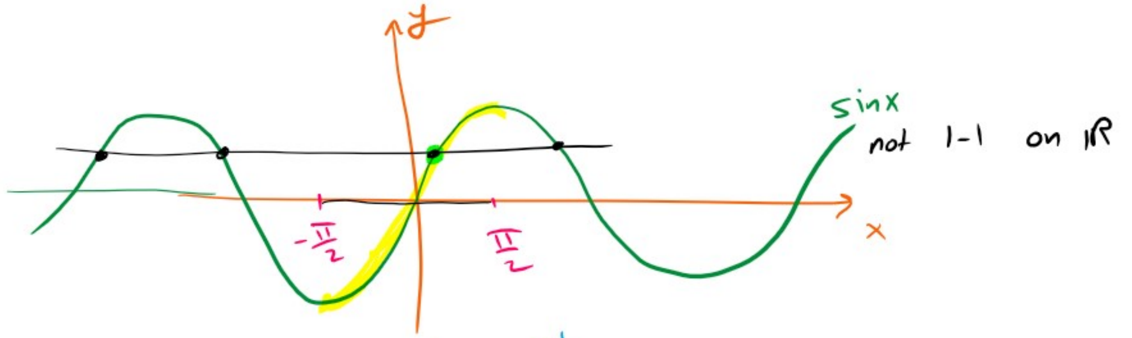
7.6 Inverse Trigonometric functions

Monday, December 13, 2021 12:38 PM

$f = \sin x, \ \cos x, \ \tan x, \ \cot x, \ \sec x, \ \csc x$

مواليد فقط

1-1 functions



Q. How to find f^{-1} ?

A. We need to make f 1-1

Q. How to make f 1-1?

A. we take sub-domain

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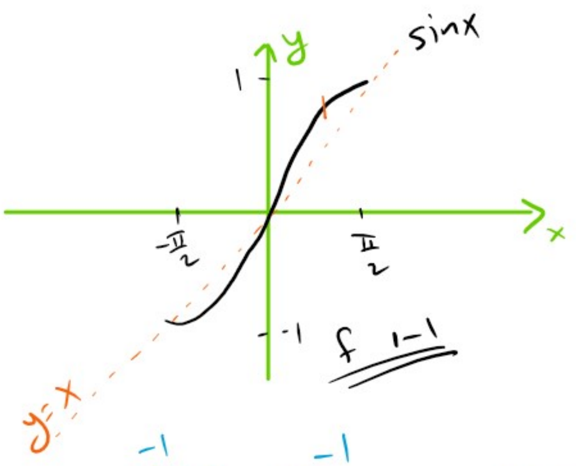
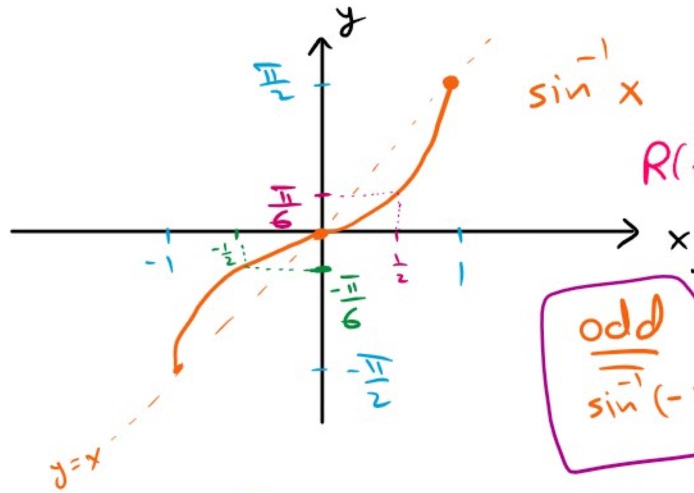
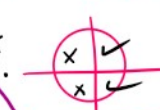
1) $f(x) = \sin x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}] \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}



