L 22
17-8-2021

ENEE2360 Analog Electronics

T11:

Operational Amplifiers

Instructor : Nasser Ismail

1

Operational Amplifiers

- Early Operational Amplifiers were constructed with vacuum tubes and were used in analog computers to perform mathematical operations.
- Even as late as 1965, vacuum tube operational amplifiers were still in use and cost in the range of $75.
- These days, they are linear Integrated circuits (IC) that use low voltage dc supplies , they are reliable and inexpensive
- The operational amplifier has become so cheap in price (often less than $1.00 per unit) and it can be used in so many applications

Early Vacum Tube Operational Amplifiers

pins

The Philbrick Operational Amplifier (1952)

From "Operational Amplifier", by Tony van Roon: http://www.uoguelph.ca/~antoon/gadgets/741/741.html

Operational Amplifiers

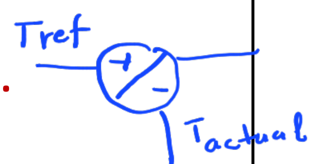
Operational was used as a descriptor early-on because this form of amplifier can perform operations of :

- **Adding signals**
- **Subtracting signals**
- **Integrating signals,** $\int x(t)dt$
- **Differentiation of signals,**

The applications of operational amplifiers (shortened to op amp) have grown beyond those listed above.

What can you do with Op amps?

- You can make music louder when they are used in stereo equipment.
- You can amplify the heartbeat by using them in medical cardiographs.
- You can use them as comparators in &cooling heating systems.



- You can use them for Math operations
- And many other applications in all fields of engineering

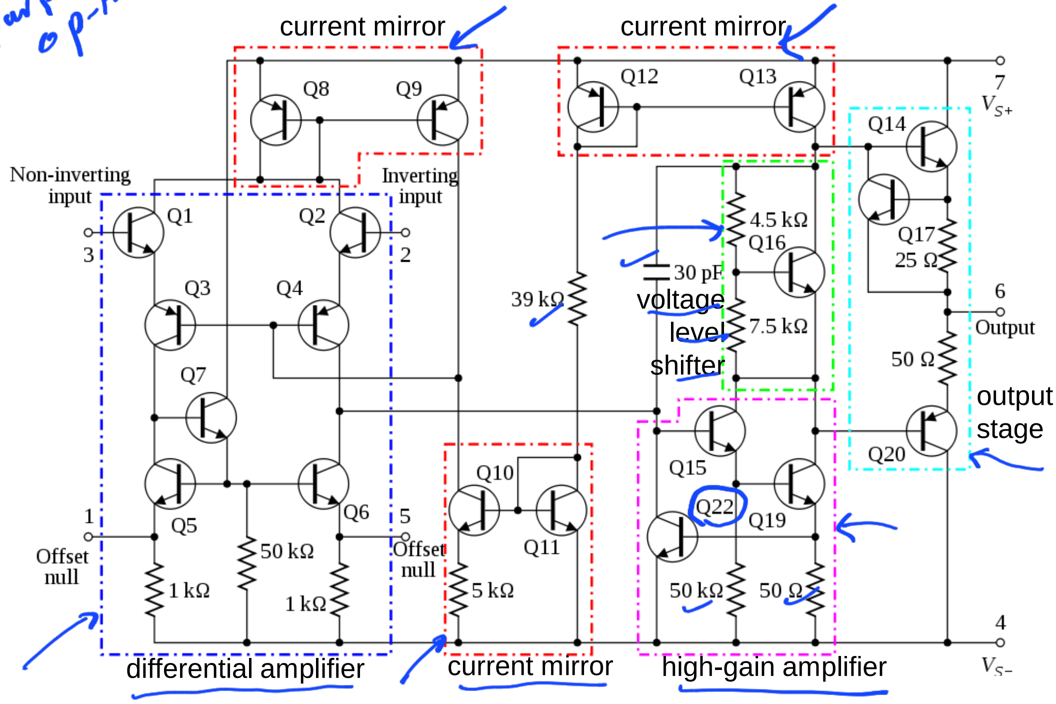
Operational Amplifiers → ?

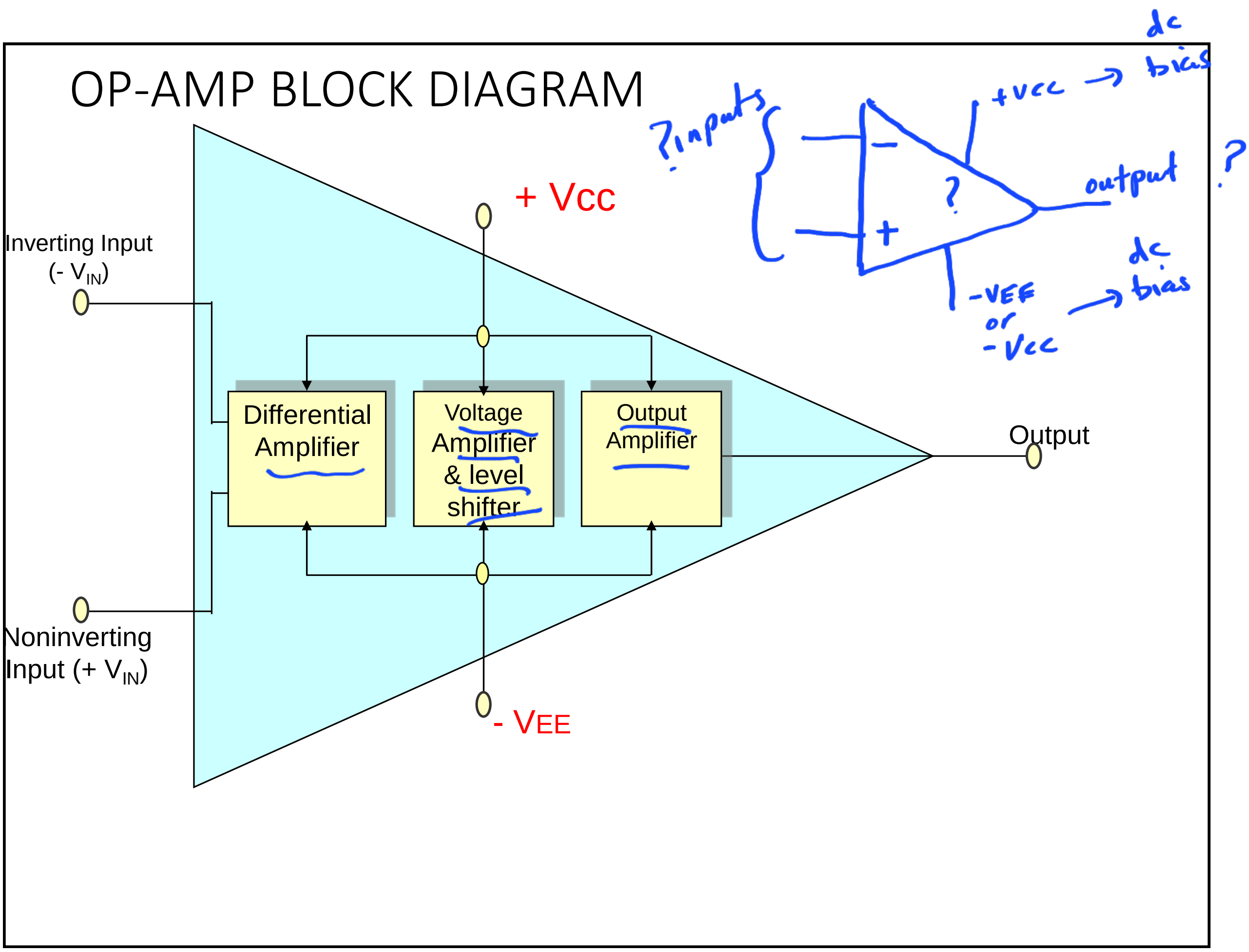
- In this course we will be concerned with ***how to use the op amp as a device.***

- The internal configuration (design) is beyond the scope of our study and can be covered in an advanced electronics course.
- The complexity is illustrated in the following block diagram and detailed circuit.

The simplest General purpose Op-Amp

741 Op-Amp Schematic




OP-AMP BLOCK DIAGRAM
+ Vcc
Inverting Input
(- V_{IN})
Differential
Amplifier
Voltage
Amplifier
& level
shifter
Output
Amplifier
Output
Noninverting
Input (+ V_{IN})
- VEE
?inputs
+Vcc → dc bias
output ?
-VEE
or
-Vcc → dc bias

OP-AMP CHARACTERISTICS

1. Very high input impedance (in mega ohms)
2. Very high gain (> 100,000)
3. Very low output impedance (in ohms)

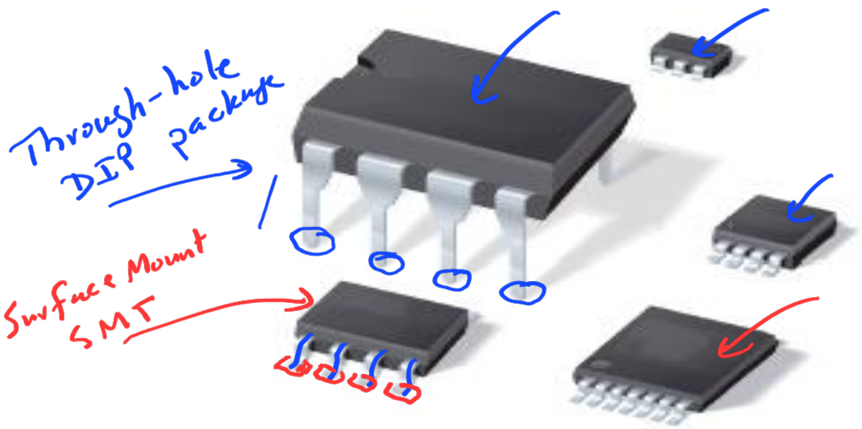
* OP-AMP is a differential, voltage amplifier with high gain.

Operational Amplifiers

Fortunately, we do not have to ***assemble*** a circuit with so many transistors and resistors in order to get and use the op amp

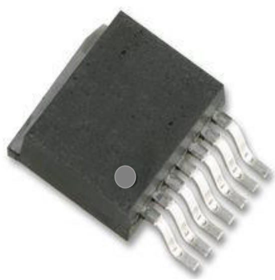
The circuit in the previous slide is usually encapsulated into a dual in-line pack (DIP)

For a single LM741, the pin connections for the chip are shown below.



Packaging Types

(a) Op Amp 741
8-pins DIP package



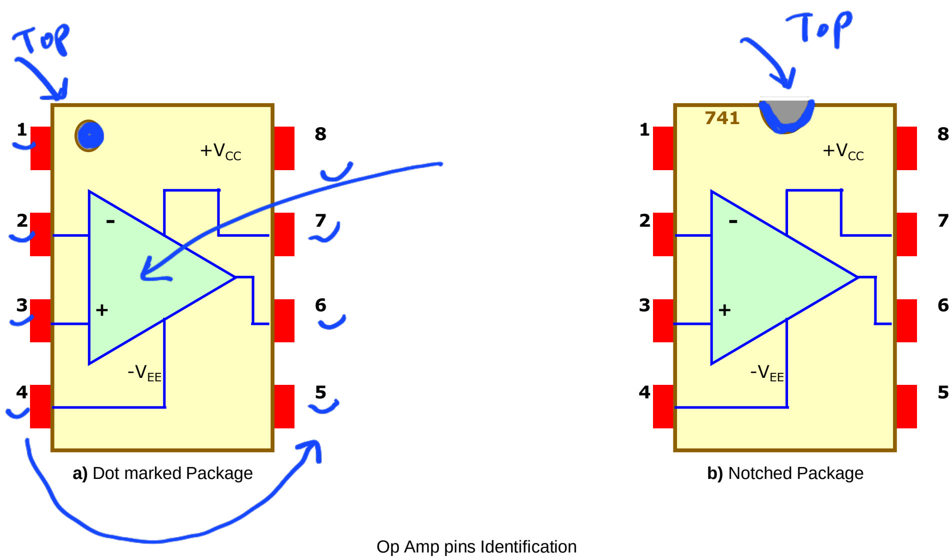
(b) OPA547FKTWT

DIP SMT package

(c) TO-5 metal can
8-Leads package

Op Amp packages

OP-AMP pins identification

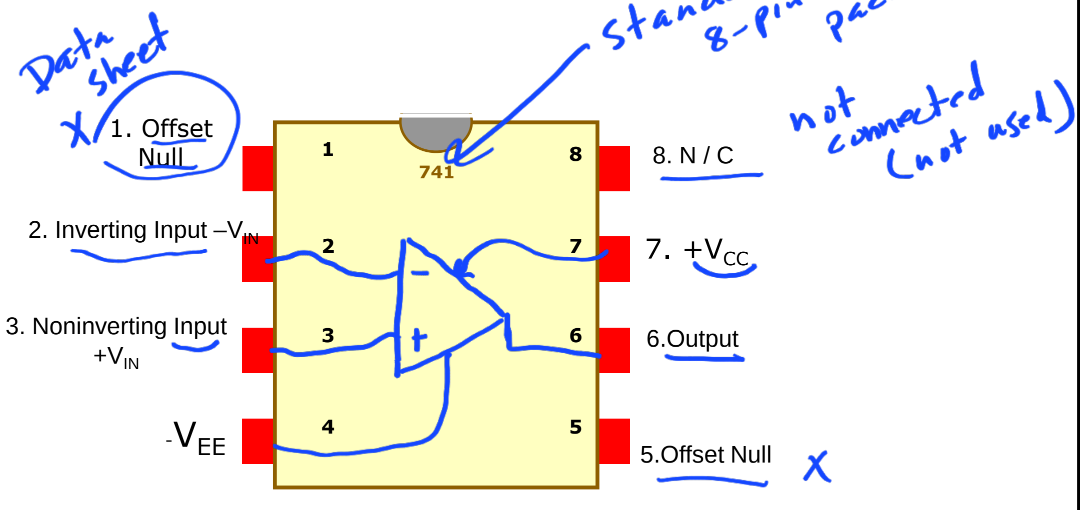


a) Dot marked Package

b) Notched Package

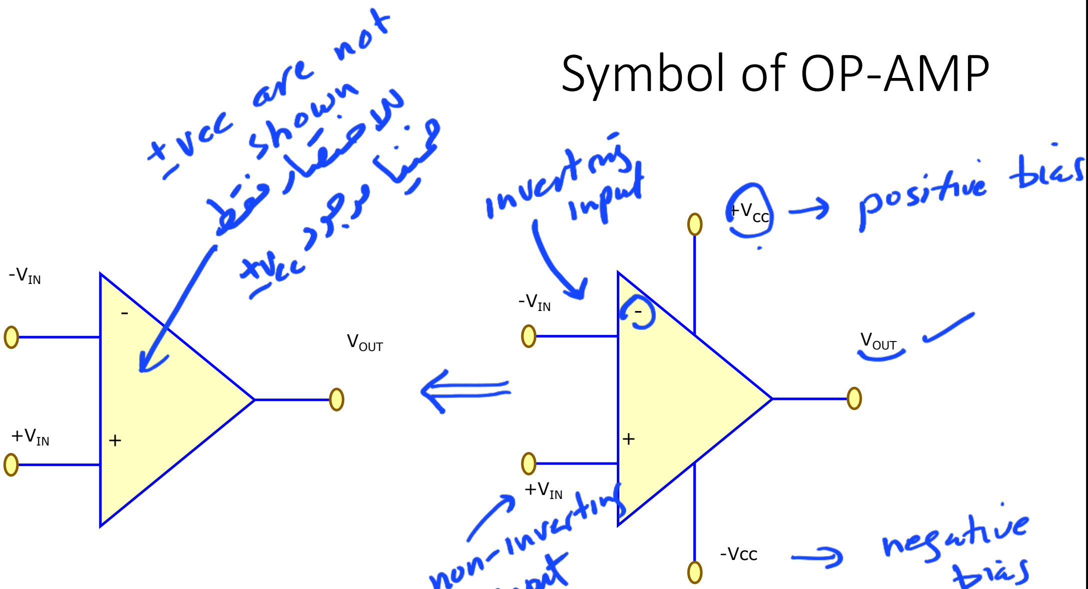
Op Amp pins Identification

What are these pins?



Op Amp pins Description

Symbol of OP-AMP



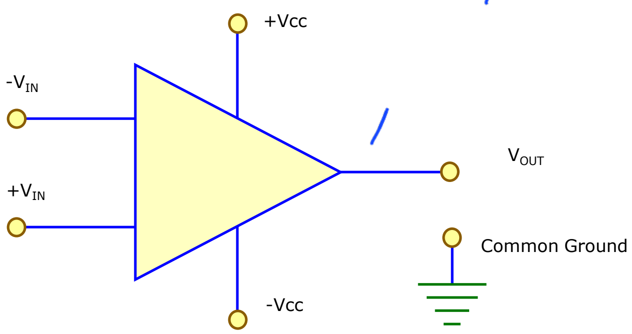
(a) Without power connection **(b)** With power connection

Op Amp Schematic Symbols

Most Op Amps require dual power supply with common ground

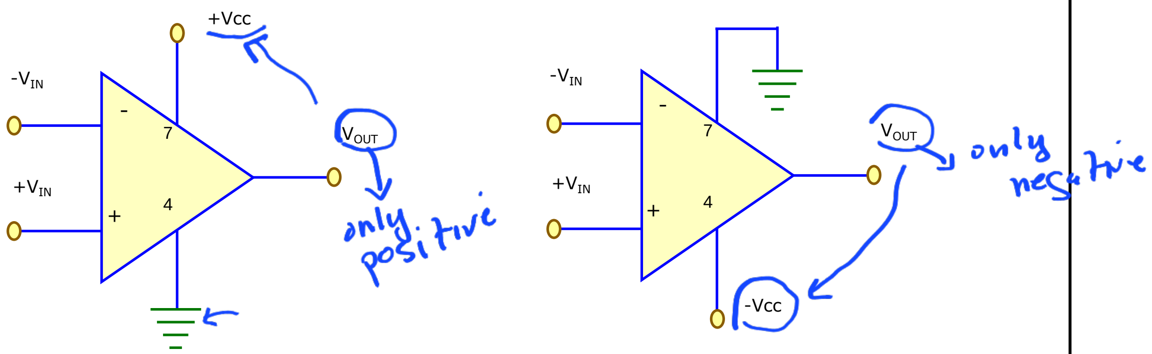
Positive Supply +Vcc to pin7 in 741 opamp

Negative Supply -Vcc to pin4 in 741 opamp



Dual Supply Voltages connection

Some Op Amps work on single supply also (with some restrictions)



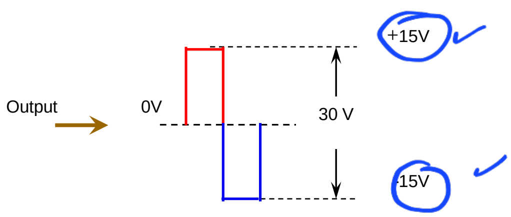
(a) Single Positive Voltage

Single Negative Voltage

Single Supply Voltages connection

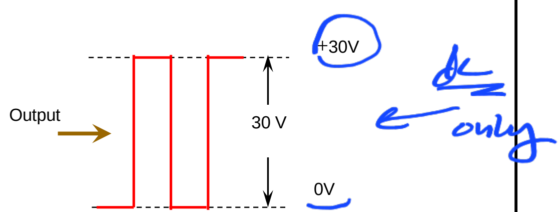
Advantage of dual power supply

Using dual power supply will let the op amp to output true AC voltage.



Op Amp powered from Dual supply

can be
ac or dc



Op Amp powered from Single supply

What is dual power supply?

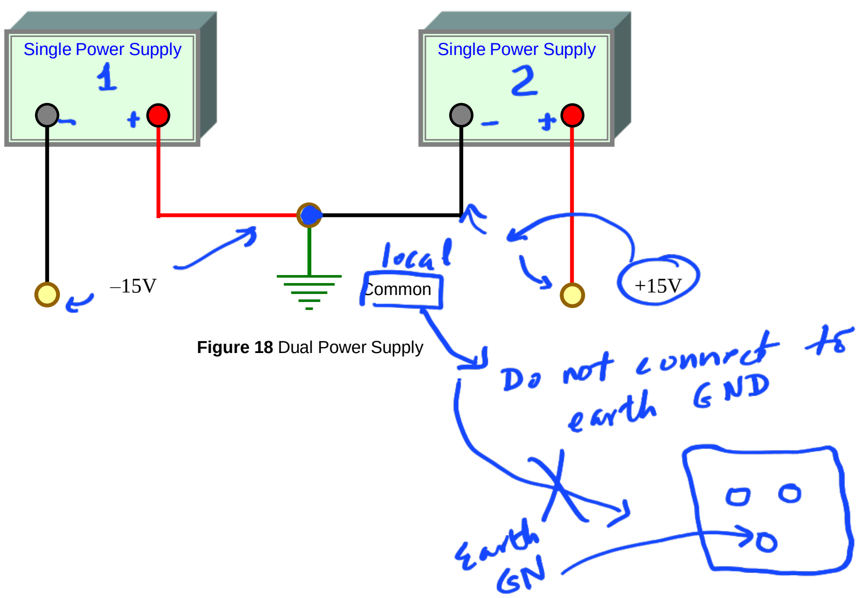
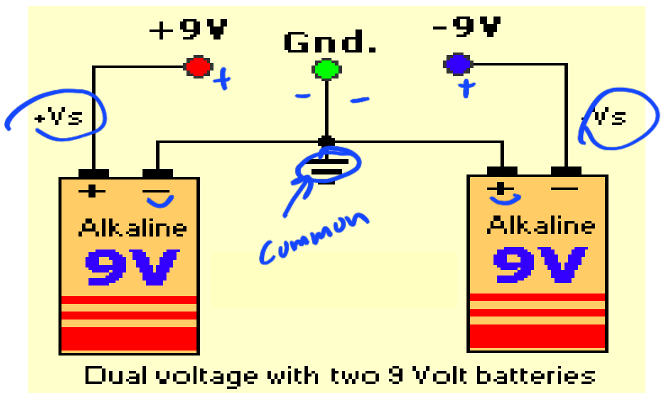


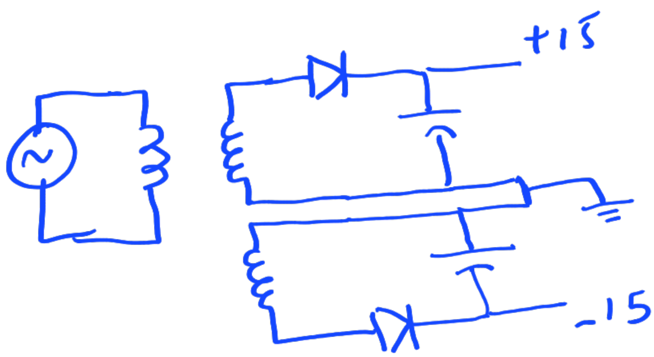
Figure 18 Dual Power Supply

How can you make a dual power supply using two 9V batteries?

What is the voltage between + of first battery and – of second battery?

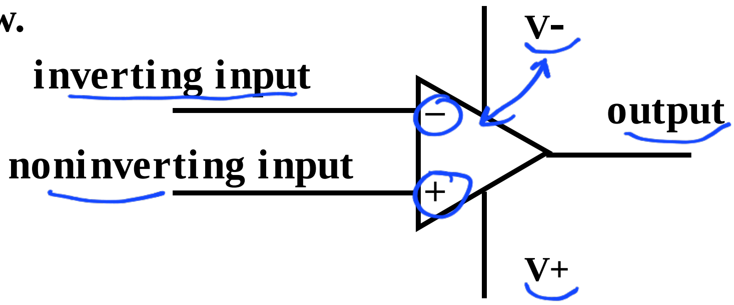


Dual voltage with two 9 Volt batteries



Operational Amplifiers

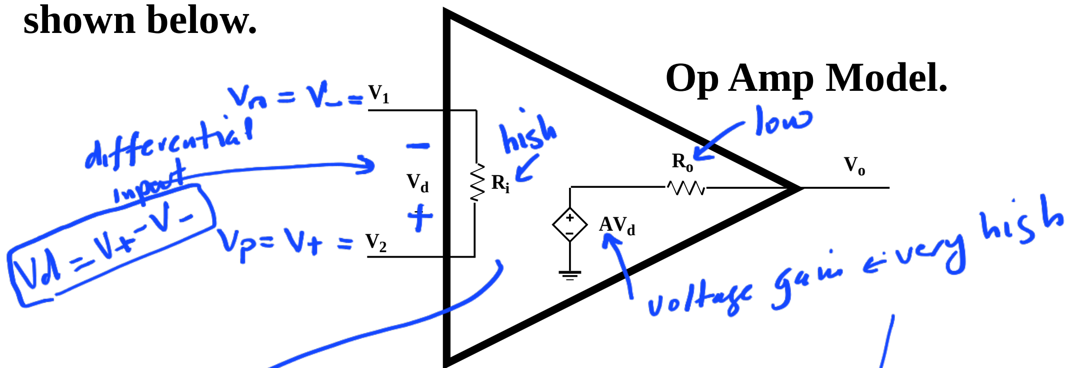
The basic op amp with supply voltage included is shown in the diagram below.



