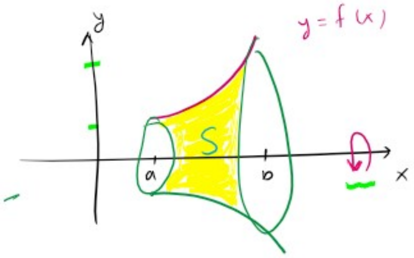
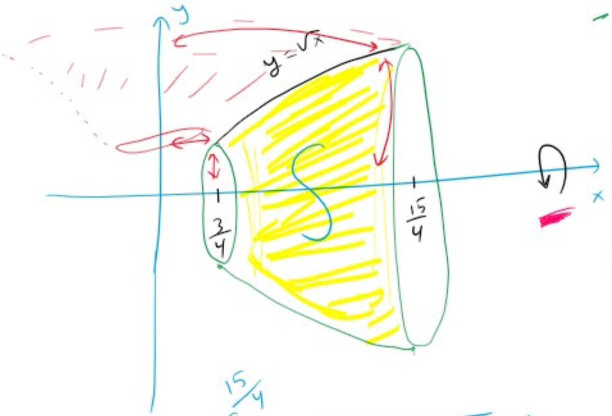
6.4
Saturday, April 17, 2021 9:26 AM

Q14 , Q24



Q14 $y=\sqrt{x}$, $\frac{3}{4} \le x \le \frac{15}{4}$, x-axis



$y=f(x) \geq 0$ on $[a,b]$

$y=f(x)$ cont, diff on $[a,b]$

$$S = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

$f=\sqrt{x}$

$f' = \frac{1}{2\sqrt{x}}$

$(f')^2 = \frac{1}{4x}$

$$S = 2\pi \int_{3/4}^{15/4} \sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx$$

$$= 2\pi \int_{3/4}^{15/4} \sqrt{x}\sqrt{\frac{4x+1}{4x}}\,dx$$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$$= \cancel{2}\pi \int_{3/4}^{15/4} \cancel{\sqrt{x}}\, \frac{\sqrt{4x+1}}{\cancel{\sqrt{4}}\,\cancel{\sqrt{x}}}\,dx$$

$$= \pi \int_{3/4}^{15/4} \sqrt{4x+1}\,dx$$

$u = 4x+1$

$du = 4\,dx$

$x=\frac{3}{4} \Rightarrow u = \cancel{4}(\frac{3}{\cancel{4}})+1 = 4$

$x=\frac{15}{4} \Rightarrow u = \cancel{4}(\frac{15}{\cancel{4}})+1 = 16$

$$= \pi \int_4^{16} \sqrt{u}\,\frac{du}{4}$$

$$= \frac{\pi}{4}\int_4^{16} u^{\frac{1}{2}}\,du = \frac{\pi}{4}\left.\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right|_4^{16}$$

$$= \frac{\pi}{\cancel{4}_2}\,\frac{\cancel{2}}{3}\left.\sqrt{u^3}\right|_4^{16} = \frac{\pi}{6}\left(\sqrt{(16)^3} - \sqrt{(4)^3}\right)$$

$$= \frac{\pi}{6}\left(16\sqrt{16} - 4\sqrt{4}\right)$$

$$\begin{array}{r} 40 \\ 24 \\ \hline 64 \end{array}$$

$$= \frac{\pi}{\ } \left(16(4) - 4(2)\right)$$

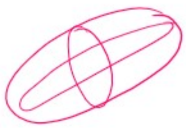
$= \frac{\pi}{6}\left(16\,(4) - 4\,(2)\right)$

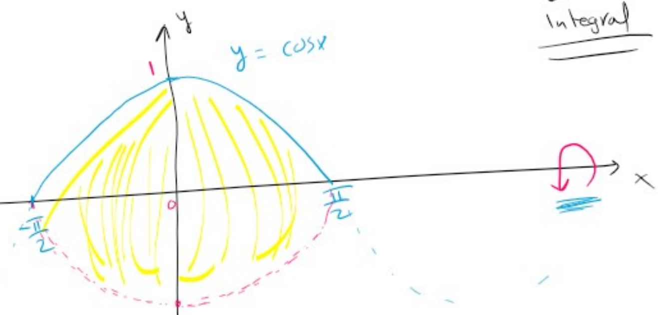
$= \frac{\pi}{6}\left(64 - 8\right)$

$= \frac{\pi}{6}\left(56\right) = \boxed{\frac{28\pi}{3}}$ ✓

مساحه
مطلوب

Q24 $y = \cos x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, x-axis, Find SA only, write Integral





$$S = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

$y = \cos x$

$y' = -\sin x$

$(y')^2 = \sin^2 x$

$$= 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sqrt{1+\sin^2 x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$x = -\frac{\pi}{2} \Rightarrow u = -1$

$x = \frac{\pi}{2} \Rightarrow u = 1$

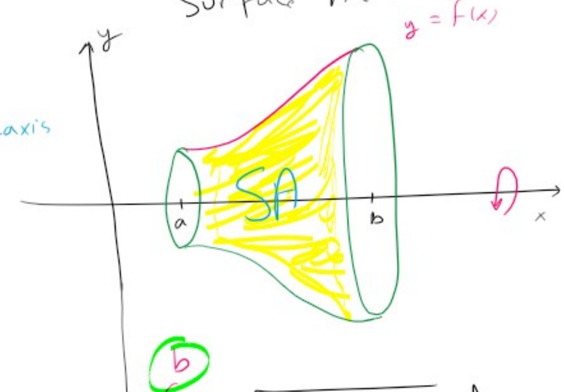
$$= 2\pi \int_{-1}^{1} \sqrt{1+u^2}\,du$$??