$\Rightarrow$



$R(f^{-1}) = [-\frac{\pi}{2}, \frac{\pi}{2}]$



odd

$\sin^{-1}(-x) = -\sin^{-1} x$

$f^{-1}(x) = \sin^{-1} x = \arcsin x$ on $[-1, 1]$

$D(f) = R(f^{-1})$

$R(f) = D(f^{-1})$

$f^{-1}(x) = \sin^{-1} x = \arc\sin x$ on $[-1,1]$

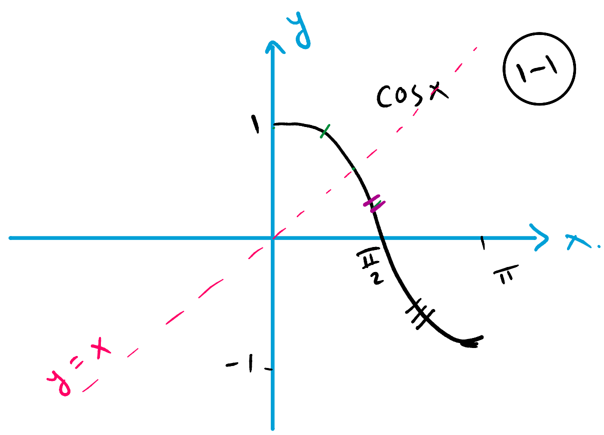
$D(f) = R(f^{-1})$
$R(f) = D(f^{-1})$

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\frac{1}{2} = -\frac{\pi}{6}$

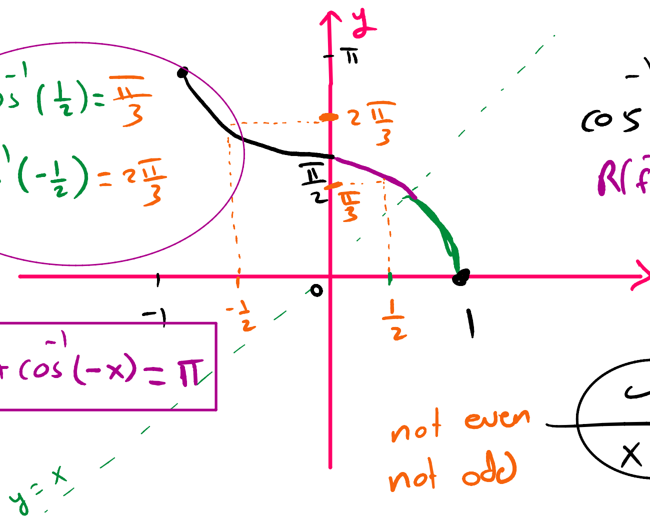
✓ $\sin^{-1}(x) + \sin^{-1}(-x) = 0$

(2) $f(x) = \cos x$ on $[0,\pi] \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}



$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

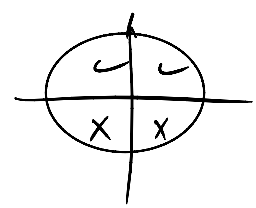


$\cos^{-1} x$

$R(f^{-1}) = [0,\pi]$

$\boxed{\cos^{-1}(x) + \cos^{-1}(-x) = \pi}$

not even
not odd

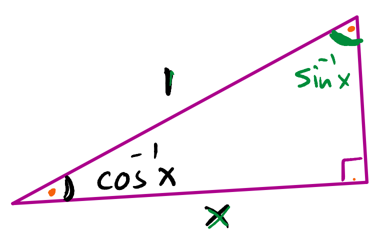


$f^{-1}(x) = \cos^{-1} x = \arc\cos x$ on $[-1,1]$

$\boxed{\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}}$

زاوية $\cos^{-1} x \Rightarrow$ x مجاور زاوية

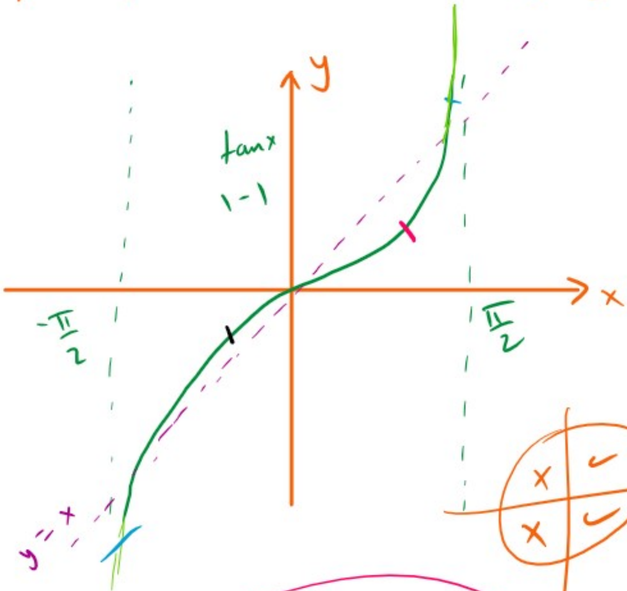
زاوية $\sin^{-1} x \Rightarrow$ x مقابل زاوية

