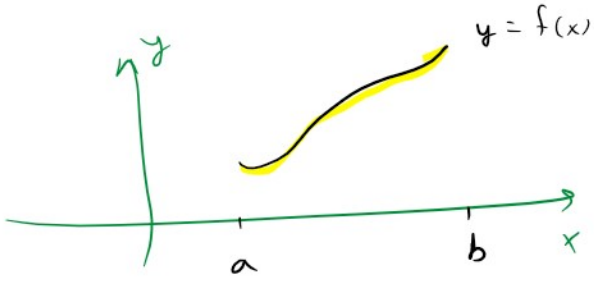
6.3

Saturday, April 17, 2021 9:26 AM

Q2 , Q10

length curve



Q2 $y = x^{\frac{3}{2}}$, $x = 0$ to $x = 4$

$y = \sqrt{x^3}$ cont on $[\overset{a}{0}, \overset{b}{4}]$

$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$ cont. con $[0,4]$

✓ f, f' cont. on $[a,b]$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Poly $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

poly of degree n

$n \in \mathbb{Z}^+$

$$\left(\frac{dy}{dx}\right)^2 = \frac{9}{4} x$$

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx = \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$u = 1 + \frac{9}{4}x$ | $x = 0 \Rightarrow u = 1$

$du = \frac{9}{4} dx$ | $x = 4 \Rightarrow u = 10$

$$L = \int_1^{10} \sqrt{u}\, \frac{4}{9} du = \frac{4}{9} \int_1^{10} u^{\frac{1}{2}} du = \frac{4}{9} \left. \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right|_1^{10}$$

$$= \frac{4}{9} \frac{2}{3} \left. \sqrt{u^3} \right|_1^{10} = \frac{8}{27}\left(\sqrt{1000} - 1\right) = \frac{8}{27}\left(10\sqrt{10} - 1\right)$$

مسافة موجبة

$\sqrt{100 \times 10}$

$10\sqrt{10}$

Q10 $y = \int^{x} \sqrt{3t^4 - 1}\, dt$, $\underset{a}{-2} \le x \le \underset{b}{-1}$

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Q10 $y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt$, $-2 \le x \le -1$ (a, b)

$$\frac{dy}{dx} = \sqrt{3x^4 - 1} \quad \boxed{1}$$ → معرفة

$$\left(\frac{dy}{dx}\right)^2 = 3x^4 - 1$$

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_{-2}^{-1} \sqrt{\cancel{1} + 3x^4 - \cancel{1}}\, dx$$

$$= \int_{-2}^{-1} \sqrt{3x^4}\, dx$$

$$= \sqrt{3} \int_{-2}^{-1} |x^2|\, dx = \sqrt{3} \int_{-2}^{-1} x^2\, dx$$

$$= \sqrt{3}\, \frac{x^3}{3} \Big|_{-2}^{-1} = \frac{\cancel{\sqrt{3}}}{\sqrt{3}\cancel{\sqrt{3}}} \left((-1)^3 - (-2)^3 \right) = \frac{1}{\sqrt{3}} (-1 - -8)$$

$(-1 + 8)$

$$= \frac{7}{\sqrt{3}}$$ مسافة طول curve

موجب

$3x^4 - 1 \ge 0$

$3x^4 \ge 1$

$x^4 \ge \frac{1}{3}$

$\sqrt{x^4} \ge \sqrt{\frac{1}{3}}$

$|x^2| \ge \sqrt{\frac{1}{3}}$

$x^2 \ge \frac{1}{\sqrt{3}}$

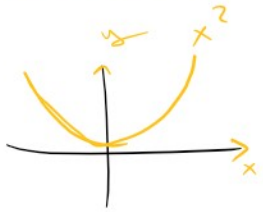
$\sqrt{x^2} \ge \sqrt{\frac{1}{\sqrt{3}}}$

