Discussion 7.2

23 Find y' if $y = \ln(\ln x)$

$$y' = \frac{\frac{1}{x}}{\ln x} = \left(\frac{1}{x}\right)\left(\frac{1}{\ln x}\right) = \frac{1}{x \ln x}$$

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46 Evaluate $\int_2^{16} \frac{dx}{2x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x = 2 \Rightarrow u = \ln 2$

$x = 16 \Rightarrow u = \ln 16$

$$\int_{\ln 2}^{\ln 16} \frac{du}{2\sqrt{u}}$$

$$= \sqrt{u}\,\Big|_{\ln 2}^{\ln 16} = \sqrt{\ln 16} - \sqrt{\ln 2}$$

$$= \sqrt{4 \ln 2} - \sqrt{\ln 2}$$

$$= 2\sqrt{\ln 2} - \sqrt{\ln 2}$$

$$= \sqrt{\ln 2}$$

$\ln 16 = \ln 2^4 = 4 \ln 2$

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64 Use logarithmic differentiation to find y' if

$$y = \frac{\theta \sin\theta}{\sqrt{\sec\theta}} \Rightarrow \ln y = \ln\left(\frac{\theta \sin\theta}{\sqrt{\sec\theta}}\right)$$

$$\ln y = \ln(\theta \sin\theta) - \ln(\sec\theta)^{\frac{1}{2}}$$

$$= \ln\theta + \ln\sin\theta - \frac{1}{2}\ln\sec\theta$$

$$\frac{y'}{y} = \frac{1}{\theta} + \frac{\cos\theta}{\sin\theta} - \frac{1}{2}\frac{\sec\theta\tan\theta}{\sec\theta}$$

$$y' = \frac{\theta\sin\theta}{\sqrt{\sec\theta}}\left[\frac{1}{\theta} + \cot\theta - \frac{1}{2}\tan\theta\right]$$