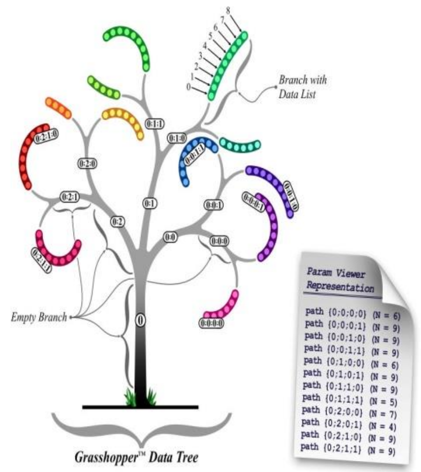
BIRZEIT UNIVERSITY

FACULTY OF ENGINEERING AND TECHNOLOGY

COMPUTER SCIENCE DEPARTMENT

COMP2321

Data Structures

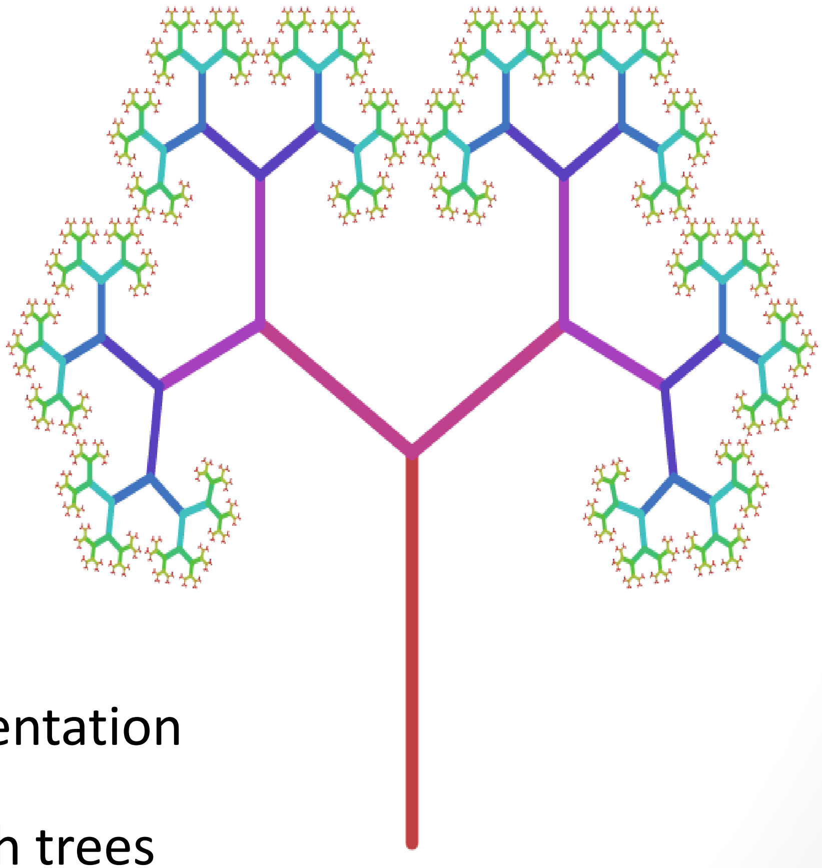


Chapter 4

Trees 1

Trees

- What is a Tree?
- Tree terminology
- Why trees?
- What is a general tree?
- Consrtucting trees
- Binary trees
- Binary Search tree implementation
- Application of Binary Search trees

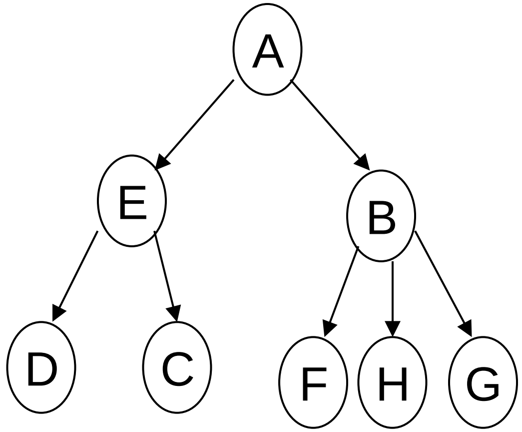


What is a Tree?

- A tree, is a finite set of nodes together with a finite set of directed edges that define parent-child relationships. Each directed edge connects a parent to its child. Example:

Nodes={A,B,C,D,E,f,G,H}

Edges={(A,B),(A,E),(B,F),(B,G),(B,H),
(E,C),(E,D)}

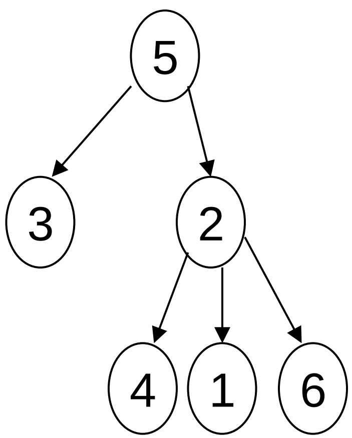


- A directed **path** from node m_1 to node m_k is a list of nodes $m_1, m_2, \ldots, m_k$ such that each is the parent of the next node in the list. The length of such a path is k - 1.
- Example: A, E, C is a directed path of length 2.

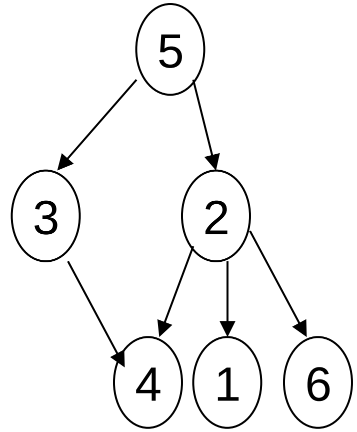
What is a Tree?

