

Experiment 4

①

RC Circuit

First, let

$$x(t) = V_0 u(t);$$

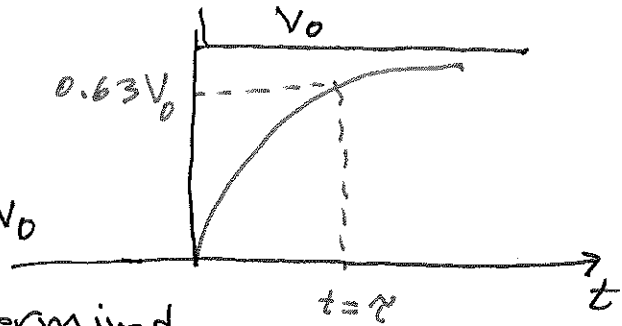
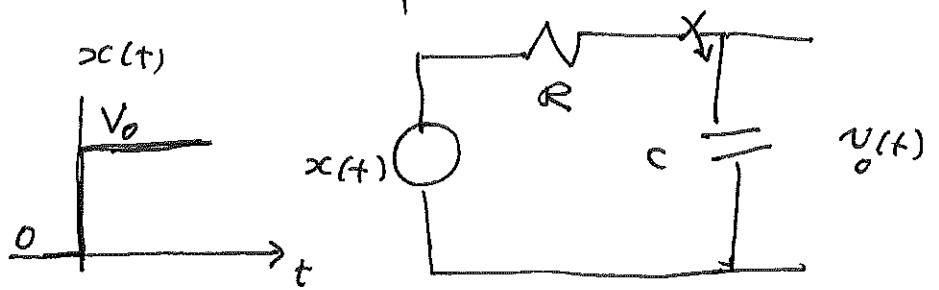
the output is

$$v(t) = V_0 (1 - e^{-t/RC}) = V_0 (1 - e^{-t/\tau})$$

τ : time constant.

when $t = \tau$,

$$v(\tau) = V_0 (1 - e^{-1}) = 0.63 V_0$$

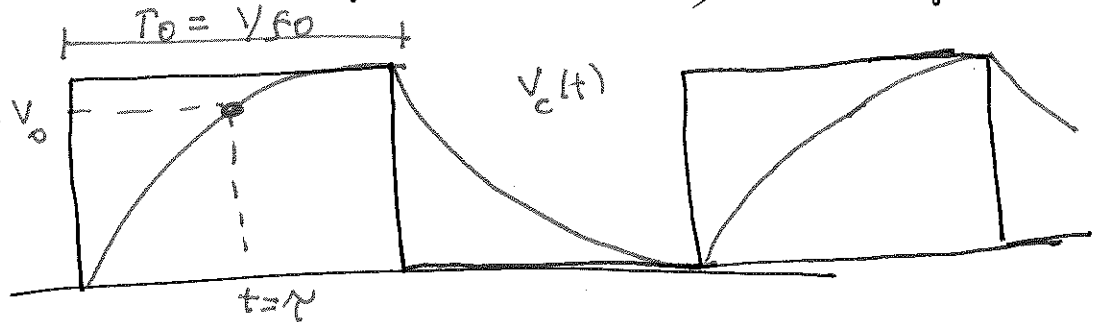


From this τ can be determined.

Limiting values; when $t \rightarrow \infty$, $v_0 \Rightarrow V_0$

when the input is a square function, the output looks like this

$f_0 = 50 \text{ Hz}$.

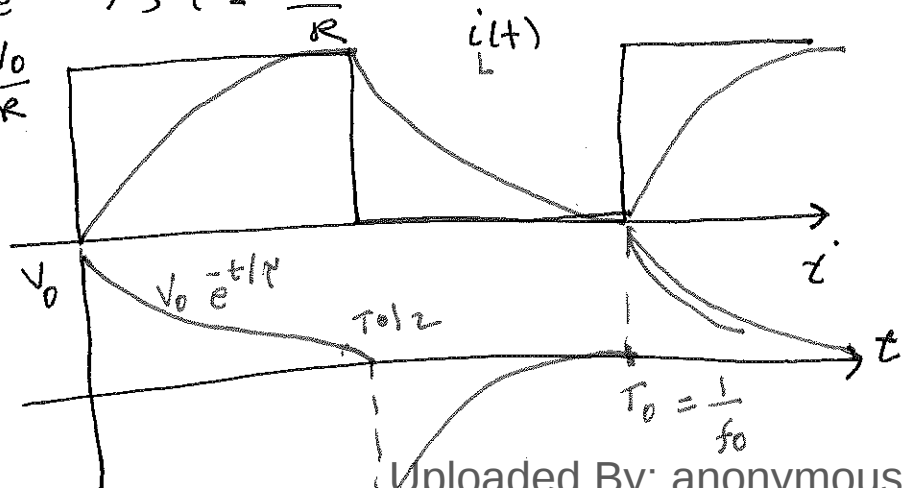
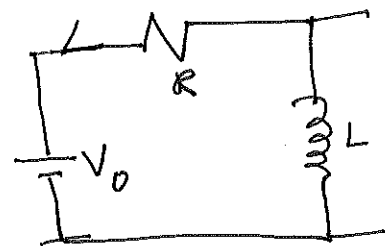


