7.7 :- Hyperbolic Functions

[4] By using $\boxed{\cosh^2 x - \sinh^2 x = 1}$ find the values of the remaining five hyperbolic functions

- $\cosh x = \frac{13}{5}$, $x > 0$ $\longrightarrow$ $\sinh^2 x = \cosh^2 x - 1$

$$= \left(\frac{13}{5}\right)^2 - 1$$

$$\sinh^2 x = \frac{144}{25} \longrightarrow \sinh x = \pm\frac{12}{5}$$

$\boxed{\sinh x \cancel{=} -\frac{12}{5}}$ reject

$\boxed{\sinh x = \frac{12}{5}}$ Accept $x>0$

- $\sinh x = \frac{12}{5}$
- $\tanh x = \frac{12}{13}$
- $\operatorname{csch} x = \frac{5}{12}$
- $\operatorname{sech} x = \frac{5}{13}$
- $\coth x = \frac{13}{12}$

[6] Rewrite the expression in terms of exponentials

$$\sinh(2\ln x) = \frac{e^{2\ln x} - e^{-2\ln x}}{2}$$

$$= \frac{x^2 - x^{-2}}{2} = \frac{x^4 - 1}{2x^2}$$

$\boxed{10}$ $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$

$= \ln(e^{x}) + \ln(e^{-x})$

$= x - x = 0$

Another solution

$\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$

$= \ln\left[(\cosh x + \sinh x)(\cosh x - \sinh x)\right]$

$= \ln\left[\cosh^2 x - \sinh^2 x\right]$

$= \ln[1] = 0$

Find the derivative of y with respect x

$\boxed{15}$ $y = 2\sqrt{x} \tanh\sqrt{x}$

$y' = \cancel{2\sqrt{x}} \cdot \operatorname{sech}^2(\sqrt{x}) \cdot \frac{1}{\cancel{2\sqrt{x}}} + \tanh\sqrt{x} \cdot \frac{\cancel{2}\,1}{\cancel{2}\sqrt{x}}$

$= \operatorname{sech}^2(\sqrt{x}) + \frac{\tanh\sqrt{x}}{\sqrt{x}}$

$\boxed{18}$ $y = \ln(\cosh x)$

$y' = \frac{1}{\cosh x} \cdot \sinh x = \tanh x$

$\boxed{21}$ $y = \ln\cosh x - \frac{1}{2}\tanh^2 x$

$y' = \frac{1}{\cosh x} \cdot \sinh x - \frac{1}{2}\, 2\tanh x \cdot \operatorname{sech}^2 x$

$y' = \tanh x - \tanh x \operatorname{sech}^2 x$

$= \tanh x \left[1 - \operatorname{sech}^2 x\right]$

$= \tanh^3 x$

$\boxed{42}$ $\int \sinh \frac{x}{5} \, dx$

Let $u = \frac{x}{5}$

$du = \frac{1}{5} dx$

$\int 5 \sinh u \, du$

$I = 5 \cosh u + C$

$I = 5 \cosh\left(\frac{x}{5}\right) + C$

$\boxed{46}$ $\int \coth \frac{\theta}{\sqrt{3}} \, d\theta$

Let $u = \frac{\theta}{\sqrt{3}}$

$du = \frac{1}{\sqrt{3}} d\theta$

$\sqrt{3} \int \coth u \, du$

$= \sqrt{3} \int \frac{\cosh u}{\sinh u} \, du$

$= \sqrt{3} \ln |\sinh u| + C$

$= \sqrt{3} \ln \left| \sinh \frac{\theta}{\sqrt{3}} \right| + C$

$\boxed{52}$ $$\int_0^{\ln 2} \tanh 2x \, dx$$

$$= \frac{1}{2}\int_0^{\ln 2} \frac{2\sinh 2x}{\cosh 2x}\, dx$$

$$= \frac{1}{2} \ln|\cosh 2x| \Big]_0^{\ln 2}$$

$$= \frac{1}{2}\left[\ln\left|\frac{e^{2x}+e^{-2x}}{2}\right|\right]_0^{\ln 2}$$

$$= \frac{1}{2}\left[\ln\left|\frac{4+1/4}{2}\right| - \ln\left|\frac{2}{2}\right|\right]$$

$$= \frac{1}{2}\left[\ln\left(\frac{17}{8}\right) - 0\right] = \frac{1}{2}\ln\left(\frac{17}{8}\right).$$

$\boxed{54}$ $$\int_0^{\ln 2} 4e^{-\theta}\sinh\theta \, d\theta$$

$$\int_0^{\ln 2} 4 \; e^{-\theta}\left(\frac{e^{\theta}-e^{-\theta}}{2}\right) d\theta$$

$$= 4\int_0^{\ln 2} \frac{1-e^{-2\theta}}{2}\, d\theta$$

$$= 2\int_0^{\ln 2}\left(1 - \frac{2e^{-2\theta}}{2}\right) d\theta$$

$$= 2\left[\theta + \frac{e^{-2\theta}}{2}\right]_0^{\ln 2}$$

$$= 2\left[\ln 2 + \frac{1}{8} - 0 - \frac{1}{2}\right]$$

$$= 2\left[\ln 2 - \frac{3}{8}\right]$$

$$= \ln 4 - \frac{3}{4}$$

$\boxed{57}$

$$\int_1^2 \frac{\cosh(\ln t)}{t}\, dt$$

let $U = \ln t$

$dU = \frac{1}{t}\, dt$

* $t=1 \longrightarrow U=0$

$t=2 \longrightarrow \ln 2$

$$= \int_0^{\ln 2} \cosh(U)\, dU$$

$$= \sinh(U)\Big]_0^{\ln 2}$$

$$= \sinh(\ln 2) - \cancel{\sinh(0)}$$

$$= \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{2 - 1/2}{2} = \boxed{\frac{3}{4}}$$

$\boxed{60}$. $\displaystyle\int_0^{\ln 10} 4\sinh^2\left(\frac{x}{2}\right) dx$

Note:

$\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

$$= \int_0^{\ln 10} 4\left[\frac{\cosh x - 1}{2}\right] dx$$

$$= 2\int_0^{\ln 10} \cosh x - 1 \, dx$$

$$= 2\Big[\sinh x - x\Big]_0^{\ln 10}$$

$$= 2\Big[\sinh(\ln 10) - \ln 10\Big]$$

$$= 2\left[\frac{e^{\ln 10} - e^{-\ln 10}}{2} - \ln 10\right]$$

$$= 10 - \frac{1}{10} - 2\ln 10$$