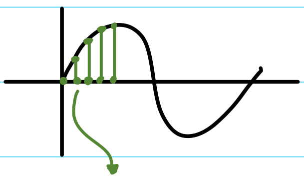
(لا تقعدوا فراغاً فان الموت يطلبكم).

التلخيص غير كافي
لهذا التشابتر

sampling of continuous - time signal

Periodic Sampling



- T_s :- Sampling period
- F_s :- Sampling Rate / Frequency (number of Samples / sec) Hz
- $\Omega_s = \frac{2\pi}{T}$ (sampling frequency) (radians / sec).
- $F_s = \frac{1}{T}$ (Sampling frequency) (samples / sec)

$F_s \uparrow \Rightarrow$ quality $\uparrow \Rightarrow$ memory $\uparrow$

$F_s \downarrow \Rightarrow$ quality $\downarrow \Rightarrow$ memory $\downarrow$

1 * Higher $F_s \rightarrow$ Higher quality

لما تزيد عدد العينات ، أنت تلتقط تفاصيل أدق

وبالتالي الجودة تتحسن

2 * Higher $F_s \rightarrow$ More Memory Storage

لإنك بتخزن نقاط أكثر في نفس الفترة الزمنية ،

حجم البيانات بيزداد وبالتالي تحتاج مساحة أكبر

سواء للذاكرة أو القرص

مثال :-

← لو تسجل صوتك عند 8KHz (8000 عينة/ثانية)

الجودة متوسطة ، والحجم صغير

← لو تسجل صوتك عند 44.1KHz (44100 عينة/ثانية)

الجودة ممتازة لكن حجم أكبر

* To determine the best F_s , Nyquist Rate is presented.

Nyquist Rate $\geq$ 2 (maximum frequency of the Singal)

$F_s \geq 2\,BW$

* if $F_s <$ Nyquist Rate $\rightarrow$ Aliasing in Sampling $\rightarrow$ *لما ناخذ عينات لإشارة لكن بمعدل قليل جدًا ، فالإشارة بعد أخذ العينات بتكون مش صحيحة وفيها تشويه

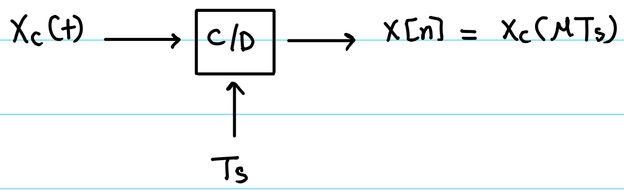
* إذا كان تردد أخذ العينات F_s أقل من ضعف أعلى تردد موجود في الإشارة (Nyquist Rate)

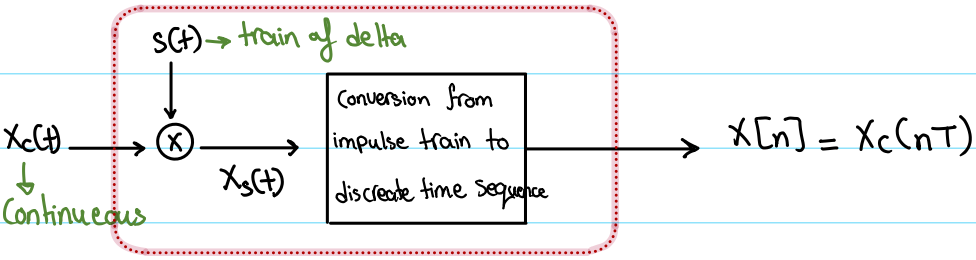
بالتالي راح يصير عنا تداخل بين الترددات (overlap)

في المجال الترددي وهذا بيولد Aliasing ..