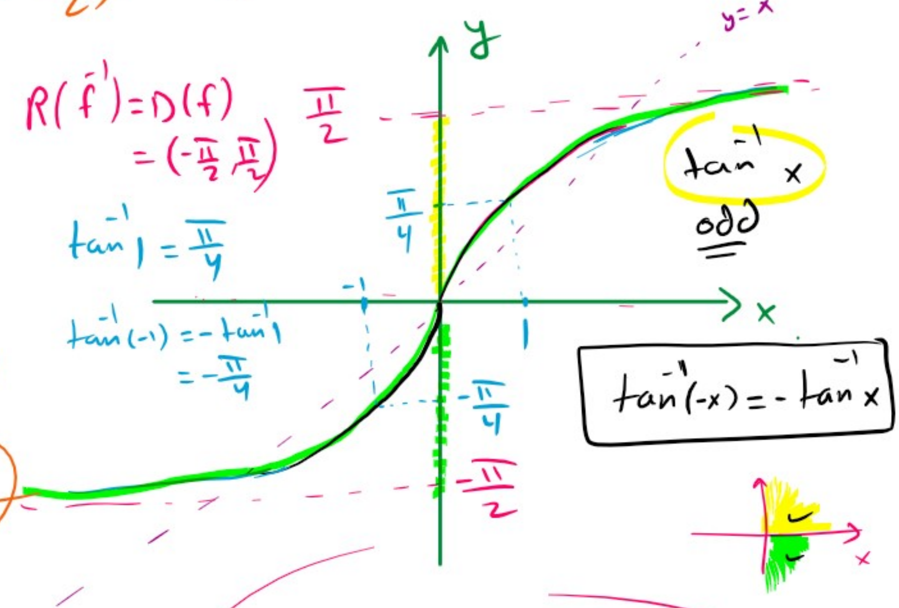


(3) $f(x) = \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}

y=x

$f(x) = \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}





$R(f^{-1}) = D(f) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1} 1 = \frac{\pi}{4}$

$\tan^{-1}(-1) = -\tan^{-1} 1 = -\frac{\pi}{4}$

$\tan^{-1} x$ odd

$\boxed{\tan^{-1}(-x) = -\tan^{-1} x}$

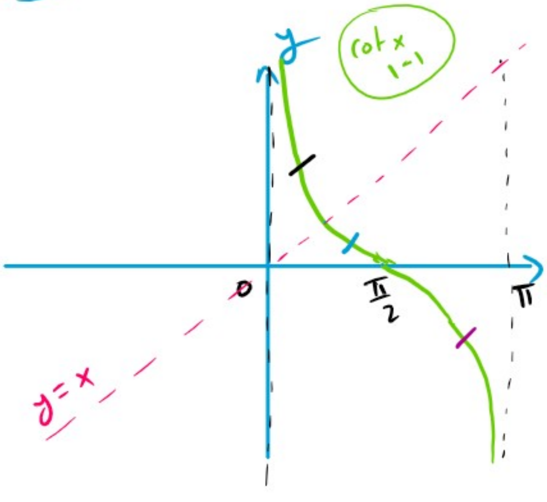
$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$ and $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

$y = \frac{\pi}{2}$ is H. Asy.

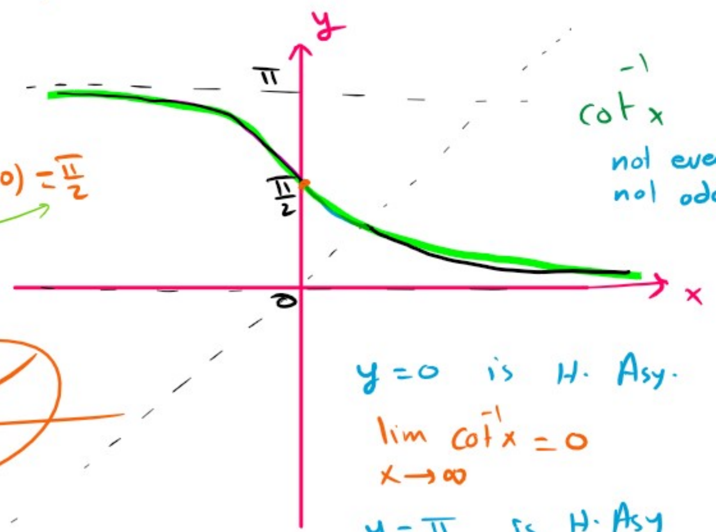
$y = -\frac{\pi}{2}$ is H. Asy.

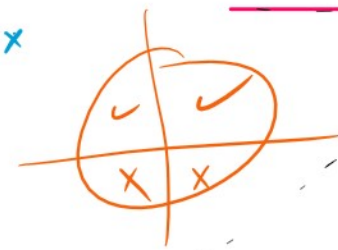
$f^{-1}(x) = \tan^{-1} x = \text{arc}\tan x$ on $\mathbb{R}$

$\boxed{4}$ $f(x) = \cot x$ on $(0, \pi) \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}



$\cot^{-1}(0) = \frac{\pi}{2}$





$\cot^{-1} x$ not even not odd

$y = 0$ is H. Asy.