- A tree satisfies the following properties:

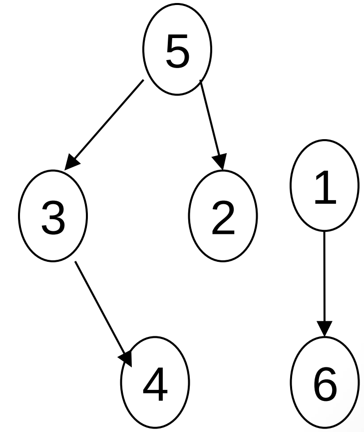
1. It has **one designated node**, called the **root**, that has **no parent**.
2. Every node, except the root, **has exactly one parent**.
3. A node may **have zero** or more children.
4. There is a **unique directed** path from the root to each node.



tree



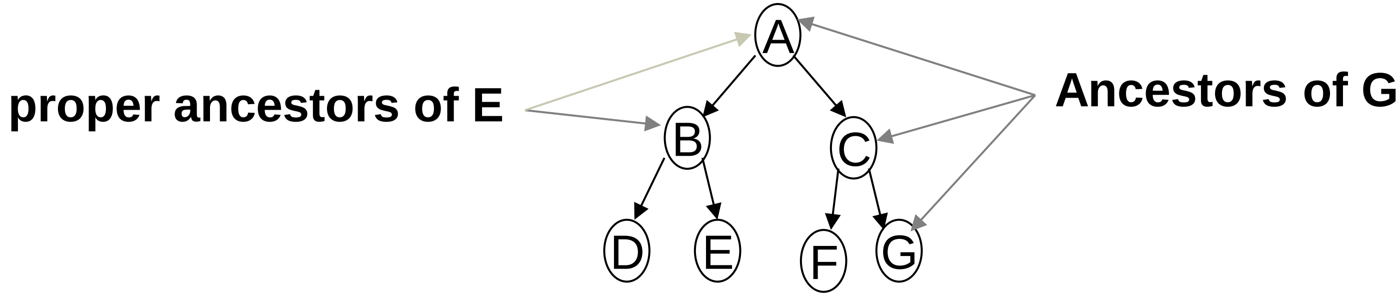
Not a tree



Not a tree

Tree Terminology

- Ordered tree: A tree in which the children of each node **are linearly ordered** (usually from left to right).

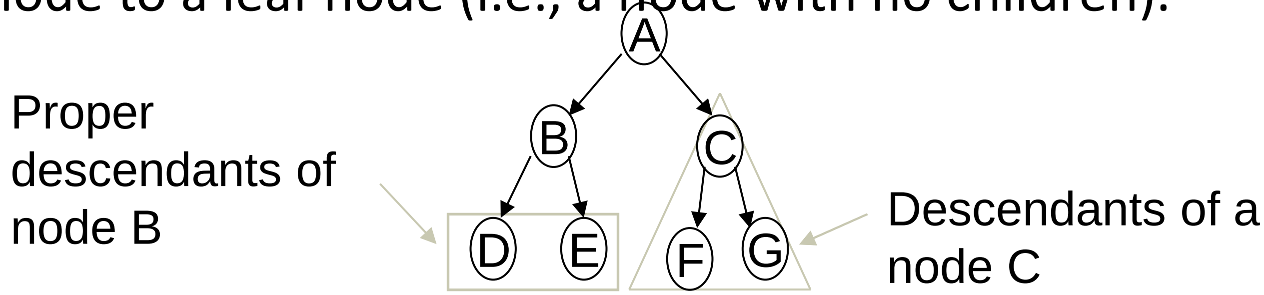


An Ordered Tree

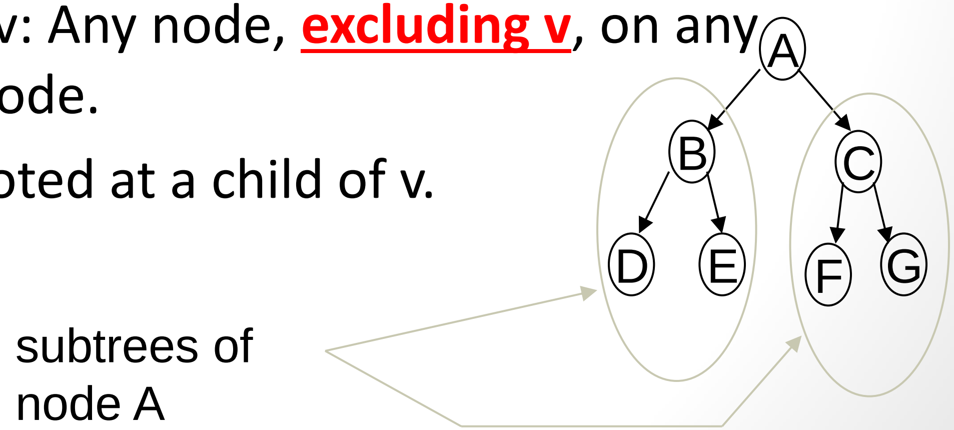
- **Ancestor** of a node v: Any node, **including v itself**, on the path from the root to the node.
- **Proper ancestor** of a node v: Any node, **excluding v,** on the path from the root to the node.

Tree Terminology (Contd.)

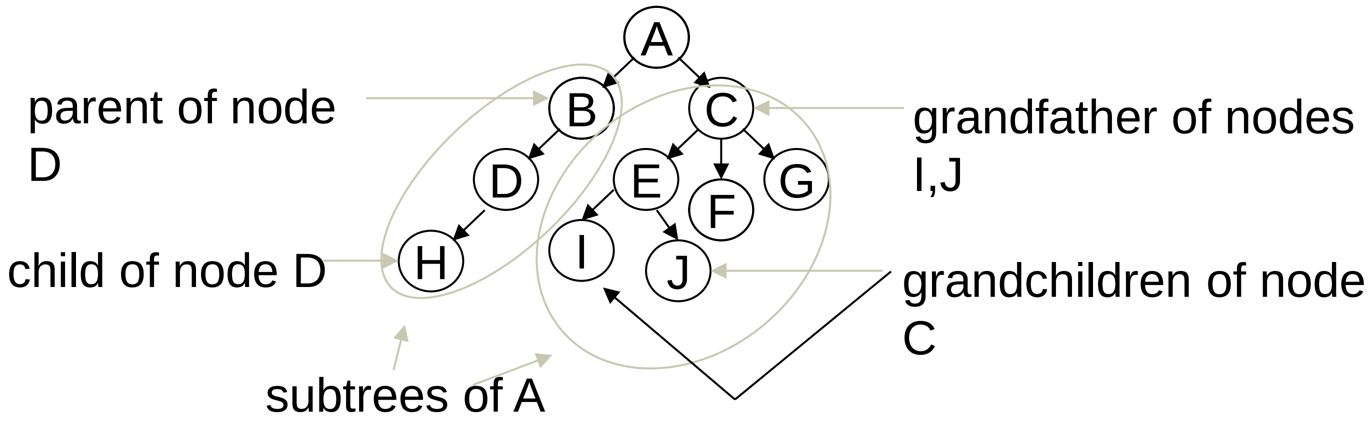
- **Descendant** of a node v: Any node, **including v itself**, on any path from the node to a leaf node (i.e., a node with no children).