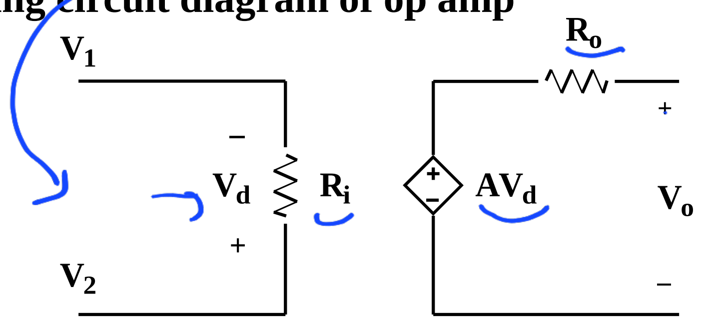
In most cases only the two inputs and the output are shown for the op amp.
However, one should keep in mind that supply voltage is required, and a ground.

Operational Amplifiers Model

A model of the op amp, with respect to the symbol, is shown below.

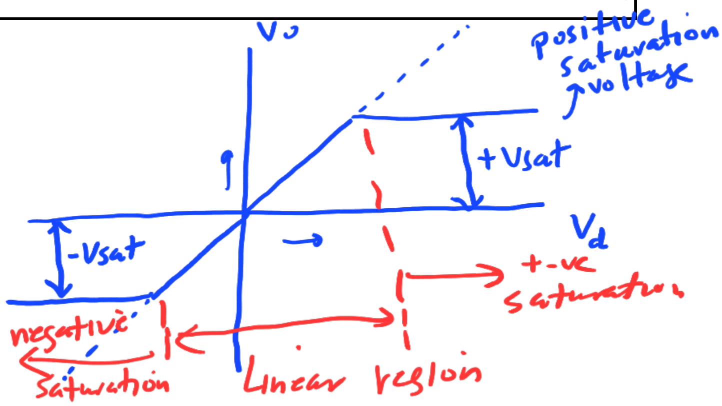


Working circuit diagram of op amp

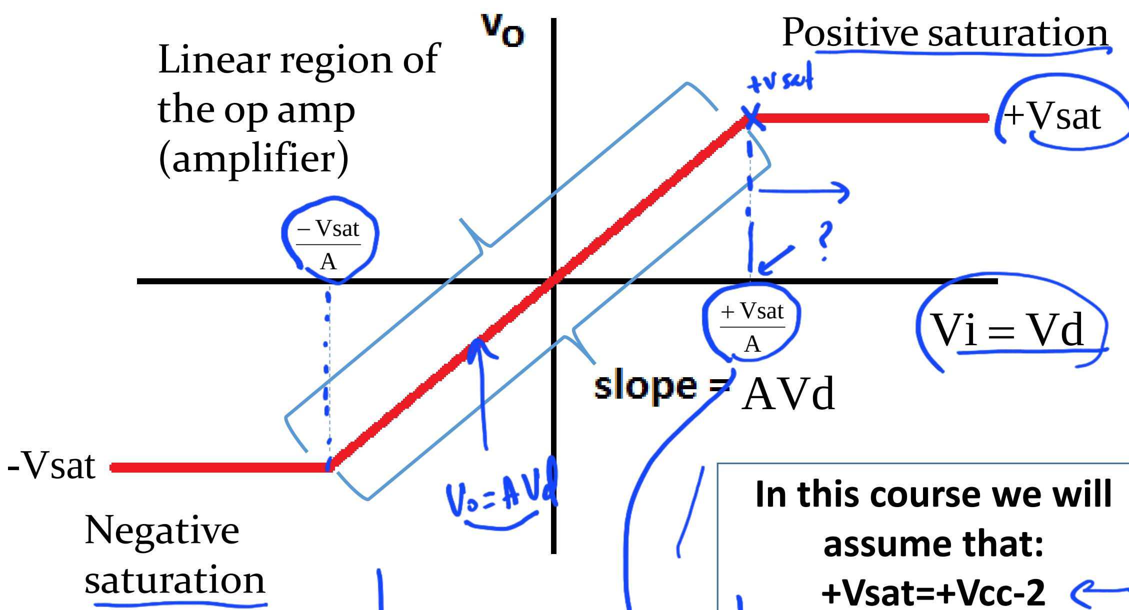


$$+V_{sat} \cong +V_{cc} - 2$$
$$-V_{sat} \cong -V_{cc} + 2$$

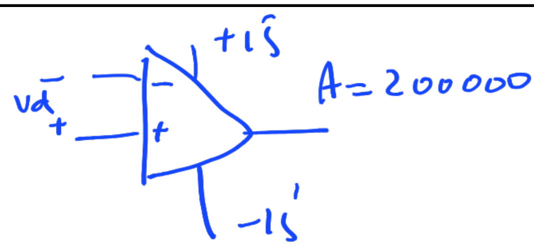
to be used in this course



Voltage Transfer Characteristic



In this course we will assume that:
+Vsat=+Vcc-2
-Vsat=-Vcc+2
In reality value must be taken from data sheets



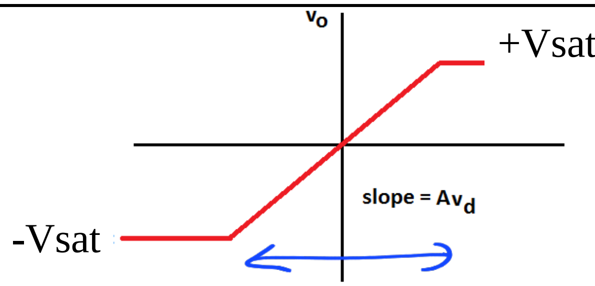
$\pm V_{sat} = \pm 13$

$$\frac{+V_{sat}}{A} = \frac{13}{200000} = 65\,\mu V$$

$$\frac{-V_{sat}}{A} = \quad = -65\,\mu V$$

Output Voltage

- Real Op Amp

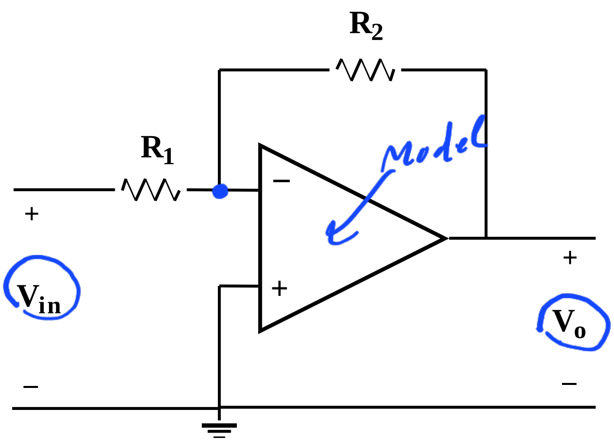


Mode of operation	Voltage Range	Output Voltage
Positive Saturation	**$A\,V_d > +Vsat$**	**$v_o = +Vsat =\sim +Vcc-2$**
Linear Region	**$-Vsat < A\,V_d < +Vsat^+$**	**$v_o = A\,V_d$**
Negative Saturation	**$A\,V_d < -Vsat$**	**$v_o = -Vsat =\sim -Vcc+2$**

The voltage produced by the dependent voltage source inside the op amp is limited by the voltage applied to the positive and negative rails and +/-Vsat level which is approximated by the formulas above

Operational Amplifiers
Analysis (Exact)

As an application of the previous model, consider the following configuration.
Find V_o as a function of V_{in} and the resistors R_1 and R_2.



Op amp functional circuit.

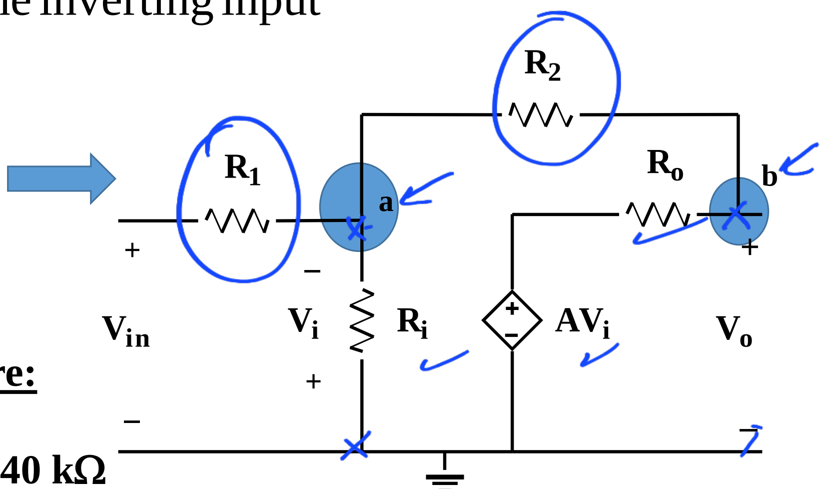
Operational Amplifiers

$Vi = Vd = V_p - V_n = V_+ - V_-$ differential input voltage

$V_p = V_+$ voltage at the non - inverting input

$V_n = V_-$ voltage at the inverting input

Equivalent circuit

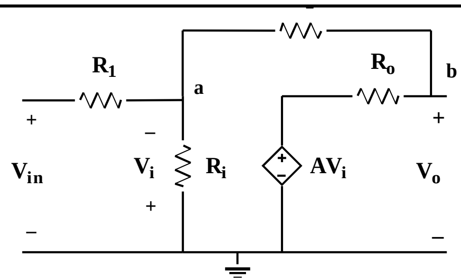


Component values are:

R_1 = 10 kΩ **R_2 = 40 kΩ**
R_o = 50 Ω
A = 100,000 **R_i = 1 meg Ω**

Operational Amplifiers

Exact solution
We can write the following equations for nodes a and b.



R_1 = 10 kΩ R_2 = 40 kΩ
R_o = 50 Ω
A = 100,000 R_i = 1 meg Ω

KCL at A

$$\frac{V_{in}+V_i}{R_1}=\frac{-V_i}{R_i}-\frac{V_i+V_o}{R_2} \qquad (1)$$

KCL at B

$$\mathrm{V_o}=R_o\left[\frac{-(V_i+V_o)}{R_2}\right]+AV_i \qquad (2)$$

Operational Amplifiers

Equation 1 simplifies to;

R_1 = 10 kΩ **R_2 = 40 kΩ**
R_o = 50 Ω
A = 100,000 **R_i = 1 meg Ω**

$$\frac{V_{in}+V_i}{10k}=\frac{-V_i}{1000k}-\frac{V_i+V_o}{40k}$$

$$-25V_o - 126V_i = 100V_{in} \qquad (3)$$

Equation 2 simplifies to;

$$V_o = 50\left[\frac{-(V_i+V_o)}{40k}\right]+100{,}000V_i$$

$$(4.005.10^5)V_o-(4.10^9)V_i=0 \qquad (4)$$

Operational Amplifiers

From Equations (3) and (4) we find; **

$$\boxed{V_o = -3.99V_{in}} \qquad (5)$$

This is an expected answer.

Fortunately, we are not required to do elaborate circuit analysis, as above, to find the relationship between the output and input of an op amp. Simplifying the analysis is our next consideration.

Operational Amplifiers Models

For most operational amplifiers,
R_i is 1 Meg Ω or larger and
R_o is around 50 Ω or less.
The open-loop gain, A, is greater than 100,000.

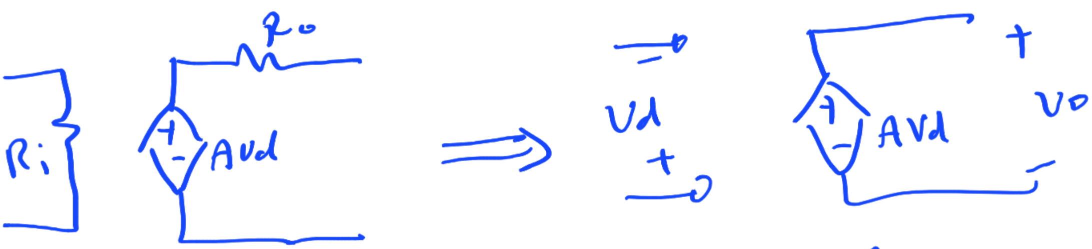
Ideal Op Amp Model:

The following assumptions are made for the ideal op amp.

$$1.\ Infinite\ \ open-loop\ \ gain; \Rightarrow A \cong \infty$$

$$2.\ Zero\ \ output\ \ ohms; \Rightarrow R_o = 0$$

$$3.\ Infinite\ \ input\ \ ohms; \Rightarrow R_i = \infty$$



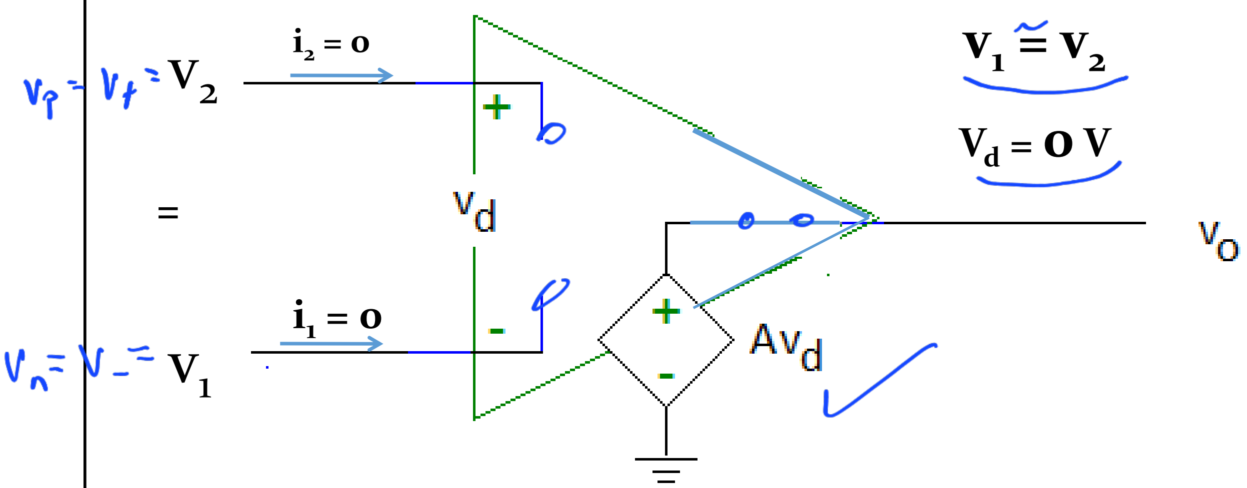
op-amp ideal-Model

** To be used in this course

Ideal Op Amp Model

Because Ri is equal to ∞Ω,

the voltage across Ri is 0 V.



$$V_1 \cong V_2$$

$$V_d = 0\ V$$

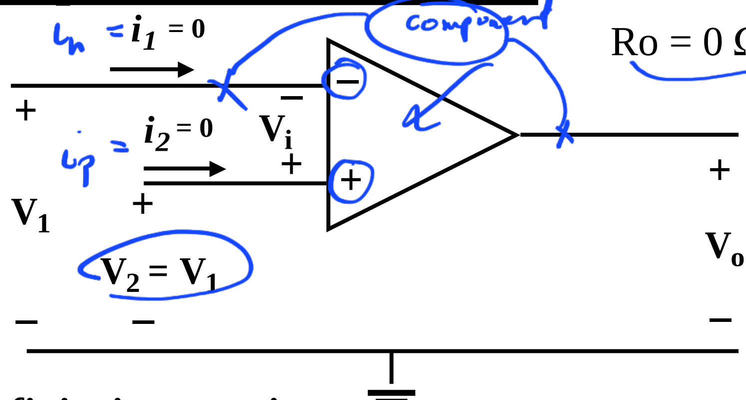
Important Note:

Ideal Op Amp

Only Ideal Op Amp Model will be used from now on:

Ri = ∞ Ω

Ro = 0 Ω



(a) $i_1 = i_2 = 0$**: Due to infinite input resistance.**

(b) V_i **is negligibly small;** $V_1 = V_2$.

V −

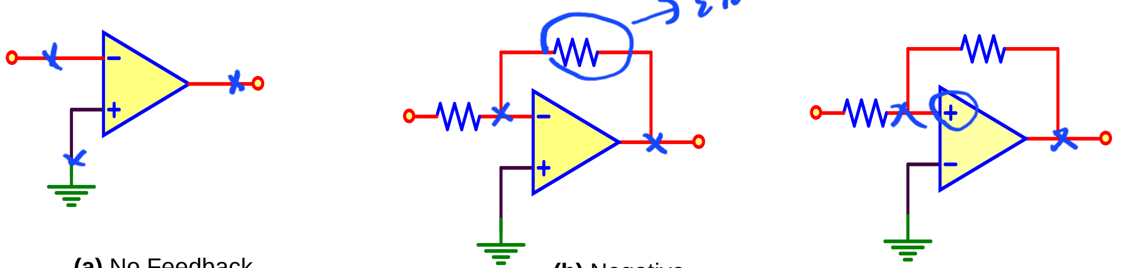
The op amp forces the voltage at the inverting input terminal to be equal to the voltage at the noninverting input terminal if there is some component connecting the output terminal to the inverting input terminal.

V+

applications

OP-AMP CONFIGURATIONS

Element



(a) No Feedback (open loop comparator circuit)

+Vsat or -Vsat

(b) Negative Feedback

1)

(c) Positive Feedback

- No feedback : Open loop (used in comparators)
- Negative feedback : Feedback to the inverting input (Used in amplifiers)
- Positive feedback : Feedback to the non inverting input (Used in oscillators) and Schmitt triggers (comparators with hysterisis)

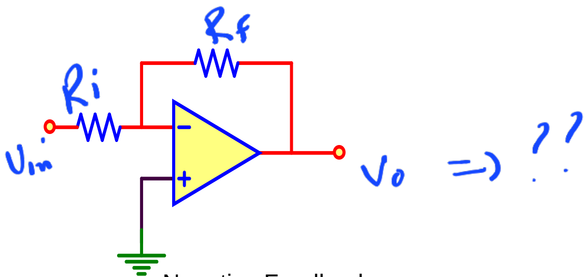
**

OP-AMPS WITH NEGATIVE FEEDBACK

The two basic amplifier circuits with negative feedback are:

- The non-inverting Amplifier.
- The inverting Amplifier

(Note: Negative feedback is used to limit the gain)



Negative Feedback

Example #2: Closed Loop Gain Av $= \frac{V_o}{V_s}$
(inverting amplifier)

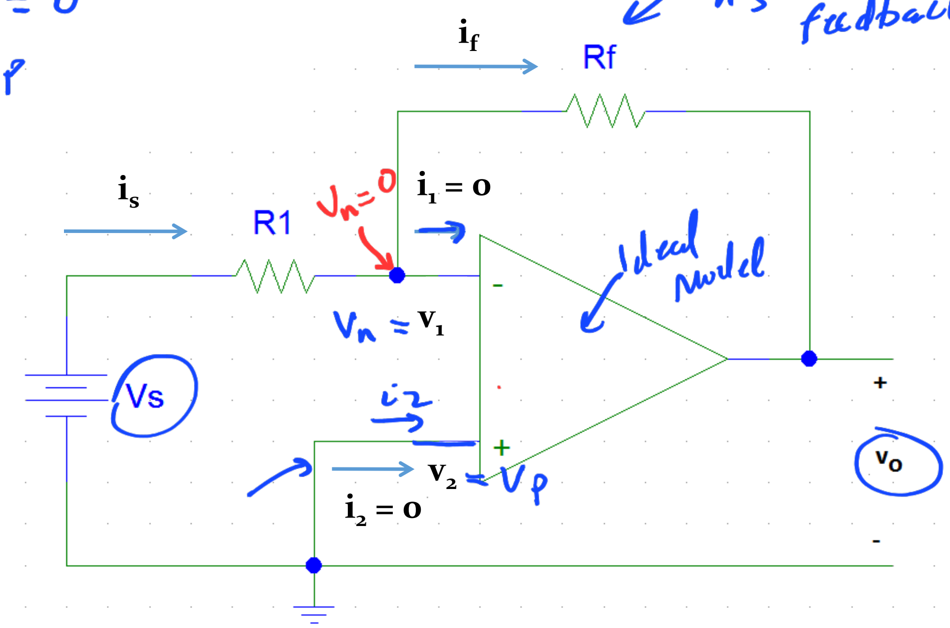
$$\begin{cases} i_1 = i_2 = 0 \\ V_n = V_p \end{cases}$$

but

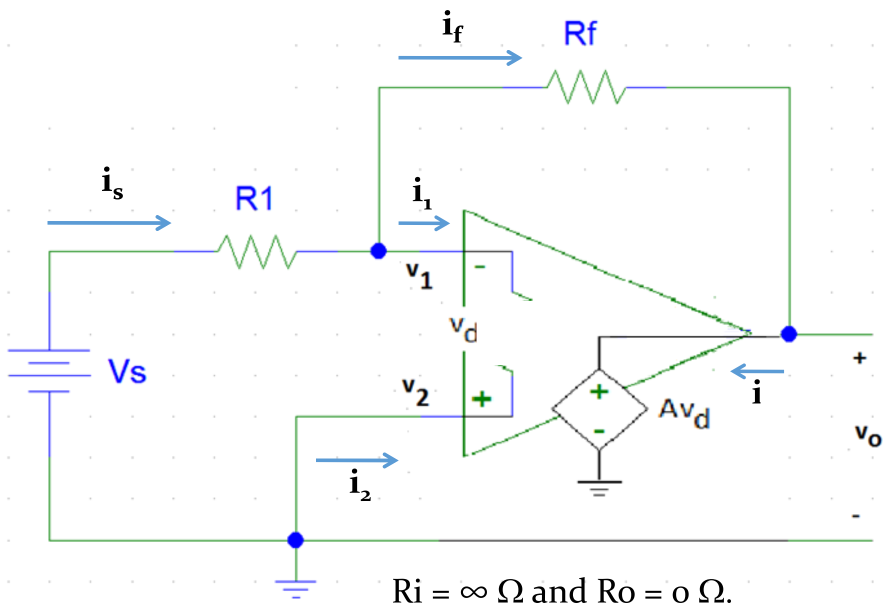
$V_p = 0$

$\therefore V_n \approx 0$

negative feedback

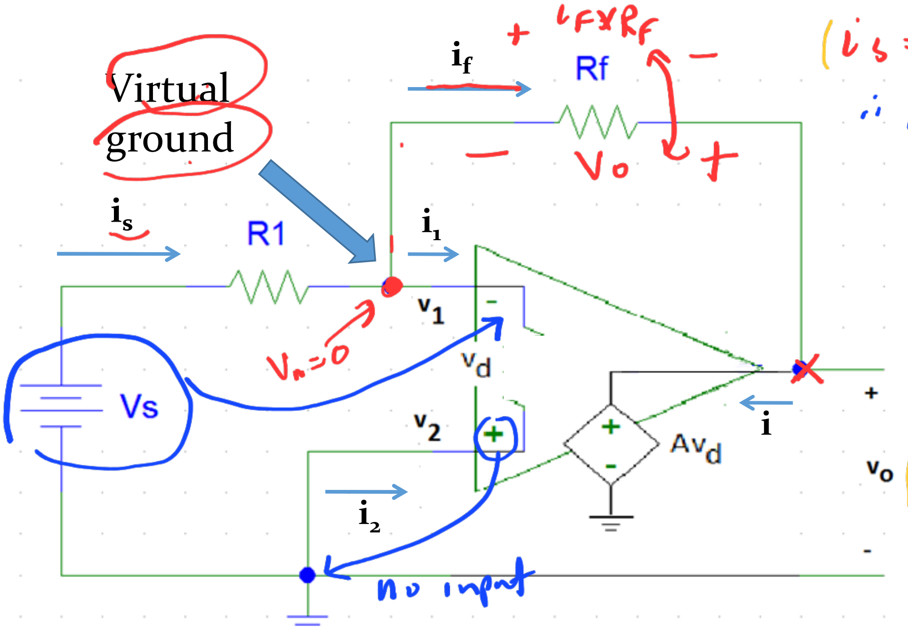


Example #2 (con't)



Ri = ∞ Ω and Ro = 0 Ω.

The op amp outputs a voltage Vo such that V1 = V2.
V2=Vp=0 ➔ V1=Vn=0
(virtual ground – potential equal 0 even though it is not connected really to ground)



$$V_1 = V_2 = 0$$

$$(i_s = \cancel{i_1}^{\,0} + i_f)$$

$$\therefore\ i_s = i_f$$

$$i_s = \frac{V_s - \cancel{V_1}^{\,0}}{R_1}$$

$$i_s = \frac{V_s}{R_1} = i_f$$

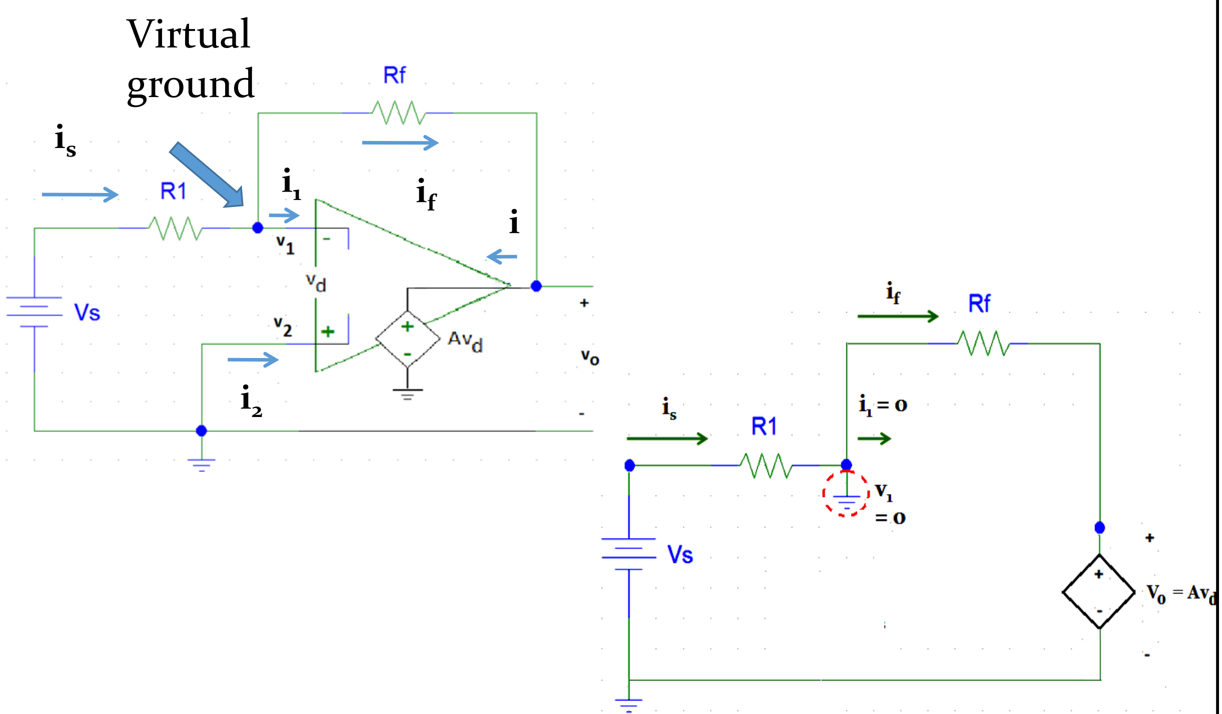
$$V_o = -i_f R_f$$

$$\boxed{V_o = -\frac{V_s}{R_1} R_f}$$

$$V_o = -\frac{R_f}{R_1} \cdot V_s \Rightarrow \boxed{A_v = \frac{V_o}{V_s} = -\frac{R_F}{R_1}}$$

Inverting Amplifier Gain

Example #2 (con’t)



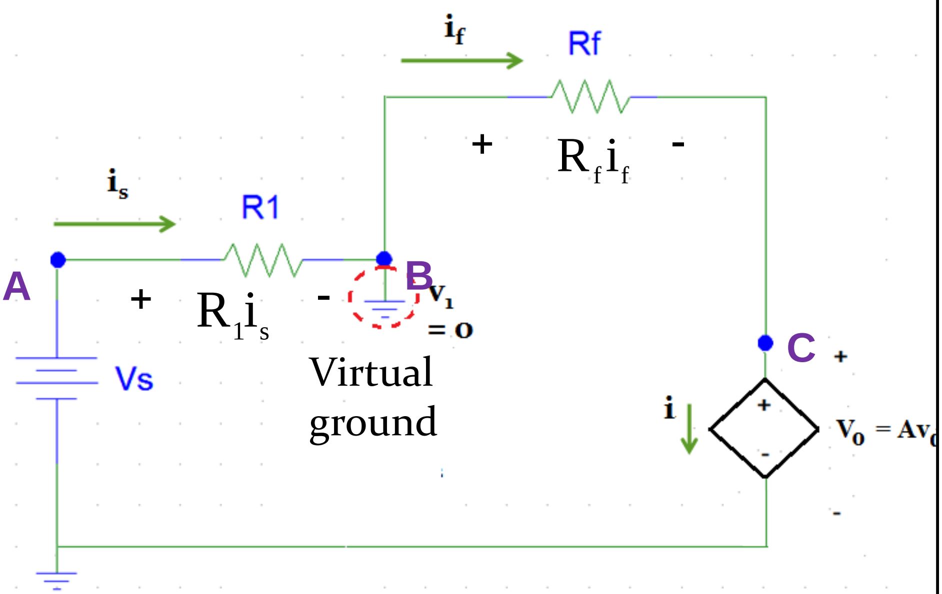
Example #2: Closed Loop Gain

$$V_2 = 0V \Rightarrow V_1 = 0V$$

$$V_S = R_1 i_s$$

$$v_o = -R_f i_f$$

$$i_s = i_f = i$$



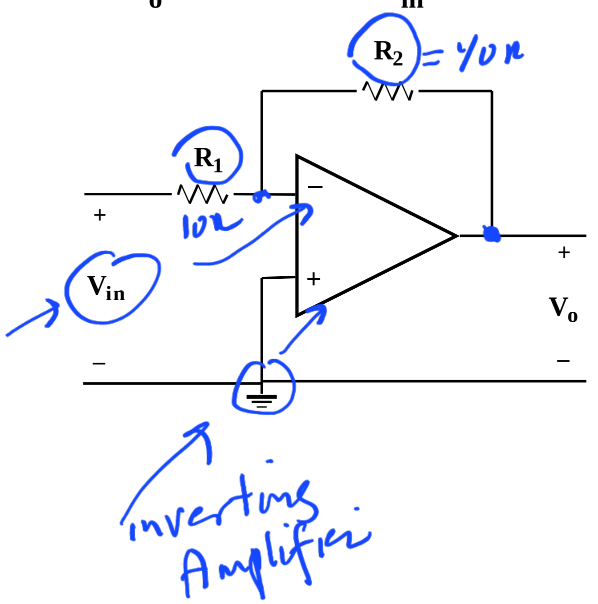
$$A_V = v_o / V_s = \frac{-R_f i_f}{R_1 i_s}$$

$$A_V = -R_f / R_1$$

This circuit is known as an inverting amplifier.

Inverting Amplifier (previous example of slide 30)

Find V_o in terms of V_{in} for the following configuration.



$$V_0 = \frac{-R_2}{R_1} V_{in}$$

With R_2 = 40 kΩ and R_1 = 10 kΩ, we have

$$\boxed{V_o = -4V_{in}}$$

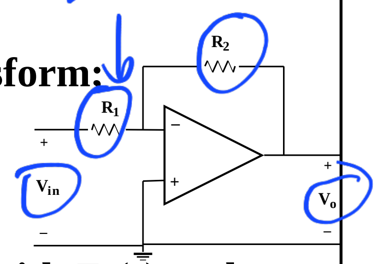
Earlier we got →

$$\boxed{\boldsymbol{V_o = -3.99V_{in}}}$$

$\frac{0.01}{4} \times 100\% = 0.25\%$ error

Inverting Op Amp:

$Z_{in}(s)$ $Z_{fb}(s)$



When V_i = 0 in and we apply the Laplace Transform:

$$\frac{V_0(s)}{V_{in}(s)} = \frac{-R_2}{R_1}$$

In fact, we can replace R_2 with $Z_{fb}(s)$ and R_1 with $Z_1(s)$ and we have the important expression;

$$\frac{V_0(s)}{V_{in}(s)} = \frac{-Z_{fb}(s)}{Z_{in}(s)}$$

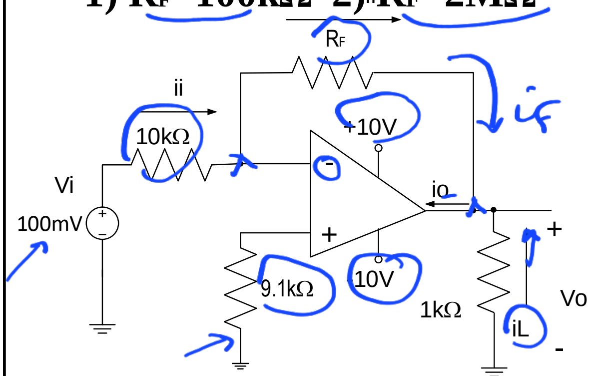
At this point in circuits we are not able to appreciate the use of this equation. We will revisit this at a later point in circuits but for now we point out that judicious selections of $Z_{fb}(s)$ and $Z_{in}(s)$ leads to important applications in

- **Analog Compensators in Control Systems**
- **Analog Filters**
- **Application in Communications**

Example

Find the value of Vo and Io and verify if the opamp is in linear or saturation mode for two values of feedback resistor; assume Io(max)=20 mA:

1) R_F=100kΩ 2) R_F=2MΩ



$$V_p = V(+) = 0V$$

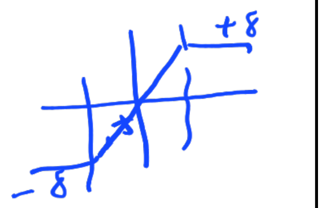
$$i_1 = \frac{V_1}{R_1}$$

$$i_i = i_f = \frac{V_i}{10k} = \frac{100\,\text{mV}}{10\,\text{k}\Omega} = 10\,\mu\text{A}$$

$$1)\ V_o = -\frac{R_F}{10\,\text{k}\Omega} V_i = -10\,V_i = -1V$$

$$V_o > -V_{sat}$$

Linear mode



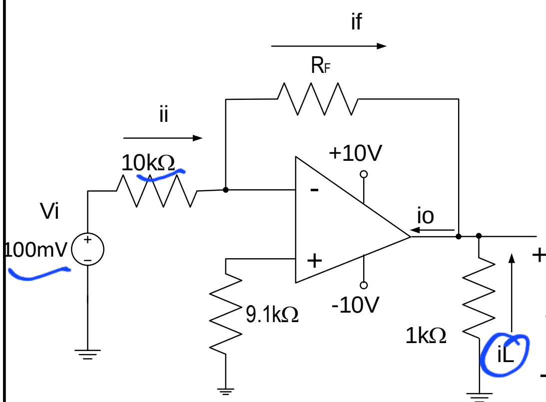
Important

Io(max) is few mA for most opamps which limits the values of resistors to be used to kohm range

Example

Find the value of Vo and Io and verify if the opamp is in linear or saturation mode for two values of feedback resistor:

1) R_F=100kΩ 2) R_F=2MΩ



$$i_o = i_f + i_L$$

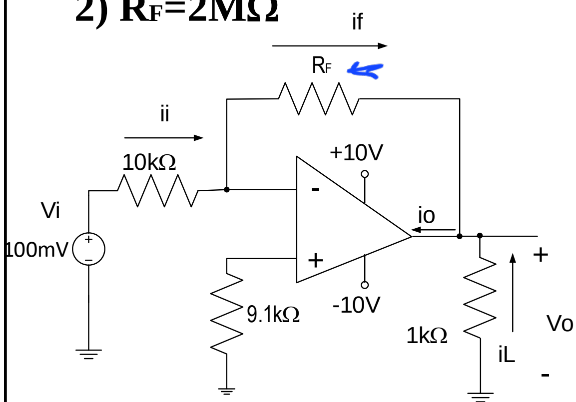
$$i_L = \frac{1V}{1k} = 1mA$$

$$i_o = 10\mu A + 1mA = 1.01mA < Io(max)$$

20mA

Example continued

2) R_F=2MΩ



$$i_i = i_f = \frac{V_i}{10k} = \frac{100mV}{10k} = 10\,\mu A$$

$$2) V_o = -\frac{R_F}{10k} V_i = -200 V_i = -20V$$

X

$-V_{sat} = -8V$

$$V_o < -V_{sat}$$

Saturation mode ⇒ V_o is limited to - V_{sat}

$$\boxed{\therefore V_o = -V_{sat} = -8V}$$

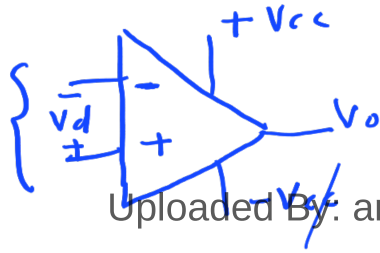
$$i_L = \frac{8V}{1k} = 8mA$$

$$i_o = 10\,\mu A + 8mA = 8.01mA < Io(max)$$

20mA

End of L22

op-Amp

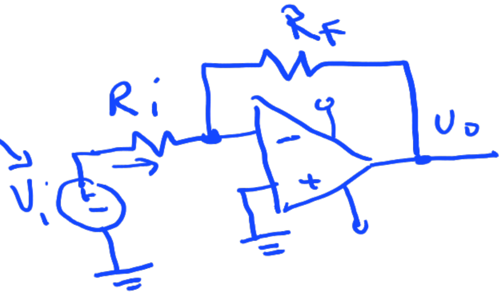


L23 18/8/2021

Negative Feed back

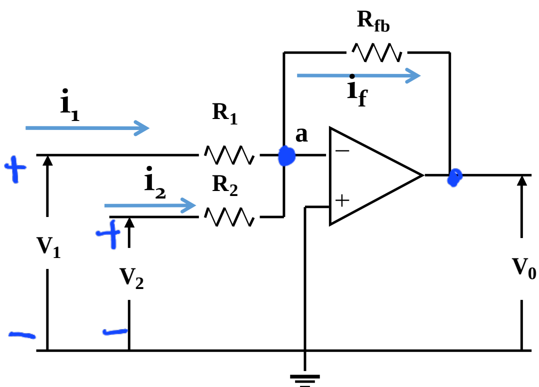
1. Inverting Amplifier

$$\boxed{V_o = -\frac{R_F}{R_i} V_i}$$



Inverting Adder or Summing Amplifier

Summing Amplifier: This is an application of inverting amplifier



$$V_p = V_{(+)} = 0V$$

$$i_1 = \frac{V_1}{R_1} \qquad i_2 = \frac{V_2}{R_2}$$

$$V_o = -R_f i_f$$

$$i_f = i_1 + i_2$$

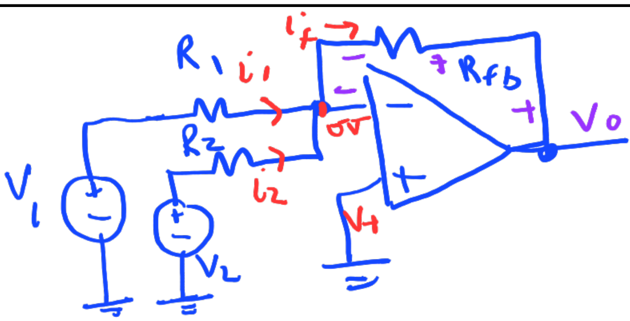
$$V_o = -R_f [i_1 + i_2]$$

$$V_0 = -\left[\left(\frac{R_{fb}}{R_1}\right)V_1 + \left(\frac{R_{fb}}{R_2}\right)V_2\right]$$

If $R_1 = R_2 = R_{fb}$ then,

$$V_0 = -[V_1 + V_2]$$

Therefore, we can add signals with an op amp



$V_+ = 0$; $V_- = V_+ = 0$

$i_1 = \frac{V_1}{R_1}$; $i_2 = \frac{V_2}{R_2}$;

$i_f = i_1 + i_2$

$V_o = -i_f R_{fb} = -(i_1 + i_2) R_{fb}$

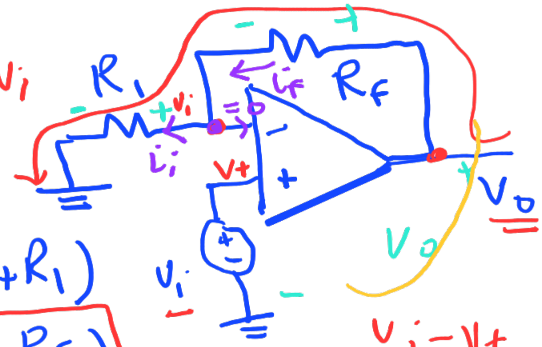
$$\boxed{V_o = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) R_{fb}}$$

if $R_1 = R_2 = R_{fb} \Rightarrow \boxed{V_o = -(V_1 + V_2)}$ *

2) Non-Inverting

$V_+ = V_-$; but $V_+ = V_i$; $\therefore V_- = V_i$

$i_i = \frac{V_i}{R_1} = i_F$



$V_o = i_F R_F + i_i R_1 = i_F(R_F + R_1)$

$$\boxed{V_o = \frac{V_i}{R_1}(R_F + R_1) = V_i\left(1 + \frac{R_F}{R_1}\right)}$$

$V_i = V_+$

$$\boxed{V_o = V_+\left(1 + \frac{R_F}{R_1}\right)}$$

***

The non-inverting op amp.

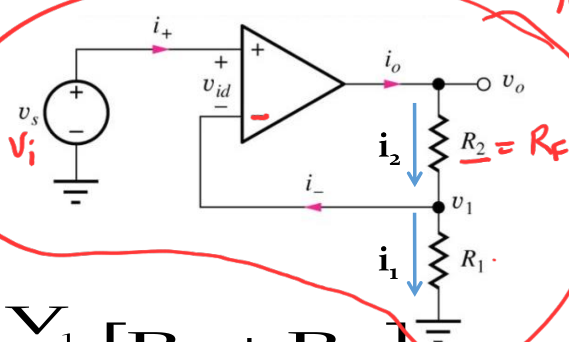
The non-inverting op amp has the input voltage connected to its (+) terminal while no voltage at the negative terminal

non-inverting Amplifier

$$V_n = V_1 = V_p = V_s$$

$$i_+ = i_- = 0$$

$$V_o = V_{R1} + V_{R2}$$



$$V_{R1} = V_1 = i_1 R_1$$

$$V_{R2} = i_2 R_2$$

$$V_o = \frac{V_1}{R_1}[R_1 + R_2]$$

but

which gives,

$$i_1 = i_2 = \frac{V_1}{R_1}$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right)V_s$$

Example: Non-inverting Amplifiers

Example: Find V_0 for the following op amp configuration.

$$V_x = \frac{6k}{6k+2k}4V$$

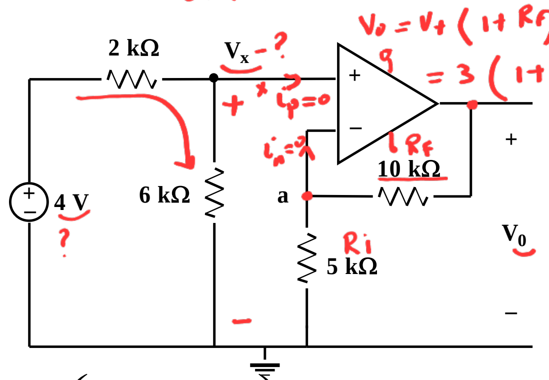
$$V_x = 3V$$

$$V_o = \left(1+\frac{R_F}{R_i}\right)V_x$$

$V_+ = V_x = \frac{6K}{6K+2K} \times 4V = 3V$

$V_o = V_+ (1 + R_F/R_i)$

$= 3\left(1+\frac{10K}{5K}\right) = 9V$



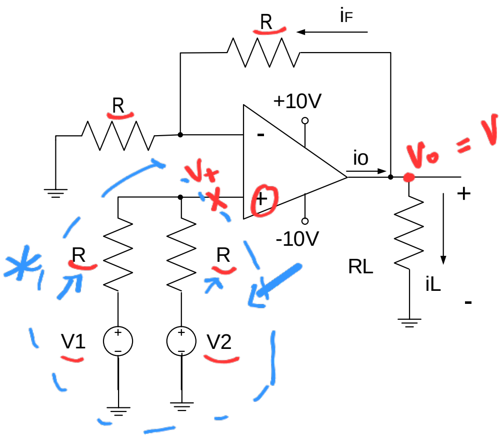
$$V_o = \left(1+\frac{10k}{5k}\right)3V$$

$$V_o = 9V$$

$= -V_{cc}+2$ $\quad$ $+V_{cc}-2$

$$\text{Make sure that: } -V_{sat} < V_o < +V_{sat}$$

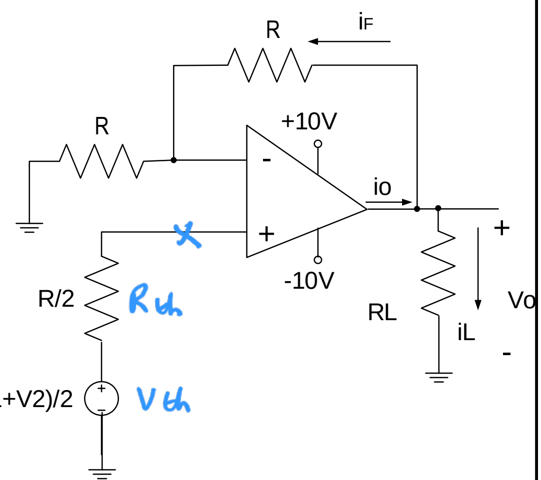
Non-Inverting Adder



$V_o = V_+\left(1 + \frac{R}{R}\right)$

==>

Thevenin's equivalent at + terminal



$$V_{TH} = \left(\frac{R}{R+R}\right)V_1 + \left(\frac{R}{R+R}\right)V_2$$

$$= \left[\frac{V_1 + V_2}{2}\right]$$

$$V_+ = V_{TH} \quad \Leftarrow \quad (\text{since } i_+ = 0)$$

$$V_o = \left(1 + \frac{R}{R}\right)V_+$$

$$\Rightarrow V_o = \underbrace{\left(1 + \frac{R}{R}\right)}_{2}\underbrace{\left[\frac{V_1 + V_2}{2}\right]}_{V_{th} = V_+}$$

$$V_o = \cancel{2}\left[\frac{V_1 + V_2}{\cancel{2}}\right]$$

adder

$$\boxed{V_o = \left[V_1 + V_2\right]}$$

superposition

kill V_1

kill V_2

$V_{th}' = \frac{R}{R+R} V_2$

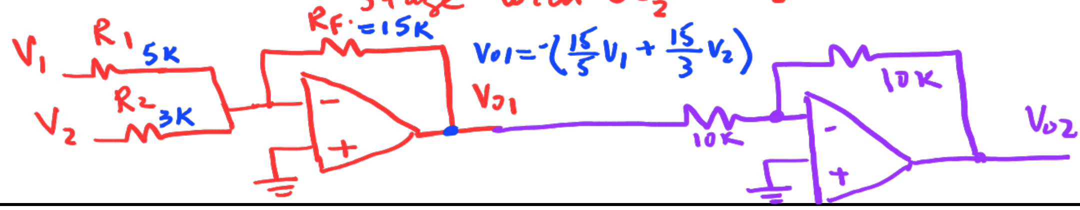
$V_{th}'' = \frac{R}{R+R} V_1$

$V_{th} = V_+ = (V_1 + V_2)\frac{1}{2}$

→ Design An Amplifier that have two input voltages V_1 & V_2 such that the output $V_o = 3V_1 + 5V_2$ (use inverting Amplifiers)

Solution → with inverting Amplifier $V_{o_1} = -(3V_1 + 5V_2)$

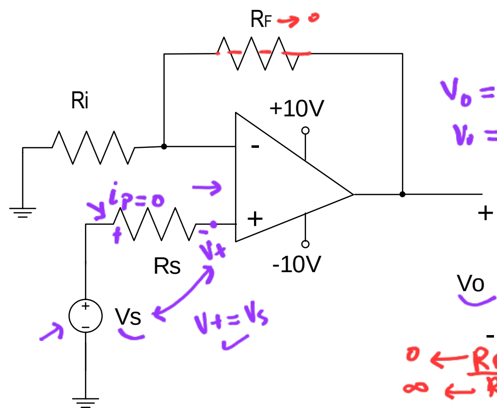
→ we can have a second stage with $V_{o_2} = -V_{in_2}$, $V_{o1} = V_{in\,2}$



$V_{o1} = -\left(\frac{15}{5}V_1 + \frac{15}{3}V_2\right)$

$V_{o2} = \frac{-10K}{10K} V_{o1} = -V_{o1} = 3V_1 + 5V_2$

Non-Inverting Amplifier with unity Gain (Buffer)

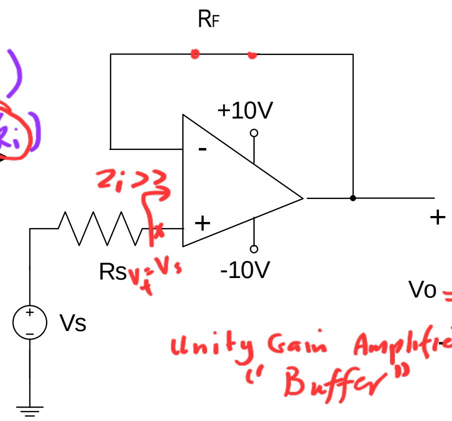


$V_o = V_+\left(1 + \frac{R_F}{R_i}\right)$

$V_o = V_s\left(1 + R_F/R_i\right)$ ==>

$V_o = V_s$?

