Ch1
Functions

Notes:

Exp:	Example		
Ch:	Chapter		
iff:	if and only if ($\iff$)		
$\forall$:	for all	$\perp$:	Perpendicular (normal)
$\exists$:	there exist	$\parallel$:	Parallel
$\in$:	belongs to	R(f):	Range of f
D:	Domain (values of x)	Th:	Theorem
R:	Range (values of y)	$\uparrow$:	Increasing
$\mathbb{R}$:	Real numbers	$\downarrow$:	Decreasing
D(f):	Domain of f	Q.:	Question
Def:	Definition	A.:	Answer

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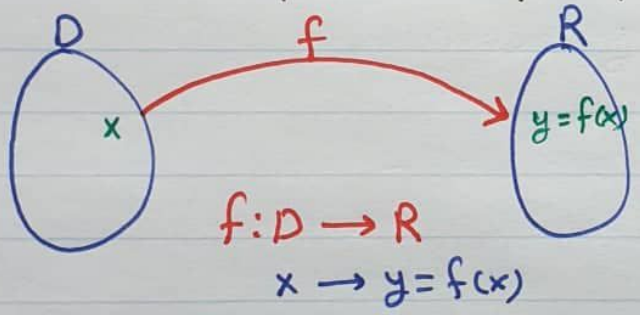
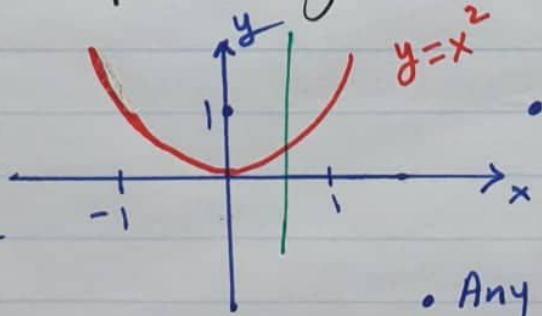
* What is a function f from D to R ?



A function f from D to R is a rule that assigns to each point $x \in D$ a unique point $y=f(x)$ in R.

Exp① Is $y = x^2$ representing a function?

Yes ⇒ Every x in the domain has only one image $y = f(x)$ in the range



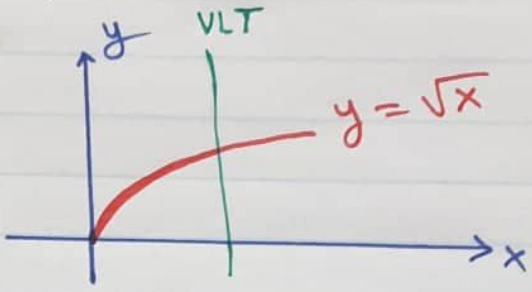
- We can use the Vertical line Test (VLT):
- Any VL crosses $y=x^2$ at most once

② Find D and R.

D = $(-\infty, \infty) = \mathbb{R}$ and R = $[0, \infty)$

Exp ① Is $y = \sqrt{x}$ function?

Yes since it crosses any VL at most once



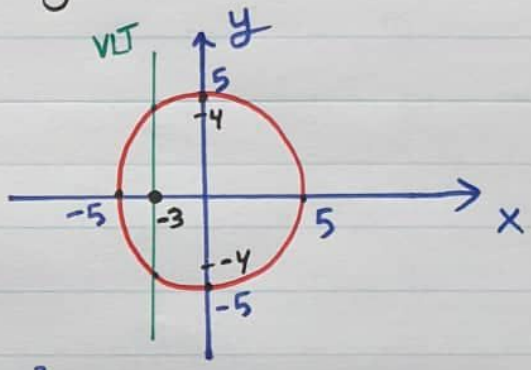
② Find D(f) and R(f)

$D(f) = [0, \infty) = R(f)$

↳ possible values of x ↳ possible values of y

Exp Is $x^2 + y^2 = 25$ representing function?