A/D ⇝ Convert from Analog to Digital

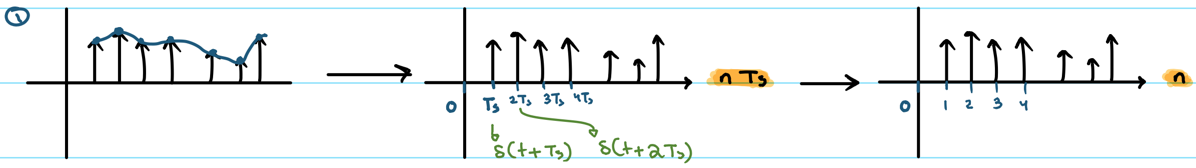
C/D ⇝ Convert from Cont. to Discreate.





sampling process → two stages

1. Impulse train Modulator. "multiplying the Cont. Signal by train of impulses".
2. Normalization "Convert the impulse train the Discreat sequence by divide by T_s in time domain



$$\sum_{n=-\infty}^{\infty} \delta(t+nT_s)$$

$$X_s(t) = X_c(t) \cdot S(t) \qquad \Longrightarrow \text{normalization } \frac{nT_s}{T_s}$$

$$= X_c(t) \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \qquad\qquad x(t).\delta(t) = x(0)\,\delta(t)$$

$$= \sum_{n=-\infty}^{\infty} X_c(nT)\,\delta(t-nT_s). \quad \rightsquigarrow \text{Sifting property.}$$

→ The Fourier transform of aperiodic impulse train

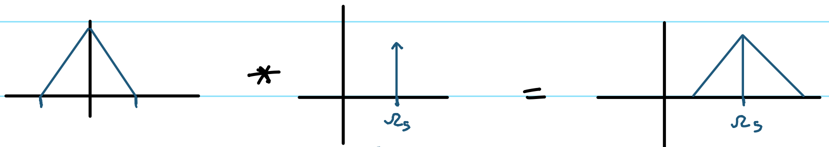
$$S(j\Omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\Omega - n\Omega_s) \rightarrow ①$$

$$X_s(j\Omega) = \frac{1}{2\pi}\, X_c(j\Omega) * S(j\Omega)$$

substitute ①.

$$X_s(j\Omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X_c(j(\Omega - n\Omega_s)) \rightarrow \text{Periodically Repeated Copies of } X_c(j\omega)$$

* Copies of $X_c(j\Omega)$ are shifted by integer Multiplies of sampling frequency Ω_s