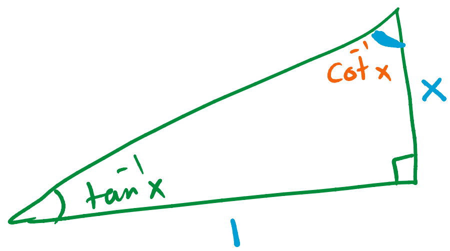
$\lim_{x \to \infty} \cot^{-1} x = 0$

$y = \pi$ is H. Asy

$f^{-1}(x) = \cot^{-1} x = \text{arc}\cot x$ on $\mathbb{R}$

$y = \pi$ is H. Asy

$\lim_{x \to -\infty} \cot^{-1} x = \pi$

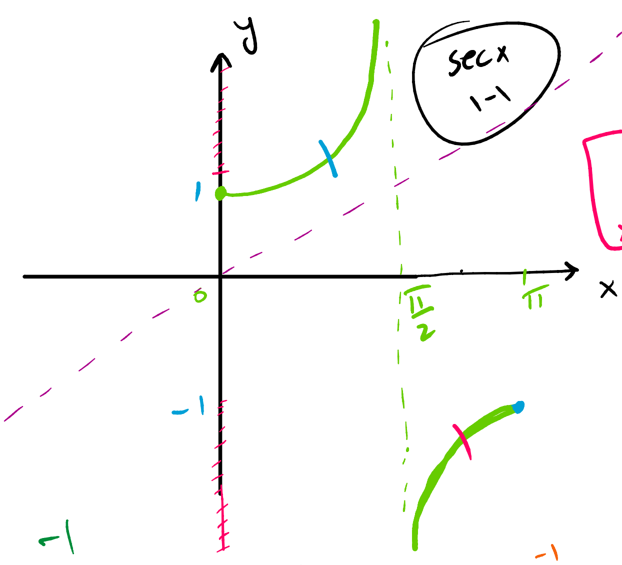


$$\boxed{\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}}$$

$\tan^{-1} 0 + \cot^{-1} 0 \overset{?}{=} \frac{\pi}{2}$

$0 + \frac{\pi}{2} = \frac{\pi}{2}$ ✓

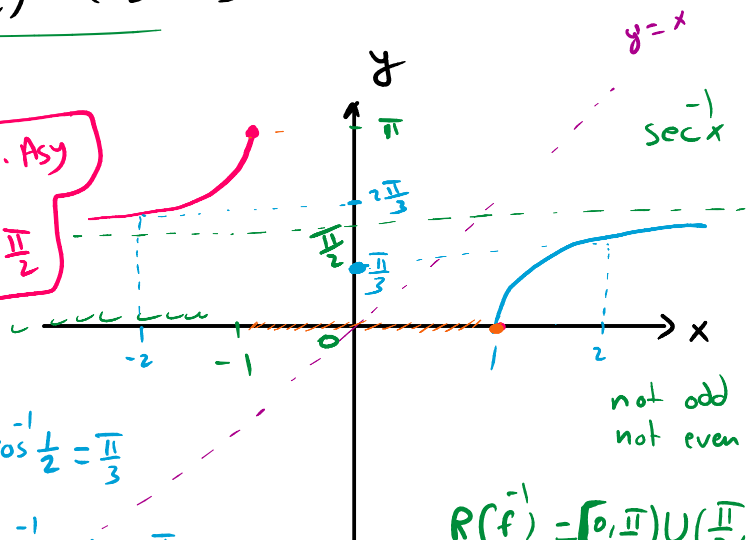
5) $f(x) = \sec x$ on $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}



sec x 1-1

$y = \frac{\pi}{2}$ H. Asy

$\lim_{x \to \infty, -\infty} \sec^{-1} x = \frac{\pi}{2}$



$\sec^{-1} 2 = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$

$\sec^{-1} -2 = \cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$

$\sec^{-1} 1 = \cos^{-1} 1 = 0$

$\sec^{-1}(-1) = \cos^{-1}(-1) = \pi$

not odd
not even

$R(f^{-1}) = [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

$\boxed{\sec^{-1} \frac{1}{2} \text{ undefined}}$

$f^{-1}(x) = \sec^{-1} x = \text{arc}\sec x$ on $(-\infty, -1] \cup [1, \infty)$

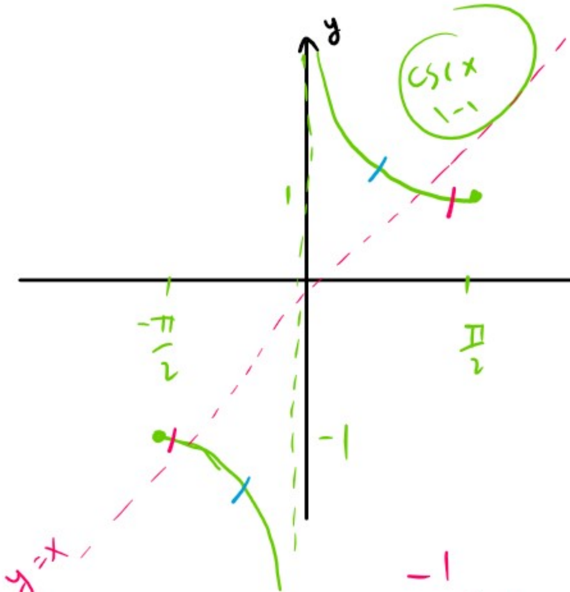
$D(f^{-1}) = R(f)$

$$\boxed{\sec^{-1} x = \cos^{-1} \frac{1}{x}} = \frac{\pi}{2} - \sin^{-1} \frac{1}{x} \checkmark$$

$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$ ✓

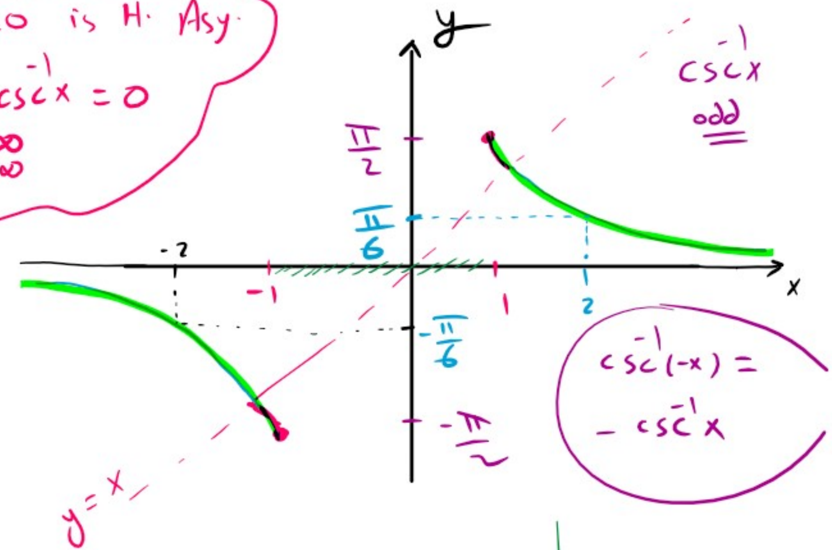
6) $f(x) = \csc x$ on $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}] \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}

(6) $f(x) = \csc x$ on $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}] \Rightarrow f$ 1-1 $\Rightarrow f$ has f^{-1}



$y = 0$ is H. Asy.