- No since it crosses the VL in two points



- This means when $x = -3 \Rightarrow$ $x = -3$ has two images ± 4 since $(-3)^2 + y^2 = 25 \Rightarrow y^2 = 16 \Rightarrow y = \pm 4$

Exp Find Domain and Range of the function $f(x) = \sqrt{4 - x^2}$

Domain: $4 - x^2 \geq 0 \Rightarrow x^2 \leq 4 \Rightarrow \sqrt{x^2} \leq \sqrt{4}$

possible values of x

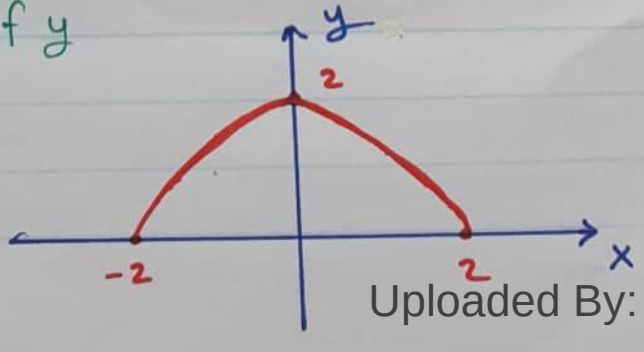
$\Rightarrow |x| \leq 2$

$-2 \leq x \leq 2$

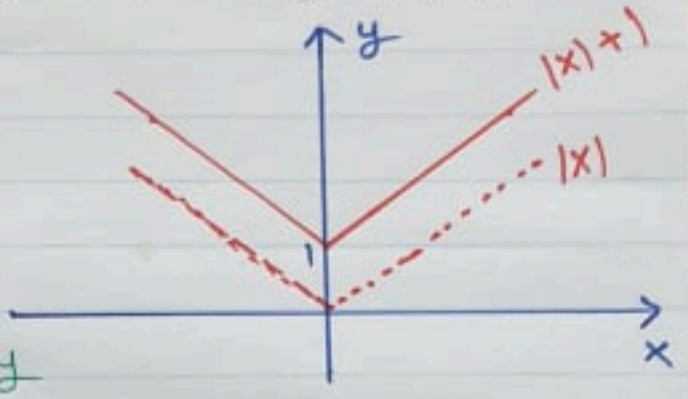
$D = [-2, 2]$

Range: $f(x) \geq 0 \Rightarrow R = [0, \infty)$

possible values of y



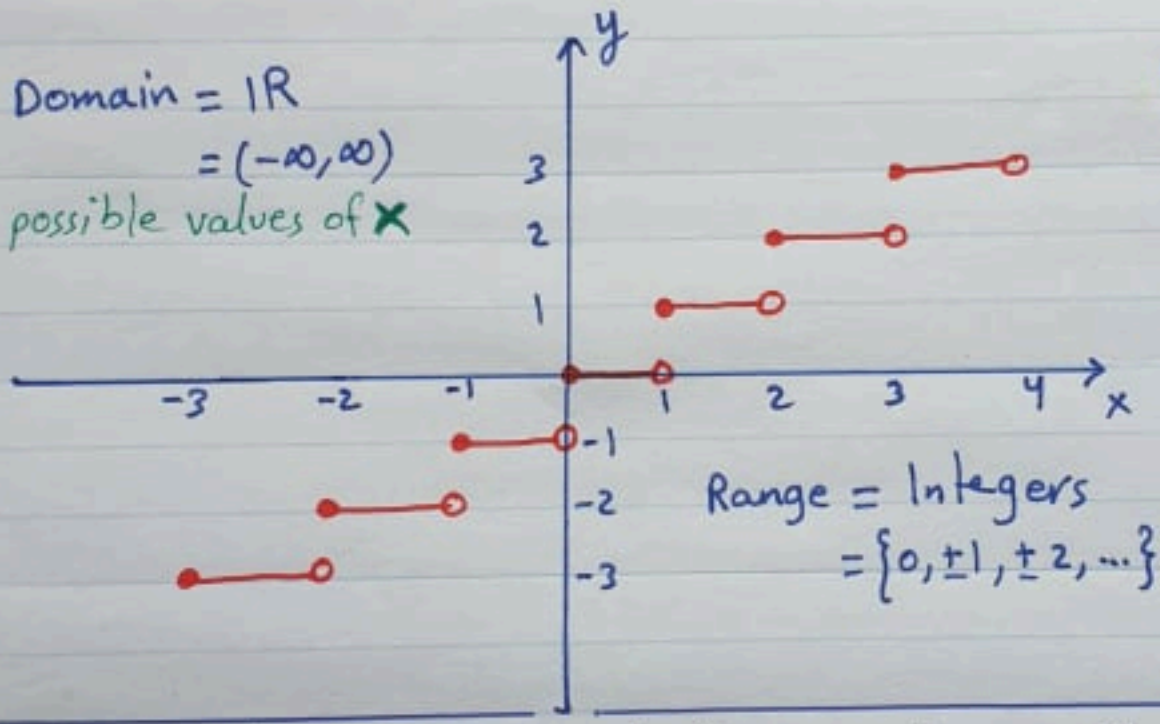
Exp Find Domain and Range of $f(x) = |x| + 1$