$0 \leftarrow \frac{R_F}{R_i} = 0$
$\infty \leftarrow R_i$



Unity Gain Amplifier "Buffer"

$$V_o = V(-)$$

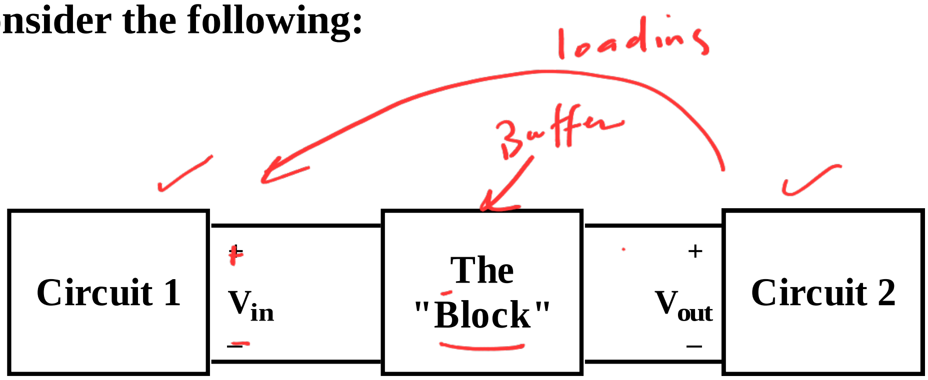
$$V(-) = V(+) = Vs$$

$$V_o = Vs$$

$$\frac{V_o}{Vs} = 1$$

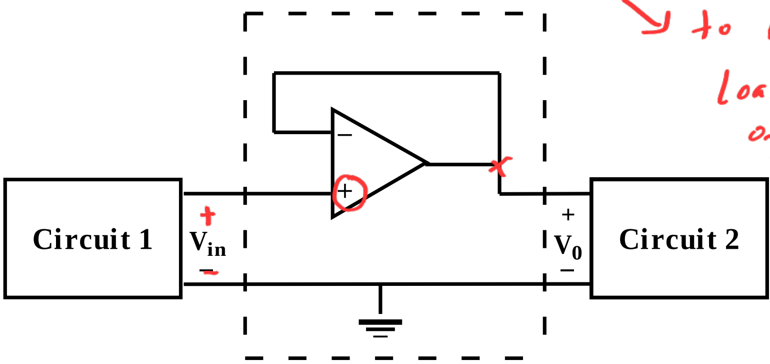
Buffer or Isolation Amplifier or Voltage Follower

- Applications arise in which we wish to connect one circuit to another without the first circuit loading the second.
- This requires that we connect to a "block" that has infinite input impedance and zero output impedance.
- An operational amplifier does a good job of approximating this.
- Consider the following:



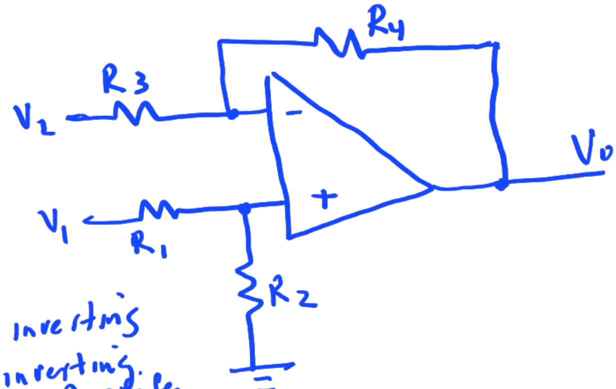
Buffer or Isolation Amplifier or Voltage Follower

The Block → Buffer → to reduce / Eliminate loading effect of circuit 2 on Gain of circuit 1



Circuit isolation with an op amp.

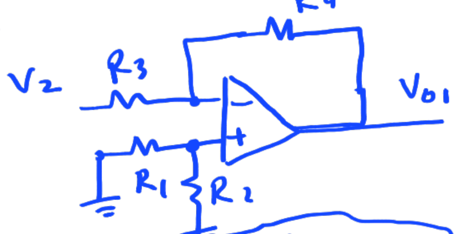
It is easy to see that: $V_0 = V_{in}$



mix of inverting & non-inverting Amplifier

superposition

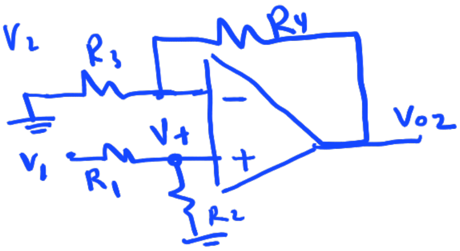
1. Kill V_1 → inverting Amp



$$\boxed{V_{o1} = -\frac{R_4}{R_3} \cdot V_2}$$

2) kill V_2

non-inverting amplifier



$$\boxed{V_{o2} = \left(1+\frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right)V_1}$$

$$V_+ = \frac{R_2}{R_1+R_2}\cdot V_1$$

$$V_o = V_{o1} + V_{o2} \quad *$$

$$\boxed{V_o = \left(\frac{R_3+R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right)V_1 - \frac{R_4}{R_3}V_2}$$

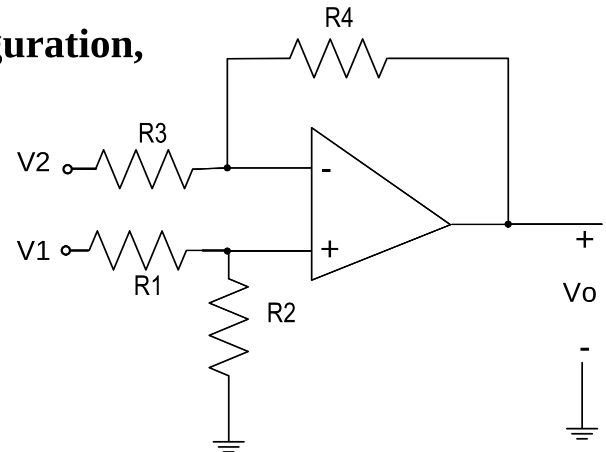
$$a = \left(\frac{R_3+R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right)$$

$$b = \frac{R_4}{R_3}$$

$$\boxed{V_o = aV_1 - bV_2}$$

Difference Amplifier (Subtractor)

For the following op amp configuration, in order to find Vo we can use Superposition:



1) **Short V2 and find contribution of V1 to Vo ==> non-inverting amp**

$$V_o = V_{o1}$$

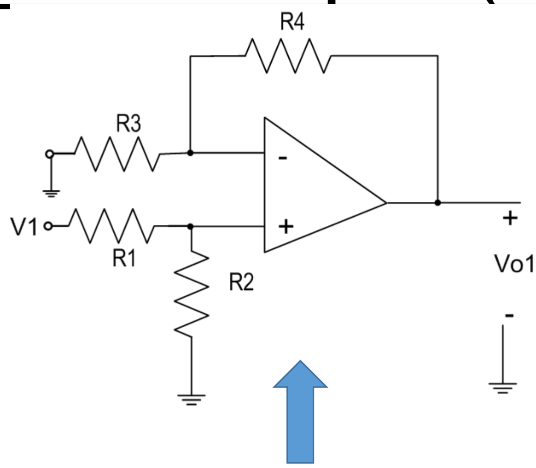
2) **Short V1 and find contribution of V2 to Vo ==> inverting amp**

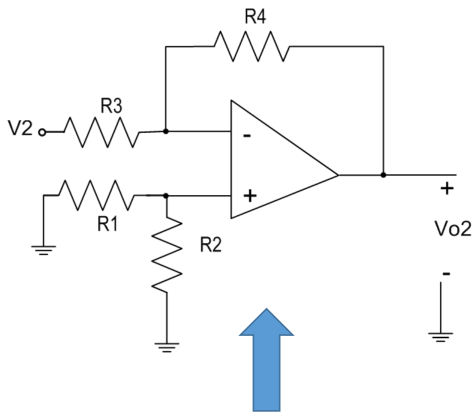
$$V_o = V_{o2}$$

3) **The total output is found by summing the two results above**

$$V_o = V_{o1} + V_{o2}$$

Difference Amplifier (Subtractor)





Non-Inverting Amplifier

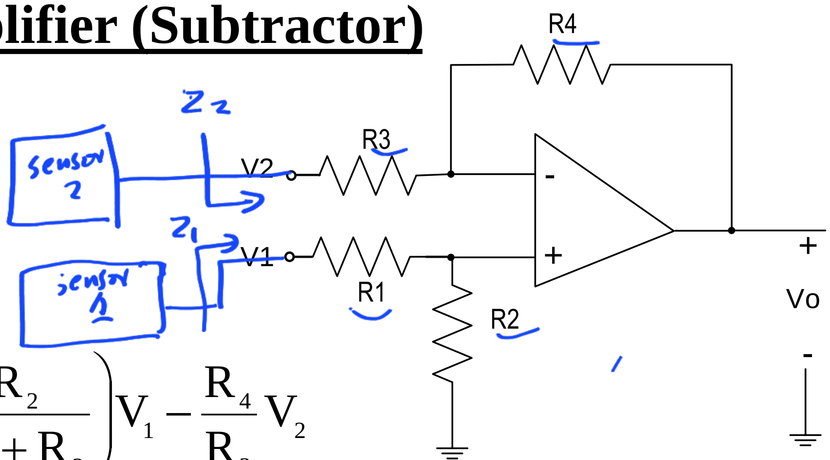
$$V_{o1} = \left(1 + \frac{R_4}{R_3}\right) V_+$$

$$V_{o1} = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right) V_1$$

Inverting Amplifier

$$V_{o2} = -\frac{R_4}{R_3} V_2$$

Difference Amplifier (Subtractor)



$$V_o = \left(1+\frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right)V_1 - \frac{R_4}{R_3}V_2$$

$$V_o = \overset{m}{a}V_1 - \overset{m}{b}V_2$$

$$a = \left(1+\frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right) = \frac{R_2}{R_3} = \frac{mR}{R} = m$$

$$b = \frac{R_4}{R_3} = \frac{mR}{R} = m$$

let $R_1 = R_3 = R$

$R_2 = R_4 = mR$

$$\boxed{\therefore\ V_o = m(V_1 - V_2)}$$

under this condition this is a difference amplifier **

$\left.\begin{matrix} V_1 \\ V_2 \end{matrix}\right\}$ signals from sensors which is low in value ⟹ Volts

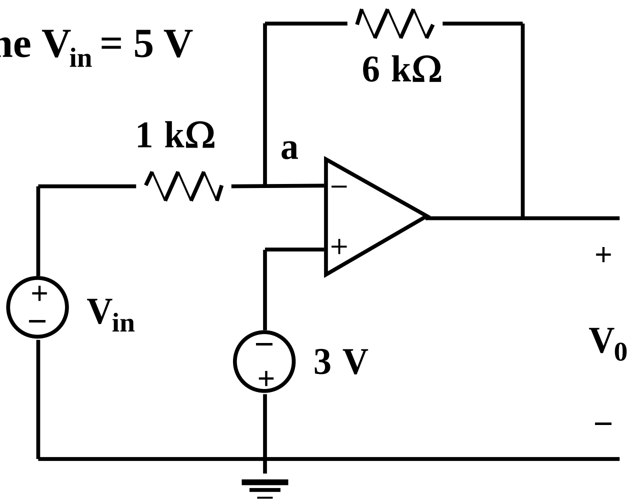
mV's

$V_o = m(V_1 - V_2)$

amplification factor « Gain »

<u>Example</u> **Consider the op amp configuration below.**

Assume V_{in} = 5 V

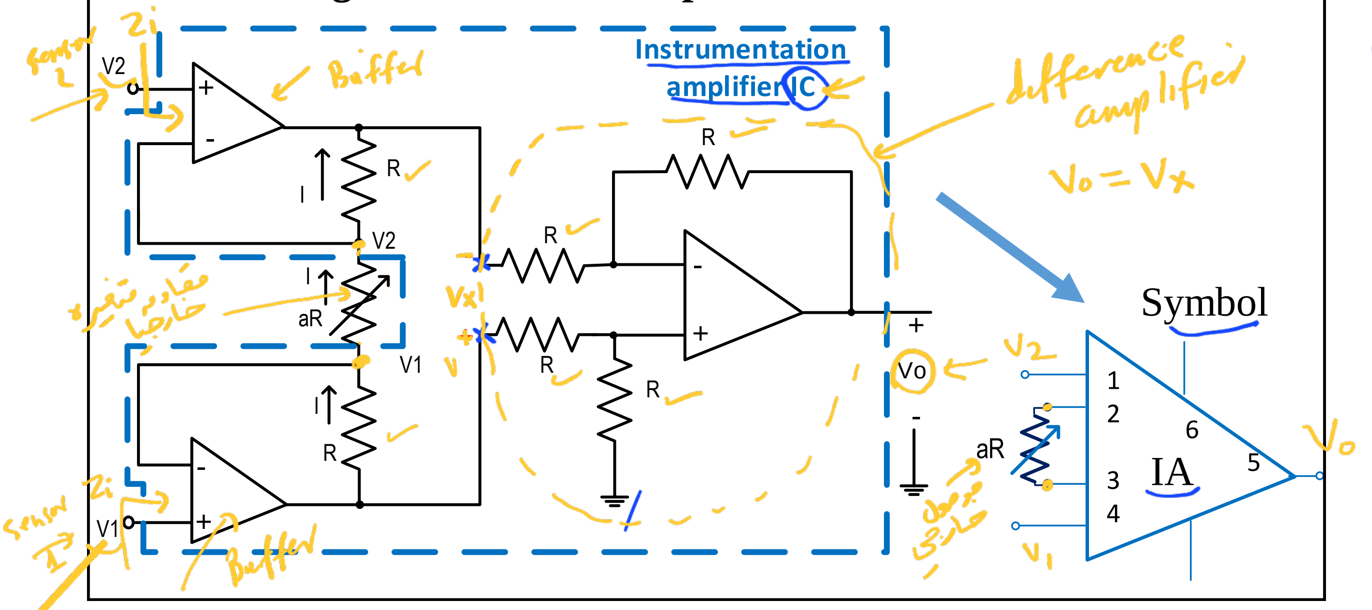


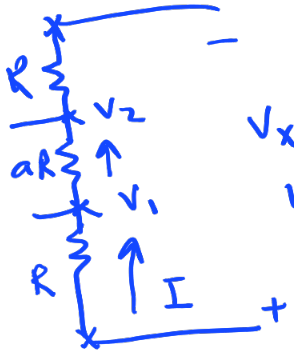
$$V_0 = (1+\frac{6k}{1k})(-3) - (\frac{6k}{1k})(5)$$
$$= -21 - 30 = -51V$$

Since V_0 = -51 V **(op amp will saturate and Vo will be limited to -Vsat)**

Instrumentation Amplifier

- The previous difference amplifier has low input impedance and it is difficult to vary the gain "m"
- The instrumentation amplifier solves this problem by adding a buffer stage and a difference amplifier stage to solve the disadvantages of difference amplifier





for $V_1 > V_2$

$V_x = I(R + aR + R)$

$V_x = I(2R + aR) = I(2 + a)R$

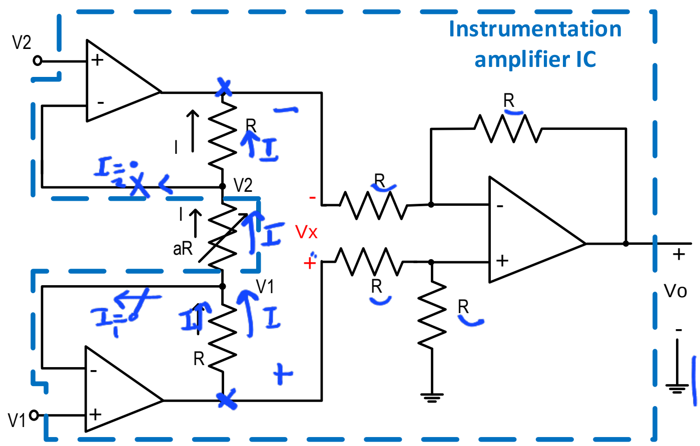
$\boxed{I = \frac{V_1 - V_2}{aR}}$

$V_x = \frac{V_1 - V_2}{aR}(2 + a)R$

$= (V_1 - V_2)\frac{(2 + a)}{a}$

$\boxed{V_o = V_x = (V_1 - V_2)\left(1 + \frac{2}{a}\right)}$

Instrumentation Amplifier



assume $V_1 > V_2$

$$I = \frac{V_1 - V_2}{aR}$$

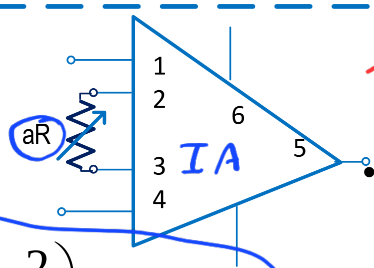
$$V_x = (R + R + aR)I$$

$$V_x = (R + R + aR)\frac{V_1 - V_2}{aR}$$

$$V_x = \left(\frac{(2 + a)R}{aR}\right)(V_1 - V_2)$$

$$** \quad \boxed{V_x = \left(1 + \frac{2}{a}\right)(V_1 - V_2)}$$

$$\boxed{V_O = mV_x}$$



for $m = 1$

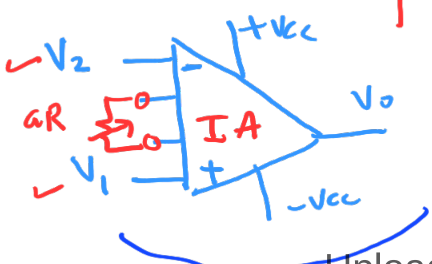
$$V_o = V_x = \left(1 + \frac{2}{a}\right)(V_1 - V_2)$$

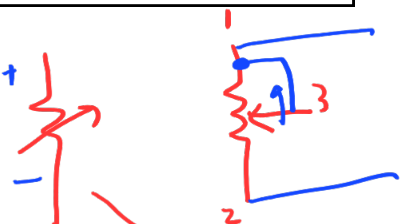
- Only by varying the value of potentiometer aR, the output can be adjusted
- R is an internal resistance given in data sheet of "IA" IC

L24 19/8/2021

Instrumentation Amplifier "I"

$$\boxed{V_o = \left(1 + \frac{2}{a}\right)(V_1 - V_2)} \; **$$





aR, R = const

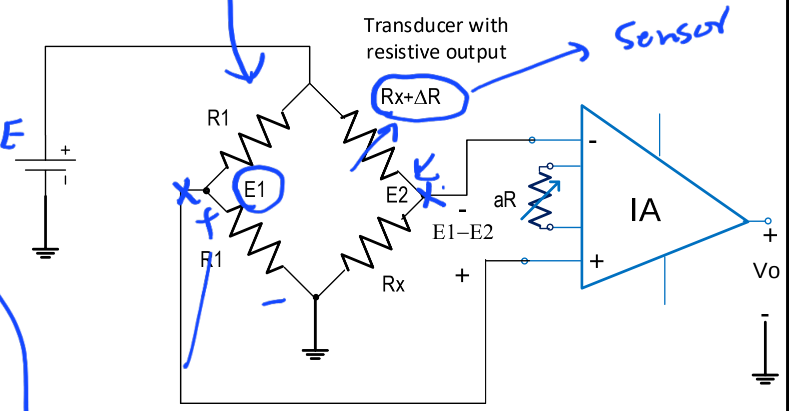
a → 0

a → 1

Wheatstone Bridge

Example: Instrumentation Amplifier (self study)

$$V_o = \boxed{V_x = \left(1+\frac{2}{a}\right)\left(E_1 - E_2\right)}$$



$$E_1 = \frac{R_1}{R_1 + R_1}E = \frac{1}{2}E$$

$$E_2 = \frac{R_x}{R_x + R_x + \Delta R_x}E$$

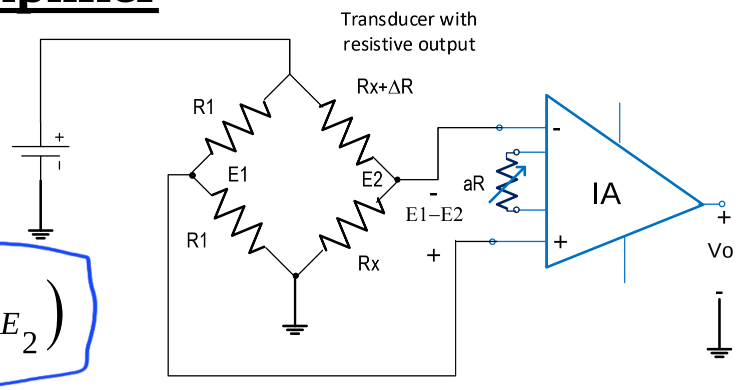
$$E_1 - E_2 = \frac{1}{2}E - \frac{R_x}{2R_x + \Delta R_x}E$$

$$= \frac{2R_x + \Delta R_x}{2\left(2R_x + \Delta R_x\right)}E - \frac{2R_x}{2\left(2R_x + \Delta R_x\right)}E$$

$$E_1 - E_2 = \frac{E}{2}\left(\frac{2R_x + \Delta R_x - 2R_x}{2R_x + \Delta R_x}\right)$$

$$= \frac{E}{2}\left(\frac{\Delta R_x}{2R_x + \Delta R_x}\right)$$

Instrumentation Amplifier



assume $\Delta R_x << R_x$

$$V_o = E1 - E2 = \left(1 + \frac{2}{a}\right)\left(E_1 - E_2\right)$$

$$V_o \; \alpha \; \Delta R_x \qquad = \left(1 + \frac{2}{a}\right)\frac{E}{4R_x}\left(\Delta R_x\right)$$

let $R_1 = 10k$; $E = 10V$; $\Delta R_x = 10$;

$aR = 50.12$ ohm

(potentiomater is set to 50.12 ohms)

$$a = \left(\frac{aR}{R}\right) = \left(\frac{50.12}{10000}\right) = 0.005013$$

$$\left(1 + \frac{2}{a}\right) = 400$$

$$V_o = (400)\left(\frac{10}{4}\right) * \frac{10}{10000} = 1\,V$$

Example: Use an instrumentation amplifier in order to get an output $V_o = 5V$ if $V_1 = 15mV$ (sensor 1), $V_2 = 5mV$ (sensor 2) & given that internal IA resistance $= 20k\Omega$

$V_o = \left(1+\frac{2}{a}\right)(V_1 - V_2)$

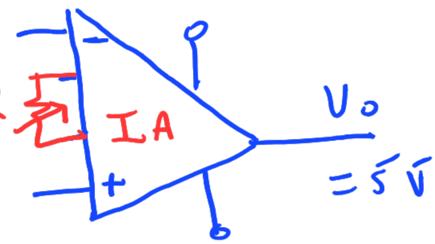
$5 = \left(1+\frac{2}{a}\right)(15m - 5m)$

$5 = \left(1+\frac{2}{a}\right)(10mV)$

$1+\frac{2}{a} = \frac{5V}{10mV} = \frac{5000mV}{10mV} = 500$

$V_2 = 5mV$

$? \rightarrow aR$



$15mV = V_1$

$= 5V$

$1+\frac{2}{a} = 500$

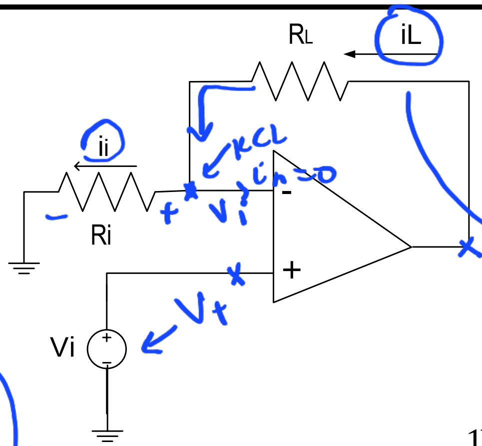
$\frac{2}{a} = 499$

$a = \frac{2}{499}$

$aR = \frac{2}{499} \times 20k$

$= 80.16\Omega$

Voltage to Current converter



$V_+ = V_-$

$i_p = i_n = 0$

replace R_L by current measuring Device

PMMC

$\pm 1mA = I_M$

$$i_i = \frac{V_i}{R_i}$$

$$i_i = i_L$$

let $V_i = 1V$; Ri = 1k

$$i_L = \frac{1V}{1k} = +1mA$$

$$i_L = \frac{1V}{1k} = +1mA$$

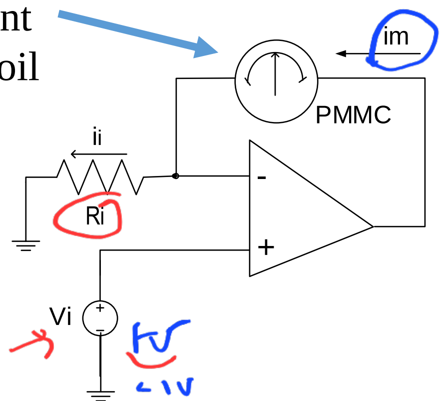
let $V_i = -1V$; Ri = 1k

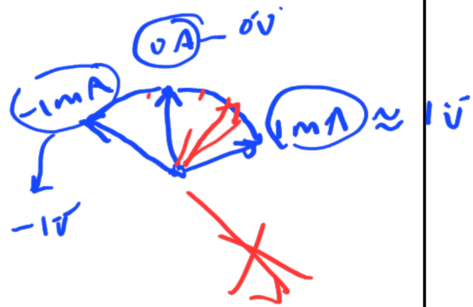
$$i_L = \frac{-1V}{1k} = -1mA$$

Here we converted ± 1V to ± 1mA

Voltage to Current converter

PMMC: Permanent magnet moving coil





$$i_i = \frac{V_i}{R_i}$$

$$i_i = i_m$$

$$\text{let } V_i = \pm 1V; Ri = 1k$$

$$i_m = \frac{1V}{1k} = \pm 1mA$$

Current to Voltage converter

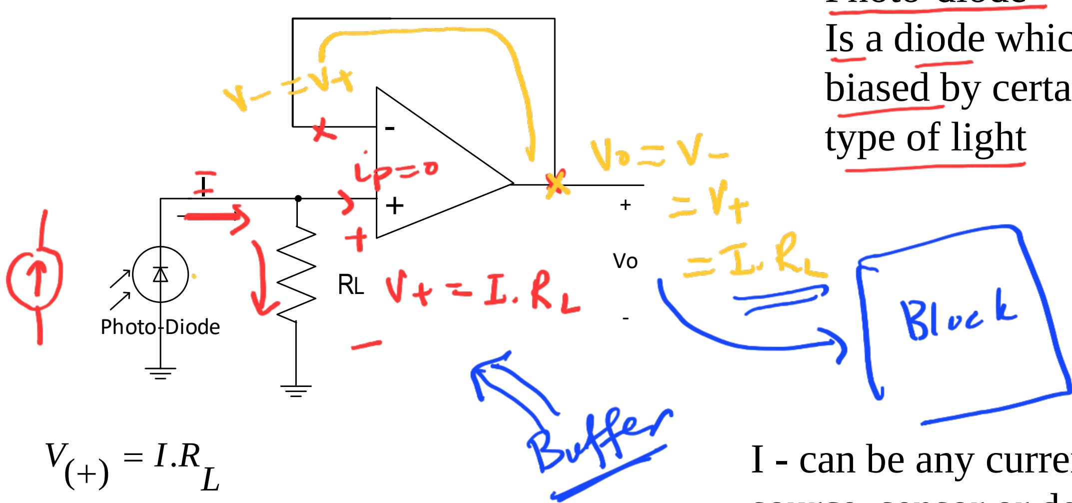


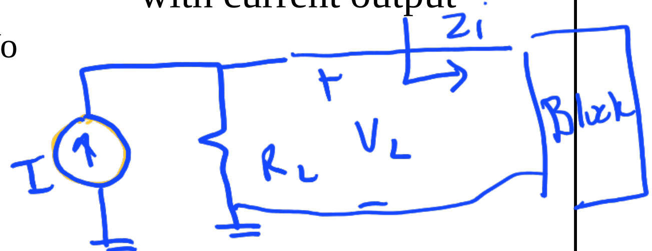
Photo-diode
Is a diode which is biased by certain type of light

$$V_{(+)} = I.R_L$$

$$V_O = V_{(-)} = V_{(+)} = I.R_L$$

Here we converted current I to voltage Vo

I - can be any current source, sensor or device with current output



So far ; circuits with Negative feedback

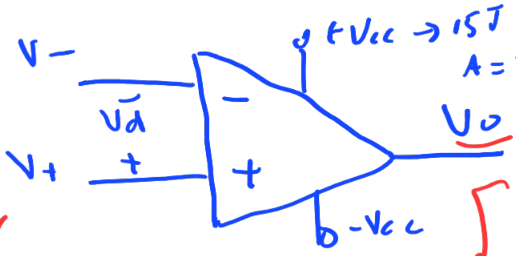
- inverting
- non-inverting
- Difference
- Instrumentation

