P3.2 at $\theta_i = \theta_t = 0$ (1)

$$r_s = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$\lim_{\substack{\theta_i \to 0 \\ \theta_t \to 0}} r_s = \frac{n_i - n_t}{n_i + n_t} = -\left(\frac{n_t - n_i}{n_t + n_i}\right)$$

$$\lim_{\substack{\theta_i \to 0 \\ \theta_t \to 0}} r_p = \lim_{\substack{\theta_i \to 0 \\ \theta_t \to 0}} \frac{n_i \cos\theta_t - n_t \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{n_i - n_t}{n_i + n_t}$$

$$= -\left(\frac{n_t - n_i}{n_t + n_i}\right)$$

$$\lim_{\substack{\theta_i \to 0 \\ \theta_t \to 0}} t_s = \lim_{\theta_i, \theta_t \to 0} \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{2 n_i}{n_i + n_t}$$

$$\lim_{\substack{\theta_i \to 0 \\ \theta_t \to 0}} t_p = \lim_{\theta_i, \theta_t \to 0} \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} = \frac{2 n_i}{n_i + n_t}$$

Why is there no difference between (p) & (s) for normal incidence?

for

n_i	n_t
air 1	glass 1.5

$$r_s = r_p = -\left(\frac{1.5-1}{1.5+1}\right) = -\left(\frac{0.5}{2.5}\right) = -0.2$$

$$t_s = t_p = \frac{2(1)}{1.5+1} = \frac{2}{3.5} = \frac{4}{7}$$

---

in general.

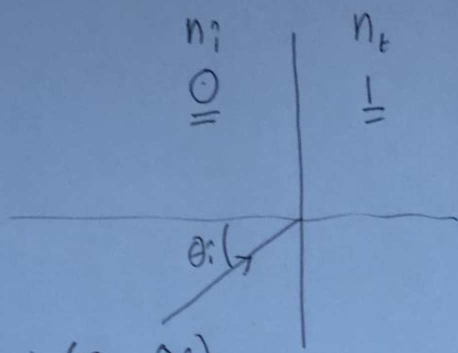
$$R_s = \frac{(n_t - n_i)^2}{(n_t + n_i)^2} \qquad T_s = \frac{4n_i^2}{(n_i + n_t)^2}$$

$$R_s + T_s = ?$$

$$= \frac{(n_t - n_i)^2 + 4n_i^2}{(n_t + n_i)^2} = \frac{n_t^2 + 2n_i n_t + n_i^2}{(n_t + n_i)^2}$$