- Domain $= (-\infty, \infty)$
  possible values of x
- Range $= [1, \infty)$
  possible values of y

Exp Find Domain and Range of the function $f(x) = \lfloor x \rfloor$, where $\lfloor x \rfloor$ is the greatest Integer function or Floor function

Domain $= \mathbb{R}$
$= (-\infty, \infty)$
possible values of x

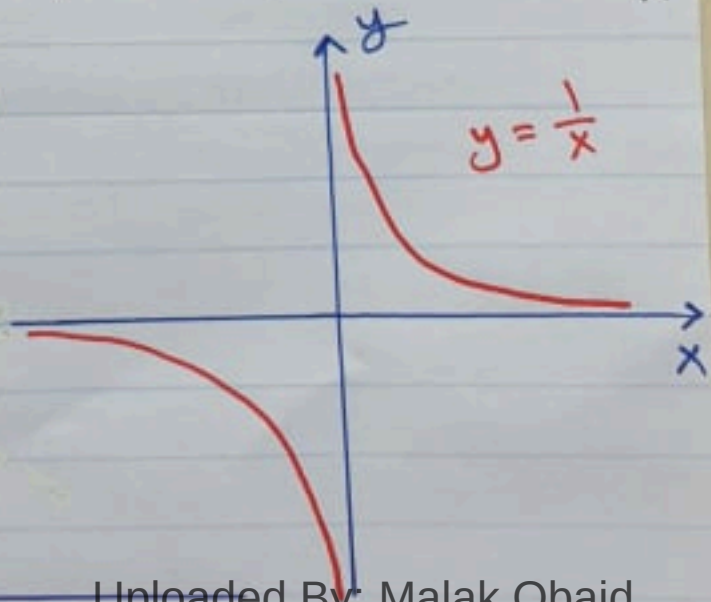


Range = Integers
$= \{0, \pm 1, \pm 2, \ldots\}$

$\lfloor 3 \rfloor = 3$
$\lfloor 3.1 \rfloor = 3$
$\lfloor 3.5 \rfloor = 3$
$\lfloor 3.9 \rfloor = 3$
$\lfloor -3 \rfloor = -3$
$\lfloor -3.1 \rfloor = -4$
$\lfloor -3.9 \rfloor = -4$

Exp Find Domain and Range of the function $f(x) = \frac{1}{x}$

Domain $= (-\infty, 0) \cup (0, \infty)$
$= \mathbb{R} \setminus \{0\}$
possible values of x



Range $= (-\infty, 0) \cup (0, \infty)$
$= \mathbb{R} \setminus \{0\}$
possible values of y