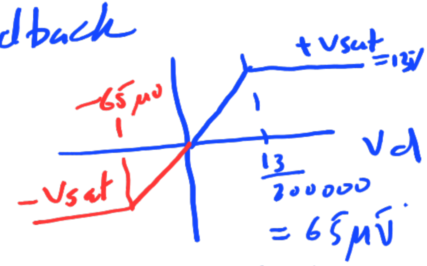
Amplifiers

- Voltage to current
- current to voltage

converters

2) Open loop ⇒ no feedback



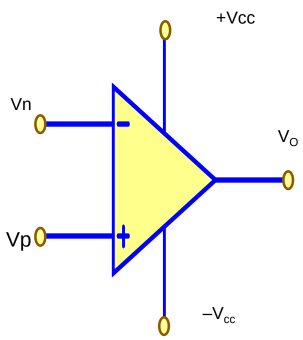


Comparator

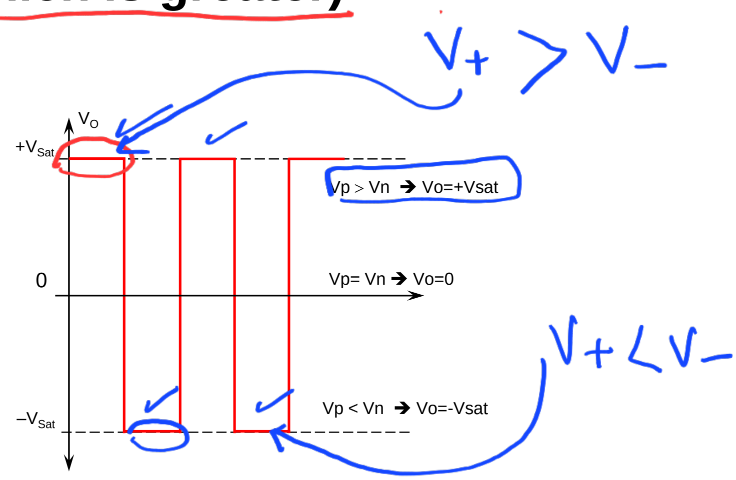
if $V_D = V_+ - V_- > 65\mu V \Rightarrow V_o = +V_{sat}$

if $V_D < -65\mu V \Rightarrow V_o = -V_{sat}$

OP AMP as a Comparator (compares 2 voltages and produces a signal to indicate which is greater)



(a) Comparator Circuit



(b) Comparator Output

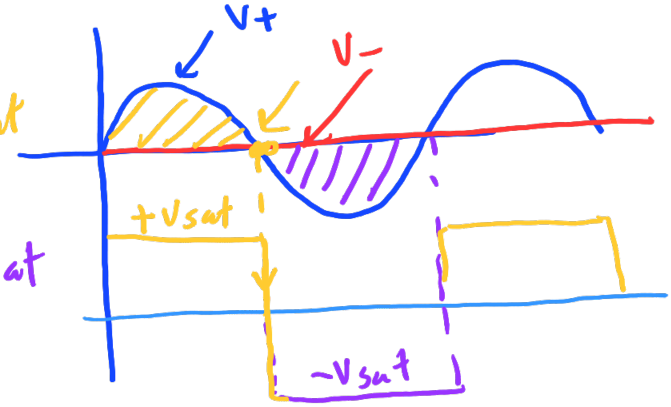
Applications of Comparators

- Analog to digital converters (ADC)
- Counters (e.g. count pulses that exceed a certain voltage level).
- Cross Over Detectors

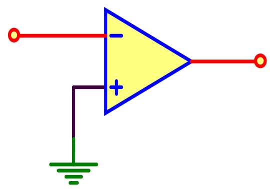
comparator

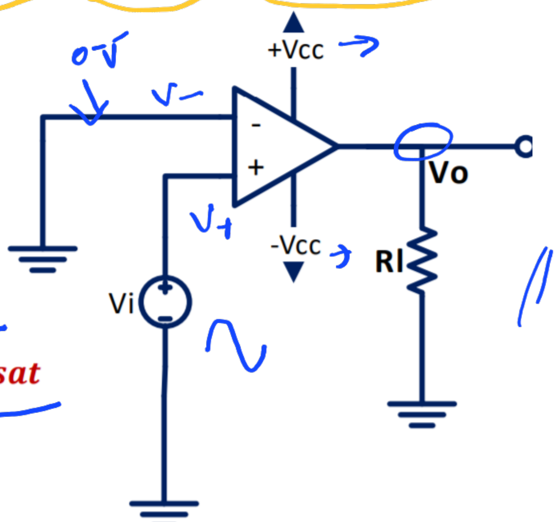
$V_+ > V_- \Rightarrow V_o = +V_{sat}$

$V_+ < V_- \Rightarrow V_o = -V_{sat}$



Comparator : Zero –Level detector





Exact analysis:

When $vd > 65\mu V$; $V_O = +V_{sat}$

When $vd < -65\mu V$; $V_O = -V_{sat}$

0V

Approximate analysis

When $vd > 0V$; $V_O = +V_{sat}$

When $vd < 0V$; $V_O = -V_{sat}$

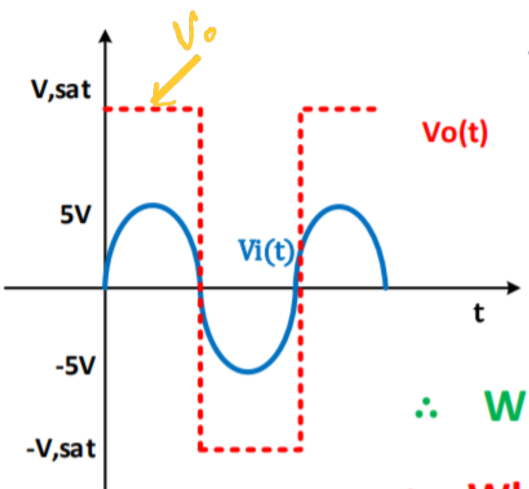
$V_d = V_+ - V_- \Rightarrow V_+ > V$

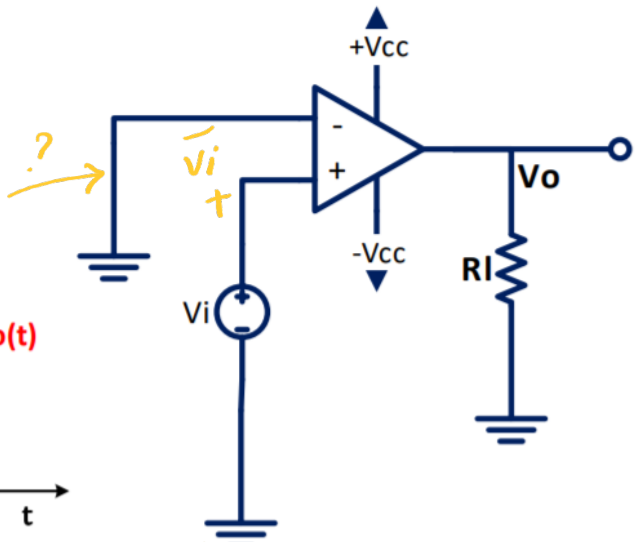
$\Rightarrow V_+ < V_-$

Comparator : Zero –Level detector

$V_i\,(t) = 5\sin\omega t$ v

$\pm\ V_{sat} = \pm 13$ v





Vd

$\therefore$ When $V_i > 0V$; $V_O = +V_{sat}$

$\therefore$ When $V_i < 0V$; $V_O = -V_{sat}$

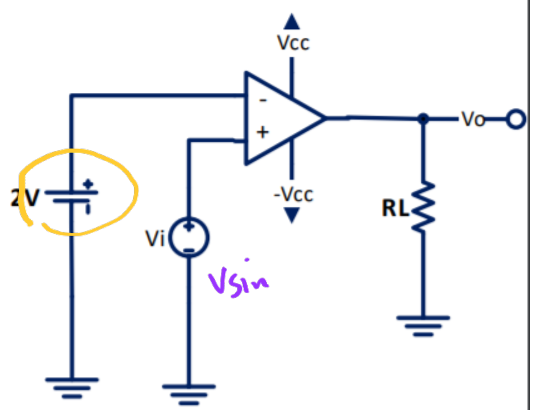
Non Zero –Level detector

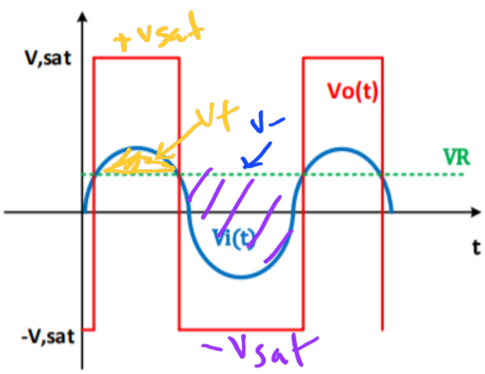
$V_i\,(t) = 5\sin\omega t$ v

$\pm\ V_{sat} = \pm 13$ v

When $V_i > 2V$; $V_O = +V_{sat}$

When $V_i < 2V$; $V_O = -V_{sat}$





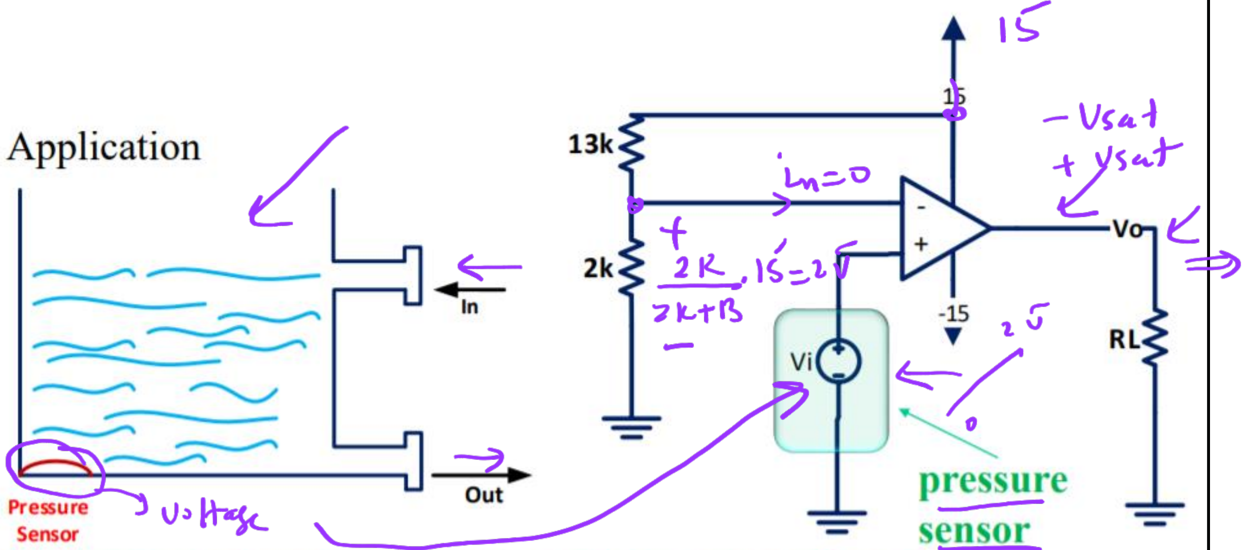
$V_{sin} - 2 > 0 \rightarrow V_{sin} > 2$

When $vd > 0V$; $V_O = +V_{sat}$

When $vd < 0V$; $V_O = -V_{sat}$

$V_{sin} - 2 < 0 \rightarrow V_{sin} < 2$

Practical Non Zero –Level detector



The pressure sensor generates a voltage proportional to the water level in the tank

V(-) = 2V

When water level reaches the maximum allowable lever

➡ Vi = 2V

When V_i >2V ; V_O=+ V_{sat}

When V_i <2V ; V_O= - V_{sat}

Voltage–Level detector with LEDs:

When V_i >2V ; V_O= + V_{sat}

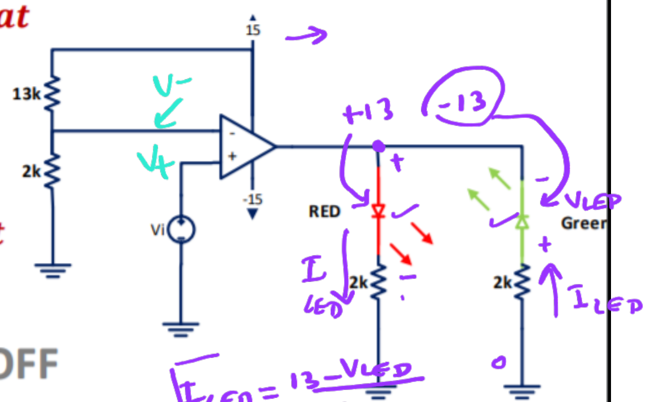
∴ Red LED is ON

∴ green LED is OFF

When V_i <2V ; V_O= - V_{sat}

∴ green LED is ON

∴ Red LED is OFF



$I_{LED} = \frac{13 - V_{LED}}{2k}$

$V_{LED} \simeq 2-3V$

take $V_{LED} = 3V$

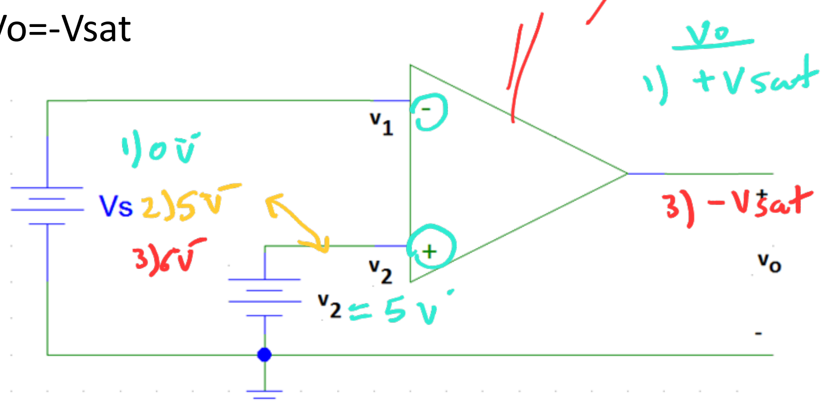
$I_{LED} = \frac{13-3}{2k} = 5mA$

When $V_i = 2V$; $V_O = 0$

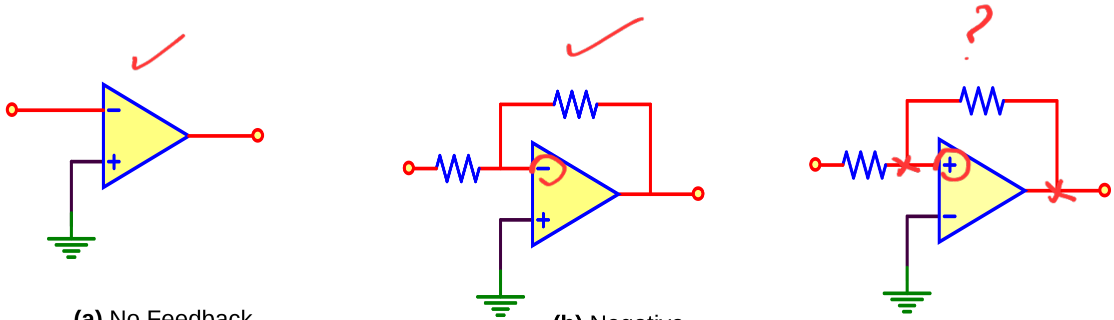
∴ green LED and the Red LED are OFF

Example

- Given how an op amp functions, what do you expect Vo to be if v2 = 5V when:
  1. Vs = 0V?
     Answer Vo=+Vsat
  2. Vs = 5V?
     Answer Vo=0 (practically impossible to have both V1=V2)
  3. Vs = 6V?
     Answer Vo=-Vsat



OP-AMP CONFIGURATIONS



(a) No Feedback (open loop comparator circuit)

(b) Negative Feedback

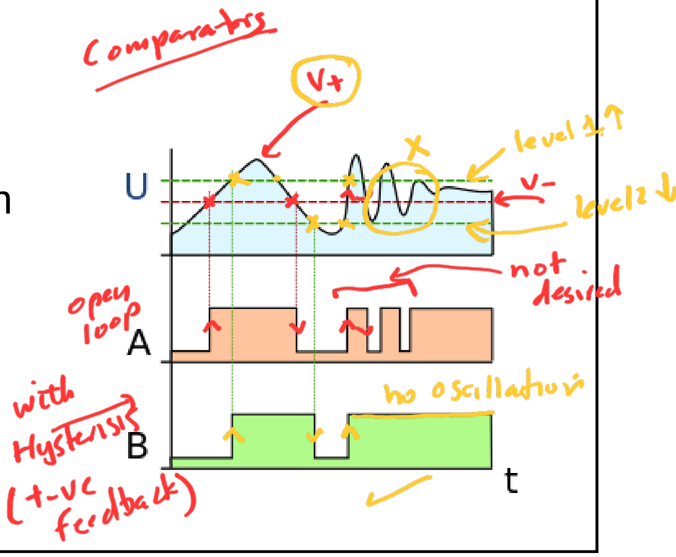
(c) Positive Feedback

- No feedback : Open loop (used in comparators)
- Negative feedback : Feedback to the inverting input (Used in amplifiers)
- Positive feedback : Feedback to the non inverting input (Used in oscillators) and Schmitt triggers (comparators with hysterisis)

a **Schmitt trigger** is a comparator circuit with hysteresis,

Schmitt trigger devices are typically used in signal conditioning applications to remove noise from signals used in digital circuits, particularly mechanical switch bounce.
They are also used in closed loop negative feedback configurations to implement relaxation oscillators, used in function generators and switching power supplies.

The output of a Schmitt trigger (B) and a comparator (A), when a noisy signal (U) is applied.
The green dotted lines are the circuit's switching thresholds.
The Schmitt trigger tends to remove noise from the signal.

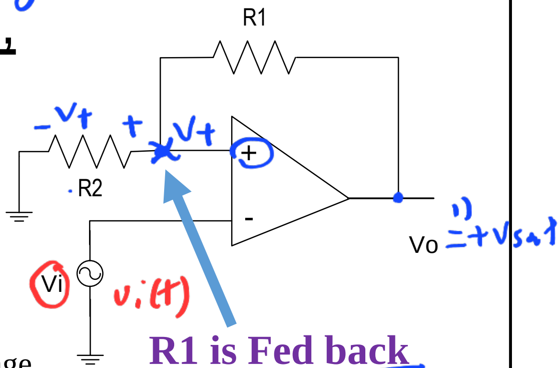


comparator with Hysterisis

Schmitt trigger

This is a comparator circuit and the output is $\boxed{Vo = \pm Vsat}$ **

Analysis: step 1



R1 is Fed back from output to (+) input

This is called positive feedback

let $V_O = +Vsat$

$$V_+ = \frac{R_2}{R_1 + R_2}(+Vsat) = V_{UT} \text{ - upper threshold voltage}$$

in order for Vo to be + Vsat

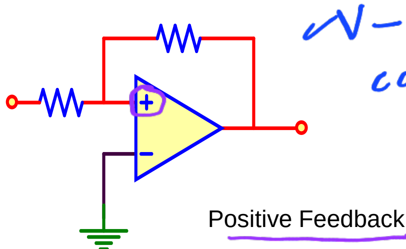
$Vd > 0$

$Vd = V(+) - V(-) > 0$

$$\frac{R_2}{R_1 + R_2} + Vsat > V_i$$

when $V_{UT} > V_i \Rightarrow Vo = +Vsat$

But when $V_i > V_{UT} \Rightarrow$ Vo switches to $-$ Vsat

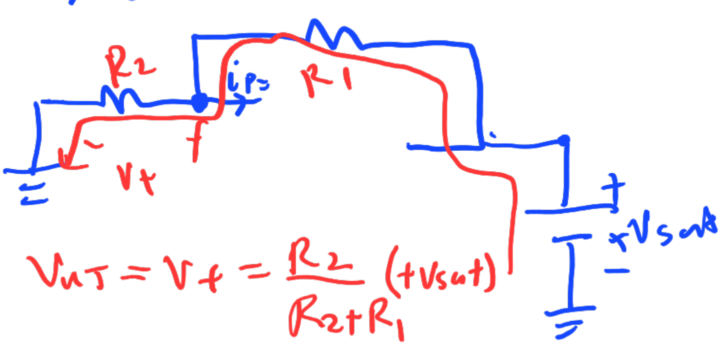


Positive Feedback

$V_- \neq V_+$ cannot be used

$i_p = i_n = 0$

1) let $V_o = +V_{sat}$



$$V_{UT} = V_f = \frac{R_2}{R_2 + R_1}(+V_{sat})$$

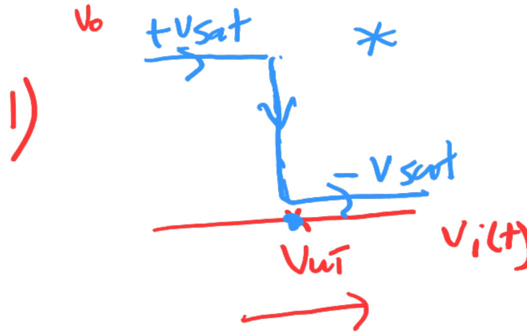
if $V_+ > V_-$

$\boxed{V_{UT} > V_i(t)} \Rightarrow V_o = +V_{sat}$

$\rightarrow$ as long as $V_i(t) < V_{UT}$ $V_o = +V_{sat}$

$\rightarrow$ if $V_i(t)$ increases such that $V_i(t) > V_{UT}$ $V_o = -V_{sat}$

1)

