the relative permeability μ_r of the stator is about 3800, so the stator reluctance is

$$\mathcal{R}_{\text{stator}} = \frac{l_{\text{stator}}}{\mu_{\text{stator}} A_{\text{stator}}} = \frac{0.60\ \text{m}}{(3800)\left(4\pi\times10^{-7}\ \text{H/m}\right)(0.05\ \text{m})(0.05\ \text{m})} = 50.3\ \text{kA}\cdot\text{t/Wb}$$

At a flux density of 1.2 T, the relative permeability μ_r of the rotor is 3800, so the rotor reluctance is

$$\mathcal{R}_{\text{rotor}} = \frac{l_{\text{rotor}}}{\mu_{\text{stator}} A_{\text{rotor}}} = \frac{0.05\ \text{m}}{(3800)\left(4\pi\times10^{-7}\ \text{H/m}\right)(0.05\ \text{m})(0.05\ \text{m})} = 4.2\ \text{kA}\cdot\text{t/Wb}$$

The reluctance of both air gap 1 and air gap 2 is

$$\mathcal{R}_{\text{air gap 1}} = \mathcal{R}_{\text{air gap 2}} = \frac{l_{\text{air gap}}}{\mu_{\text{air gap}} A_{\text{air gap}}} = \frac{0.0005\ \text{m}}{\left(4\pi\times10^{-7}\ \text{H/m}\right)\left(0.0018\ \text{m}^2\right)} = 221\ \text{kA}\cdot\text{t/Wb}$$

Therefore, the total reluctance of the core is

$$\mathcal{R}_{TOT} = \mathcal{R}_{stator} + \mathcal{R}_{air\ gap\ 1} + \mathcal{R}_{rotor} + \mathcal{R}_{air\ gap\ 2}$$

$$\mathcal{R}_{TOT} = 50.3 + 221 + 4.2 + 221 = 496\ \text{kA} \cdot \text{t/Wb}$$

The required MMF is

$$\mathcal{F}_{TOT} = \phi \mathcal{R}_{TOT} = (0.003\ \text{Wb})(496\ \text{kA} \cdot \text{t/Wb}) = 1488\ \text{A} \cdot \text{t}$$

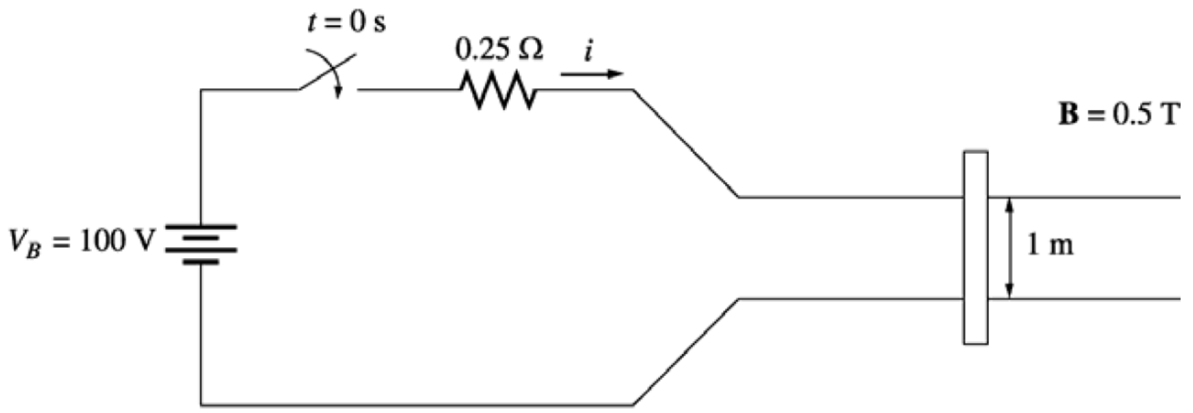
Since $\mathcal{F} = Ni$, and the current is limited to 1 A, one possible choice for the number of turns is $N = 2000$. This would allow the desired flux density to be achieved with a current of about 0.74 A.

1-21. A linear machine has a magnetic flux density of 0.5 T directed into the page, a resistance of 0.25 Ω, a bar length $l = 1.0$ m, and a battery voltage of 100 V.

(a) What is the initial force on the bar at starting? What is the initial current flow?

(b) What is the no-load steady-state speed of the bar?

(c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady-state speed? What is the efficiency of the machine under these circumstances?



