(ايجوز ان افسح لنفسي في مباح الملاهي) ؟

فقال :

(عند نفسك من الغفلة ما يكفيها) [٢] .

ch.6) structure of discrete time system

$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$, find DE?

$y[n] = b_0 x[n] + b_1 x[n-1] + a_1 y[n-1]$

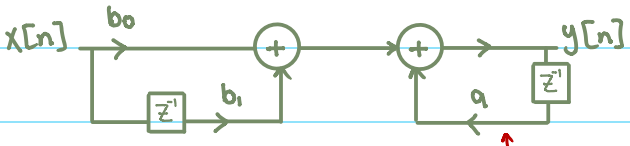
1. 

, $y[n] = x_1[n] + x_2[n]$ } addition

2. $x[n] \xrightarrow{b_0} y[n]$, $y[n] = b_0 x[n]$ } multiplication

3. $x[n] \rightarrow [z^{-1}] \rightarrow y[n]$, $y[n] = x[n-1]$ } time shifting

Direct Form I : means delay then multiply

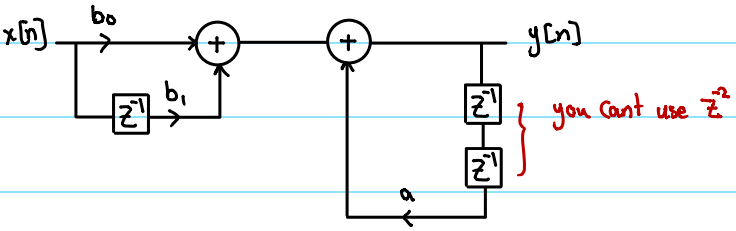


note that this is infinite loop.

- first order
- IIR (since there is feed back)

Example 8- $H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a z^{-2}}$

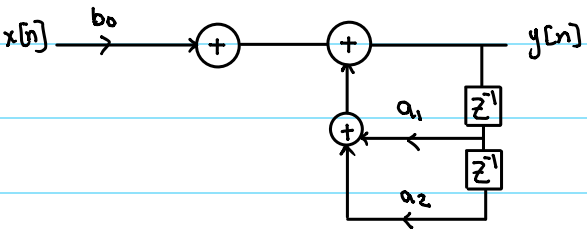
$y[n] = b_0 x[n] + b_1 x[n-1] + a y[n-2]$



you cant use z^{-2}

Example 9- $H(z) = \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}}$

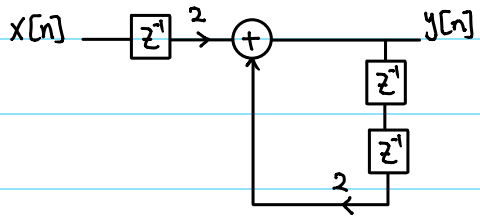
$y[n] = b_0 x[n] + a_1 y[n-1] + a_2 y[n-2]$



- 2nd order

Example:- $H(z) = \frac{2z^{-1}}{1-2z^{-2}}$

$y[n] = 2x[n-1] + 2y[n-2]$



* 2 multiplication
* 1 addition
* 2 shifting
* don't Count multiplications if it scaled by 1