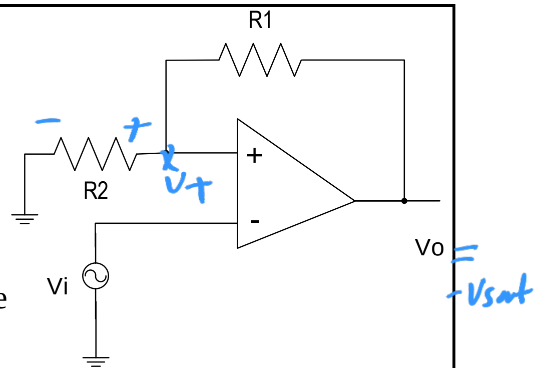


Schmitt trigger,

Analysis: step 2

let $V_O = -Vsat$

$$V_+ = \frac{R_2}{R_1 + R_2}(-Vsat) = \boxed{V_{LT}}$$ - Lower threshold voltage



in order for Vo to be – Vsat

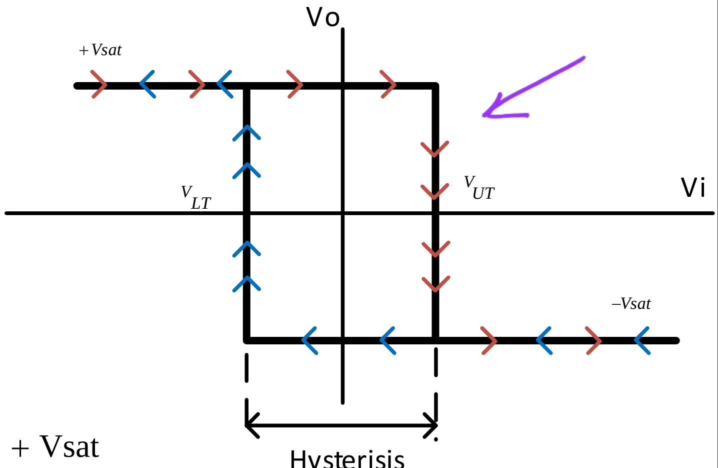
$Vd < 0$

$Vd = V(+) - V(-) < 0$

$$\frac{R_2}{R_1 + R_2} - Vsat < V_i$$

when $V_{LT} < V_i \Rightarrow Vo = -Vsat$

But when $V_i < V_{LT} \Rightarrow$ Vo switches to + Vsat



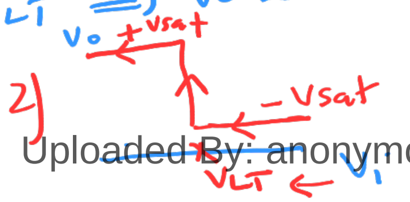
$$|V_{UT} - V_{LT}| = \text{Hysterisis}$$

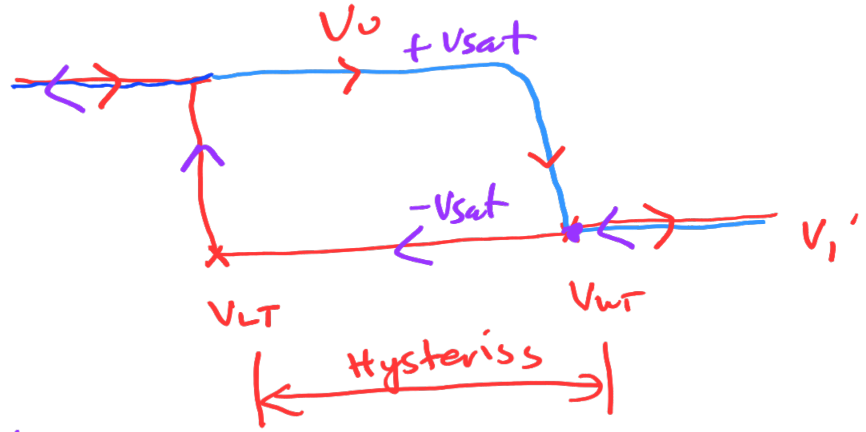
$V_+ - V_- < 0 \Rightarrow V_o = -V_{sat}$

$V_{LT} - V_i(t) < 0 \Rightarrow V_{LT} < v_i(t)$

- as long as $V_i(t) > V_{LT} \Rightarrow V_o = -V_{sat}$
- if $v_i(t)$ decreases below $< V_{LT} \Rightarrow V_o = +V_{sat}$

2)





End of L24

L25 21/8/2021

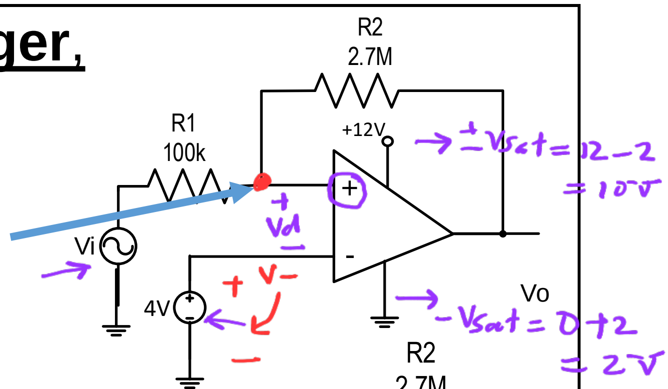
** calculate

Schmitt trigger,

Example: Find and sketch Vo(t) and the plot of Vo=f(Vi)

Solution: this is a Schmitt trigger and

∴ $\boxed{Vo = \pm Vsat}$



1) let $V_o = +Vsat$ = 10V

in order for Vo to be + Vsat

Vd > 0

Vd = V(+) − V(−) > 0

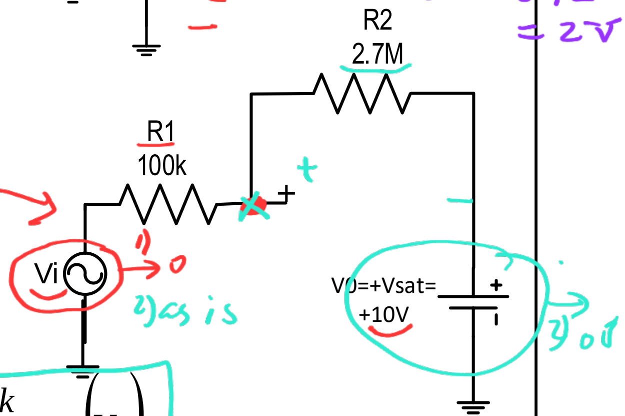
$\boxed{V(+) > V(-)}$

$\boxed{V(-) = 4V}$

superposition

$$V(+) = \frac{100k}{(2700+100)k}(+Vsat) + \frac{2700k}{(2700+100)k}(V_i)$$

$$= \frac{100k}{(2700+100)k}(\overset{10}{+Vsat}) + \frac{2700k}{(2700+100)k}(V_i) > 4V$$



Vi(t) → changing voltage

Find v_i ?

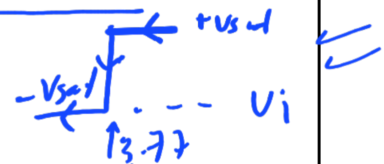
V+ V-

$$\frac{100k}{(2700+100)k}(10V)+\frac{2700k}{(2700+100)k}\left(V_i\right)>4V$$

VUT

$$\left(V_i\right)>\left[4V-\left(\frac{100k}{(2700+100)k}(10V)\right)\right]\left[\frac{(2700+100)k}{2700k}\right] ==> V_i > 3.777\,V$$

when $V_i > 3.777 \Rightarrow$ Vo $= +Vsat$; But when $V_i < 3.777 \Rightarrow$ Vo switches to $-Vsat$



2) let $V_O = -Vsat = (0+2) = 2V$

?? 4V

in order for Vo to be - Vsat ==> Vd < 0 ; ∴ V(+) < V(−)

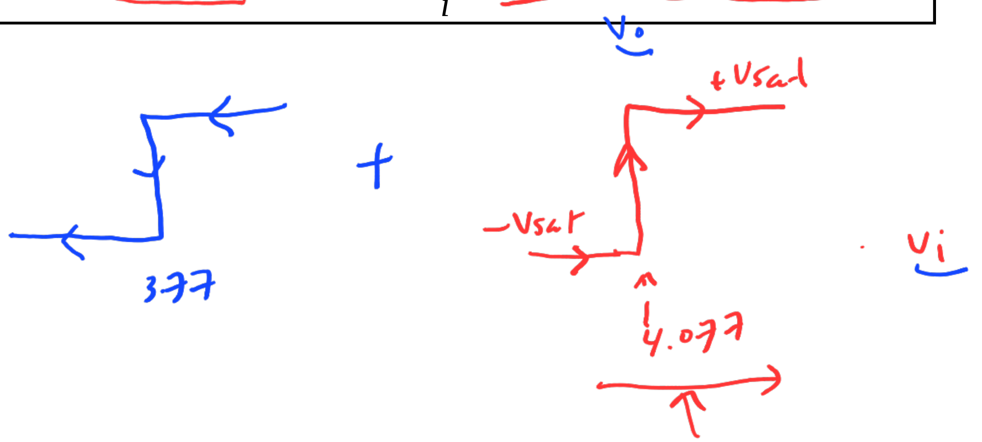
$$V(+)=\frac{100k}{(2700+100)k}\left(-Vsat\right)+\frac{2700k}{(2700+100)k}\left(V_i\right)$$

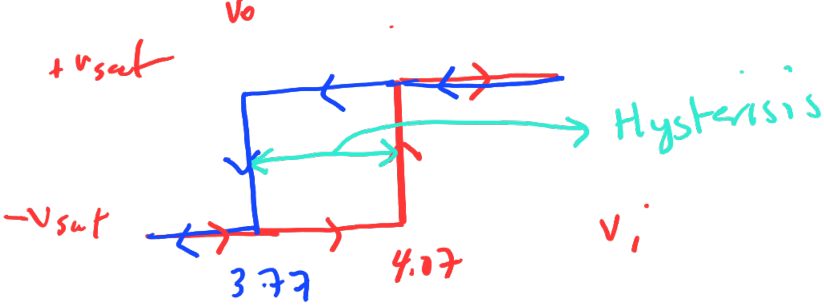
$$\frac{100k}{(2700+100)k}\left(-Vsat\right)+\frac{2700k}{(2700+100)k}\left(V_i\right)<4V$$

VLT

$$\left(V_i\right)<\left[4V-\left(\frac{100k}{(2700+100)k}(2V)\right)\right]\left[\frac{(2700+100)k}{2700k}\right] ==> V_i < 4.074\,V$$

when $V_i < 4.074\,V \Rightarrow$ Vo $= -Vsat$; But when $V_i > 4.074 \Rightarrow$ Vo switches to $+Vsat$





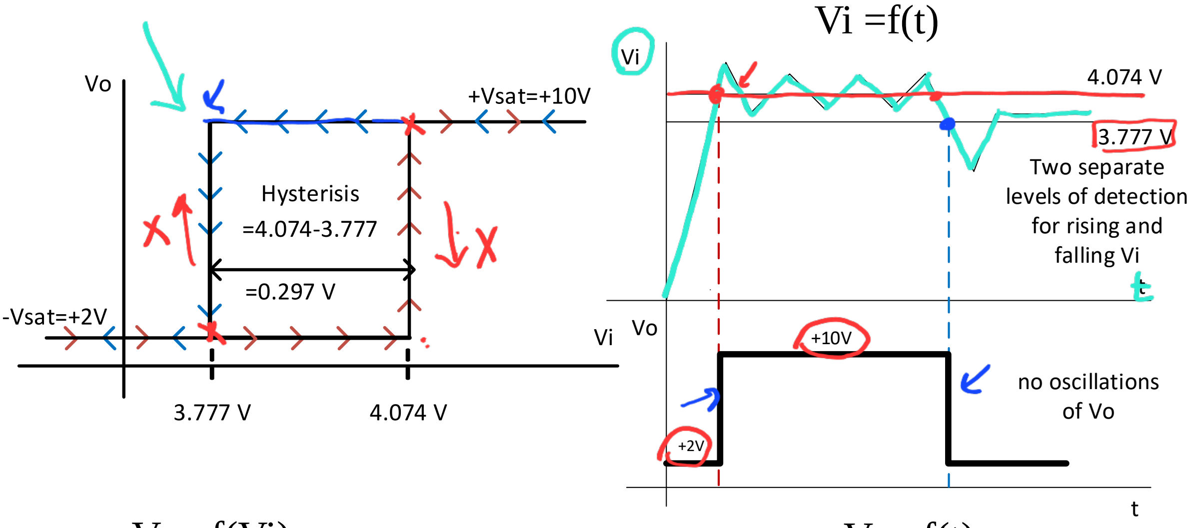
Conclusion

1) when Vi is decreasing, as long as Vi is $> 3.777V \Rightarrow$ Vo $= +Vsat$;

but when V_i becomes $< 3.777 \Rightarrow$ Vo switches to $-Vsat$

2) when Vi is increasing, as long as Vi is $< 3.777V \Rightarrow$ Vo $= -Vsat$;

but when V_i becomes $> 3.777 \Rightarrow$ Vo switches to $+Vsat$



Vo =f(Vi)

Vo =f(t)

open loop

Comparator without Hysterisis

Example: Find and sketch Vo(t) and the plot of Vo=f(Vi)

Solution: this is a comparator and

$$Vo = \pm \text{Vsat}$$

1) in order for Vo to be + Vsat

$$\text{Vd} > 0 \quad \text{and} \quad \text{Vd} = \text{V}(+) - \text{V}(-) > 0$$

$$\text{V}(+) > \text{V}(-)$$

$$\text{V}(-) = 4V ; V(+) = V_i$$

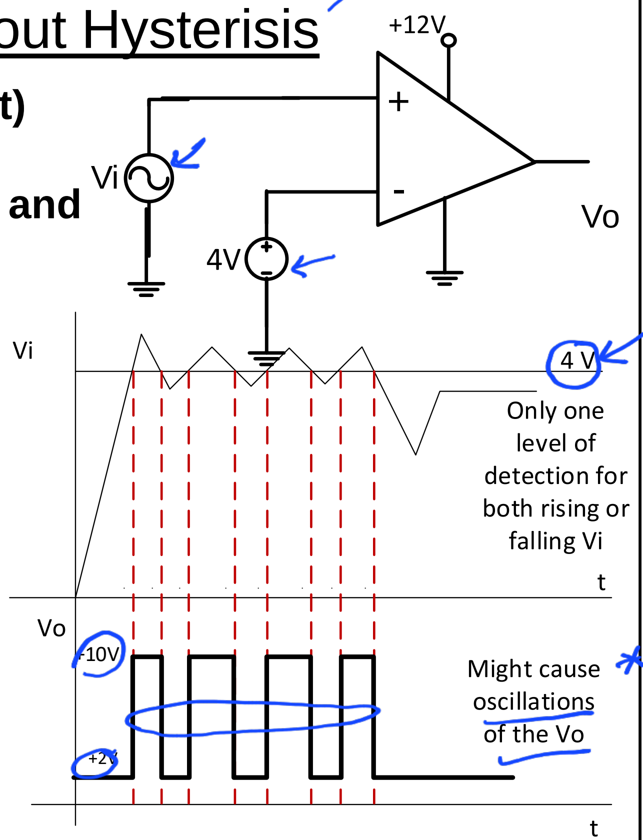
$$V_i > 4V$$

2) in order for Vo to be – Vsat = 2V

$$\text{Vd} < 0 \quad \text{and} \quad \text{Vd} = \text{V}(+) - \text{V}(-) < 0$$

$$\text{V}(+) < \text{V}(-)$$

$$V_i < 4V$$



→ $i_c(t) = C\frac{dV_c(t)}{dt}$

Integrator

$V_o(t) \propto \int V_i(t)$ → time domain

- So far, the input and feedback components have been resistors. If the feedback component used is a capacitor,, the resulting connection is called an *integrator.*
- Recall that virtual ground means that we can consider the voltage at the junction of R and X_C to be ground (since $V_+ = 0$ V) but that no current goes into ground at that point.

$$i_i = i_f = \frac{V_i(t)}{R} = i_c = C\frac{dV_c(t)}{dt}$$

$$Vc(t) = \frac{1}{C}\int_0^t i_f(t)dt$$

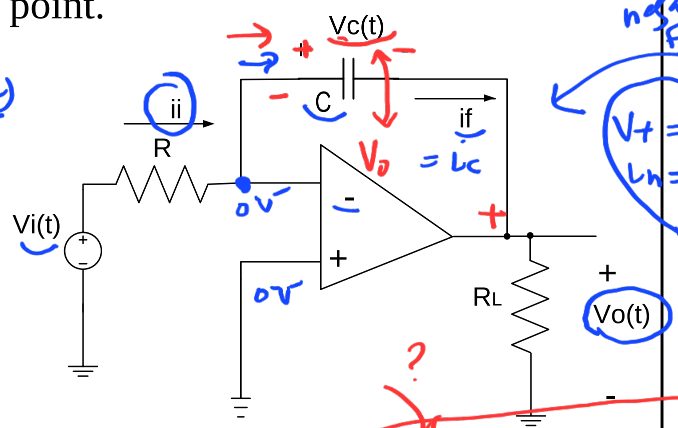
$$V_o = -Vc(t)$$

const

$$Vo = -\frac{1}{C}\int_0^t \frac{V_i(t)}{R}dt = -\frac{1}{RC}\int_0^t V_i(t)dt$$

negative feedback

$V_+ = V_-$
$i_n = i_f = 0$



?

$$i_c(t) = \boxed{C\frac{dV_c(t)}{dt} = \frac{V_i(t)}{R}}$$

Differentiator → time domain

A differentiator ,while not as useful as the circuit forms covered above, the differentiator does provide a useful operation, the resulting relation for the circuit being

$$i_i = i_f = i_C = C\frac{dV_i(t)}{dt}$$

$$Vc(t) = \frac{1}{C}\int_0^t i_f(t)dt$$

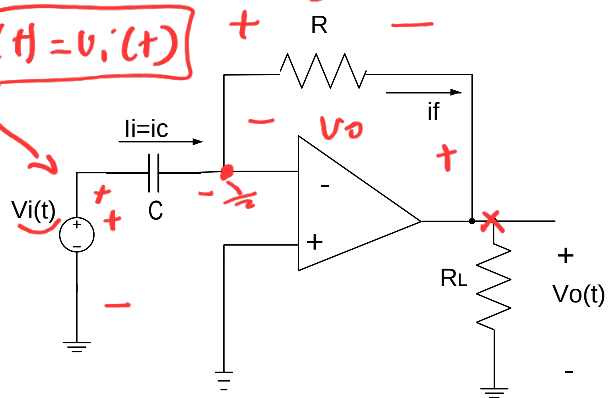
$$\boxed{V_o = -i_f(t)R}$$

const

$$Vo = -\boxed{\left(C\frac{dV_i(t)}{dt}\right)}(R) = -RC\frac{dV_i(t)}{dt}$$

$\boxed{V_c(t) = v_i(t)}$

$i_f \cdot R$



Filters → order
- First
- second
- ⋮ nth order

advantages
1) removes loading effect
2) w_c well defined

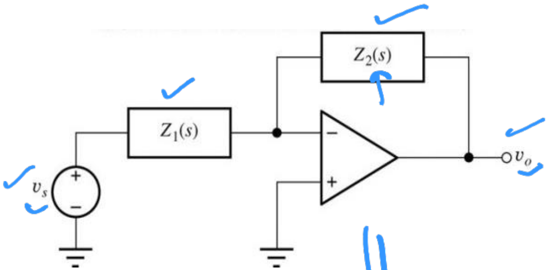
LPF, HPF, BPF, BRF

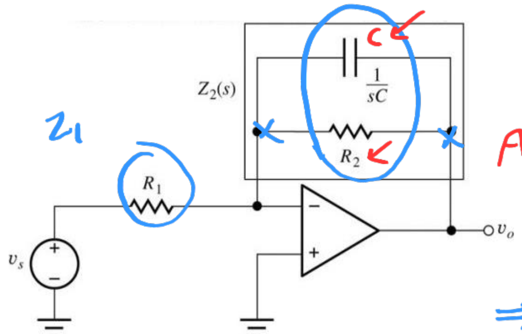
Passive type: R, L, C

Active type: R, C, Op-amp

The Active Low-pass Filter

The gain analysis of this inverting amplifier. Note s = jω.





$$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}_s(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)} \qquad \boxed{Z_1(j\omega) = R_1}$$

$$\boxed{Z_2(j\omega)} = \frac{R_2 \dfrac{1}{j\omega C}}{R_2 + \dfrac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1}$$

$$\boxed{A_v = -\frac{R_2}{R_1}\frac{1}{(1 + j\omega C R_2)}}$$

$CR_2 \to \dfrac{1}{w_c}$

$$A_v = \frac{-K}{\left(1 + \dfrac{j\omega}{\omega_c}\right)}$$

$$\Rightarrow K = \frac{R_2}{R_1} \leftarrow \text{gain (dc gain)}$$

& cut-off frequency: $\omega_c = 2\pi f_c = \dfrac{1}{R_2 C} \quad \therefore f_c = \dfrac{1}{2\pi R_2 C}$

w → rad/sec f (hertz)

w → f ? $w = 2\pi f$
$w_c = 2\pi f_c$

take the Magnitude → $A_v = \dfrac{-K}{1 + \dfrac{jw}{w_c}}$ ← complex number → phasor representation

$\boxed{|\text{mag}| \angle \text{phase}}$ or exponential

Assume $K = 1$

$$|A_v| = \frac{|-1|}{\left|1 + \frac{j\omega}{\omega_c}\right|} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

phase

$\angle$

$\omega_c = \frac{1}{R_2 C} \leftarrow$ known

$|A_v(j\omega)| \rightarrow$ drawing

Frequency Response ($\boxed{\text{Bodé Plot}}$)

$\rightarrow$ log scale ω

$\rightarrow$ Decibel scale

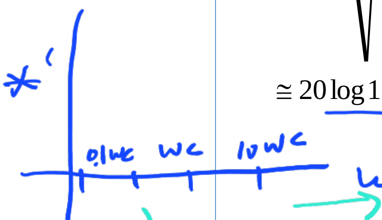
Magnitude Plot (Magnitude in decibels vs log of frequency)

$$A_{dB} = 20\log|H(j\omega)| = \frac{K}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$$

$$|H(j\omega)| = |A_v(j\omega)|$$

dB

$$|A_v(j\omega)| = \frac{K}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$$



at $\boxed{\omega = 0.1\omega_c}$

$$20\log\frac{1}{\sqrt{1+\left(\frac{0.1\omega_c}{\omega_c}\right)^2}} = 20\log\frac{1}{\sqrt{1+0.01}}$$

≈ 1

$$\cong 20\log 1 = 0dB$$

at $\omega = \omega_c$

$$20\log\frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega_c}\right)^2}} = 20\log\frac{1}{\sqrt{1+1}} = \left|\frac{V_o}{V_{in}}\right|$$

$$= 20\log|0.707| = -3dB$$

at $\omega = 10\omega_c$

$$20\log\frac{1}{\sqrt{1+\left(\frac{10\omega_c}{\omega_c}\right)^2}} = 20\log\frac{1}{\sqrt{1+100}}$$

20 log 0.1

$$= 20\log|0.1| = -20dB$$

cut-off frequency $\equiv$ corner frequency $\equiv$ -3dB frequency

@ $\omega = 100\omega_c$

$$= \frac{1}{\sqrt{1+(100)^2}}$$

$= 20\log 0.01$

$= -40dB$

ω	$\lvert A_v \rvert$
$0.1\omega_c$	0 dB
ω_c	-3 dB
$10\omega_c$	-20 dB $\leftarrow$ 0.1
$100\omega_c$	-40 dB $\leftarrow$ 0.01

$= \frac{V_o}{V_i} \Rightarrow V_o = V_i \times A_v$

$V_i = 5\sin 1000t$

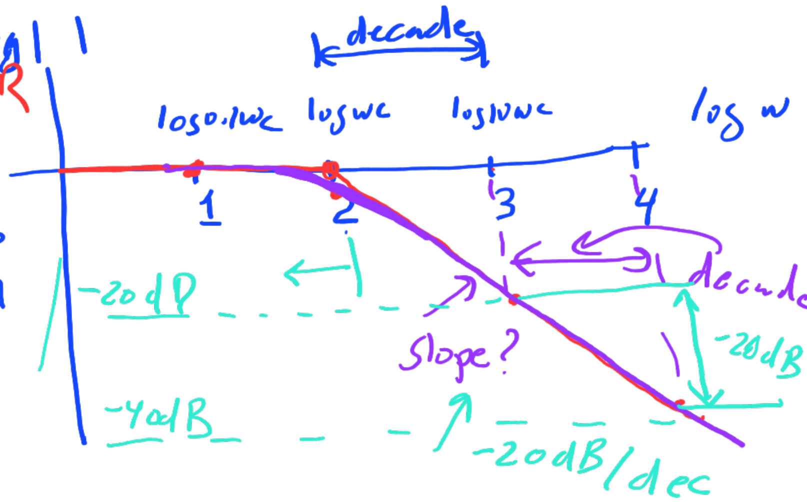
$V_o = ?$

$= 0.5\sin$

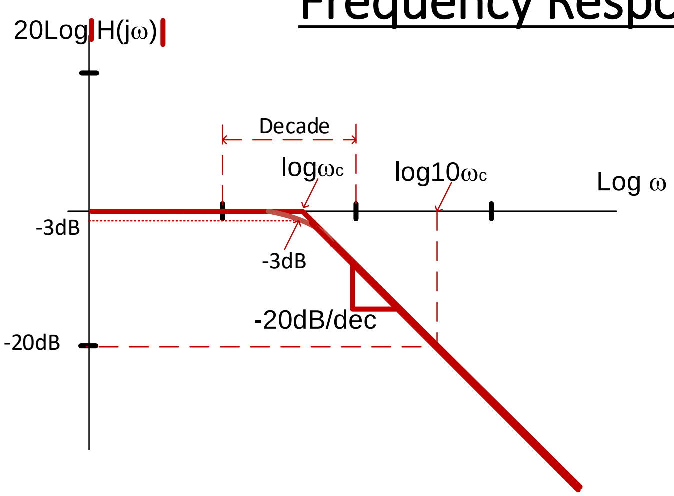
assume $\boxed{\omega_c = 100}$ → $\log 100 = 2$

$10\omega_c = 1000$ → $\log 1000 = 3$

$0.1\omega_c = 10$ → $\log 10 = 1$



Frequency Response (Bode Plot)



Summary

at $\omega = 0.1\omega_c$

$= 20\log 1 = 0dB$

at $\omega = \omega_c$

$= 20\log 0.707 = -3dB$

at $\omega = 10\omega_c$

$= 20\log 0.1 = -20dB$

at $\omega = 100\omega_c$

$= 20\log 0.01 = -40dB$

- At frequencies below ω_c, the amplifier is an inverting amplifier with gain set by the ratio of resistors R_2 and R_1.
- At frequencies above ω_c, the amplifier response "rolls off" at -20dB/decade.
- Notice that cutoff frequency and gain can be independently set.