ch8

Integrals by Trigonometric Substitution

6.4 Q14, Q24

Surface Area



Q14 $y = \sqrt{x}$, $\frac{3}{4} \le x \le \frac{15}{4}$, x-axis

$y = \sqrt{x}$ cont. on $\left[\frac{3}{4}, \frac{15}{4}\right]$

$y = \sqrt{x}$ + on $\left[\frac{3}{4}, \frac{15}{4}\right]$

$y' = \frac{1}{2\sqrt{x}}$ cont. on $\left[\frac{3}{4}, \frac{15}{4}\right]$

$$SA = 2\pi \int^{b} f(x)\sqrt{1+(f'(x))^2}\,dx$$

$y' = \frac{1}{2\sqrt{x}}$

$(y')^2 = \frac{1}{4x}$

$SA = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$

$f(x) \geq 0$

f, f' cont. on $[a,b]$

$SA = 2\pi \int_{3/4}^{15/4} \sqrt{x}\,\sqrt{1+\frac{1}{4x}}\,dx$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 2\pi \int_{3/4}^{15/4} \sqrt{x}\,\sqrt{\frac{4x+1}{4x}}\,dx$

$= \cancel{2}\pi \int_{3/4}^{15/4} \cancel{\sqrt{x}}\,\frac{\sqrt{4x+1}}{\cancel{\sqrt{4}}\,\cancel{\sqrt{x}}}\,dx$

$= \pi \int_{3/4}^{15/4} \sqrt{4x+1}\,dx$

$u = 4x+1$

$du = 4\,dx$

$x = \frac{3}{4} \Rightarrow u = \cancel{4}\left(\frac{3}{\cancel{4}}\right)+1 = 4$

$x = \frac{15}{4} \Rightarrow u = \cancel{4}\left(\frac{15}{\cancel{4}}\right)+1 = 16$

$= \pi \int_4^{16} \sqrt{u}\,\frac{du}{4}$

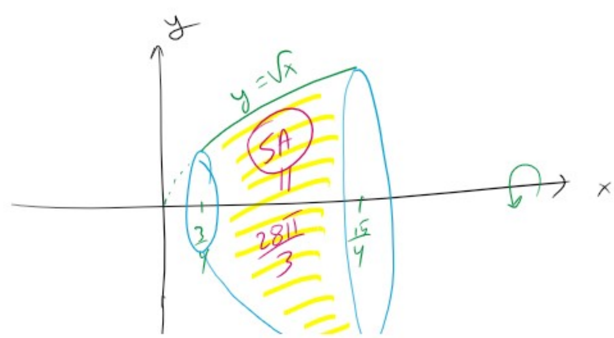
$= \frac{\pi}{4} \int_4^{16} u^{\frac{1}{2}}\,du = \frac{\pi}{4}\left.\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right|_4^{16}$

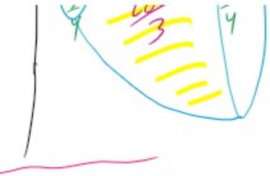
$= \frac{\pi}{\underset{2}{\cancel{4}}}\,\frac{\cancel{2}}{3}\left.\sqrt{u^3}\right|_4^{16} = \frac{\pi}{6}\left(\sqrt{(16)^3} - \sqrt{(4)^3}\right)$

$= \frac{\pi}{6}\left(16\sqrt{16} - 4\sqrt{4}\right) = \frac{\pi}{6}\left(16(4) - 4(2)\right)$

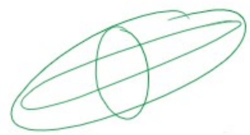
$= \frac{\pi}{6}(64-8) = \frac{\pi}{\underset{3}{\cancel{6}}}\left(\overset{28}{\cancel{56}}\right) = \frac{28}{3}\pi$

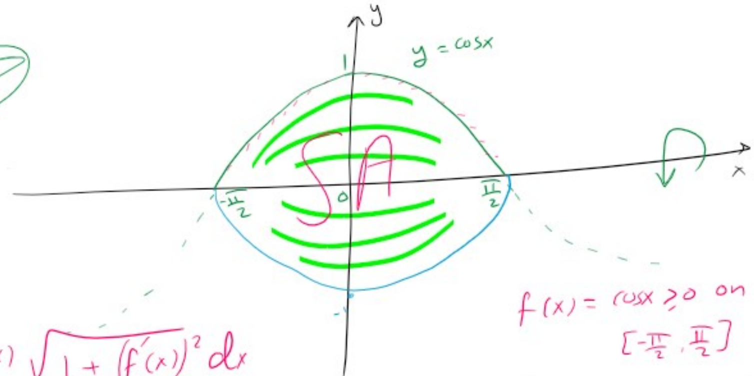
[illegible]





Q24 $y = \cos x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, x-axis





$f(x) = \cos x \ge 0$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$f'(x) = -\sin x$ cont. on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$(f'(x))^2 = \sin^2 x$

$$SA = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2}\, dx$$

$$= 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \boxed{\cos x} \sqrt{1 + \sin^2 x}\; \boxed{dx}$$

??

$$SA = 2\pi \int_{-1}^{1} \sqrt{1 + u^2}\; du$$

$u = \sin x$

$du = \boxed{\cos x \, dx}$

$x = -\frac{\pi}{2} \Rightarrow u = -1$

$x = \frac{\pi}{2} \Rightarrow u = 1$

We need to understand Ch8 to evaluate this integral

Integrals by Trigonometric Substitution