$$\boxed{= 1 \checkmark}$$

$$R_p + T_p = ?$$



$$r_s = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} \; ;$$

We get θ_t from Snell's law

$$n_i \sin \theta_i = n_t \sin \theta_t$$

ch4 Multiple Parallel Interfaces

r_s, t_s, r_p, t_p

R_s, T_s, R_p, T_p

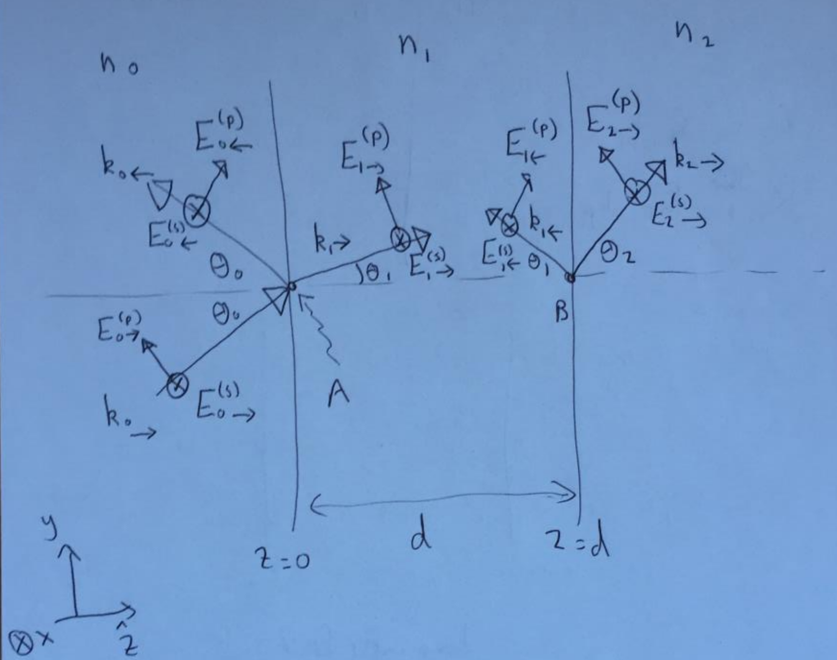
two interfaces

The goal is to find: r^{tot}, t^{tot}

R^{tot}, T^{tot}.

4.1 Double Interface / Fresnel Coeffs

2a



Assumptions: n_0, n_1, n_2 : Real (no absorption)

Note: $E_{0\leftarrow}^{(s)}$, $E_{0\leftarrow}^{(p)}$, $E_{2\rightarrow}^{(2)}$, $E_{2\rightarrow}^{(p)}$ } include light from middle layer.

(3)

$$r_s = \frac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)} \; ; \; t_s = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t+\theta_i)}$$

$$\begin{cases} r_p = \dfrac{\tan(\theta_t-\theta_i)}{\tan(\theta_t+\theta_i)} \; ; \; t_p = \dfrac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t+\theta_i)\cos(\theta_t-\theta_i)} \\ r_p = \dfrac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i} \end{cases}$$

$$r_s^{0\to1} = \frac{\sin\theta_1\cos\theta_0 - \sin\theta_0\cos\theta_1}{\sin\theta_1\cos\theta_0 + \sin\theta_0\cos\theta_1}$$

$$r_p^{0\to1} = \frac{\sin\theta_1\cos\theta_1 - \sin\theta_0\cos\theta_0}{\sin\theta_1\cos\theta_1 + \sin\theta_0\cos\theta_0}$$

$$t_s^{0\to1} = \frac{2\sin\theta_1\cos\theta_0}{\sin\theta_1\cos\theta_0 + \sin\theta_0\cos\theta_1}$$

$$t_p^{0\to1} = \frac{2\sin\theta_1\cos\theta_0}{\sin\theta_1\cos\theta_1 + \sin\theta_0\cos\theta_0}$$

$0 \to 1$ From left to middle layer.

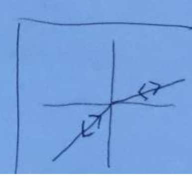
From Middle layer to first interface:

$$r_s^{0\leftarrow1} = -r_s^{0\to1} \; ?$$

$$r_p^{0\leftarrow1} = -r_p^{0\to1} \; ?$$

$$t_s^{0\leftarrow1} = \frac{2\sin\theta_0\cos\theta_1}{\sin\theta_0\cos\theta_1 + \sin\theta_1\cos\theta_0}$$

$$t_p^{0\leftarrow1} = \frac{2\sin\theta_0\cos\theta_1}{\sin\theta_0\cos\theta_0 + \sin\theta_1\cos\theta_1}$$



At second interface:

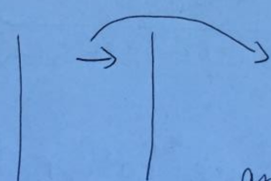
$$r_s^{1\to 2} = \frac{\sin\theta_2 \cos\theta_1 - \sin\theta_1 \cos\theta_2}{\sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2}$$

$$r_p^{1\to 2} = \frac{\sin\theta_2 \cos\theta_2 - \sin\theta_1 \cos\theta_1}{\sin\theta_2 \cos\theta_2 + \sin\theta_1 \cos\theta_1}$$

$$t_s^{1\to 2} = \frac{2\sin\theta_2 \cos\theta_1}{\sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2}$$

$$t_p^{1\to 2} = \frac{2\sin\theta_2 \cos\theta_1}{\sin\theta_2 \cos\theta_2 + \sin\theta_1 \cos\theta_1}$$