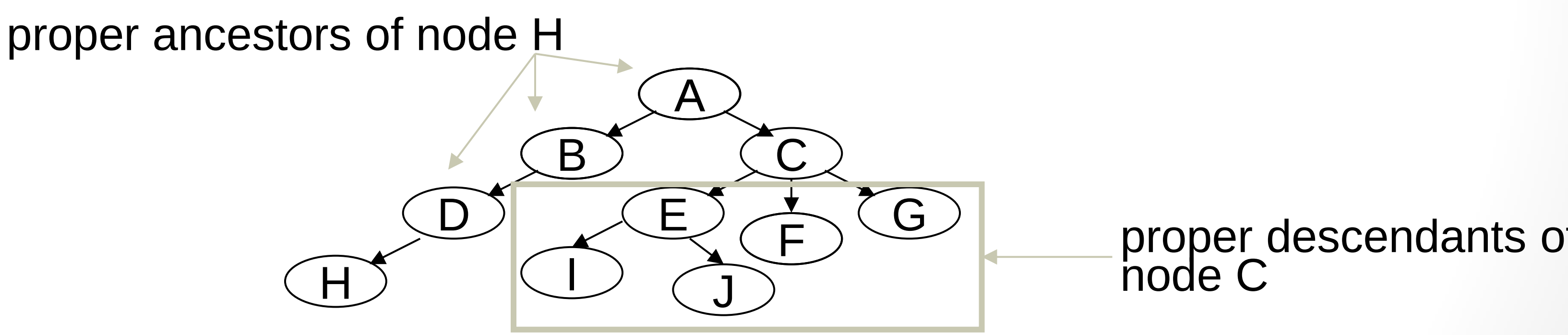


- **Proper descendant** of a node v: Any node, **excluding v**, on any path from the node to a leaf node.
- **Subtree** of a node v: A tree rooted at a child of v.



Tree Terminology (Contd.)

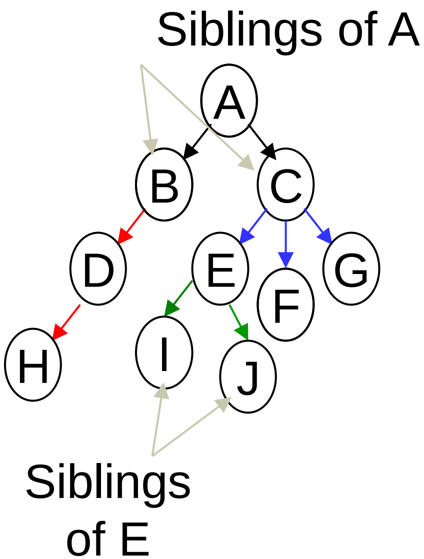




Tree Terminology

- Degree: The **number of subtrees** of a node
  - Each of node D and B has degree 1.
  - Each of node A and E has degree 2.
  - Node C has degree **3**.
  - Each of node F,G,H,I,J has degree **0**.

Tree with size of 10

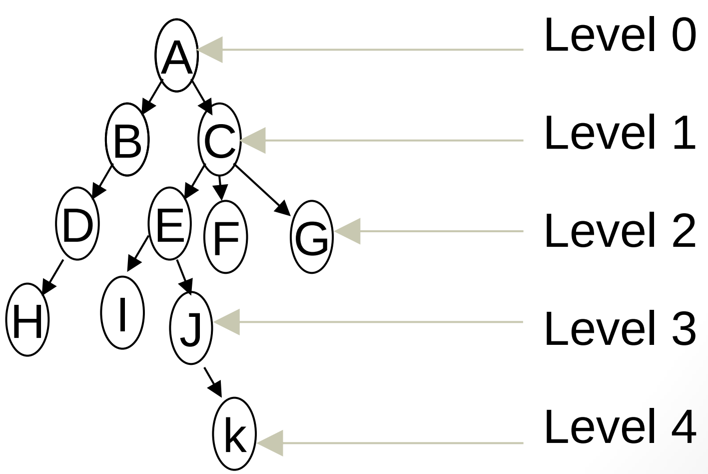


- **Leaf**: A node with degree 0.
- **Internal** or interior node: a node with degree greater than 0.
- **Siblings**: Nodes that have the same parent.
- **Size**: The number of nodes in a tree.

Tree Terminology (Contd.)

- **Level** (**or depth**) of a node v: The length of the path from the root to v.
- **Height** of a node v: The length of the **longest path from v to a leaf** node.
  - The **height of a tree** is the height of its **root mode.**
  - By definition the height of an empty tree is **-1**.

- The height of the tree is 4.
- The height of node C is 3.

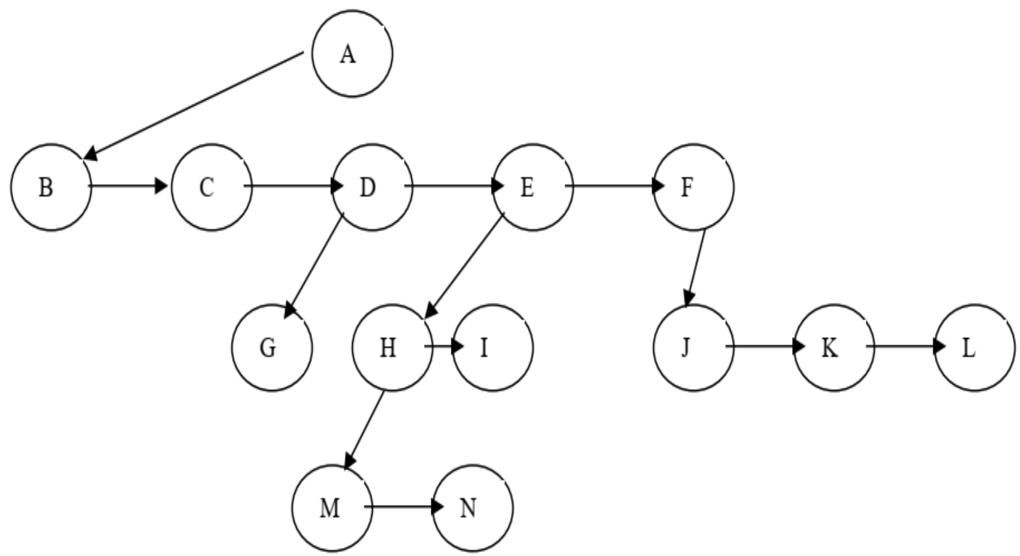


Why Trees?

- Trees are very important data structures in computing.
- They are suitable for:
  - Hierarchical structure representation, e.g.,
    - File directory.
    - Organizational structure of an institution.
    - Class inheritance tree.
  - Problem representation, e.g.,
    - Expression tree.
    - Decision tree.
  - Efficient algorithmic solutions, e.g.,
    - Search trees.
    - Efficient priority queues via heaps.