$\lim\limits_{x \to \infty, -\infty} \csc^{-1} x = 0$

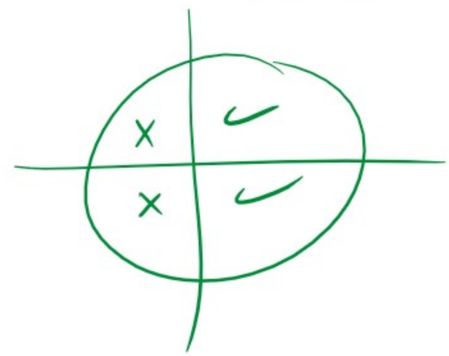


$\csc^{-1}(-x) = -\csc^{-1} x$

$\csc^{-1}(2) = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$\csc^{-1}(-2) = -\csc^{-1}(2) = -\frac{\pi}{6}$

$\csc^{-1} 0 =$ undefined



$\boxed{\csc^{-1} x = \sin^{-1}\frac{1}{x}} = \frac{\pi}{2} - \cos^{-1}\frac{1}{x}$

$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$

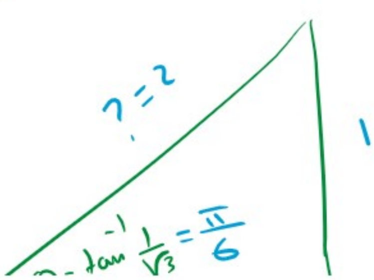
Exp $\tan^{-1}\frac{1}{\sqrt{3}}$

نزل

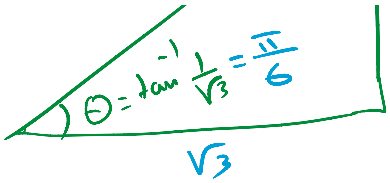
$\theta = \tan^{-1}\frac{1}{\sqrt{3}}$

$\tan\theta = \cancel{\tan}\left(\cancel{\tan^{-1}}\frac{1}{\sqrt{3}}\right)$

$\tan\theta = \frac{1}{\sqrt{3}}$



$?^2 = 1^2 + (\sqrt{3})^2$



$$?^2 = 1^2 + (\sqrt{3})^2$$

$$?^2 = 1 + 3$$

$$?^2 = 4$$

$$? = 2$$

$\sin\theta = \frac{1}{2}$

$\Downarrow$

$\theta = \frac{\pi}{6}$

$$\theta = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

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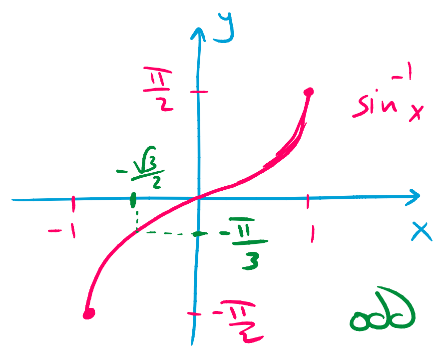
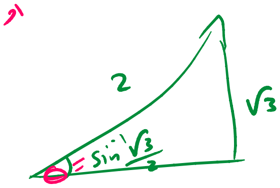
(12) $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\frac{\sqrt{3}}{2}$

$60 = \frac{\pi}{3}$



odd



$? = 1$

$\cos\theta = \frac{1}{2}$

$$\cot\left(-\frac{\pi}{3}\right)$$

$$\frac{\cos\frac{-\pi}{3}}{\sin\frac{-\pi}{3}} = \frac{\cos\frac{\pi}{3}}{-\sin\frac{\pi}{3}} = \frac{\frac{1}{\cancel{2}}}{-\frac{\sqrt{3}}{\cancel{2}}} = -\frac{1}{\sqrt{3}}$$

✓