@ $\omega = 1000$ $A_{v_{dB}} = \left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}} = 0.1$

$\frac{v_o}{5} = 0.1$

$v_o = 5 \times 0.1 = 0.5$

End of L25

Active Low-pass Filter: Example

$20\log|A_v| = 40\ dB$

$\omega_c = \frac{1}{?R_2 C} = 2\pi f_c$

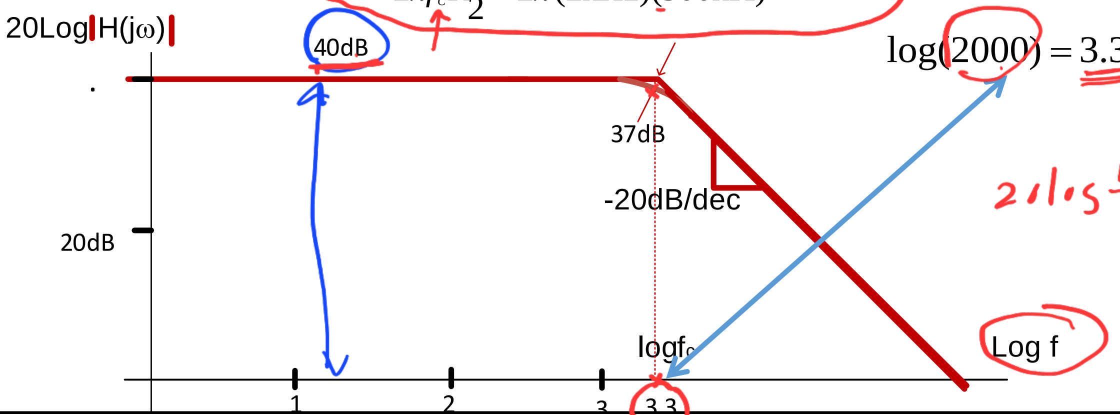
- **Problem**: Design an active low-pass filter
- **Given Data**: A_v= 40 dB, R_{in}= 5 kΩ, f_c = 2 kHz
- **Assumptions:** Ideal op amp, specified gain represents the desired low-frequency gain.
- **Analysis**: $|A_v| = 10^{40\text{dB}/20\text{dB}} = 100$

  Input resistance is controlled by R_1 and voltage gain is set by R_2 / R_1.

  The cutoff frequency is then set by C.

$$R_1 = R_{in} = 5\text{k}\Omega \quad \text{and} \quad |A_v| = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 500\text{k}\Omega$$

$$C = \frac{1}{2\pi f_c R_2} = \frac{1}{2\pi(2\text{kHz})(500\text{k}\Omega)} = 159\text{pF}$$



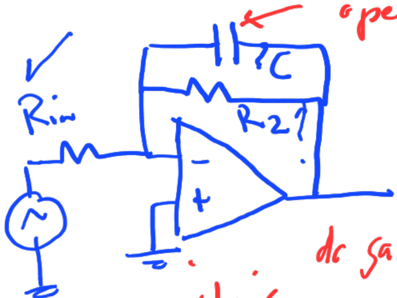
$\log(2000) = 3.3$

$20\log\frac{10}{\sqrt{2}} = 37\,dB$

or

max -3dB

L26 23/8/2021



Inverting

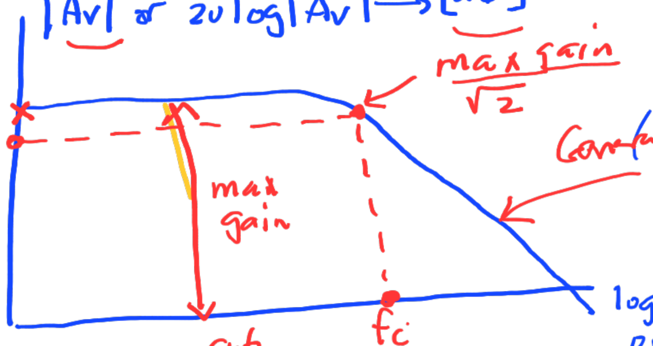
$\left(-\frac{R_2}{R_{in}}\right)$ dc gain

dc gain $K = -\frac{R_2}{R_1}$

$20\log|A_v| = 40\ dB$

LP Filter

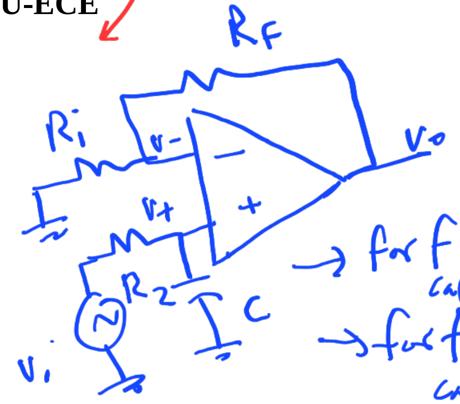
$|A_v|$ or $20\log|A_v| \rightarrow$ [dB]



$\log|A_v| = \frac{40}{20} = 2 \Rightarrow |A_v| = 10^2 = 100$

$100 = \frac{R_2}{R_{in}}$

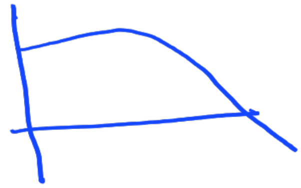
$R_2 = 100 R_{in} = 500k\Omega$



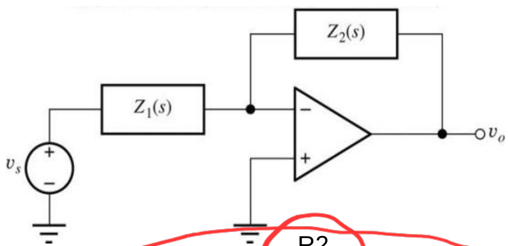
→ for $f = 0 \Rightarrow$ cap-open $\boxed{V_o = \left(1 + \frac{R_F}{R_i}\right) \cdot V_i}$

→ for $f \approx \infty \Rightarrow V_o = 0$ cap short

K =



The Active High-pass Filter



The gain analysis of this inverting amplifier.

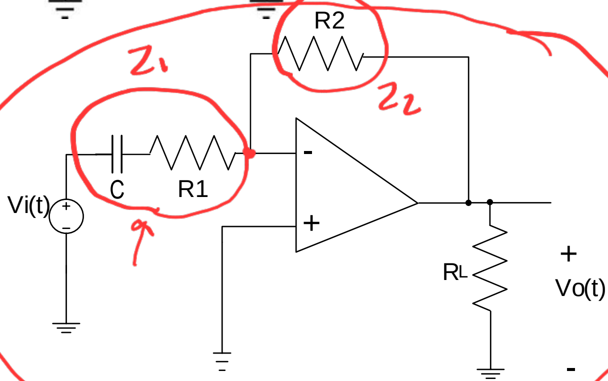
$$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}_s(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)}$$

$$Z_2(j\omega) = R_2$$

$$Z_1(j\omega) = R_1 + \frac{1}{j\omega C}$$

$$A_v = -\frac{R_2}{\left(R_1 + \frac{1}{j\omega C}\right)}$$

$$A_v = -\frac{R_2 / R_1}{\left(1 + \frac{1}{j\omega R_1 C}\right)}$$



$$A_v = \frac{-K}{1 + \frac{\omega_c}{j\omega}} \quad **$$

$$|A_v| = \frac{K}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$K = \frac{R_2}{R_1} \quad \text{high frequency gain}$$

$$\omega_c = 2\pi f_c = \frac{1}{R_1 C} \therefore f_c = \frac{1}{2\pi R_1 C}$$

1st order LPF

$$A_v = \frac{K}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

1-st order HPF

$$|A_v| = \frac{K}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$$

assume K = 1

for $\omega = \omega_c \Rightarrow 20\log|A_v| = 20\log\frac{1}{\sqrt{1+1}} = -3\,dB$

for $\omega = 0.1\omega_c \Rightarrow 20\log\frac{1}{\sqrt{1+\left(\frac{\omega_c}{0.1\omega_c}\right)^2}} = 20\log\frac{1}{\sqrt{101}} \cong 20\log 0.1 = -20\,dB$

for $\omega = 0.01\omega_c \Rightarrow 20\log\frac{1}{\sqrt{10001}} = 20\log 0.01 = -40\,dB$

for $\omega = 10\omega_c \Rightarrow 20\log\frac{1}{\sqrt{1.001}} \cong 20\log 1 = 0\,dB$

Frequency Response (Bode Plot)

Magnitude Plot (Magnitude in decibels vs log of frequency)

$$A_{dB} = 20\log|H(j\omega(|$$

$$|H(j\omega(| = |A_V(j\omega j|$$

$$= \frac{K}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$$

at $\omega = 0.1\omega_c$

$$20\log\frac{1}{\sqrt{1+\left(\frac{\omega_c}{0.1\omega_c}\right)^2}} = 20\log\frac{1}{\sqrt{1+100}}$$

$$\cong 20\log \cancel{0.01}\ 0.1 = -20dB$$

at $\omega = \omega_c$

$$20\log\left|\frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega_c}\right)^2}}\right| = 20\log\left|\frac{1}{\sqrt{1+1}}\right|$$

$$= 20\log|0.707| = -3dB$$

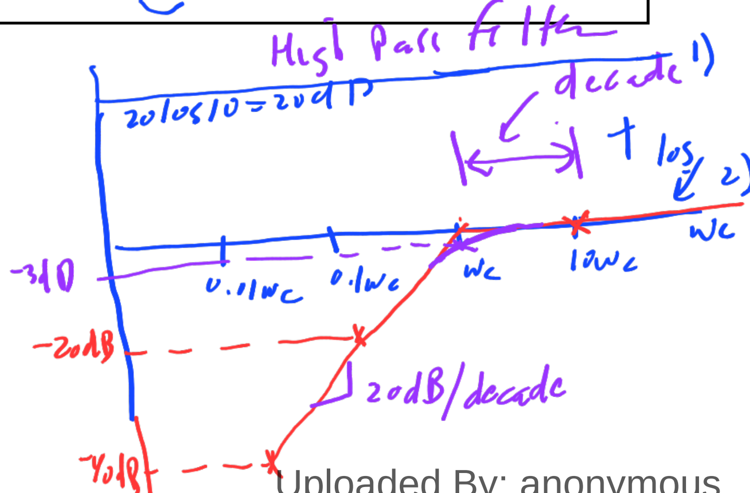
at $\omega = 0.01\omega_c$

$$20\log\left|\frac{1}{\sqrt{1+\left(\frac{\omega_c}{0.01\omega_c}\right)^2}}\right| = 20\log\left|\frac{1}{\sqrt{1+10000}}\right|$$

$$= 20\log 0.01 = -40dB$$

ω	$20\log|A_v|$ [dB] ($K=1$)	$K=10$
$10\omega_c$	0 dB	20 dB
ω_c	-3 dB	17 dB
$0.1\omega_c$	-20 dB	0 dB
$0.01\omega_c$	-40 dB	-20 dB

+20dB



20log10 − 20log100 ← for K = 10

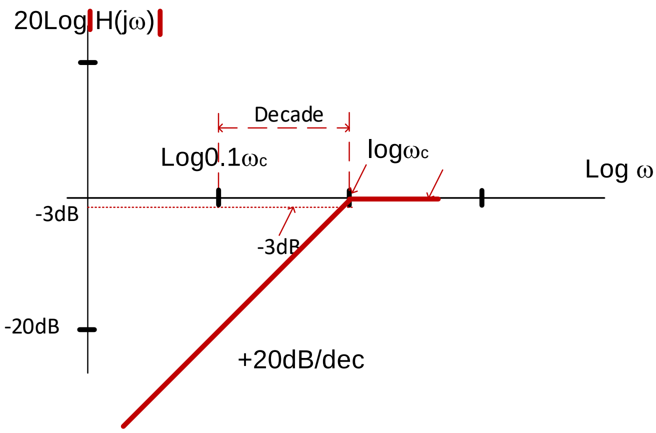
$20\log\frac{10}{100}$ = 20dB − 40dB
= −20dB

for w = 0.1wc ⟹ $20\log\frac{10}{\sqrt{101}}$ ≈ 20log1
= 0 dB

w = 0.01wc ⟹ $20\log\frac{10}{100}$
= −20dB

w = wc ⟹ $20\log\frac{10}{\sqrt{2}}$
= 17 dB

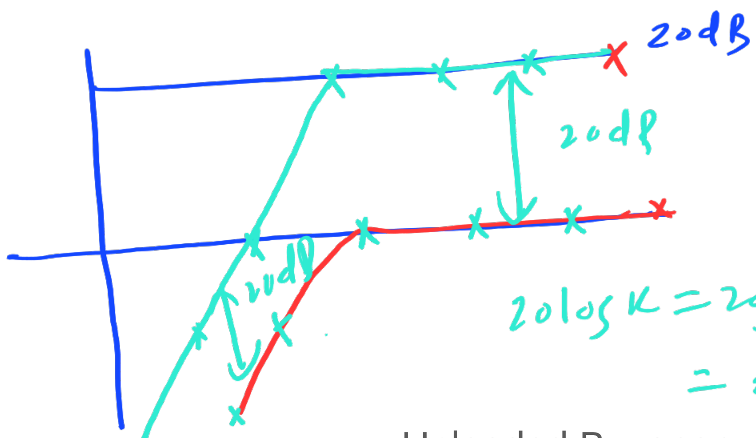
Frequency Response (Bode Plot)



Summary

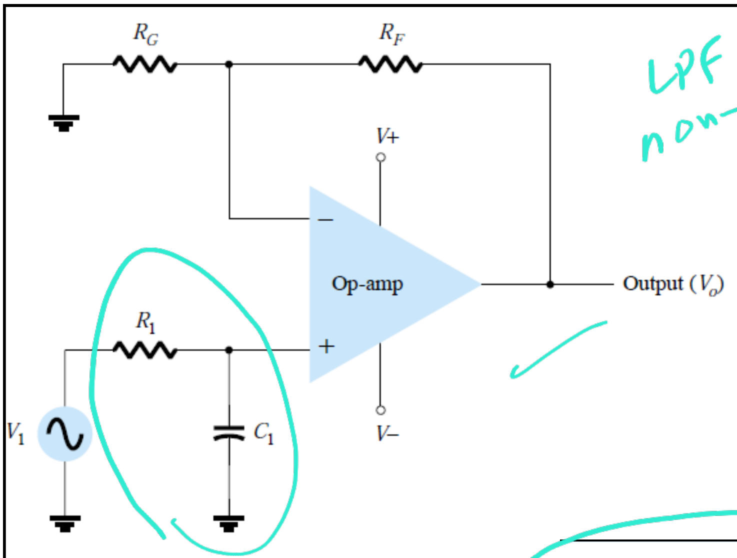
$$\text{at } \omega = 0.1\omega_c$$
$$= 20\log 0.1 = -20dB$$
$$\text{at } \omega = \omega_c$$
$$= 20\log 0.707 = -3dB$$
$$\text{at } \omega = 0.01\omega_c$$
$$= 20\log 0.01 = -40dB$$
$$\text{at } \omega = 0$$
$$= 20\log 1 = 0dB$$

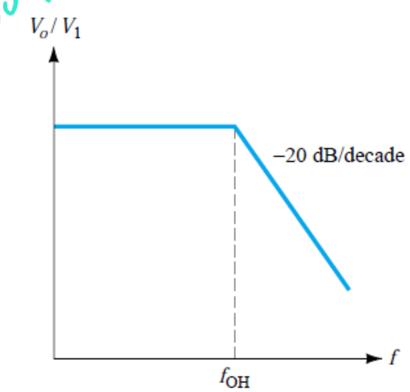
- At frequencies above ω_c, the amplifier is an inverting amplifier with gain set by the ratio of resistors R_2 and R_1.
- At frequencies below ω_c, the amplifier response "rolls off" at -20dB/decade.
- Notice that cutoff frequency and gain can be independently set.



20logK = 20log10
= 20dB

LPF non-inverting



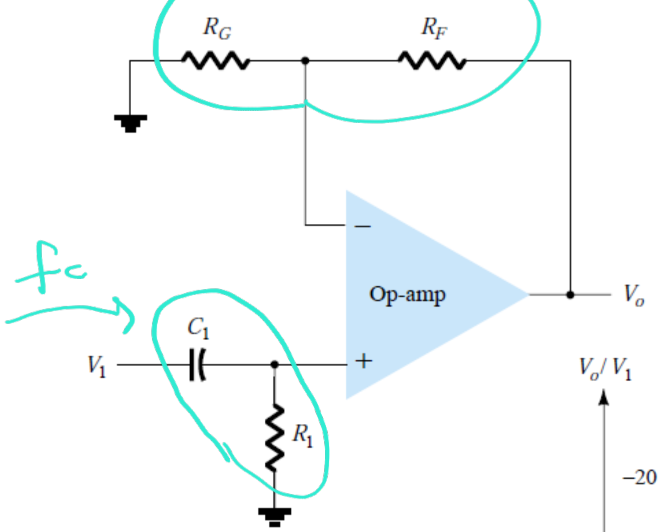


$$A_V = 1 + \frac{R_F}{R_G} = K$$

at a cutoff frequency of

$$f_c = \boxed{f_{OH} = \frac{1}{2\pi R_1 C_1}}$$

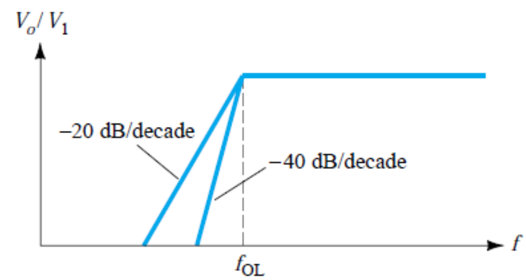
High-Pass Active Filter



High pass

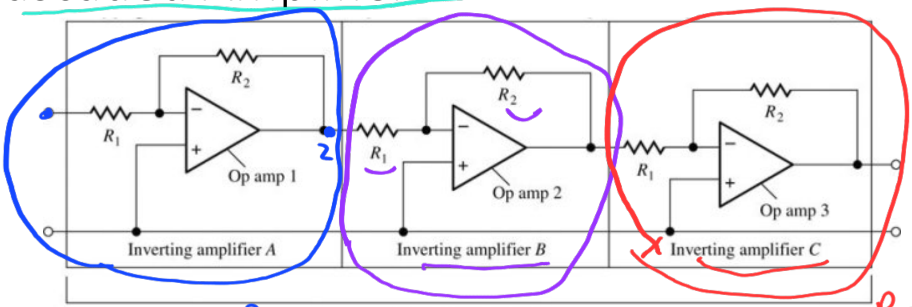
$$f_c = f_{OL} = \frac{1}{2\pi R_1 C_1}$$

$$K = 1 + \frac{R_F}{R_G}$$



$A_v = A_{v1} \cdot A_{v2} \cdot A_{v3}$

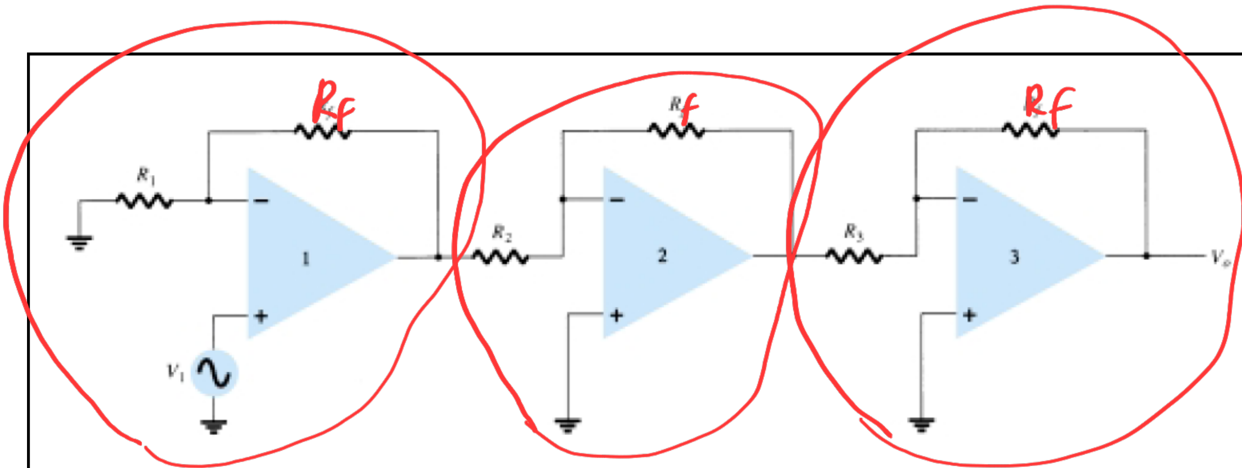
Cascaded Amplifiers



Three-stage amplifier

$A_{v1} = -\frac{R_2}{R_1}$ $A_{v2} = -\frac{R_2}{R_1}$ $A_{v3} = -\frac{R_2}{R_1}$

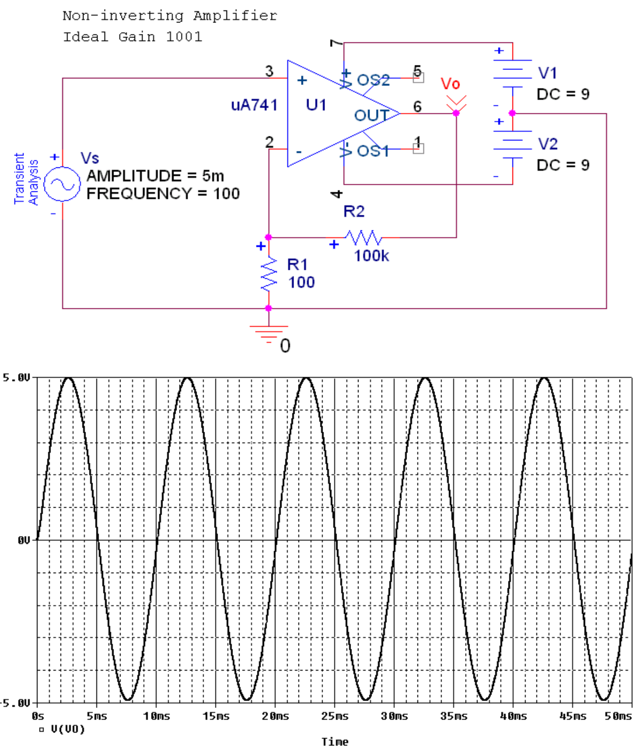
- Connecting several amplifiers in cascade (output of one stage connected to the input of the next) can meet design specifications not met by a single amplifier.
- Each amplifer stage is built using an op amp with parameters A, R_{id}, R_o, called open loop parameters, that describe the op amp with no external elements.
- A_v, R_{in}, R_{out} are closed loop parameters that can be used to describe each closed-loop op amp stage with its feedback network, as well as the overall composite (cascaded) amplifier.
- The gain of each stage can be calculated separately, then the total gain is found by multiplying the resulting gains