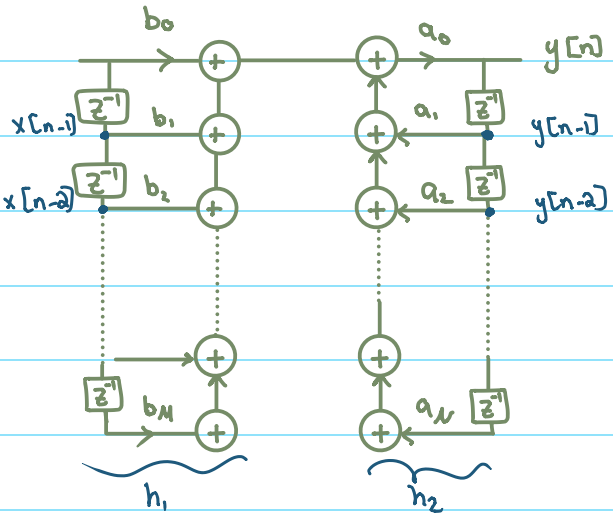
$y[n] = x[n] * h[n]$

$= x[n] * (h_1[n] * h_2[n])$

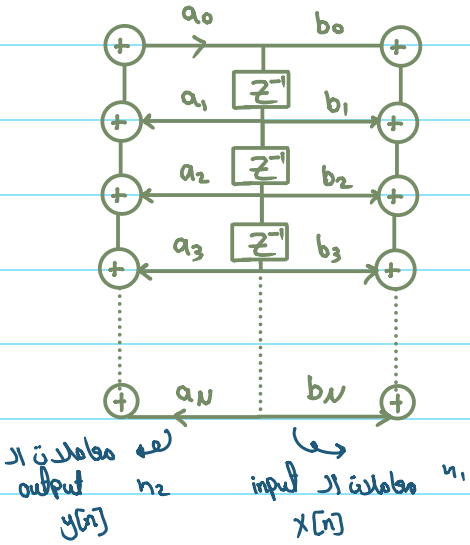
$= x[n] * (h_2[n] * h_1[n])$

Direct Form II : Means share the delay between the input and output

Direct form I



Direct form II

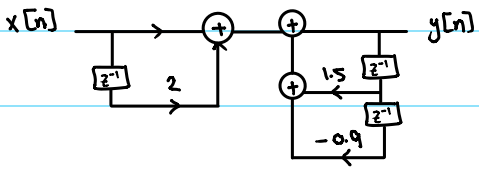


* الفكرة في Direct form II إنه اختصرنا فيها عدد z^{-1} ولكن هذا لا يعني إنه الوقت قلّ .

example: $H(z)=\frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$, Draw Direct form I , Direct form II.

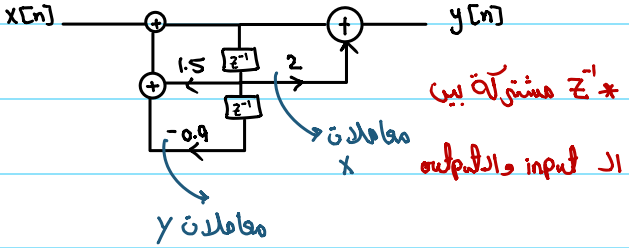
$y[n] = x[n] + 2x[n-1] + 1.5y[n-1] - 0.9y[n-2]$

1) Direct form I



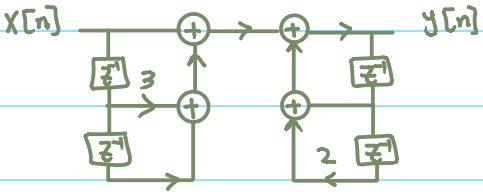
- 3 adders , 3 multiplier , 3 delay
- 2nd order

2) Direct form II



* z^{-1} مشترك بين الـ input والـ output

Examples- From Direct form I find :- 1) DE , 2) H(z) , 3) Direct form II



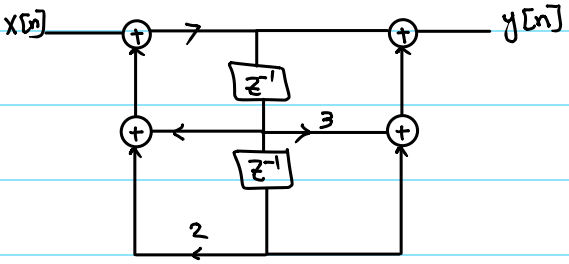
1) DE $\rightarrow y[n] = x[n] + 3x[n-1] + x[n-2] + y[n-1] + 2y[n-2]$

2) $H(z) \rightarrow Y(z) = X(z) + 3z^{-1}X(z) + z^{-2}X(z) + z^{-1}Y(z) + 2z^{-2}Y(z)$

$Y(z)(1 - z^{-1} - 2z^{-2}) = X(z)[1 + 3z^{-1} + z^{-2}]$

$\frac{Y(z)}{X(z)} = \frac{(1+3z^{-1}+z^{-2})}{(1-z^{-1}-2z^{-2})} = H(z)$

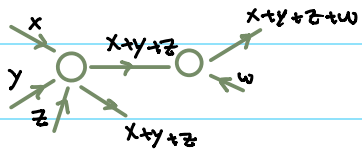
3) Direct form II

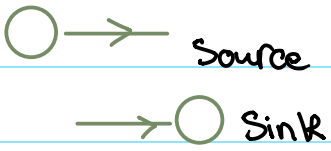


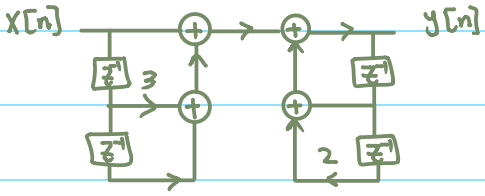
- we have 2 multiplications and 4 additions
- in direct form I , it Requires 4 Storage Register , However, in direct form II it Requires 2.