a periodic

$\delta(\Omega - \Omega_s)$

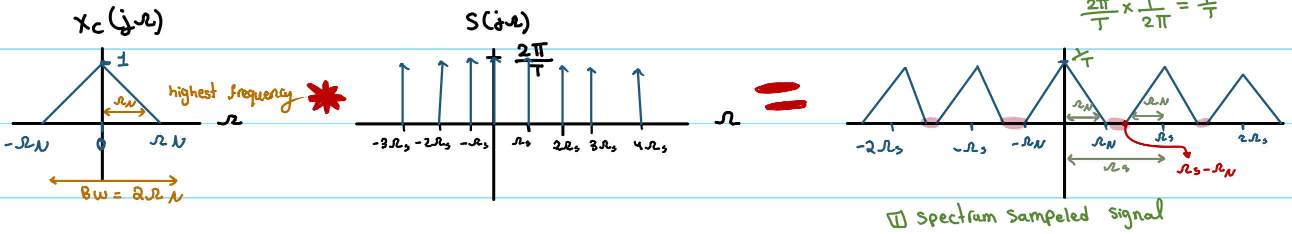
many copies of the signal spectrum

periodically Repeated copies of $X_c(j\Omega)$

More Details :-

$$X(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$$

$\frac{2\pi}{T} \times \frac{1}{2\pi} = \frac{1}{T}$



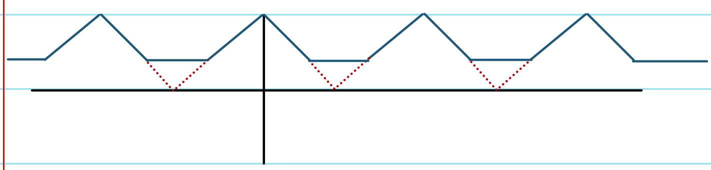
① spectrum sampeled signal

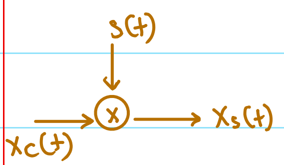
if $\Omega_s > 2\Omega_N$

or $\Omega_s - \Omega_N > \Omega_N$

$\Omega_s > 2\Omega_N$

② if $\Omega_s < 2\Omega_N \Rightarrow$ Aliasing





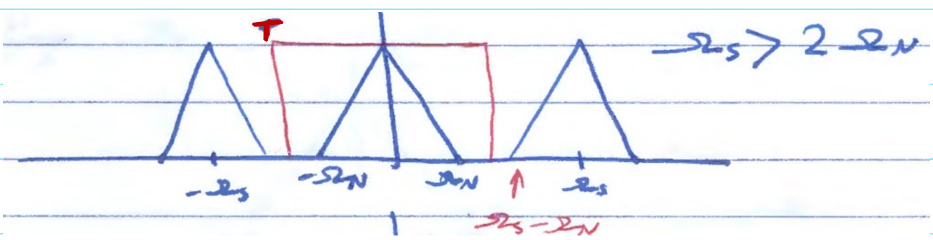
in time domain

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$$

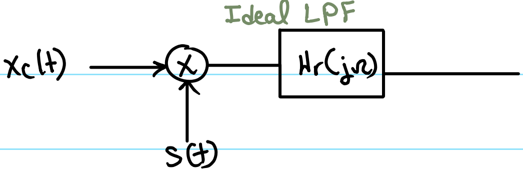
Q. How to get the Original Signal ?

A. By multiplying $X_s(t)$ with analog low pass filter in frequency domain, then we can get the Original Signal $X_c(t)$.

* Note that, it's Analog Not digital, since digital is periodic and we only need one signal no more. However, Analog low pass filter is non periodic



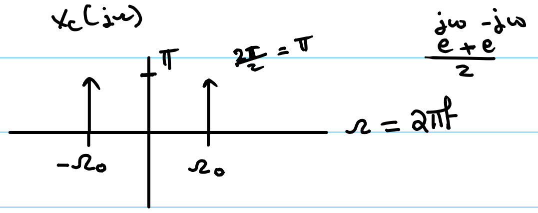
the Cut of frequency $\Omega_N < \Omega_c < \Omega_s - \Omega_N \Rightarrow$ we often take $\Omega_c = \frac{\Omega_s}{2}$

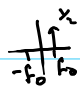


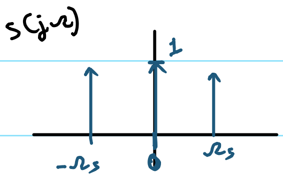
$x_r(t) = x_c(t)$ iff $\Omega_s \geq 2\Omega_N$

Example:- Assume $X_c(t) = \cos(\Omega_0 t)$ * periodic → in time domain

Aperiodic → in frequency domain



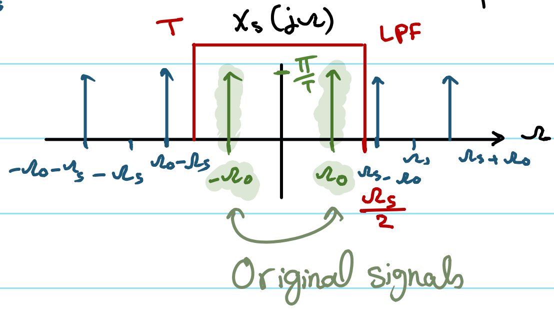




$$X_r(t) = \frac{1}{T}\left(X_c(j\Omega) * S(j\Omega) \right)$$

$$= \frac{1}{T} \times 1 \times \pi = \pi/T \rightsquigarrow \text{magnitude}$$

① $\Omega_s \geq 2\Omega_0$

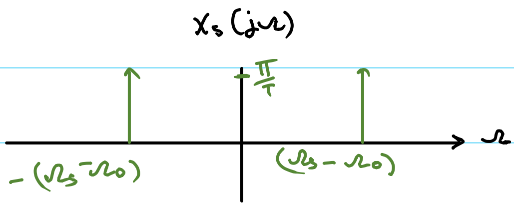


② $\Omega_s < 2\Omega_0$ → Aliasing.