SOLUTION

(a) The current in the bar at starting is

$$i = \frac{V_B}{R} = \frac{100\ \text{V}}{0.25\ \Omega} = 400\ \text{A}$$

Therefore, the force on the bar at starting is

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) = (400\ \text{A})(1\ \text{m})(0.5\ \text{T}) = 200\ \text{N, to the right}$$

(b) The no-load steady-state speed of this bar can be found from the equation

$$V_B = e_{\text{ind}} = vBl$$

$$v = \frac{V_B}{Bl} = \frac{100 \text{ V}}{(0.5 \text{ T})(1 \text{ m})} = 200 \text{ m/s}$$

(c) With a load of 25 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_L = F_{\text{ind}} = ilB$$

$$i = \frac{F_L}{Bl} = \frac{25 \text{ N}}{(0.5 \text{ T})(1 \text{ m})} = 50 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100 \text{ V - } (50 \text{ A})(0.25 \ \Omega) = 87.5 \text{ V}$$

and the velocity of the bar will be

$$v = \frac{V_B}{Bl} = \frac{87.5 \text{ V}}{(0.5 \text{ T})(1 \text{ m})} = 175 \text{ m/s}$$

The *input* power to the linear machine under these conditions is

$$P_{\text{in}} = V_B i = (100 \text{ V})(50 \text{ A}) = 5000 \text{ W}$$

The *output* power from the linear machine under these conditions is

$$P_{\text{out}} = e_{in} i = (87.5 \text{ V})(50 \text{ A}) = 4375 \text{ W}$$

Therefore, the efficiency of the machine under these conditions is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{4375 \text{ W}}{5000 \text{ W}} \times 100\% = 87.5\%$$

1-22. A linear machine has the following characteristics:

$B = 0.5$ T into page $\quad\quad R = 0.25\ \Omega$

$l = 0.5$ m $\quad\quad V_B = 120$ V

(a) If this bar has a load of 20 N attached to it opposite to the direction of motion, what is the steady-state speed of the bar?

(b) If the bar runs off into a region where the flux density falls to 0.45 T, what happens to the bar? What is its final steady-state speed?

(c) Suppose V_B is now decreased to 100 V with everything else remaining as in part *(b)*. What is the new steady-state speed of the bar?

(d) From the results for parts *(b)* and *(c)*, what are two methods of controlling the speed of a linear machine (or a real dc motor)?

SOLUTION

(a) With a load of 20 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_L = F_{\text{ind}} = ilB$$

$$i = \frac{F_L}{Bl} = \frac{20\text{ N}}{(0.5\text{ T})(0.5\text{ m})} = 80\text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 120 \text{ V} - (80 \text{ A})(0.25 \ \Omega) = 100 \text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{100 \text{ V}}{(0.5 \text{ T})(0.5 \text{ m})} = 400 \text{ m/s}$$

(b) If the flux density drops to 0.45 T while the load on the bar remains the same, there will be a speed transient until $F_L = F_{\text{ind}} = 20 \text{ N}$ again. The new steady state current will be

$$F_L = F_{\text{ind}} = ilB$$

$$i = \frac{F_L}{Bl} = \frac{20 \text{ N}}{(0.45 \text{ T})(0.5 \text{ m})} = 88.9 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 120 \text{ V} - (88.9 \text{ A})(0.25 \ \Omega) = 97.8 \text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{97.8 \text{ V}}{(0.45 \text{ T})(0.5 \text{ m})} = 433 \text{ m/s}$$

(c) If the battery voltage is decreased to 100 V while the load on the bar remains the same, there will be a speed transient until $F_{\text{app}} = F_{\text{ind}} = 20 \text{ N}$ again. The new steady state current will be

$$F_L = F_{\text{ind}} = ilB$$

$$i = \frac{F_L}{Bl} = \frac{20 \text{ N}}{(0.45 \text{ T})(0.5 \text{ m})} = 88.9 \text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100 \text{ V} - (88.9 \text{ A})(0.25 \ \Omega) = 77.8 \text{ V}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100 \text{ V} - (88.9 \text{ A})(0.25\ \Omega) = 77.8 \text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{77.8 \text{ V}}{(0.45 \text{ T})(0.5 \text{ m})} = 344 \text{ m/s}$$

(d) From the results of the two previous parts, we can see that there are two ways to control the speed of a linear dc machine. *Reducing* the flux density B of the machine *increases* the steady-state speed, and *reducing* the battery voltage V_B *decreases* the stead-state speed of the machine. Both of these speed control methods work for real dc machines as well as for linear machines.

Many Thanks

for

Your Attention!

Reference

- Instructor's Solutions Manual to accompany Electric Machinery Fundamentals by Stephen Chapman, 5th Ed., McGraw-Hill, Inc., 2012.