1.6 : Random variables of the continuous type.

Def : let X denote a random variable (r.v) with one dimensional space A which consists of an interval (continuous set) or a union of intervals. let $f(x)$ be a non-negative function, such that $\int_A f(x)\,dx = 1$. When a probability set function $P(A)$, $A \subset A$ can expressed in terms of $f(x)$ such that $P(A) = P_r(X \in A) = \int_A f(x)\,dx$, then X is said to be continuous random variable and $f(x)$ is called p.d.f of X.

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example 1: X : distance between bad records.

X has the following p.d.f : $f(x) = \begin{cases} \frac{1}{\mu_0} e^{-\frac{x}{\mu_0}} & , x > 0 \\ 0 & , \text{elsewhere.} \end{cases}$

$\rightsquigarrow A = \{x : 0 < x < \infty\}$

$\rightsquigarrow A$: continuous set. So X continuous Random variable.

↳ interval or union intervals.

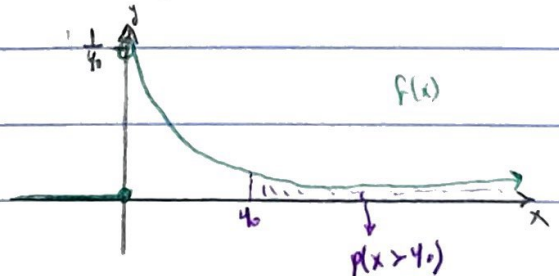
$\rightsquigarrow$ you can check that $f(x) \geq 0$ and $\int_0^\infty f(x)\,dx = 1$.

Find the prob. that the distance between two bad Record is greater than μ_0.

$$\Rightarrow P(X > \mu_0) = \int_{\mu_0}^{\infty} \frac{1}{\mu_0} e^{-\frac{x}{\mu_0}}\,dx = \frac{1}{\mu_0}\left(\frac{e^{-\frac{x}{\mu_0}}}{\ominus \frac{1}{\mu_0}} \Big|_{\mu_0}^{\infty} \right)$$

$$= e^{-\frac{\mu_0}{\mu_0}} \Big|_{\mu_0}^{\infty} \quad \bullet = e^{-1} - 0 = \frac{1}{e} = 0.3679$$

So $P(X > \mu_0) = \frac{1}{e} = 0.367$

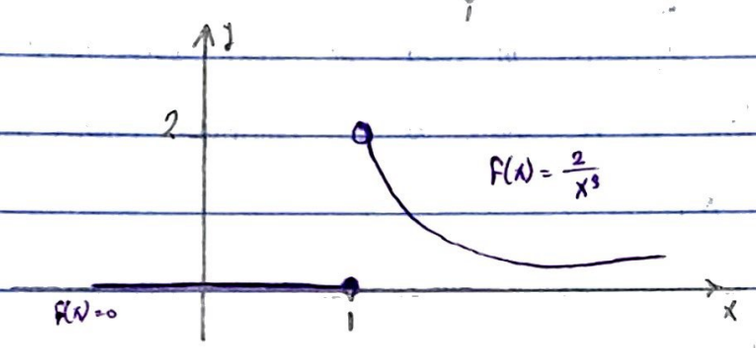


Example 2: X is a r.v with p.d.f, $f(x) = \begin{cases} \frac{2}{x^3}, & 1 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$

Find the distribution function:

$\rightsquigarrow A = \{x : 1 < x < \infty\} \rightsquigarrow A$ continuous set $\rightsquigarrow X$ cont. r.v.

$\rightsquigarrow f(x) \geq 0$ and $\int_1^\infty f(x)\,dx = 1$



$\rightsquigarrow$ Recall: $F(x) = Pr(X \leq x) \rightsquigarrow X$ r.v.

$\rightsquigarrow$ Remark: $F(x) = \int_{-\infty}^{x} f(w)\,dw$, X cont. r.v.

So $F(x) = \begin{cases} 0, & x < 1 \\ \int_1^x \frac{2}{w^3}\,dw, & x > 1 \end{cases} = \begin{cases} 0, & x < 1 \\ 1 - \frac{1}{x^2}, & x > 1 \end{cases}$

sketch: