جامعة بيرزيت
BIRZEIT UNIVERSITY

Communications and Digital Data Networks ENEE3401

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Midterm Exam
Second Semester 2024-2025

Problem 1: 25 Points

A baseband digital communication system uses the signals $s_1(t)$ and $s_2(t)$ to represent the equally probable binary digits 1 and 0, respectively,

$$s_1(t) = \begin{cases} \frac{2t}{\tau} & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$$

$$s_2(t) = \begin{cases} 2 & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$$

8 a. Find the energies in $s_1(t)$ and $s_2(t)$
9 b. Find the probability of error in AWGN with PSD = $N_0/2$.
8 c. Draw the block diagram of the optimum receiver.

a. $E_1 = \int_0^{\tau} s_1(t)^2 dt = \int_0^{\tau} \left[\frac{2t}{\tau}\right]^2 dt = \frac{4\tau}{3}$ 3+1

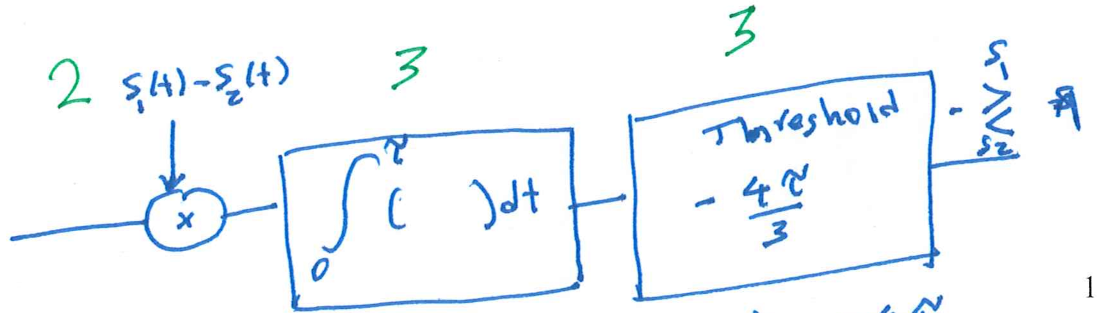
$E_2 = \int_0^{\tau} s_2(t)^2 dt = \int_0^{\tau} (2)^2 dt = 4\tau$ 3+1

b. $P_b = Q\left(\sqrt{\frac{\int_0^{\tau}(s_1-s_2)^2 dt}{2N_0}}\right)$ 3

$\int_0^{\tau} (s_1(t)-s_2(t))^2 dt = \int_0^{\tau} \left[\frac{2t}{\tau} - 2\right]^2 dt = \frac{4\tau}{3}$ 3

$P_b = Q\left(\sqrt{\frac{4\tau/3}{2N_0}}\right) = Q\left(\sqrt{\frac{2\tau}{3N_0}}\right)$ 3

c.



Threshold $= \frac{1}{2}(E_1 - E_2) = \frac{1}{2}\left(\frac{4\tau}{3} - 4\tau\right) = -\frac{4\tau}{3}$

Problem 2: 25 Points

Consider the two bases functions $\varphi_1(t)$ and $\varphi_2(t)$ defined as:

$$\varphi_1(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau \\ 0, & otherwise \end{cases} \qquad \varphi_2(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau/2 \\ -\frac{1}{\sqrt{\tau}}, & \tau/2 \le t \le \tau \end{cases}$$

The signal that represents digit 1 in AWGN with PSD $N_0/2$ is given as:

$$s_1(t) = \begin{cases} A, & 0 \le t \le \tau/2 \\ 2A, & \tau/2 \le t \le \tau \end{cases}; \qquad \text{where } s_2(t) = -s_1(t).$$

a. Find the signal space representation of $s_1(t)$ and $s_2(t)$ in the $\emptyset_1(t), \emptyset_2(t)$ plane

b. Find the probability of error in additive white Gaussian noise with PSD $N_0/2$ for the two signals representing digits 1 and 0.