RL circuit

The D.E. due to a step input is

$$V_0 u(t) = Ri + L \frac{di}{dt}$$

$$\Rightarrow i(t) = \frac{V_0}{R} (1 - e^{-t/\tau}); \tau = \frac{L}{R}$$



Also,

$$v_L = L \frac{di}{dt}$$

$$v_R = Ri = V_0 (1 - e^{-t/\tau})$$

$$v_L = V_0 - v_R = \begin{cases} V_0 e^{-t/\tau} & 0 < t < T_0/2 \\ -V_0 e^{-t/\tau} & T_0 < t < T_0 \end{cases}$$

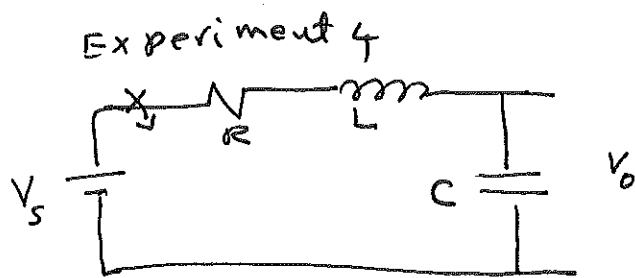
RLC circuit

D.E

$$V_s = Ri + L \frac{di}{dt} + V_0$$

$$i = C \frac{dV_0}{dt}$$

$$\Rightarrow \boxed{\frac{d^2 V_0}{dt^2} + \frac{R}{L} \frac{dV_0}{dt} + \frac{V_0}{LC} = \frac{V_s}{LC}}$$



- The solution is the sum of the final voltage $V_f = V_s$ and the natural response

$$V_0 = V_s + V_{\text{natural}}$$

$$V_0 = \begin{cases} V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} & \text{if } \alpha > \omega_0 \text{ overdamped} \\ V_s + e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t) & \text{if } \alpha < \omega_0 \text{ underdamped} \\ V_s + e^{-\alpha t} (D_1 + D_2 t) & \text{if } \alpha = \omega_0 \text{ critically damped} \end{cases}$$

$$\boxed{s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}$$

$$\alpha = \frac{R}{2L} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

attenuation undamped freq. damping freq.

⑥ critical damping when $\alpha^2 = \omega_0^2$

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \Rightarrow \boxed{R = \frac{4L}{C}}$$

$$s_1 = s_2 = -\alpha$$

⑥ over damped, s_1, s_2 are real

⑥ underdamped $s_1, s_2 = -\alpha \pm j\omega_d$ ← damped frequency

$$= -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

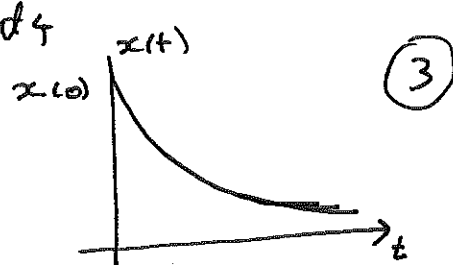
The exponential decay Experiment 4

consider the function

$$x(t) = x(0) e^{-\alpha t} u(t). \quad (1)$$

This function is plotted here

Note that as $t \rightarrow \infty$, $x(t) \rightarrow 0$



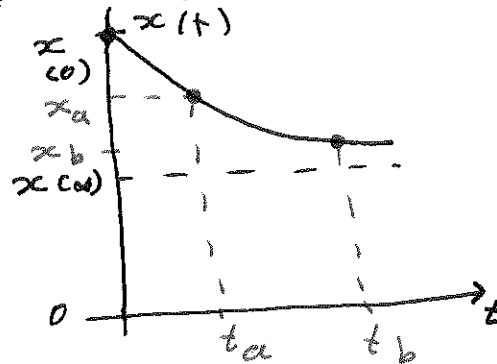
Now, consider the signal plotted below

The signal decays exponentially

reaching the limiting value

$x(\infty)$ as $t \rightarrow \infty$.

The mathematical formula for $x(t)$ is



$$x(t) = x(\infty) + (x(0) - x(\infty)) e^{-\alpha t} u(t) \quad (2)$$

How to determine α in (2)

$$x_a = x(\infty) + (x(0) - x(\infty)) e^{-\alpha t_a}$$

$$\Rightarrow e^{-\alpha t_a} = \frac{x_a - x(\infty)}{(x(0) - x(\infty))} \quad (a)$$

Also,

$$x_b = x(\infty) + (x(0) - x(\infty)) e^{-\alpha t_b}$$

$$\Rightarrow e^{-\alpha t_b} = \frac{x_b - x(\infty)}{(x(0) - x(\infty))} \quad (b)$$

Dividing (a) by (b),

$$\frac{e^{-\alpha(t_a - t_b)}}{e^{-\alpha t_b}} = \frac{x_a - x(\infty)}{x_b - x(\infty)} = e^{\alpha(t_b - t_a)}$$

$$\alpha(t_b - t_a) = \ln \left(\frac{x_a - x(\infty)}{x_b - x(\infty)} \right)$$

$$\Rightarrow \alpha = \frac{1}{t_b - t_a} \ln \left(\frac{x_a - x(\infty)}{x_b - x(\infty)} \right)$$

We shall take $(t_b - t_a) = T_d$; i.e., one period of the damping function