$\Omega_0 > \frac{\pi}{T}$

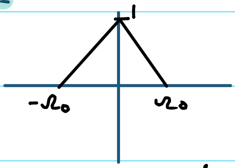
$\Omega = \frac{\omega}{T} \rightarrow \omega = \Omega T$

$x[n] = x_c[nT]$



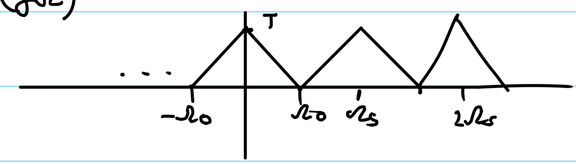
Example:-

$X_c(j\Omega)$



Ans:- $X_s(j\Omega) = \frac{1}{T} \overset{1}{X_c(j\Omega)} * \overset{1}{S(j\Omega)}$

$X_s(j\Omega)$



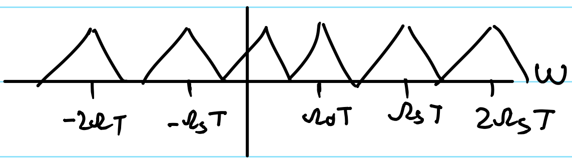
$\times T$

$\Omega = \frac{\omega}{T}$

$\downarrow$

$\omega = \Omega T$

$X_s(e^{j\omega})$



Example:- $X_c(t) = \cos(4000\pi t)$, $T = \frac{1}{6000}$ sec

Find, $X_c(j\Omega)$, $X_s(j\Omega)$, $X(e^{j\omega})$, $S(j\Omega)$

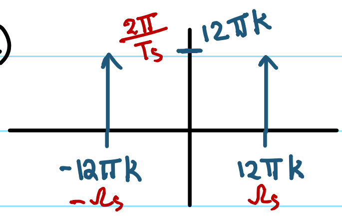
① $\Omega_0 = 4000\pi \rightarrow f_0 = \frac{\Omega_0}{2\pi} = \frac{4000\pi}{2\pi} = 2000$ Hz

② $F_s = 6000$ Hz , $T_s = \frac{1}{6000}$ sec.

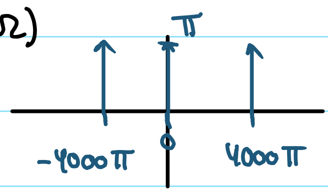
③ $\Omega_s = \frac{2\pi}{\frac{1}{6000}} = 12000\pi$

④ $\omega = T_s\Omega = \frac{4000\pi}{6000} = \frac{2}{3}\pi$

$\boxed{1}$ $S(j\Omega)$



$\boxed{2}$ $X_c(j\Omega)$



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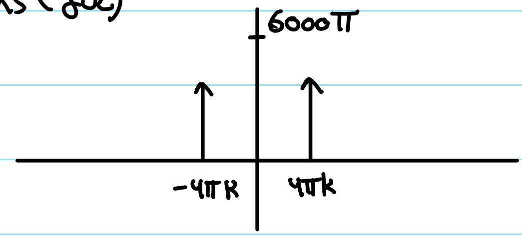
3 $X_s(j\Omega)$



$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$$

$$mag = \frac{1}{\cancel{2\pi}} \cdot \frac{\cancel{2\pi}}{T_s} \cdot \pi = \frac{\pi}{T_s}$$

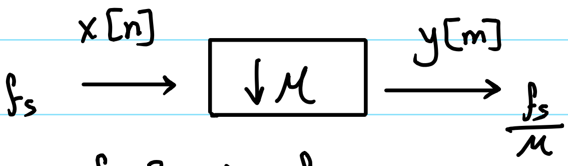
$$= \frac{\pi}{1/6000} = 6000\pi$$

Decimation:

two step process → ① low pass filtering

② down sampling by M.