$|x| \ge \frac{1}{\sqrt{\sqrt{3}}}$

Either or

موجبة $x \ge \frac{1}{\sqrt{\sqrt{3}}}$ or

سالبة $x \le -\frac{1}{\sqrt{\sqrt{3}}}$



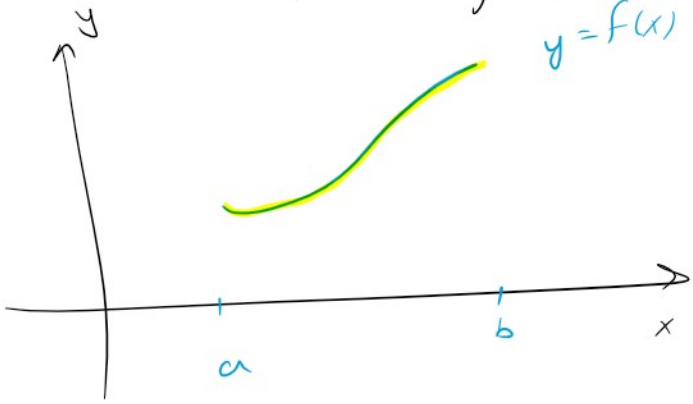
$|x| = 3 \Rightarrow x = 3$ or $x = -3$

$|x| \leq 3 \Rightarrow -3 \leq x \leq 3$

$|x| \geq 3 \Rightarrow x \geq 3$ or $x \leq -3$ ✓

6.3 Q2 , Q 10

6.3
Arc length



Q2 $y = x^{\frac{3}{2}}$, $x = 0$ to $x = 4$

$y = \sqrt{x^3}$ cont. on $[0,4]$

$y' = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$ cont. on $[0,4]$

$y = f(x)$ cont on $[a,b]$

$\boxed{y' = f'(x) \text{ cont on } [a,b]}$

$(y')^2 = \frac{9}{4} x$

$L = \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$ $\Leftarrow$ $L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

$L = \int_c^d \sqrt{1 + (g'(y))^2}\, dy$

$u = 1 + \frac{9}{4}x$, $du = \frac{9}{4} dx$

$x = 0 \Rightarrow u = 1$

$x = 4 \Rightarrow u = 1 + \frac{9}{\cancel{4}}(\cancel{4}) = 10$

$L = \int_1^{10} \sqrt{u}\, \frac{4}{9} du = \frac{4}{9} \int_1^{10} u^{\frac{1}{2}} du$

$\sqrt{1000} = \sqrt{100 \times 10}$
$= 10\sqrt{10}$

$\frac{3}{2} \quad 10 \quad 4 \quad 2 \quad \sqrt{u^3} \Big|^{10} = \frac{8}{\ } \left(\sqrt{1000} - \sqrt{1}\right)$

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$$= \frac{4}{9} \left. \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right|_1^{10} = \frac{4}{9} \cdot \frac{2}{3} \left. \sqrt{u^3} \right|_1^{10} = \frac{8}{27} \left(\sqrt{1000} - \sqrt{1} \right)$$

$$= \frac{8}{27} \left(10\sqrt{10} - 1 \right)$$

مساحة مرصبة

Q10 $\quad y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt \;,\; -2 \le x \le -1$

$$y' = \sqrt{3x^4 - 1} \cdot \boxed{1} = \sqrt{3x^4 - 1}$$ cont on

$$(y')^2 = 3x^4 - 1$$

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_{-2}^{-1} \sqrt{1 + 3x^4 - 1}\, dx$$

$$= \int_{-2}^{-1} \sqrt{3x^4}\, dx$$

$$= \sqrt{3} \int_{-2}^{-1} x^2\, dx$$

$$= \sqrt{3} \left. \frac{x^3}{3} \right|_{-2}^{-1} = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} \left((-1)^3 - (-2)^3 \right) = \frac{1}{\sqrt{3}} (-1 + 8)$$

$$= \frac{7}{\sqrt{3}}$$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{x^4} = |x^2| = x^2$

$3x^4 - 1 \ge 0$

$3x^4 \ge 1$

$\sqrt{x^4} \ge \sqrt{\frac{1}{3}}$

$\sqrt{x^4} \ge \frac{1}{\sqrt{3}}$

$|x^2| \ge \frac{1}{\sqrt{3}}$

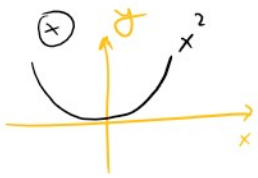
$\sqrt{x^2} \ge \sqrt{\frac{1}{\sqrt{3}}}$

$\sqrt{x^2} \ge \frac{1}{\sqrt{\sqrt{3}}}$

$|x| \ge \frac{1}{\sqrt{\sqrt{3}}}$

⊕ either $x \ge \frac{1}{\sqrt{\sqrt{3}}}$

⊖ or $x \le \frac{-1}{\sqrt{\sqrt{3}}}$



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$\sqrt{3}$

$\sqrt{3}\sqrt{3}$

$|x| = 3 \Rightarrow x = 3$ or $x = -3$

$|x| \leq 3 \Rightarrow -3 \leq x \leq 3$

$|x| \geq 3 \Rightarrow x \geq 3$ or $x \leq -3$

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