General Trees and its Implementation

- In a general tree, there is **no limit to the number** of children that a node can have.
- Representing a general tree by **linked lists**:
  - Each node **<u>has a linked list of the subtrees</u>** of that node.
  - Each element of the linked list is a subtree of the current node



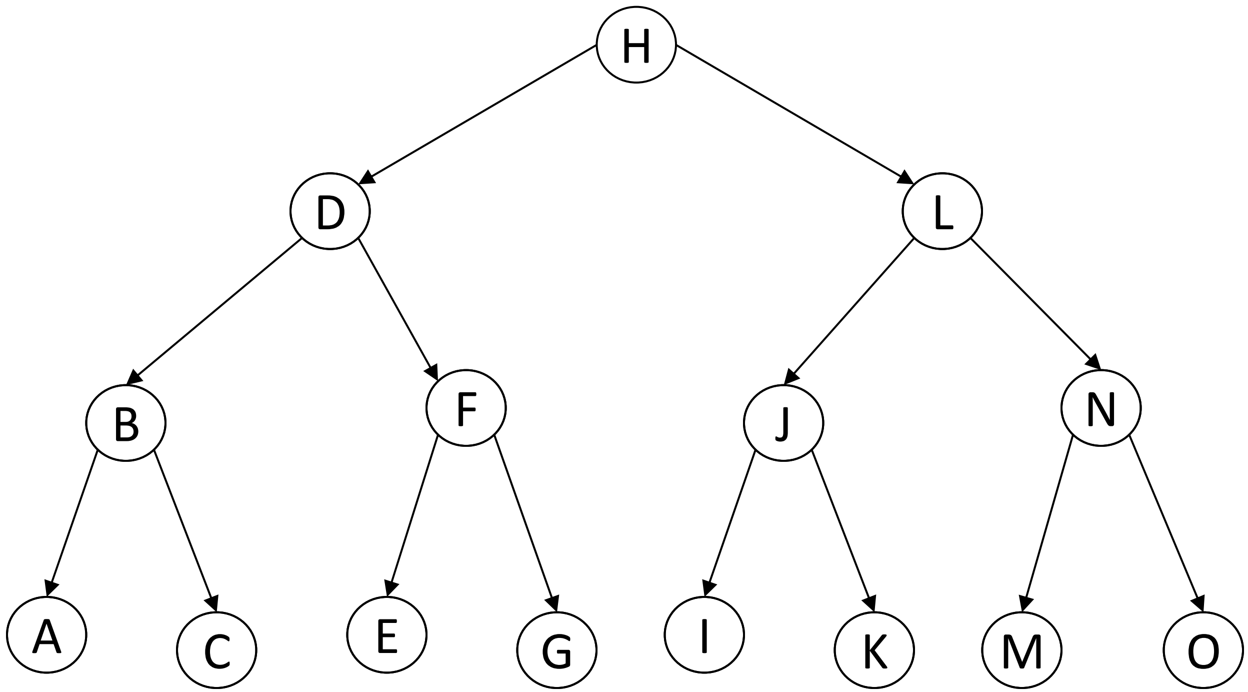
Tree Traversal

- Visiting (e.g. printing) all nodes.
- Three methods
  - Pre-order : root-Left-Right
  - In-order : Left-root-Right
  - Post-order : Left-Right-root