* $f_s\,new = \dfrac{f_{old}}{M}$



f_s: Sampling frequency.

* to ensure that the Nyquist Condition is valid, a low pass filter is applied directly befor the down Sampling, the cutoff frequency of the low pass filter should be ½ of the new Sampling Rate.

Example: $x[n] = \{1, 2, 3, 4, 5, -6, -8, 2, -3, 2\}$

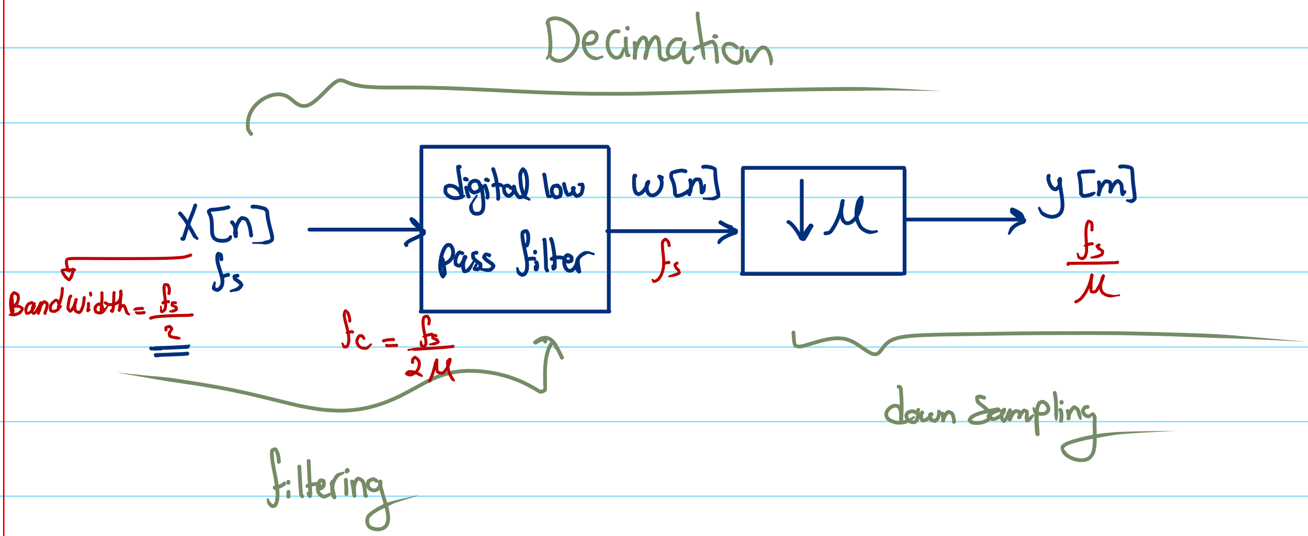
1) Down Sample by 2.