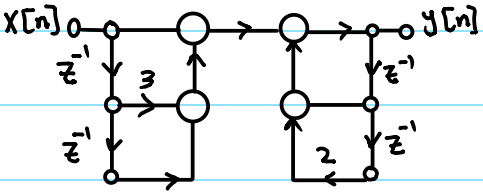
Signal flow graph

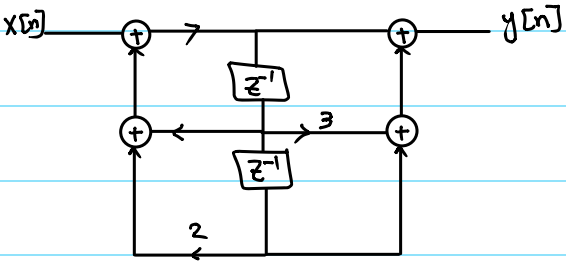
- Branch , - Node

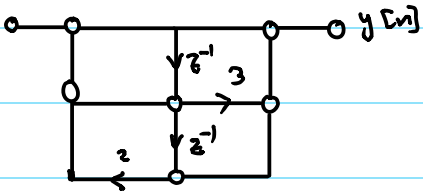




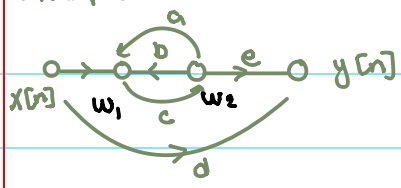








Examples-



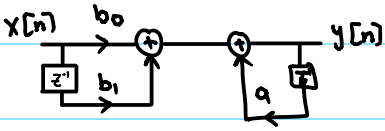
$w_1[n] = x[n] + a\,w_2[n] + b\,w_2[n]$

$w_2[n] = c\,w_1[n]$

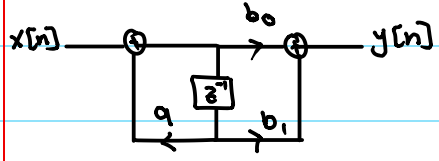
$y[n] = d\,x[n] + e\,w_2[n]$

Example- first order digital filter $H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a z^{-1}}$

$y[n] = a\,y[n-1] + b_0\,x[n] + b_1\,x[n-1]$

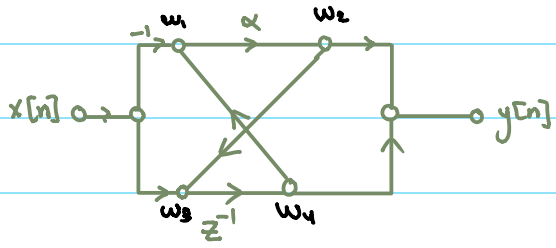


} direct form I



" direct form II"

Example:- Find the System Function H(z)?



$w_1[n] = w_4 - x[n] \quad \longleftrightarrow \quad w_1[z] = w_4[z] - X[z]$

$w_2[n] = \alpha w_1 \quad \longleftrightarrow \quad w_2[z] = \alpha w_1[z]$

$w_3[n] = w_2 + x[n] \quad \longleftrightarrow \quad w_3[z] = w_2[z] + X[z]$

$w_4[n] = w_3(n-1) \quad \longleftrightarrow \quad w_4[z] = z^{-1} w_3[z]$

$y(n) = w_4 + w_2 \quad \longleftrightarrow \quad y[z] = w_4[z] + w_2[z]$

① $w_4[z] = z^{-1} w_3[z]$

$= z^{-1}(w_2[z] + X[z])$

$w_4[z] = z^{-1} w_2[z] + z^{-1} X[z] \rightsquigarrow (*1)$

② $w_2[z] = \alpha w_1[z]$

$= \alpha(w_4(z) - X(z))$

$w_2(z) = \alpha w_4(z) - \alpha X(z) \rightsquigarrow (*2)$

↳ Substitute ①

Substitute ① and ② in $y[z]$

$w_2(z) = \alpha(z^{-1} w_2(z) + z^{-1} X(z)) - \alpha X(z)$

$w_2(z) = \alpha z^{-1} w_2(z) + \alpha z^{-1} X(z) - \alpha X(z)$

$w_2(1 - \alpha z^{-1}) = X(z)[\alpha z^{-1} - \alpha]$

$w_2 = \left[\frac{\alpha z^{-1} - \alpha}{1 - \alpha z^{-1}}\right] X(z) \rightsquigarrow (*)$

$w_4 = z^{-1} w_2 + z^{-1} X(z)$

$= z^{-1}\left[\frac{\alpha z^{-1} - \alpha}{1 - \alpha z^{-1}}\right] X(z) + z^{-1} X(z)$