$$\int_0^{\tau} \phi_1(t)^2 dt = \int_0^{\tau} \phi_2(t)^2 dt = 1 \; ; \; \int_0^{\tau} \phi_1(t)\, \phi_2(t)\, dt = 0$$

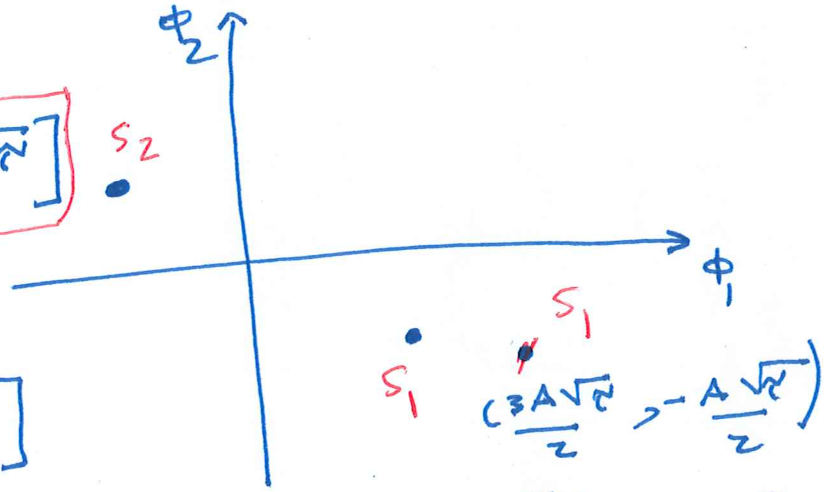
a. $s_1(t) = s_{11}\, \phi_1(t) + s_{12}\, \phi_2(t)$

5 $s_{11} = \int_0^{\tau} s_1(t)\phi_1(t)dt = \int_0^{\tau/2} \frac{1}{\sqrt{\tau}} A\, dt + \int_{\tau/2}^{\tau} \frac{1}{\sqrt{\tau}} (2A) dt = \frac{3A\sqrt{\tau}}{2}$ 3

5 $s_{12} = \int_0^{\tau} s_1(t)\phi_2(t)dt = \int_0^{\tau/2} A \cdot \frac{1}{\sqrt{\tau}} dt + \int_{\tau/2}^{\tau} 2A\left(-\frac{1}{\sqrt{\tau}}\right)dt$

$= -\frac{A\sqrt{\tau}}{2}$ 3

16 $\boxed{s_1 \equiv \left[\frac{3A\sqrt{\tau}}{2}, -\frac{A\sqrt{\tau}}{2}\right]}$



$s_2(t) = -s_1(t)$

$\Rightarrow$ $s_2 \equiv \left[-\frac{3A\sqrt{\tau}}{2}, +\frac{A\sqrt{\tau}}{2}\right]$

9 b. $P_b = Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right)$; $d_{12}^2 = \left(\frac{6A\sqrt{\tau}}{2}\right)^2 + \left(\frac{2A\sqrt{\tau}}{2}\right)^2 = 10A^2\tau$

4 $P_b = Q\left(\frac{\sqrt{10A^2\tau}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{5A^2\tau}{N_0}}\right)$ answer 2 Two methods

distance 3

or $P_b = Q\left(\sqrt{\frac{\int_0^{\tau}(s_1 - s_2)^2 dt}{2N_0}}\right)$

Problem 3: 25 Points

Consider the two bases functions $\varphi_1(t)$ and $\varphi_2(t)$ defined as:

$$\varphi_1(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau/2 \\ -\frac{1}{\sqrt{\tau}}, & \tau/2 \le t \le \tau \end{cases} \qquad \varphi_2(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$$

Let $s_1(t) = 2\emptyset_1(t) - \emptyset_2(t)$; $s_2(t) = \emptyset_1(t) + \emptyset_2(t)$; $s_3(t) = -\emptyset_1(t) + 2\,\emptyset_2(t)$;