new bandwidth = "cutoff frequency" $= \dfrac{f_s}{2(2)} = \dfrac{f_s}{4}$

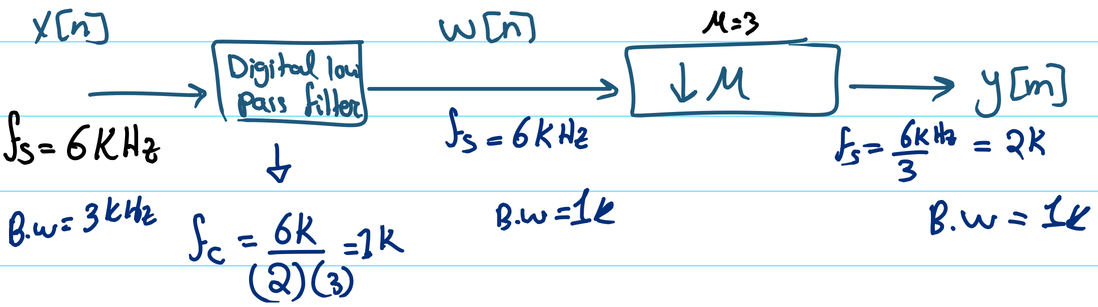
$y[m] = \{1, 3, 5, -8, -3\}$

2) Down Sample by 3. → Band width $= \dfrac{f_s}{6}$

$y[m] = \{1, 4, -8, 2\}$



Example:-



Example:- Decimation of $x[n] = \{2, 6, 4, 2, 6, 8, 4, 2, 4, 4\}$

$M = 2$, find $\underline{w}[n]$ and $\underline{y}[n]$?

$h[n] = \{1/2, 1/2\}$

gain $= 1$, $\omega_c = \frac{\pi}{2}$, for the LPF.

$w[n] = x[n] * h[n]$

	2	6	4	2	6	8	4	2	4	4	
	1	4	5	3	4	7	6	3	3	4	2
1/2	1	3	2	1	3	4	2	1	2	2	
1/2	1	3	2	1	3	4	2	1	2	2	

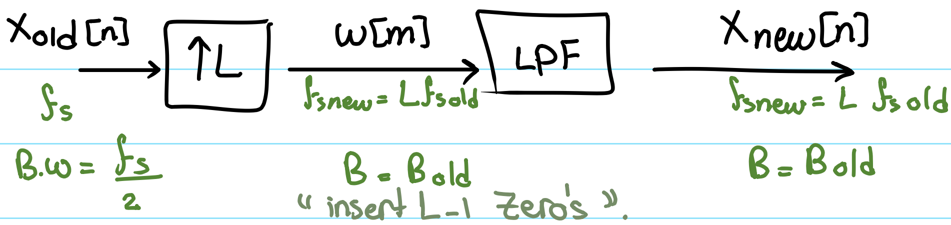
$w[n] = \{1, 4, 5, 3, 4, 7, 6, 3, 3, 4, 2\}$

$y[m] = \{1, 5, 4, 6, 3, 2\}$

* $\omega_c = \frac{2\pi f_{s\,old}}{f_{s\,new}} = \frac{\cancel{2}\pi \frac{\cancel{f_s}}{\cancel{2}M}}{\cancel{f_s}} = \frac{\pi}{M}$

interpolation

Sample Rate increase by interpolation . New Sample values need to be Calculated



Example: $x[n] = \{1, 0.9, -0.5\}$, $L=3$, find $w[n]$:

$w[n] = \{1, 0, 0, 0.9, 0, 0, -0.5, 0, 0\}$

Example: $X[n] = \{1, 2, 4, 3, -5, 6, -7, 2, 4, 3\}$ find Interpolation for :

1. $L = 2$.

$w[n] = \{1, 0, 2, 0, 4, 0, 3, 0, -5, 0, 6, 0, -7, 0, 2, 0, 4, 0, 3, 0\}$

2. $L = 3$

$w[n] = \{1, 0, 0, 2, 0, 0, 4, 0, 0, 3, 0, 0, -5, 0, 0, 6, 0, 0, -7, 0, 0, 2, 0, 0, 4, 0, 0, 3, 0, 0\}$

Note:-

the quality of the Signal will not improved after the interpolation even the Sampling.

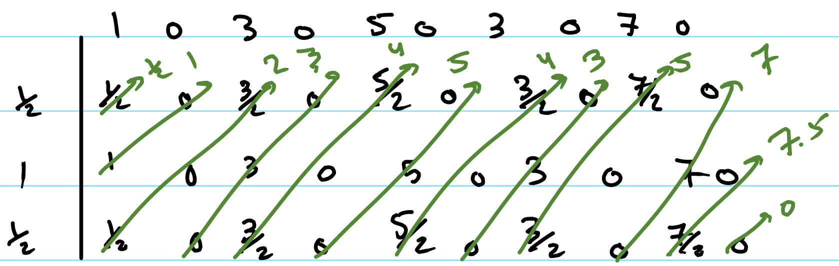
Example: interpolation of $x[n] = \{1, 3, 5, 3, 7\}$, $h[n] = \{1/2, 1/2\}$

find $w[m]$, $y[m]$?

