Problem

Let *X* and *Y* be sets, let *A* and *B* be any subsets of *X*, and let *C* and *D* be any subsets of *Y*. Determine which of the properties are true for all functions *F* from *X* to *Y* and which are false for at least one function *F* from *X* to *Y*. Justify your answers.

Exercise

If $A \subseteq B$ then $F(A) \subseteq F(B)$.

Step-by-step solution

Step 1 of 2

The given function is $f : X \to Y$.

$f(A) \subseteq f(B)$ only if for each element $y \in f(A)$, it belongs to $f(B)$ also.

It is given that $y = f(x)$ for some $x \in A$.

So, $y \in f(A)$.

But A is a subset of B, $A \subseteq B$.

Therefore, if $x \in A$ then x belongs to B also.

$x \in B$

The definition of a function states a property according to which no element of the domain set of the function can have more than one image in the co-domain set.

Therefore, $f(x)$ will always attain the same value for any particular value of $x = x_1$.

Thus, $f(x) = y$ for $x \in B$

So, y must be inside $f(B)$, $y \in f(B)$.

Since, $y \in f(A)$ and $y \in f(B)$

Step 2 of 2

But if A is not a subset of B, then there are values of x such that $x \in A$ but $x \notin B$.

Since, the domain of function $f : X \to Y$ doesn't fall in B, the co-domain may not be same as the co-domain for the domain of values that belong to B. So, if A is not a subset of B, then $f(A)$ may not be subset of $f(B)$.

Hence, it is proved that for all subsets A and B of X, if $A \subseteq B$, then $f(A) \subseteq f(B)$

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