4 a. Find the correlation coefficient between $s_2(t)$ and $s_3(t)$.
8 b. Find the average energy per signal.
4 c. Find the probability of receiving $s_3(t)$ if $s_1(t)$ was sent.

$$\int_0^\tau \phi_1(t)^2 dt = \int_0^\tau \phi_2(t)^2 dt = 1; \quad \int_0^\tau \phi_1(t)\,\phi_2(t)\,dt = 0$$

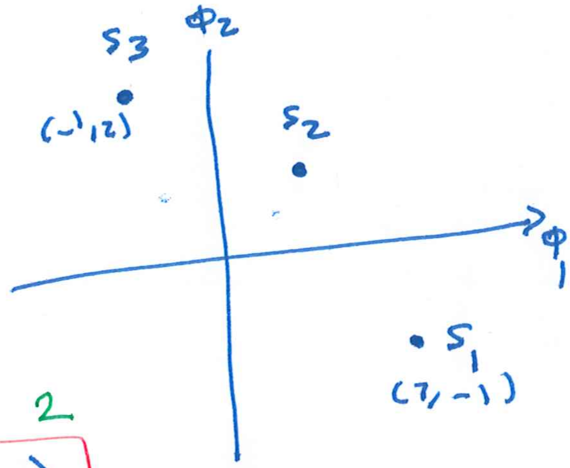
3 a. $$\rho = \frac{1}{\sqrt{E_2 E_3}} \int_0^\tau s_2(t)\, s_3(t)\, dt \Rightarrow \rho = \frac{1}{\sqrt{2*5}} \int_0^\tau [\phi_1 + \phi_2][-\phi_1 + 2\phi_2]\,dt$$

2 $$E_2 = \int_0^\tau [\phi_1(t) + \phi_2(t)]^2 dt = 2 \qquad \Rightarrow \rho = \frac{1}{\sqrt{10}}[1]$$

2 $$E_3 = \int_0^\tau [-\phi_1 + 2\phi_2]^2 dt = 5$$

$$\boxed{\rho = \frac{1}{\sqrt{10}}}$$ 2

b. $$E_{av} = \frac{5 + 2 + 5}{3} = \frac{12}{3} = 4$$



c. $$p(s_3|s_1) = Q\left(\frac{d_{13}}{\sqrt{2N_0}}\right)$$ 3

3 $$d_{13} = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$ 2

$$p(s_3|s_1) = Q\left(\frac{3\sqrt{2}}{\sqrt{2N_0}}\right) = \boxed{Q\left(\frac{3}{\sqrt{N_0}}\right)}$$

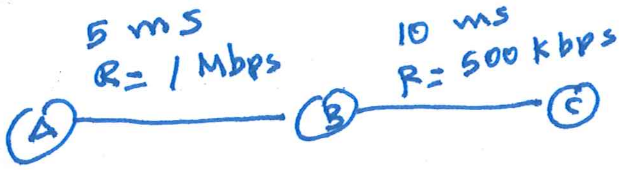
Problem 4: 25 Points

A message of 6,000 bytes is to be sent from Host A to Host C using packet switching through an intermediate node B (i.e., two hops: A → B → C). The message is divided into packets of 1,500 bytes each, with no header overhead for simplicity. Packets are forwarded as soon as they are received (i.e., store-and-forward with pipelining).

The characteristics of the links are:

- Link A–B: Propagation delay = 5 ms, Data rate = 1 Mbps
- Link B–C: Propagation delay = 10 ms, Data rate = 500 kbps

Calculate the total time from when the first bit of the message leaves Host A until the last bit of the message is received at Host C.



- Time to transmit one packet over AB

$$T_1 = \frac{1500 * 8}{1,000,000} = 12 \text{ ms}$$

Time to transmit one packet over BC

$$T_2 = \frac{1500 * 8}{500000} = 24 \text{ ms}$$

$$T_{total} = 24 * 4 + 10 + 17$$

$$= 123$$