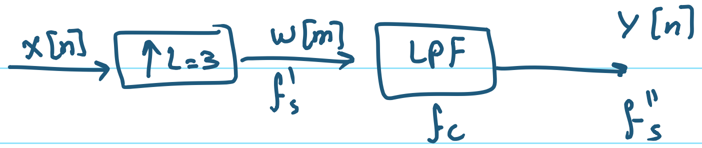
gain = 2 , $w_c = \pi/2$, $L = 2$

$w[m] = \{1, 0, 3, 0, 5, 0, 3, 0, 7, 0\}$

$y[m] = w[m] * h[n] = \{0.5, 1, 2, 3, 4, 5, 4, 3, 5, 7, 7.5, 0\}$



Example:



$f_s = 8\,KHz$

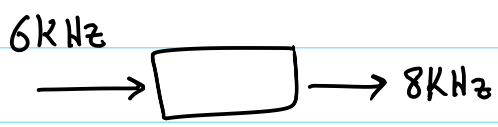
$BW = 2\,KHz$

What are the values of f_c and f'_c ?

$f''_s = f'_s = L\,f_s = (3)(8\,KHz) = 24\,KHz$

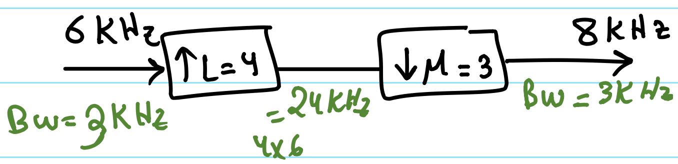
$f_c = BW = 2\,KHz$ or $\frac{f_s}{2}$ if the BW is not given.

Sampling Rate conversion by non integer factor:



$\frac{8}{6} = \frac{4}{3}$

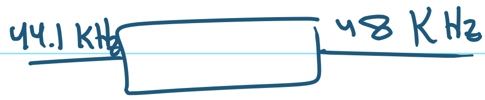
in this case L is non integer so, we need to do the following steps.



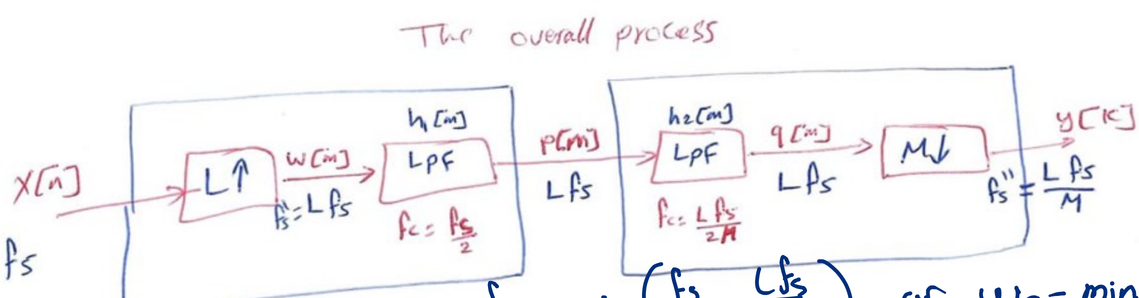
the overall process is interpolation since the Ratio is $\frac{4}{3}$ more than 1

for best quality, do up sampling then down sampling.

Example:-



$\frac{48k}{44.1k} = \frac{\cancel{48000}^{160}}{\cancel{44100}_{147}} = \frac{160}{147} = \frac{L}{M}$



$f_c = \min\left(\frac{f_s}{2}, \frac{Lf_s}{2M}\right)$ or $w_c = \min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$