Tree Traversal

Preorder

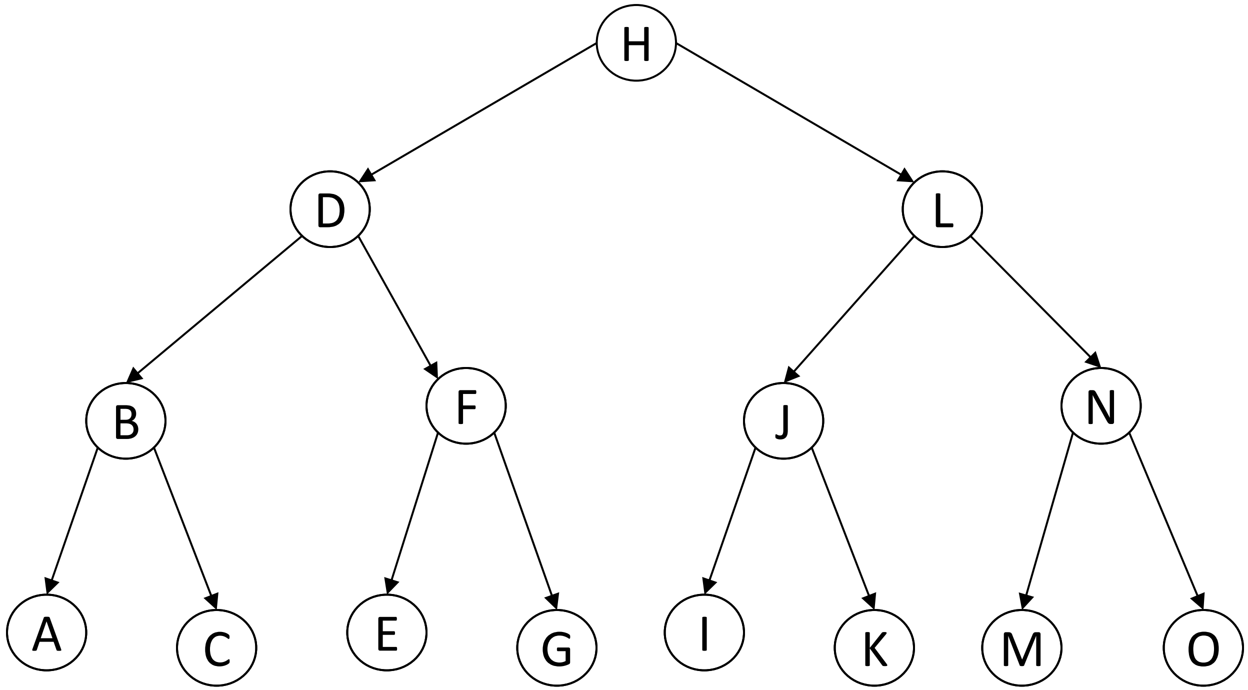
root-L-R



H	D	B	A	C	F	E	G	L	J	I	K	N	M	O

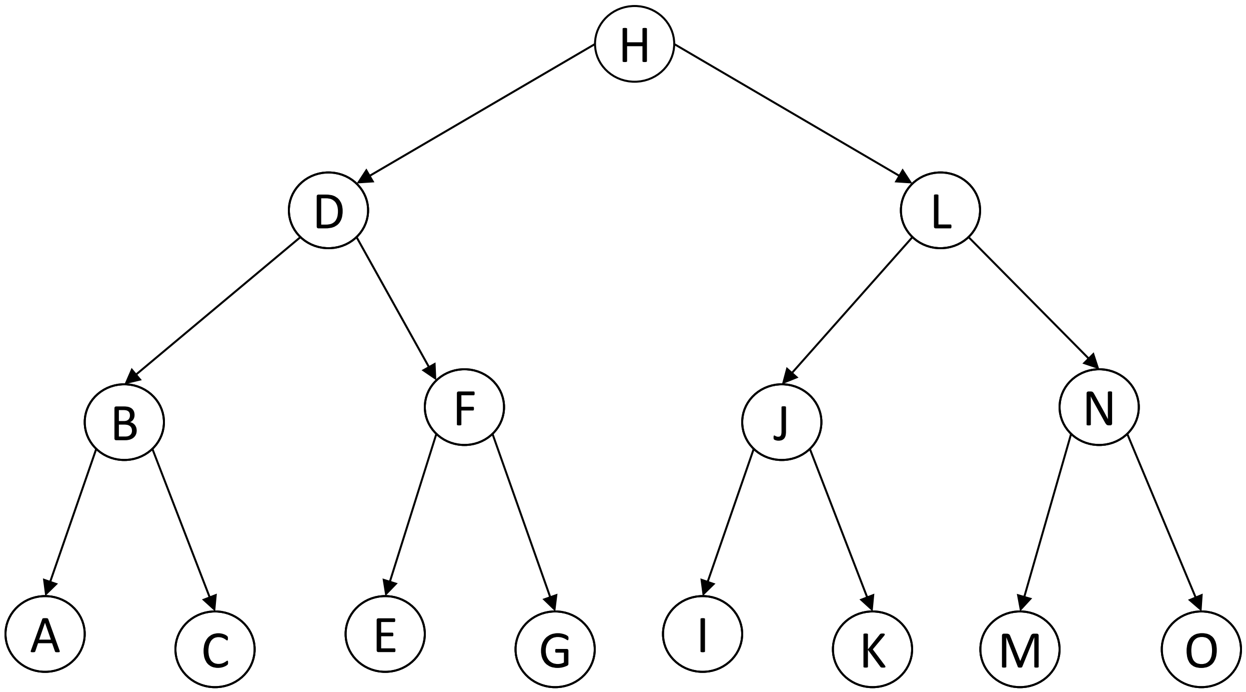
In-order

L-root-R



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

Post-order **L-R-root**



A	C	B	E	G	F	D	I	K	J	M	O	N	L	H

InOrder(root) visits nodes in the following order:

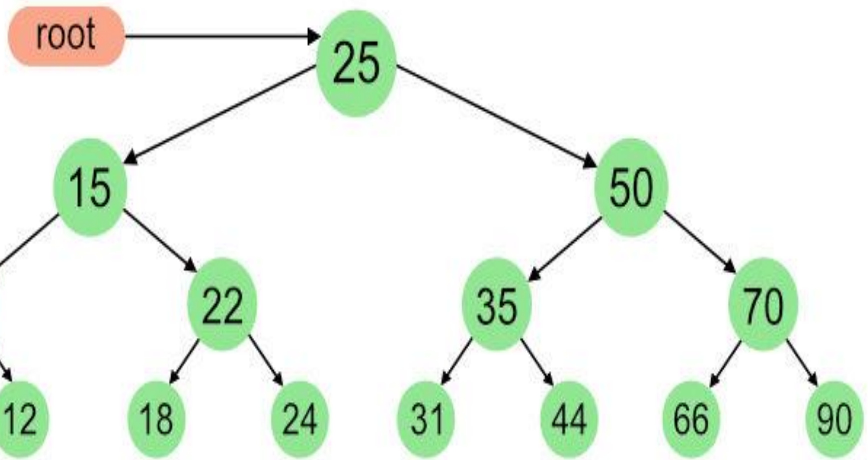
4, 10, 12, 15, 18, 22, 24, 25, 31, 35, 44, 50, 66, 70, 90

A Pre-order traversal visits nodes in the following order:

25, 15, 10, 4, 12, 22, 18, 24, 50, 35, 31, 44, 70, 66, 90

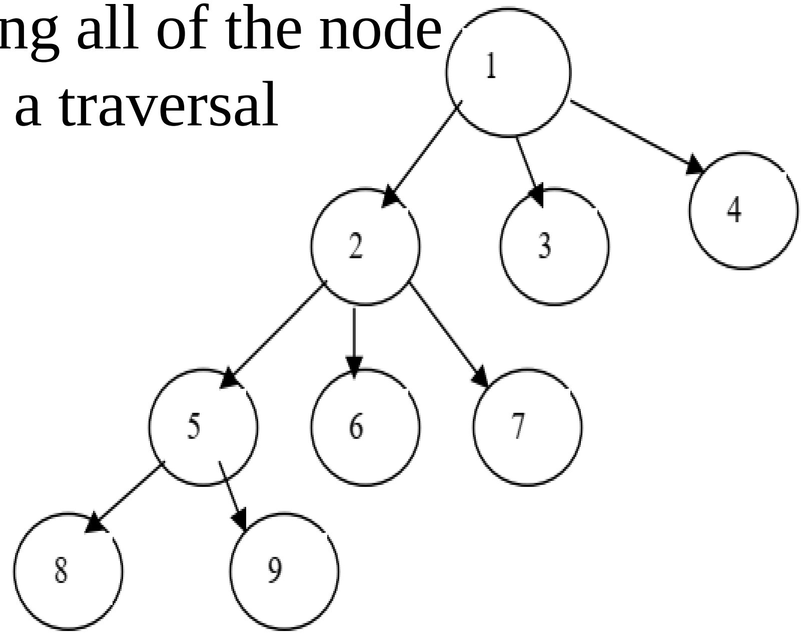
A Post-order traversal visits nodes in the following order:

4, 12, 10, 18, 24, 22, 15, 31, 44, 35, 66, 90, 70, 50, 25



Tree Traversal

- Any process for visiting all of the node in some order is called a traversal



1) Pre order
root ,left , right
1, 2, 5, 8, 9,6,7, 3, 4
2) Post order
left , right , root
8, 9, 5, 6, 7, 2, 3, 4, 1
3) In order
left , root , right
8, 5, 9, 2, 6, 7, 1, 3, 4

1) Pre order

root ,left, right

F, B, A,D, C, E,G,I, H

2) Post order

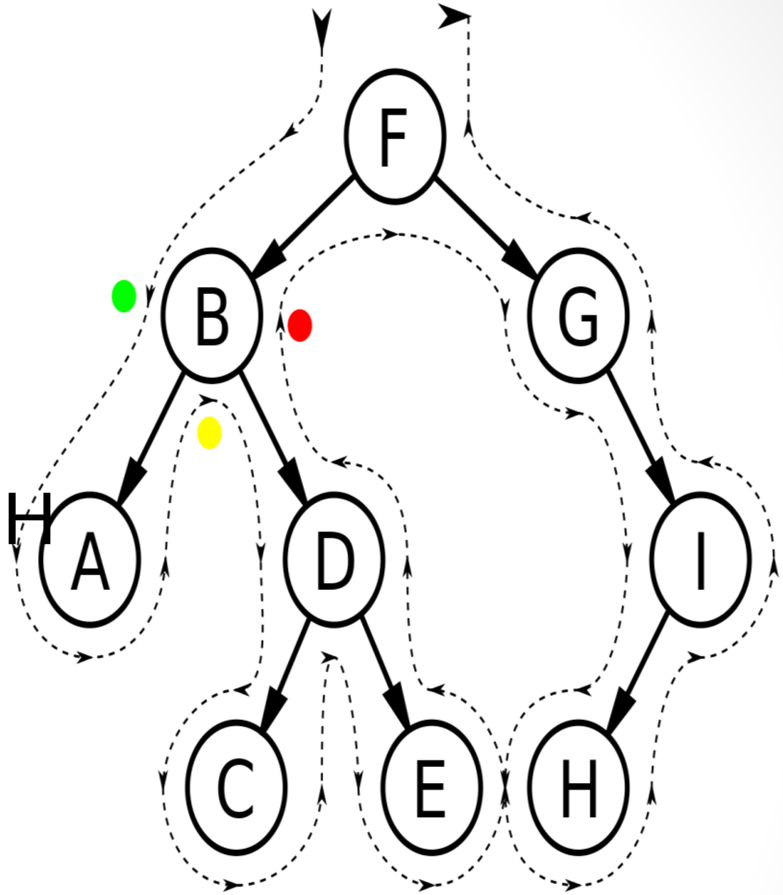
left , right , root

A, C, E, D, B, H, I, G,F

3) In order

left , root , right

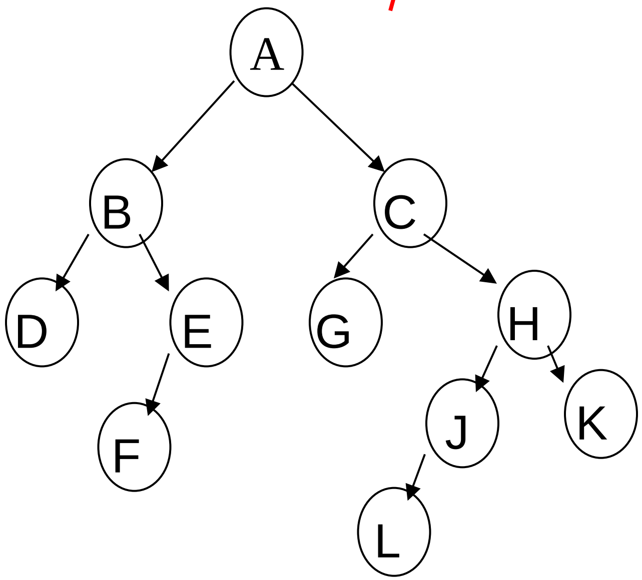
A, C, D, E, B, F, H, I, G



Construct the binary tree whose traversal are given below

- In order : D B F E A G C L J H K
- Pre order : A B D E F C G H J L K

- In order : (D B F E) A (G C (L J H K))
- Pre order : A B D E F C G H J L K



Exercise (very Important)

Construct the binary tree whose traversal are given below

Preorder	H	D	B	A	C	F	E	G	L	J	I	K	N	M	O
Postorder	A	C	B	E	G	F	D	I	K	J	M	O	N	L	H

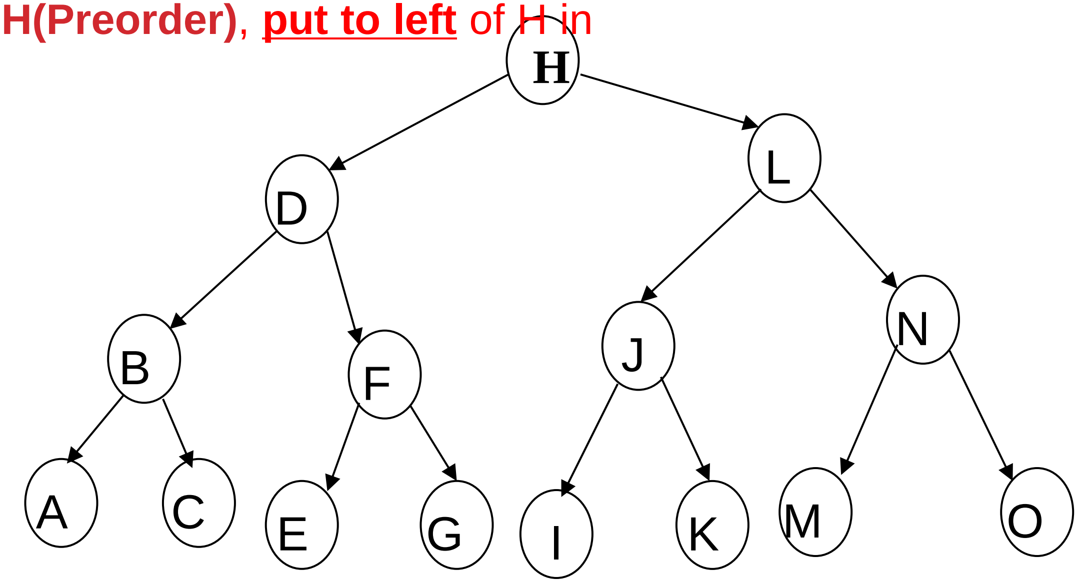
N1=predecessor of root in postorder (right)
N2=successor of root in preorder (left)

Preorder	H	D	B	A	C	F	E	G	L	J	I	K	N	M	O

Postorder	A	C	B	E	G	F	D	I	K	J	M	O	N	L	H

The left of node **H (postorder)** ,**<u>put on right</u>** of H in tree

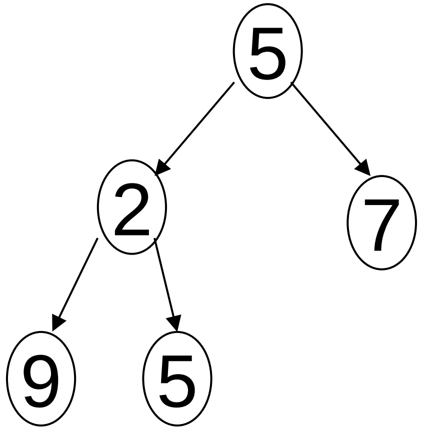
The right of node **H(Preorder)**, **<u>put to left</u>** of H in tree



Binary Trees

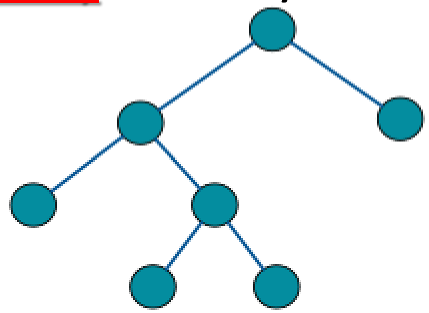
- A binary tree is a tree in which node can **have no more than two** children
- Thus, a binary tree is either:
  1. An empty tree, or
  2. A tree consisting of a root node and at most two non-empty binary subtrees.

Example:

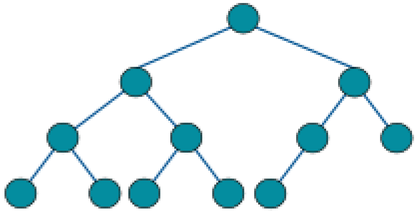


Types of Binary Trees

- A full binary if every node has 0 or 2 children.



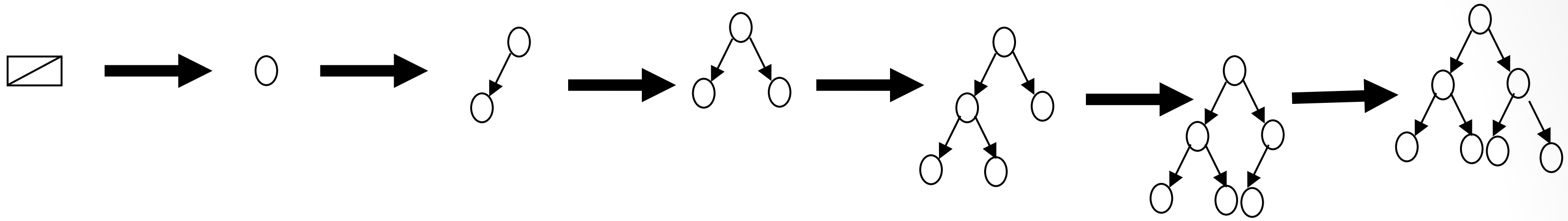
- A complete binary every level, *except possibly the last*, is completely filled, and all nodes in the last level are as far left as possible.



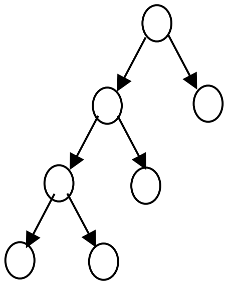
Binary Trees (Contd.)

BIRZEIT UNIVERSITY

- Example showing the growth of a complete binary tree:

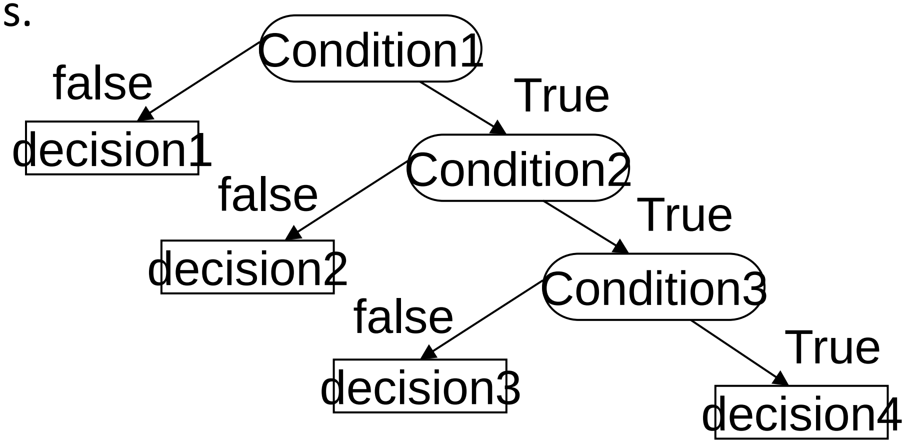


- **Full, but not complete**: Huffman coding example

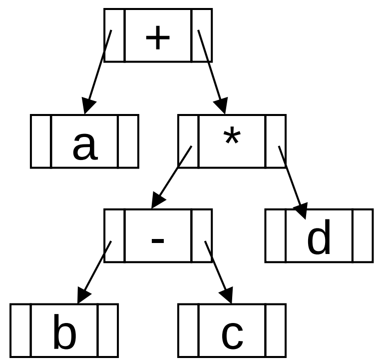


Application of Binary Trees

- Binary trees have many important uses. Two examples are:
  1. Binary decision trees.
     - Internal nodes are conditions. Leaf nodes denote decisions.



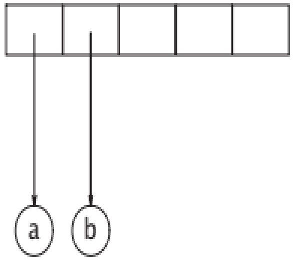
  2. Expression Trees



Constructing an Expression Tree

Since we already have an algorithm to convert **infix to postfix**, we can generate expression trees from the two common types of input

We read our **expression one symbol at a time**. If the symbol is an operand, we create a **one-node tree** and **push it onto a stack**. If the symbol is an operator, we pop two trees T_1 and T_2 from the stack (T_2 is popped first) and form a new tree whose root is the operator and whose left and right children are T_1 and T_2, respectively. This new tree is then pushed onto the stack.



As an example, suppose the input is

a b + c d e + * *

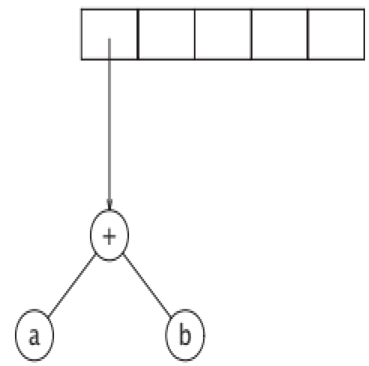
The first two symbols are operands, so we create one-node trees and push them onto a stack.

As an example, suppose the input is

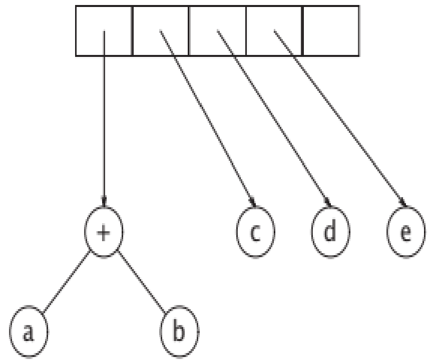
a b + c d e + * *

Next, a + is read, so two trees are popped, a new tree is formed, and it is pushed onto the stack

a b + c d e + * *

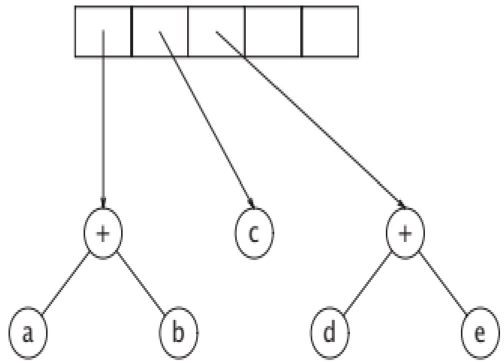


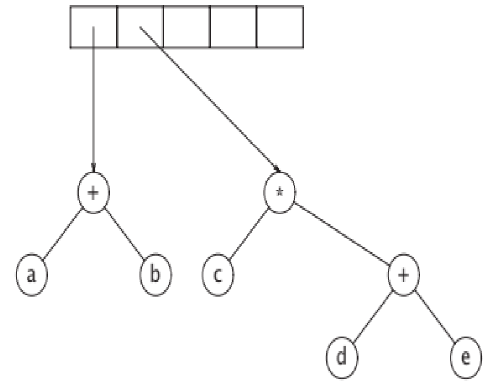
Next, c, d, and e are read, and for each a one-node tree is created and the corre s pushed onto the stack



Now a + is read, so two trees are merged

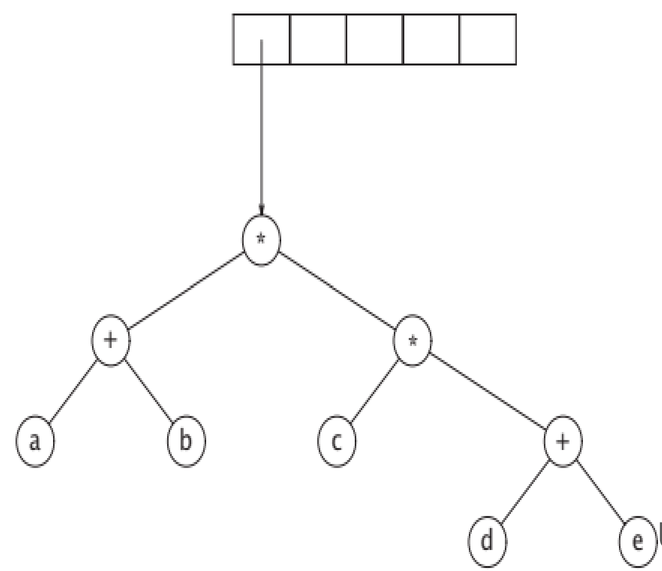
a b + c d e + * *





Continuing, a * is read, so we pop two trees and form a new tree with a * as root.

Finally, the last symbol is read, two trees are merged, and the final tree is left on the stack.

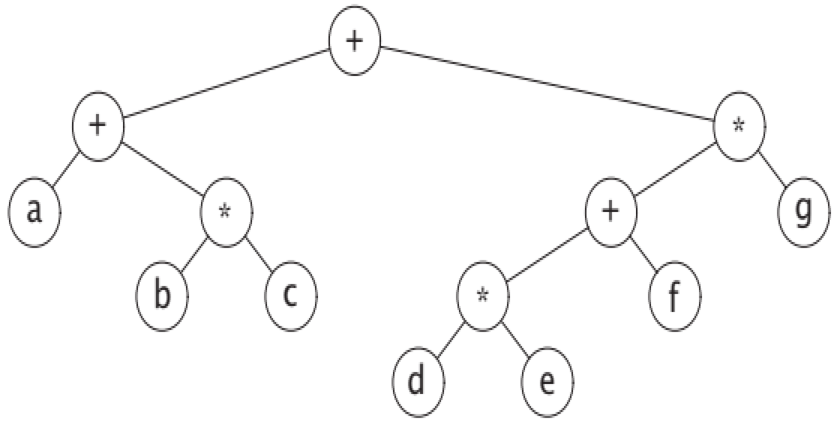


Construct the binary tree whose traversal are given below
(**solved at class**)

Post order a b c* + d e * f + g * + **Postfix**

Construct the binary tree whose traversal are given below
(**solved at class**)

Pre order + + a * b c * + * d e f g **Prefix**



Pre order + + a * b c * + * d e f g **Prefix**

Post order a b c* + d e * f + g * + **Postfix**

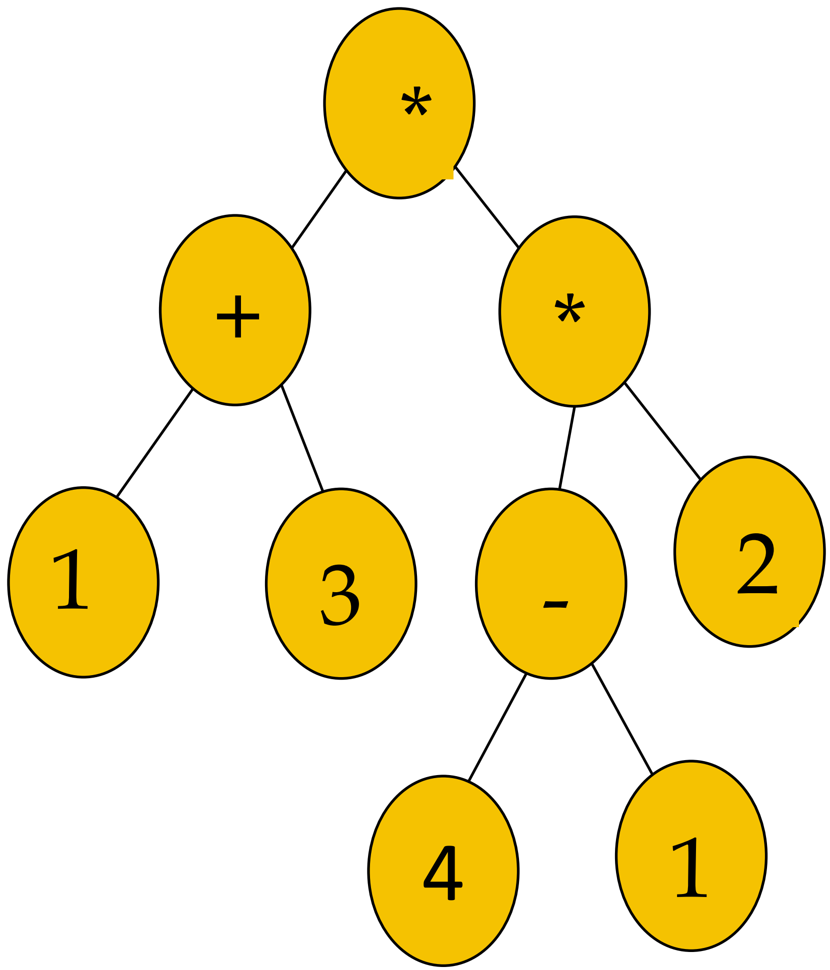
In order (a + (b * c)) + (((d * e) + f) * g). **Infix**

Construct expression tree? Excercise (Try at home)

A) preorder : * + 1 3 * - 4 1 2

B) postorder : 1 3 + 4 1 - 2 * *

Traversing an expression tree Excercise (Try at home :answer)



preorder (1^{st} touch)
* + 1 3 * - 4 1 2

postorder (last touch)
1 3 + 4 1 - 2 * *

inorder (2^{nd} touch)
1 + 3 * 4 - 1 * 2