For the s-polarized case:



transmission of incident wave and reflection of backward wave

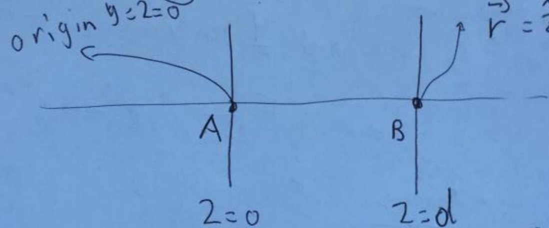
Using Fresnel coefficients:

$$E^{(s)}_{1\rightarrow} = t_s^{0\rightarrow 1} E^{(s)}_{0\rightarrow} + r_s^{0\leftarrow 1} E^{(s)}_{1\leftarrow} \qquad (4,4)$$

Similarly

$$E^{(s)}_{0\leftarrow} = r_s^{0\rightarrow 1} E^{(s)}_{0\rightarrow} + t_s^{0\leftarrow 1} E^{(s)}_{1\leftarrow} \qquad (4,5)$$

Due to thickness of middle region, we need to worry about phase shifts.



at $(y, z) = (0, d)$ the adjusted field is $E^{(s)}_{1\rightarrow} e^{i \vec{k}_{1\rightarrow} \cdot \vec{r}} = E^{(s)}_{1\rightarrow} e^{i k_1 d \cos\theta_1}$

$$E^{(s)}_{2\rightarrow} = t_s^{1\rightarrow 2} E^{(s)}_{1\rightarrow} e^{i k_1 d \cos\theta_1} \qquad (4.6)$$

Similarly (6)

$$E_{1\leftarrow}^{(s)} = E_{1\rightarrow}^{(s)} e^{ik_1 d\cos\theta_1} r_s^{1\rightarrow 2} e^{ik_1 d\cos\theta_1} \quad (4.7)$$

a transmission through middle layer
+ reflection from second interface
+ a transmission through middle layer back to first interface.

Example 4.1 We have $\underline{\underline{4}}$ unknowns for s-polar.

$E_{0\leftarrow}^{s}$; $E_{1\rightarrow}^{s}$; $E_{1\rightarrow}^{s}$; $E_{2\rightarrow}^{s}$

Find $t_s^{tot} = \frac{E_{2\rightarrow}^{s}}{E_{0\rightarrow}^{s}}$?

(4.8) 1) $E_{1\rightarrow}^{s} = \frac{E_{2\rightarrow}^{s}}{t_s^{1\rightarrow 2}} e^{-ik_1 d\cos\theta_1}$ from (4.6)

(4.9) 2) Sub into 4.7

$$E_{1\leftarrow}^{s} = E_{1\rightarrow}^{s} e^{ik_1 d\cos\theta_1} r_s^{1\rightarrow 2} e^{ik_1 d\cos\theta_1}$$

$$= \frac{E_{2\rightarrow}^{s}}{t_s^{1\rightarrow 2}} e^{-ik_1 d\cos\theta_1} e^{ik_1 d\cos\theta_1} r_s^{1\rightarrow 2} e^{ik_1 d\cos\theta_1}$$

Sub 4.8 + 4.9 into 4.4

$$E^{s}_{1\to} = t_s^{0\to1} E^{s}_{0\to} + r_s^{0\leftarrow1} E^{s}_{1\leftarrow}$$

$$\downarrow$$

$$\frac{E^{s}_{2\to}}{t_s^{1\to2}} e^{-ik_1 d\cos\theta_1} = t_s^{0\to1} E^{s}_{0\to} + r_s^{0\leftarrow1} E^{(s)}_{2\to} \frac{r_s^{1\to2} e^{ik_1}}{t_s^{1\to2}}$$

$$\boxed{t_s^{tot} = \frac{E^{s}_{2\to}}{E^{s}_{0\to}} = \frac{t_s^{0\to1} e^{ik_1 d\cos\theta_1} t_s^{1\to2}}{1 - r_s^{0\leftarrow1} r_s^{1\to2} e^{ik_1 d\cos\theta_1}}}$$