$= X(z)\left[\frac{\alpha z^{-2} - \alpha z^{-1}}{1 - \alpha z^{-1}} + z^{-1}\right]$

$$= X(z)\left[\frac{\alpha z^{-2} - \alpha z^{-1} + z^{-1} - \alpha z^{-2}}{1-\alpha z^{-1}}\right]$$

$$W_4 = X(z)\left[\frac{-\alpha z^{-1} + z^{-1}}{1-\alpha z^{-1}}\right]$$

$$y = w_4 + w_2$$

$$= X(z)\left[\frac{-\alpha z^{-1} + z^{-1}}{1-\alpha z^{-1}}\right] + X(z)\left[\frac{\alpha z^{-1} - \alpha}{1-\alpha z^{-1}}\right]$$

$$y[z] = X(z)\left[\frac{z^{-1} - \alpha}{1-\alpha z^{-1}}\right]$$

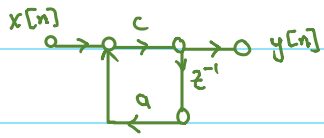
$$H(z) = \frac{z^{-1} - \alpha}{1-\alpha z^{-1}}$$

$$h[n] = (\alpha)^{n-1} u[n-1] - (\alpha)^{n+1} u[n]$$

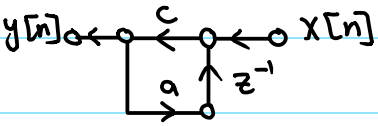
$$y[n] = \alpha y[n-1] + x[n-1] - \alpha x[n]$$

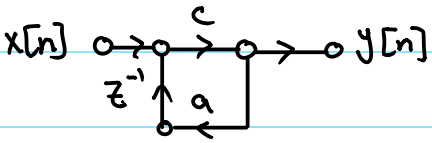
Transposed form Means multiply then delay the inverse af direct form I

1) Reverse Direction for all Branches.
2) Interchange Input and output.
3) Flip the structure the make the input the the left and the output to the Right.

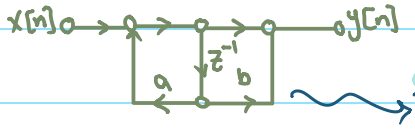
Example:- 

, Find Transpose form



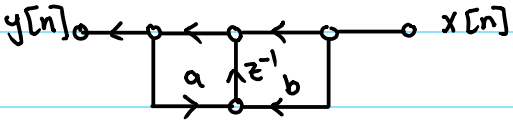


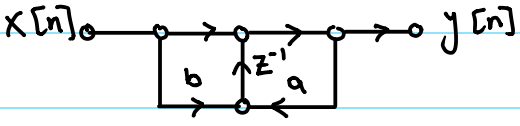
$$y[n] = (x[n] + a y[n-1])\, c$$
$$y[n] = c\,x[n] + ac\,y[n-1]$$
$$y[z] = c\,x[z] + ac\,z^{-1}\,y[z]$$
$$y[z]\,(1 - ac\,z^{-1}) = c\,x[z]$$
$$H(z) = \frac{c}{1 - ac\,z^{-1}}$$

Example:- 

, Find the transpose form.

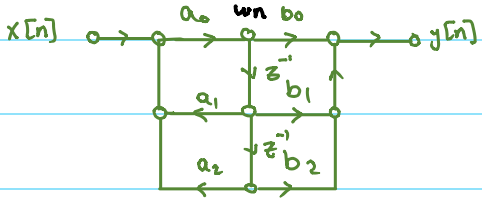
* direct form II
b → x coefficient
a → y coefficient



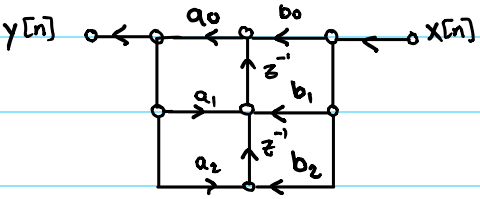


$$y[n] = x[n] + b\,x[n-1] + a\,y[n-1]$$

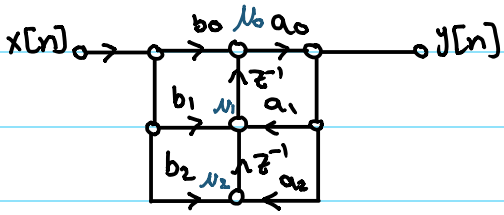
Examples- Second-order IIR System , Draw the transposed form ?



1,2)



3)



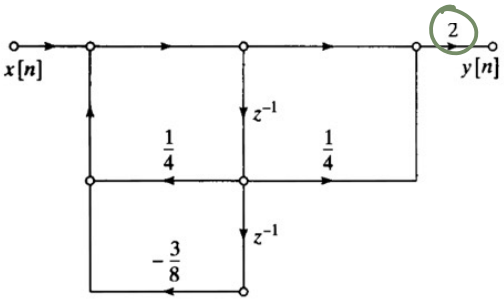
$w_0 = b_0 x[n] + w_1(n-1)$

$w_1 = b_1 x[n] + a_1 w_0 + w_2(n-1)$

$w_2 = b_2 x[n] + a_2 w_0$

ربنا تقبل منا إنك
أنت السميع العليم ..

suggested problems



system function?