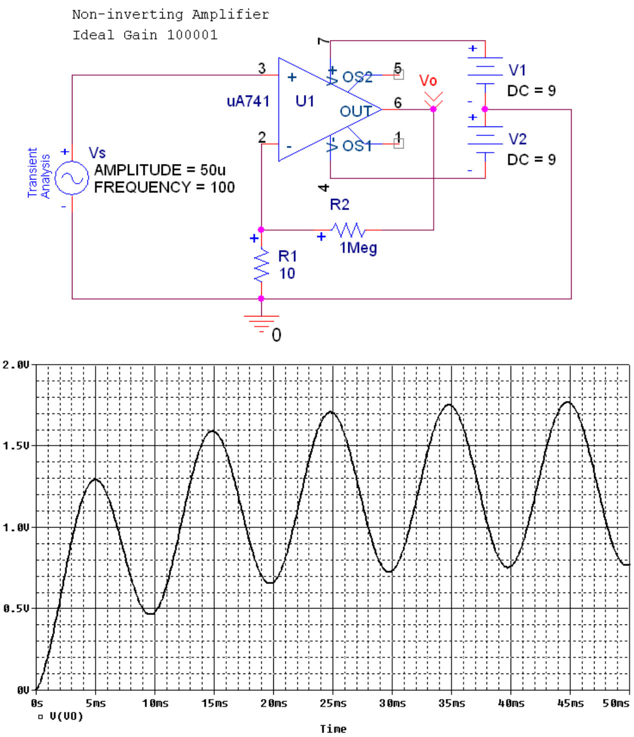


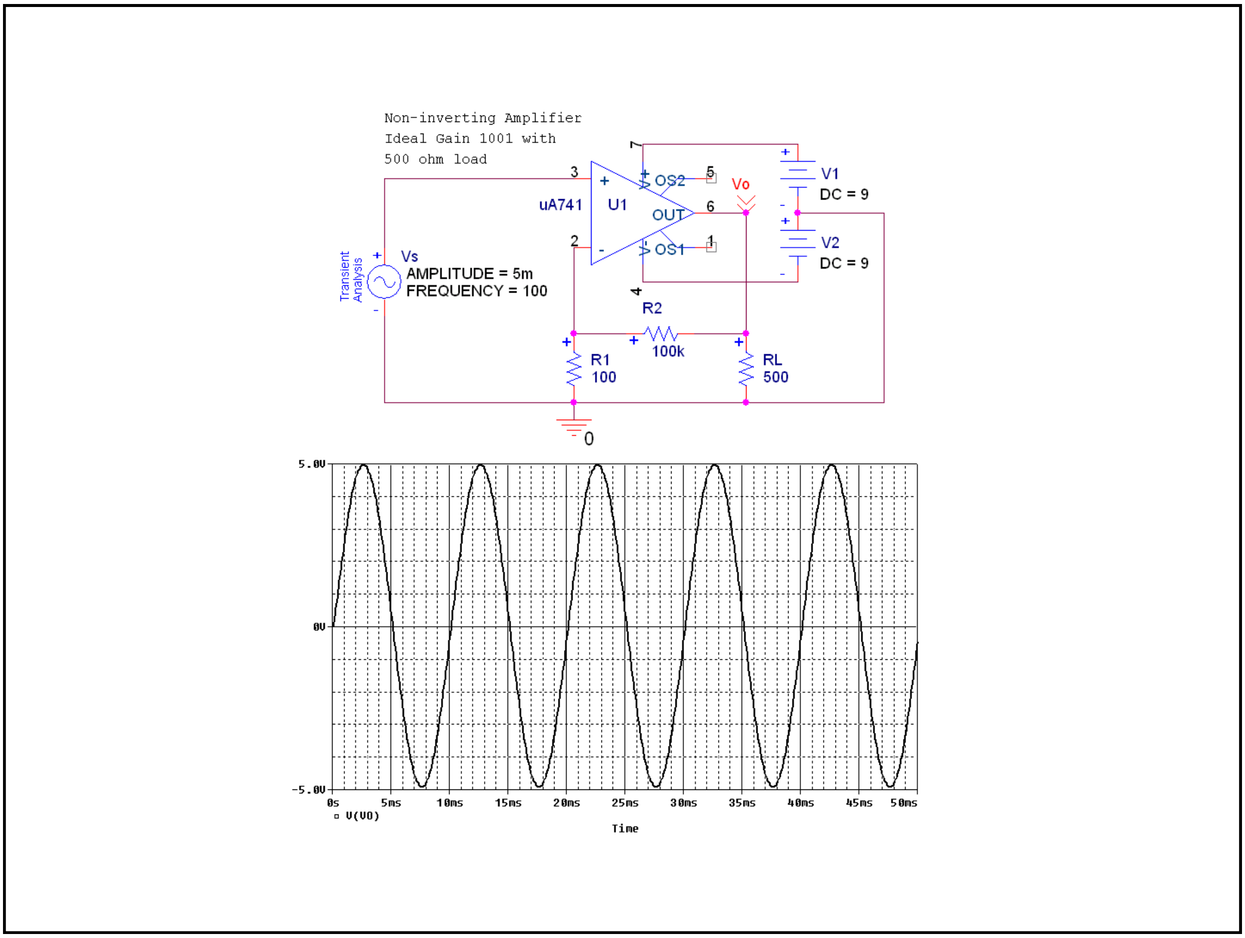
$$\boxed{A = A_1 A_2 A_3}$$

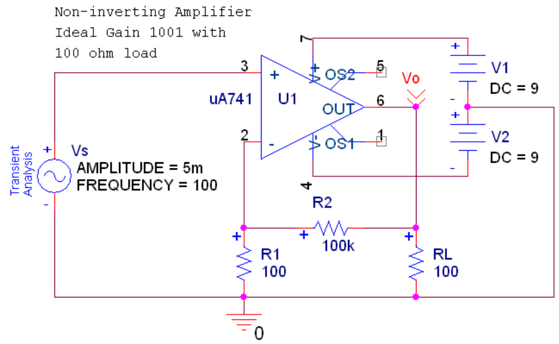
where $\boxed{A_1 = 1 + R_f/R_1}$, $A_2 = -R_f/R_2$, and $A_3 = -R_f/R_3$.

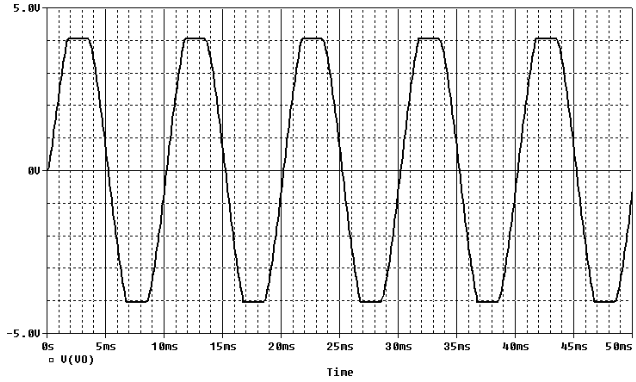
Example PSpice Simulations of Non-inverting Amplifier Circuits


Non-inverting Amplifier
Ideal Gain 1001
uA741
U1
OS2
OUT
OS1
Vo
V1
DC = 9
V2
DC = 9
Vs
AMPLITUDE = 5m
FREQUENCY = 100
Transient Analysis
R2
100k
R1
100
0
5.0V
0V
-5.0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time

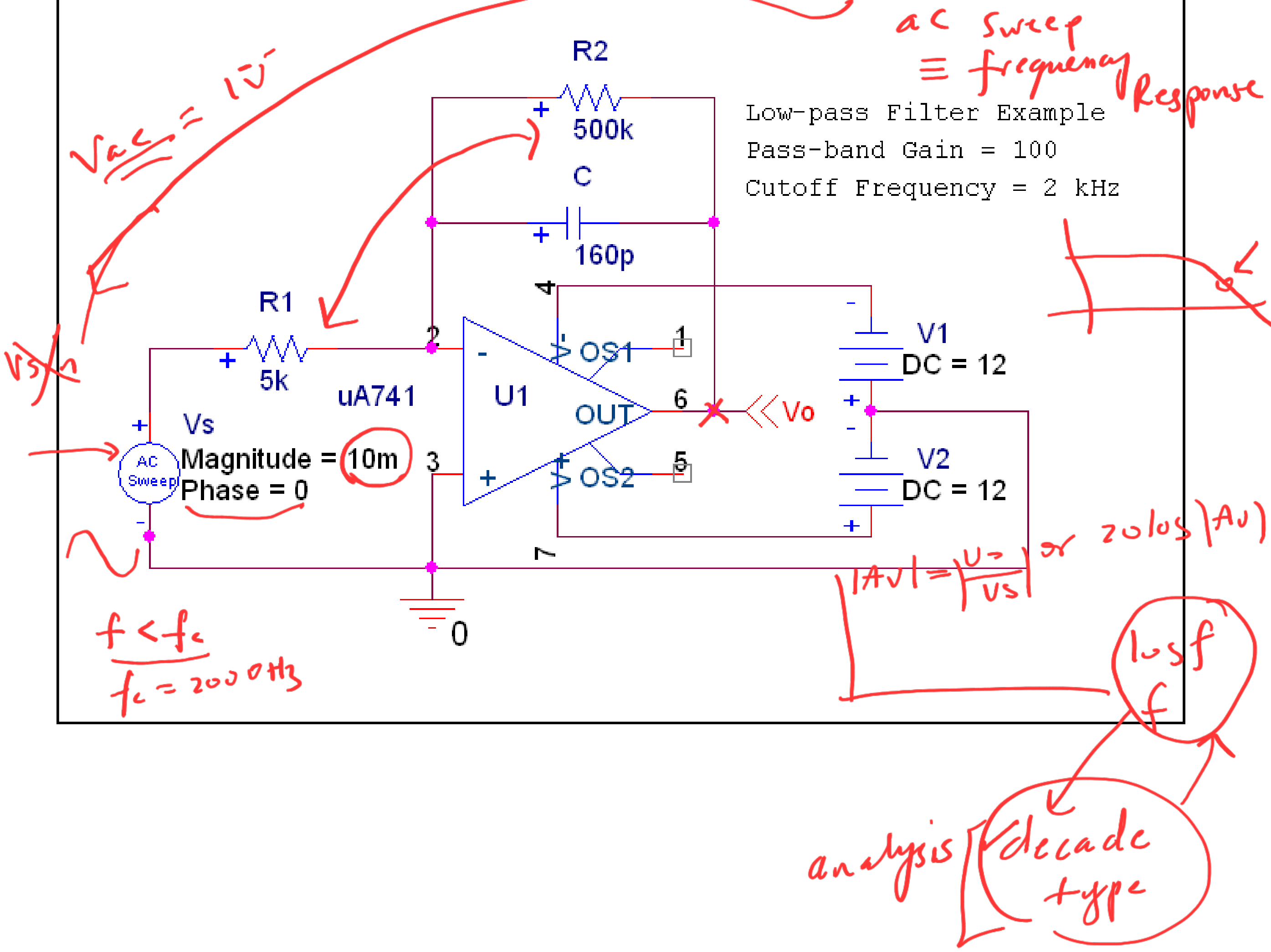

Non-inverting Amplifier
Ideal Gain 100001
uA741
U1
OUT
OS2
OS1
Vo
V1
DC = 9
V2
DC = 9
Vs
AMPLITUDE = 50u
FREQUENCY = 100
Transient Analysis
R2
1Meg
R1
10
0
2.0V
1.5V
1.0V
0.5V
0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time

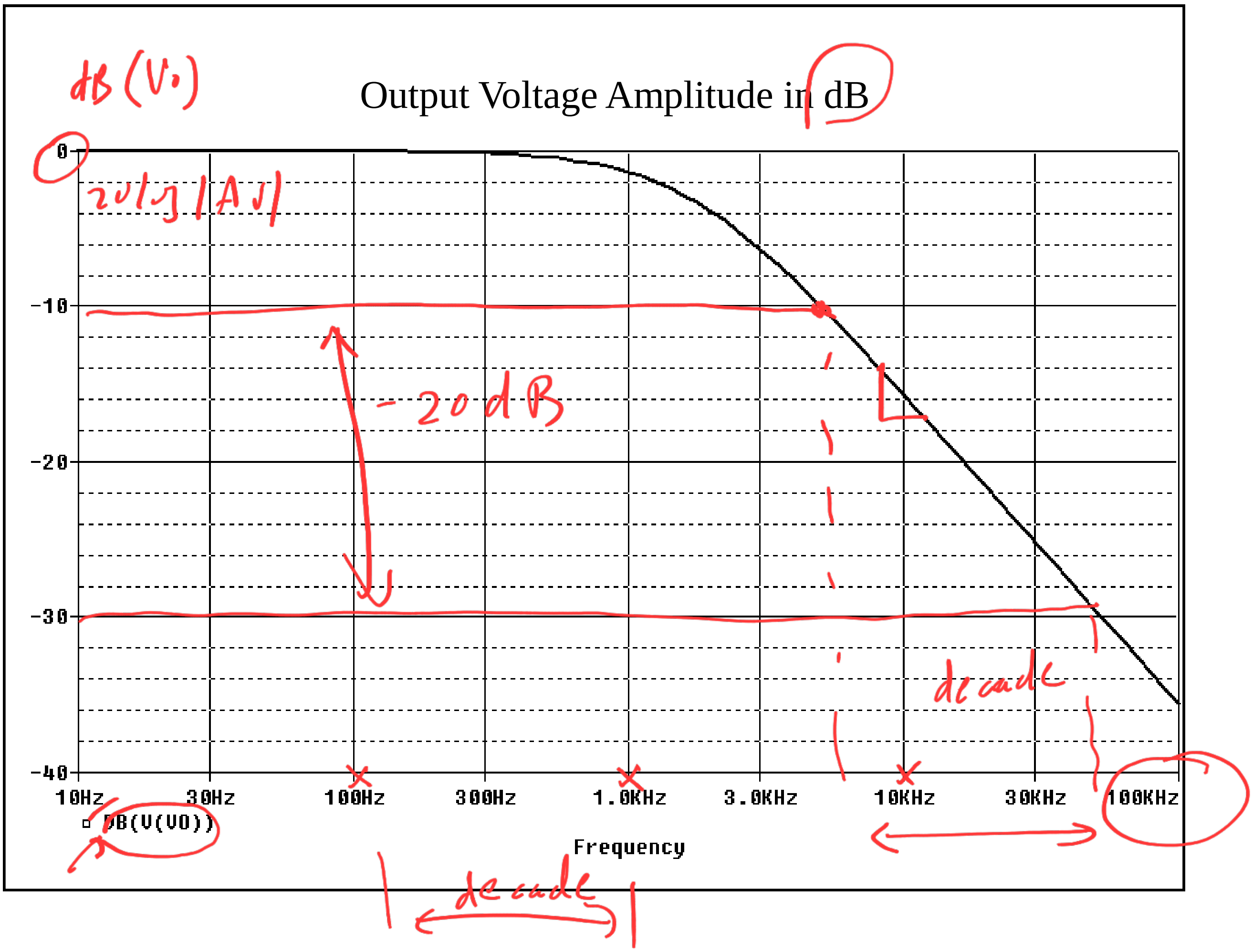

Non-inverting Amplifier
Ideal Gain 1001 with
500 ohm load
uA741
U1
OUT
OS2
OS1
Vo
V1
DC = 9
V2
DC = 9
Transient
Analysis
Vs
AMPLITUDE = 5m
FREQUENCY = 100
R2
100k
R1
100
RL
500
0
5.0V
0V
-5.0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time

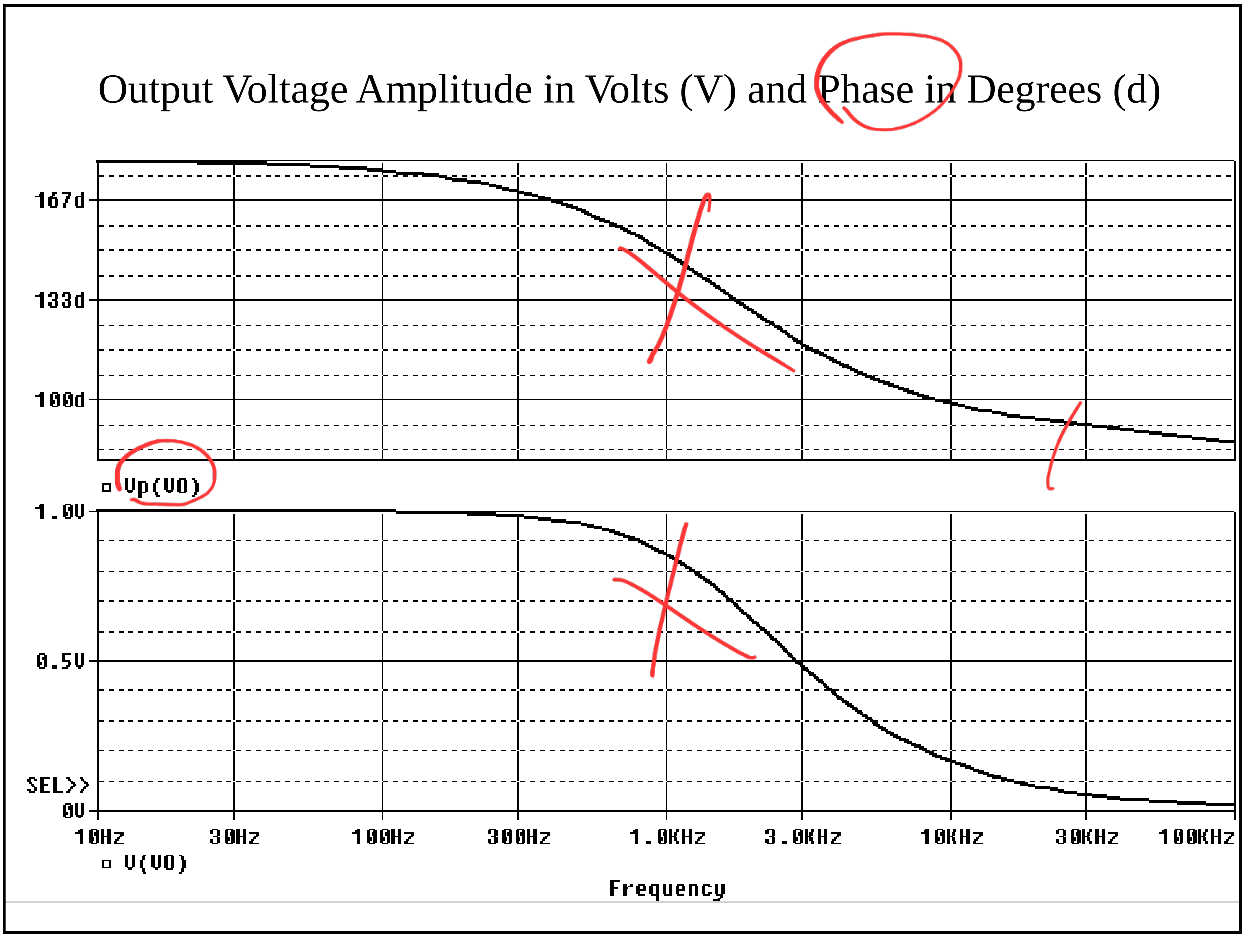

Non-inverting Amplifier
Ideal Gain 1001 with
100 ohm load
uA741
U1
OS2
OUT
OS1
Vo
V1
DC = 9
V2
DC = 9
Vs
AMPLITUDE = 5m
FREQUENCY = 100
Transient
Analysis
R2
100k
R1
100
RL
100
0


5.0V
0V
-5.0V
0s
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms
50ms
V(VO)
Time

Low-pass Filter Example PSpice Simulation



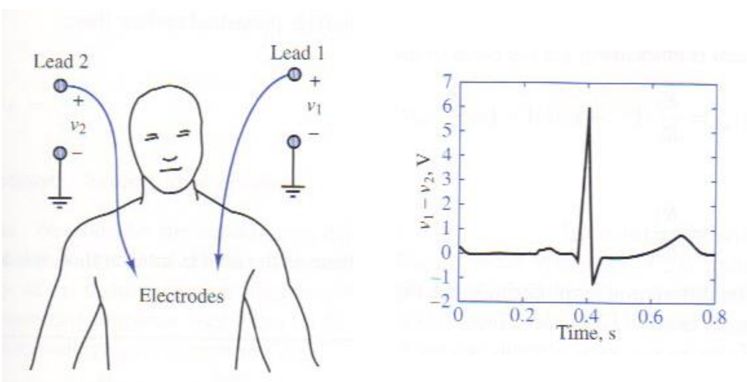

dB (Vo)
Output Voltage Amplitude in dB
0
20 log |Av|
-10
-20 dB
-20
-30
-40
10Hz
30Hz
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
30KHz
100KHz
decade
DB(V(VO))
Frequency
decade


Output Voltage Amplitude in Volts (V) and Phase in Degrees (d)
167d
133d
100d
Vp(VO)
1.0V
0.5V
SEL>>
0V
10Hz
30Hz
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
30KHz
100KHz
V(VO)
Frequency

Following Material is for Reference Only

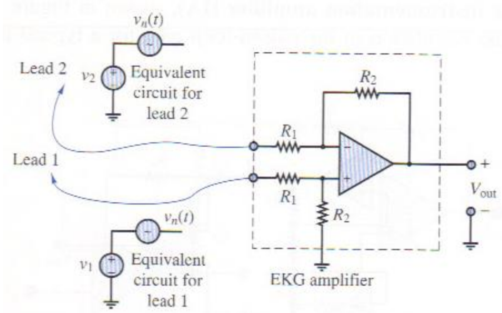
Applications of Op-Amps

- Electrocardiogram (EKG) Amplification
  - Need to measure difference in voltage from lead 1 and lead 2
  - 60 Hz interference from electrical equipment

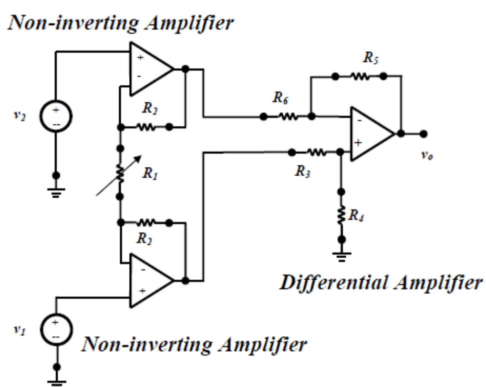


Applications of Op-Amps

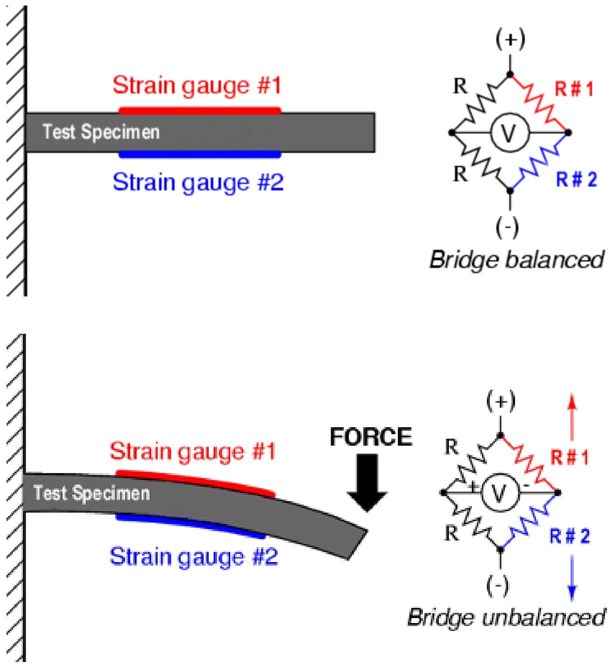
- Simple EKG circuit
  - Uses differential amplifier to cancel common mode signal and amplify differential mode signal



- Realistic EKG circuit
  - Uses two non-inverting amplifiers to first amplify voltage from each lead, followed by differential amplifier
  - Forms an "instrumentation amplifier"



Strain Gauge



Use a Wheatstone bridge to determine the strain of an element by measuring the change in resistance of a strain gauge

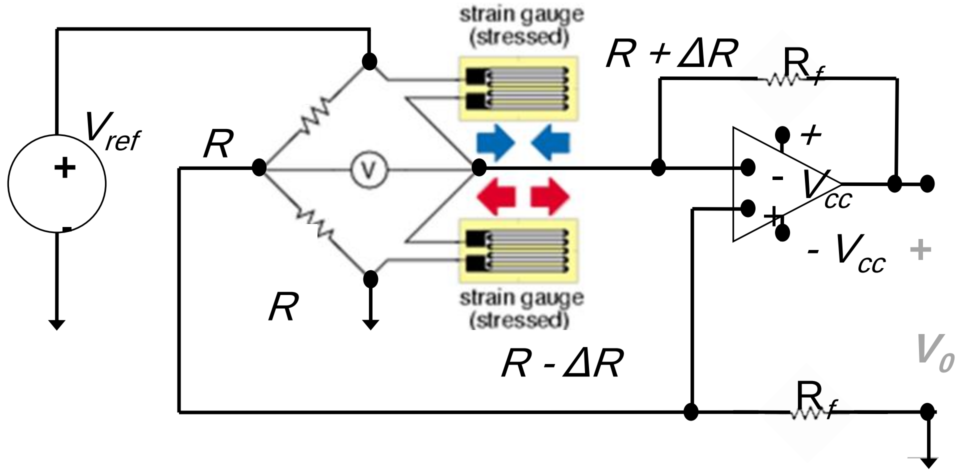
(No strain) Balanced Bridge
R #1 = R #2

(Strain) Unbalanced Bridge
R #1 ≠ R #2

Strain Gauge

Half-Bridge Arrangement

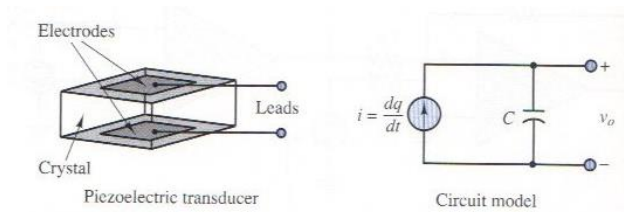
Op amp used to amplify output from strain gauge



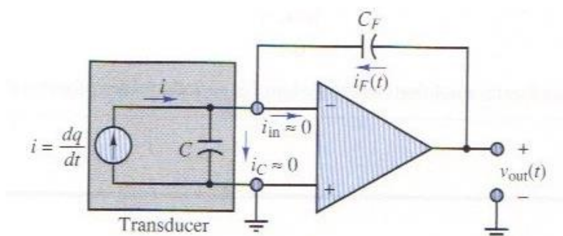
Using KCL at the inverting and non-inverting terminals of the op amp we find that ➔ $\varepsilon \sim V_o = 2\Delta R(R_f/R^2)$

Applications of Op-Amps

- Piezoelectric Transducer
  - Used to measure force, pressure, acceleration
  - Piezoelectric crystal generates an electric charge in response to deformation



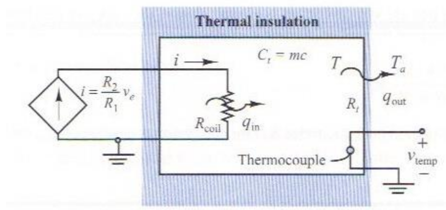
- Use Charge Amplifier
  - Just an integrator op-amp circuit



Applications of Op-Amps

- Example of PI Control: Temperature Control

- Thermal System we wish to automatically control the temperature of:



- Block Diagram of Control System: