Binary Band Pass Transmission

Binary Phase Shift Keying (BPSK)

Binary Digital Bandpass Modulation

Here, the baseband data modulates a high frequency carrier to produce a modulated signal, whose spectrum is centered on the carrier frequency. We will consider four types of bandpass transmission schemes; Amplitude Shift Keying (ASK), Phase Shift Keying (PSK), Frequency Shift Keying (FSK), and Quadri-phase Shift Keying (QPSK). For each type, we consider the generation, optimum receiver, probability of error, power spectral density, and bandwidth.

Binary Phase Shift Keying: Signal Representation

Signal Representation:

Send: $s_1(t) = Acos(2\pi f_c t)$ if the information bit is "1";

Send: $s_2(t) = Acos(2\pi f_c t + \pi)$

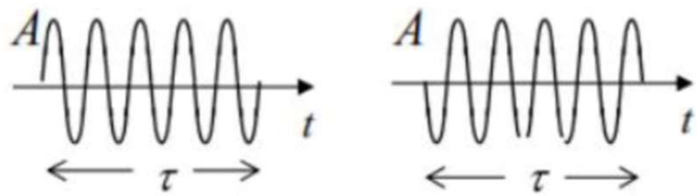
$s_2(t) = -Acos(2\pi f_c t)$ if the information bit is "0";

$\tau = nT_c$

τ: is the time allocated to transmit the binary digit.

T_c=1/f_c is the carrier period

"1" "0"



$s_1(t) = A\cos(2\pi f_c t)$ $s_2(t) = A\cos(2\pi f_c t + \pi)$

$0 \le t \le \tau$ $0 \le t \le \tau$

(τ is an integer number of $1/f_c$)

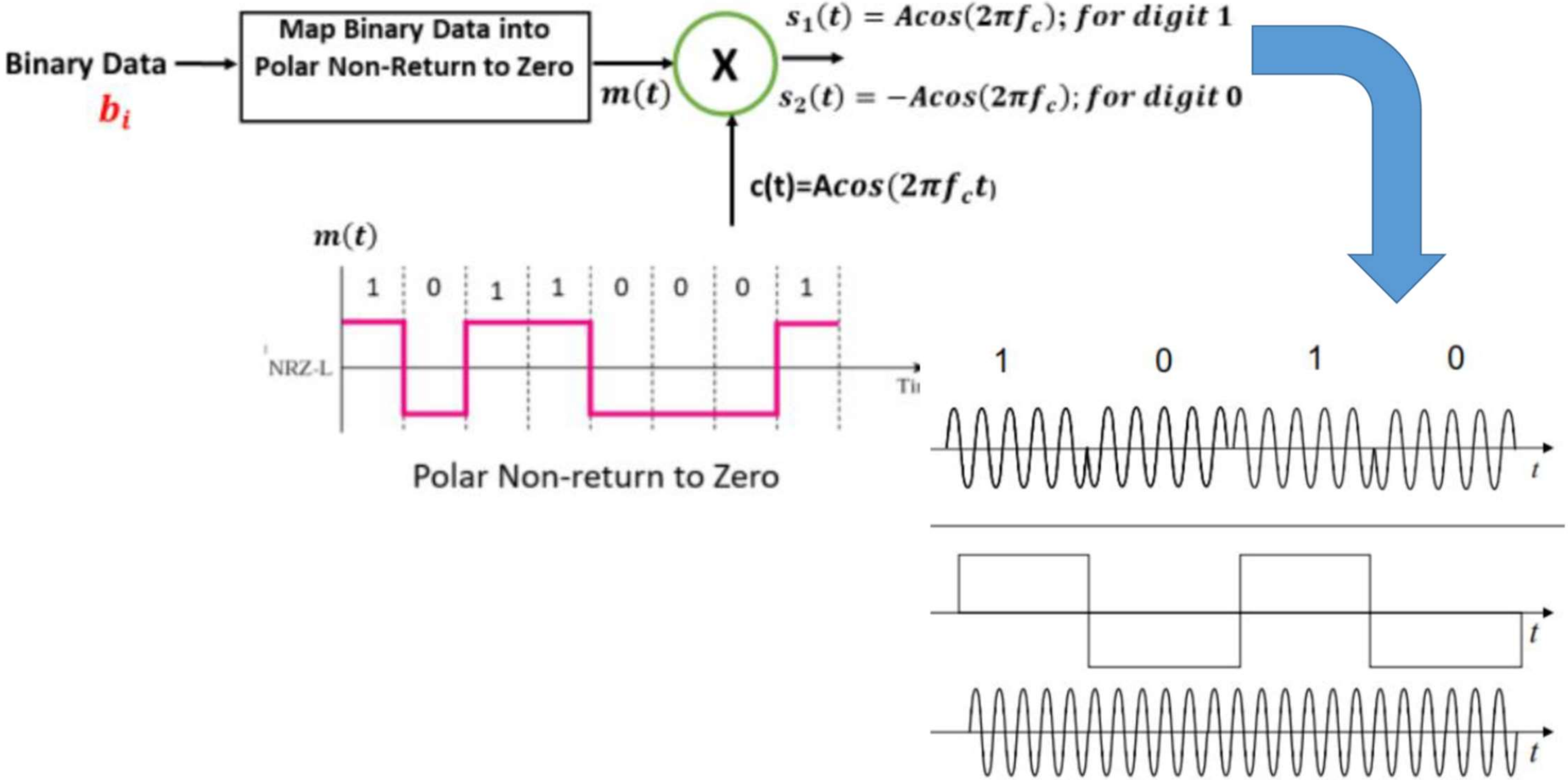
$s_2(t) = s_1(t+\pi) = -s_1(t)$

$\tau = nT_c$

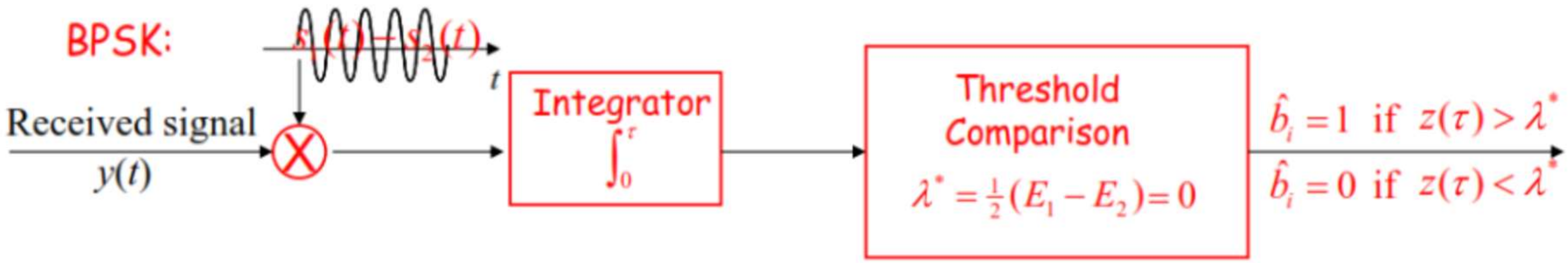
In this figure n=5

Ac

Binary Phase Shift Keying: Generation



Binary Phase Shift Keying: The Optimum Receiver



Optimum Receiver Implemented as a Correlator Followed by a Threshold Detector

Probability of Error:

$$P_b^* = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) - s_2(t))^2 dt}{2N_0}}\right)$$

Energy of $s_i(t)$: $E_1 = E_2 = \frac{1}{2}A^2\tau$

Average Energy per bit: $E_b = \frac{1}{2}(E_1 + E_2) = \frac{1}{2}A^2\tau$

$$\boldsymbol{E = \int_0^\tau (s(t))^2 dt}$$

With $\tau = nT_c$

$$\boldsymbol{E = A^2\tau/2}$$

Verify this result

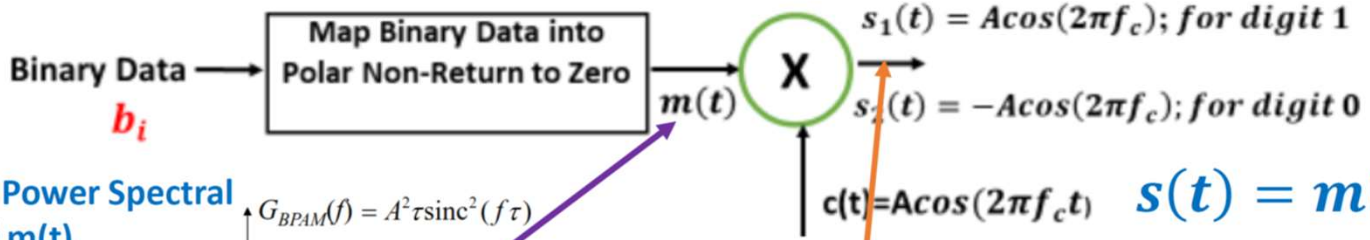
$$s_1(t) = A\cos(2\pi f_c t)$$

$$s_2(t) = -A\cos(2\pi f_c t)$$

Optimal BER:

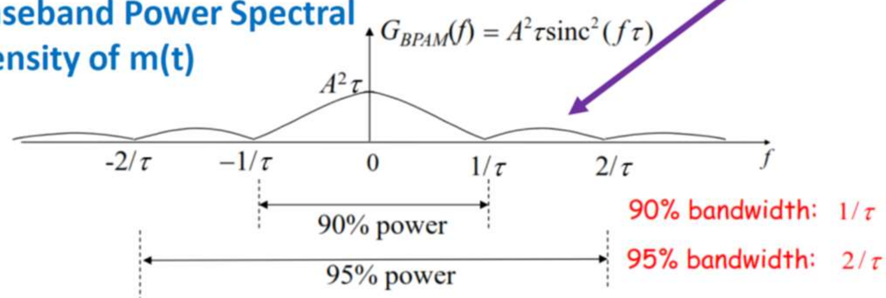
$$P_b^* = Q\left(\sqrt{\frac{A^2\tau}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Binary Phase Shift Keying: Power Spectral Density and Bandwidth



$$s(t) = m(t)\cos(2\pi f_c t)$$

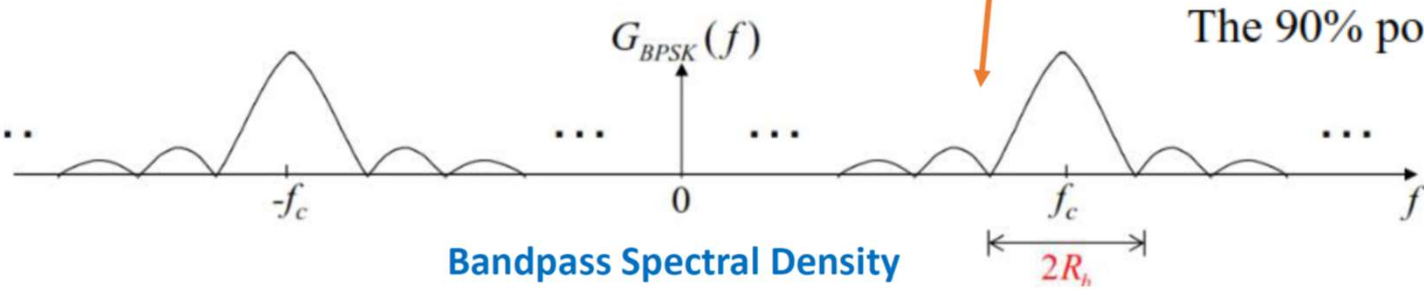
Baseband Power Spectral Density of m(t)



Bandwidth of BPSK s(t)

Power Spectral Density

$$G_{BPSK}(f) = \tfrac{1}{4}[G_{BPAM}(f - f_c) + G_{BPAM}(f + f_c)]$$



Bandpass Spectral Density

The 90% power bandwidth $= \frac{2}{\tau} = 2R_b$

(twice the data rate); Same as that of BASK

Extra Material on the Power Spectral Density

The Wiener –Khintchine Thorem:

The power spectral density $G_X(f)$ and the autocorrelation function $R_X(\tau)$ of a stationary random process $X(t)$ form a Fourier transform pairs:

$G_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$ (Fourier Transform)

$R_X(\tau) = \int_{-\infty}^{\infty} G_X(f) e^{j2\pi f\tau} df$ (Inverse Fourier Transform)

Example: Mixing of a random process with a sinusoidal signal.

- A random process $X(t)$ with an autocorrelation function $R_X(\tau)$ and a power spectral density $G_X(f)$ is mixed with a sinusoidal function $\cos(2\pi f_c t + \theta)$; θ is a r.v uniformly distribution over $(0, 2\pi)$ to form a new process

  $Y(t) = X(t)\cos(2\pi f_c t + \theta)$. Find $R_Y(\tau)$ and $G_Y(f)$
- **Solution**: We first find $R_Y(\tau)$
- $R_Y(\tau) = E\{\, Y(t)Y(t+\tau)\}$
- $= E\{X(t)\cos(2\pi f_c t + \theta) \cdot X(t+\tau)\cos(2\pi f_c t + 2\pi f_c \tau + \theta)\}$

  When $X(t)$ and θ are independent, then
- $= E\{X(t)\, X(t+T)\}E\{\cos(2\pi f_c t + \theta) \cdot \cos(2\pi f_c t + 2\pi f_c \tau + \theta)\}$
- $= R_X(\tau)E\{\frac{\cos(4\pi f_c t + 2\pi f_c \tau + 2\theta) + \cos 2\pi f_c \tau}{2}\}$
- $\boldsymbol{R_Y(\tau) = \frac{R_X(\tau)}{2} \cdot \cos 2\pi f_c \tau}$;
- The power spectral density is
- $\boldsymbol{S_Y(f) = \frac{1}{4}\{G_X(f - f_c) + G_X(f + f_c)\}}$
- Which is quite similar to the modulation property of the Fourier transform.

Binary Amplitude Shift Keying (BASK): Signal Representation

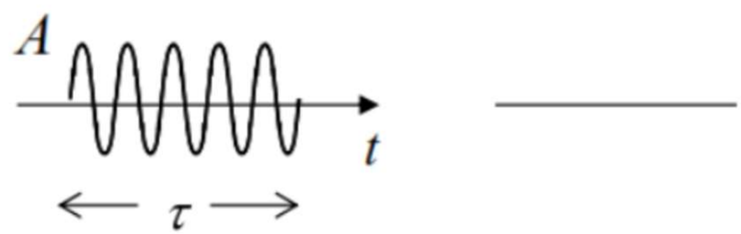
Send: $s_1(t) = Acos(2\pi f_c t)$, if the information bit is "1" $\Rightarrow E_1 = \frac{A^2\tau}{2}$

Send: $s_2(t) = 0$, if the information bit is "0"; $\Rightarrow E_2 = 0$

The average energy per bit $E_b = \frac{1}{2}(E_1 + E_2) = \frac{A^2\tau}{4}$

"1" "0"

$\tau = nT_c$
τ: is the time allocated to transmit the binary digit.
$T_c = 1/f_c$ is the carrier period
$R_b = \frac{1}{\tau}$: $Data\ rate$ bits/sec



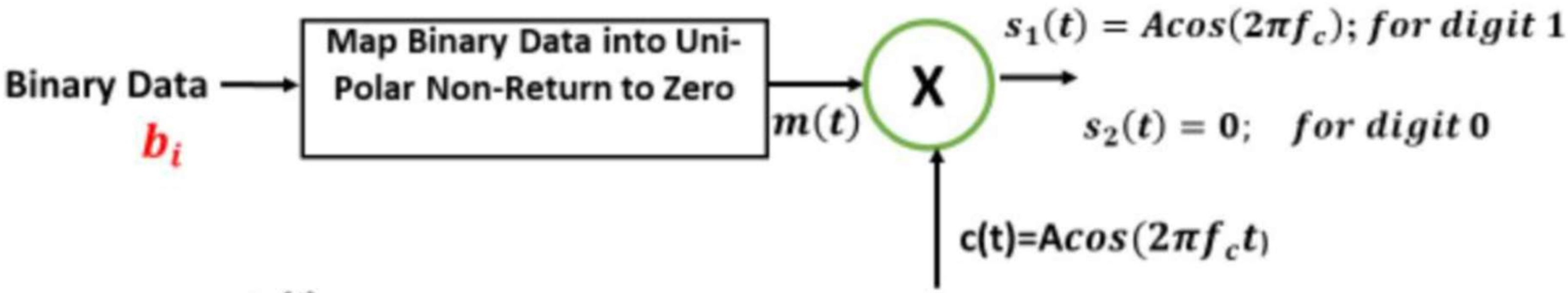
$s_1(t) = A\cos(2\pi f_c t)$
$0 \le t \le \tau$

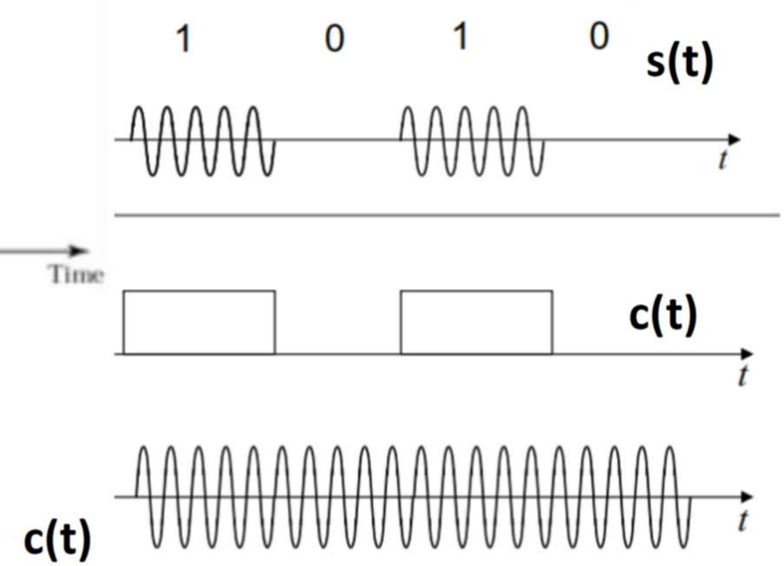
$s_2(t) = 0$

$\tau = nT_c$

In this figure n=5

Binary Amplitude Shift Keying : Generation

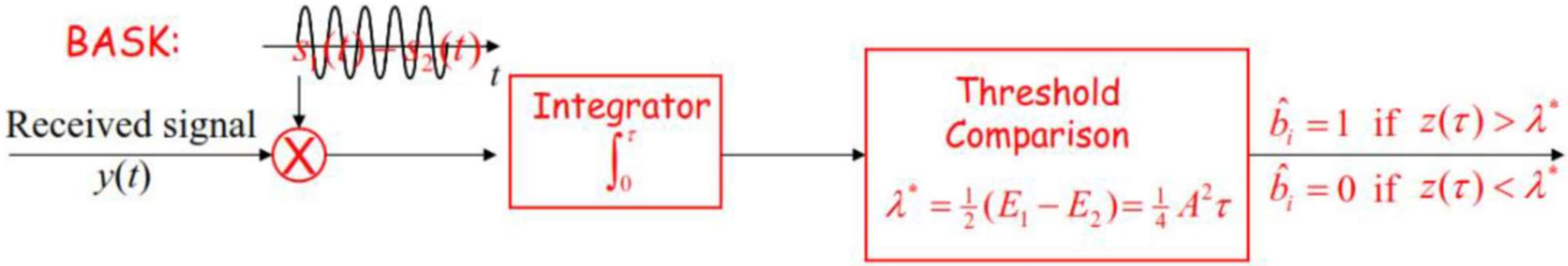




$$s(t) = m(t)c(t)$$

NOTE: BASK is also called ON-OFF Keying

Binary Amplitude Shift Keying : The Optimum Receiver



Optimum Receiver Implemented as a Correlator Followed by a Threshold Detector

Probability of Error:

$$P_b^* = Q\left(\sqrt{\frac{\int_0^{\tau}(s_1(t) - s_2(t))^2 dt}{2N_0}}\right)$$

$$E_1 = \int_0^{\tau}(s_1(t))^2 dt$$

With $\tau = nT_c$

$$E_1 = A^2\tau/2$$

$$s_1(t) = Acos(2\pi f_c t)$$

$$s_2(t) = 0,$$

$$E_1 = \frac{A^2\tau}{2}$$

$$E_2 = 0$$

$$E_b = \frac{1}{2}(E_1 + E_2) = \frac{A^2\tau}{4}$$

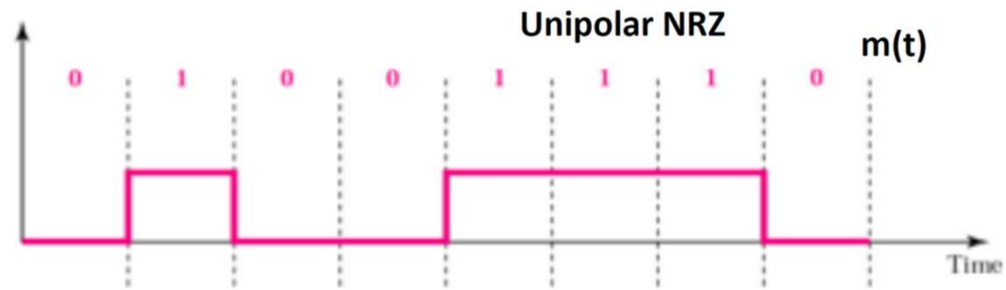
Optimal BER:

$$P_b^* = Q\left(\sqrt{\frac{A^2\tau}{4N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Binary Amplitude Shift Keying: Power Spectral Density

Let $\mathrm{m}(t)$ be the unipolar NRZ signal with autocorrelation function $R_m(\tau)$ and power spectral density $G_m(f)$.

- You can easily verify that the unipolar non-return to zero signal $m(t)$ is related to the polar non-return signal $m'(t)$ (used in the generation of the BPSK) by:



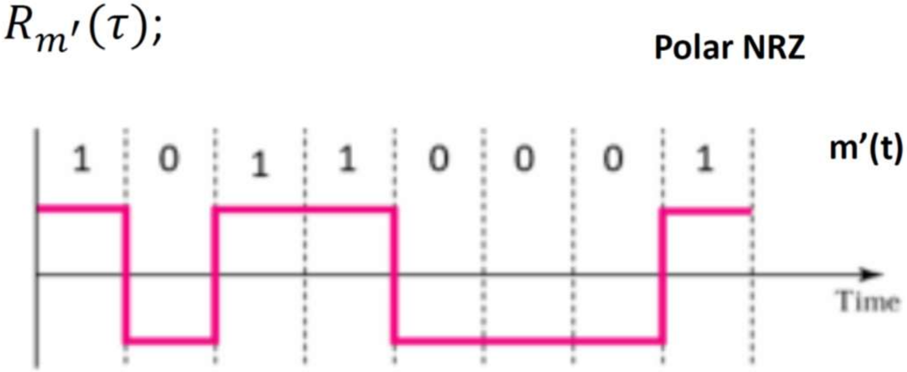
- $m(t) = \frac{1}{2}(1 + m'(t)$

The autocorrelation function of m(t) is

- $R_Y(\tau) = E\{\, m(t)m(t+\tau)\} =$

= $E\left\{\frac{1}{2}(1 + m'(t)\frac{1}{2}(1 + m'(t+\tau)\right\} = \frac{1}{4} + \frac{1}{4}R_{m'}(\tau);$

Note that for the polar-NRZ $\boldsymbol{E\{m'(t)\} = 0}$



- The power spectral density of m(t) is:

- $G_m(f) = \frac{1}{4}\delta(f) + \frac{1}{4}G_{m'}(f)$

Binary Amplitude Shift Keying: Power Spectral Density

The Wiener –Khintchine Thorem: The power spectral density $G_X(f)$ and the autocorrelation function $R_X(\tau)$ of a stationary random process $X(t)$ form a Fourier transform pairs:

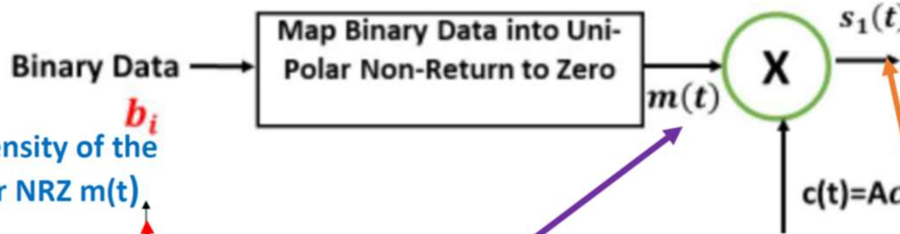
- $G_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$ (Fourier Transform)
- $R_X(\tau) = \int_{-\infty}^{\infty} G_X(f) e^{j2\pi f\tau} df$ (Inverse Fourier Transform)
- The power spectral density of m(t), the unipolar NRZ is:
- $\boldsymbol{G_m(f) = \frac{1}{4}\delta(f) + \frac{1}{4}G_{m'}(f)}$
- In the previous video we saw that if a random process $X(t)$ with an autocorrelation function $R_X(\tau)$ and a power spectral density $G_X(f)$ is mixed with a sinusoidal function $\cos(2\pi f_c t + \theta)$; θ is a r.v uniformly distribution over $(0, 2\pi)$ to form a new process

$Y(t) = X(t)\cos(2\pi f_c t + \theta)$. **Our Problem: $\boldsymbol{s(t) = m(t)\cos(2\pi f_c t + \theta)}$**

then the autocorrelation function and power spectral density of Y(t) are given by:

- $R_Y(\tau) = E\{Y(t)Y(t+\tau)\} = \frac{R_X(\tau)}{2} \cdot \cos 2\pi f_c \tau$;
- $\boldsymbol{G_Y(f) = \frac{1}{4}\{G_X(f - f_c) + G_X(f + f_c)\}}$
- **Hence,** $\boldsymbol{G_{BASK}(f) = \frac{1}{4}\{G_m(f - f_c) + G_m(f + f_c)}$

Binary Amplitude Shift Keying : Power Spectral Density and Bandwidth

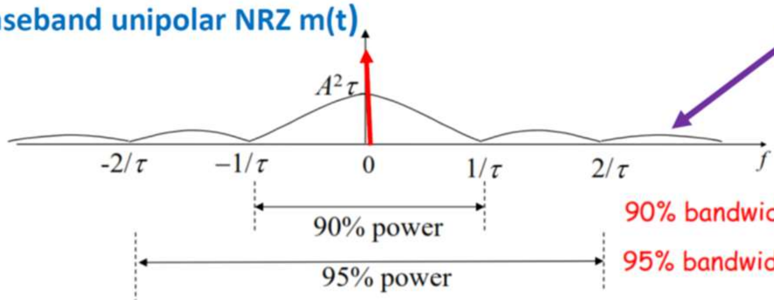


$s_1(t) = A\cos(2\pi f_c)$; *for digit 1*

$s_2(t) = 0$; *for digit 0*

$$s(t) = m(t)\cos(2\pi f_c t)$$

Power Spectral Density of the baseband unipolar NRZ m(t)



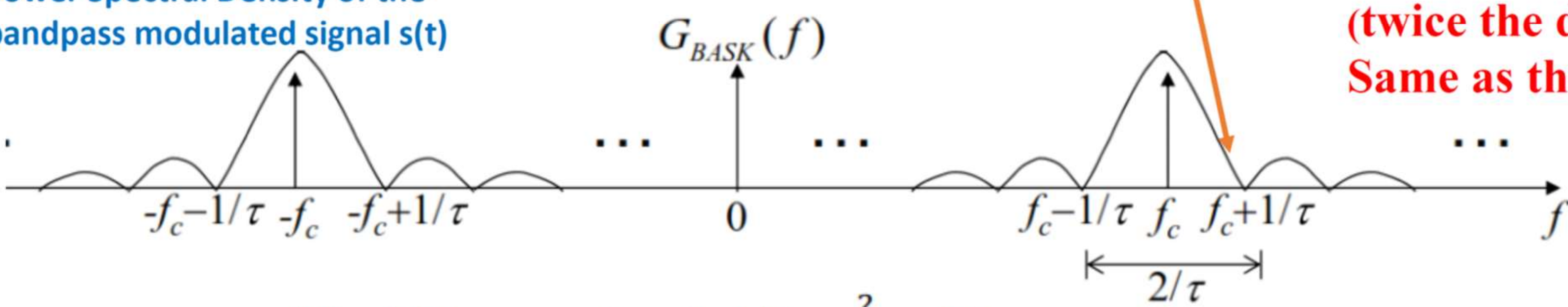
90% bandwidth: $1/\tau$

95% bandwidth: $2/\tau$

$$G_{BASK}(f) = \tfrac{1}{4}[G_{BOOK}(f - f_c) + G_{BOOK}(f + f_c)]$$

Bandwidth of BASK s(t)

Power Spectral Density of the bandpass modulated signal s(t)

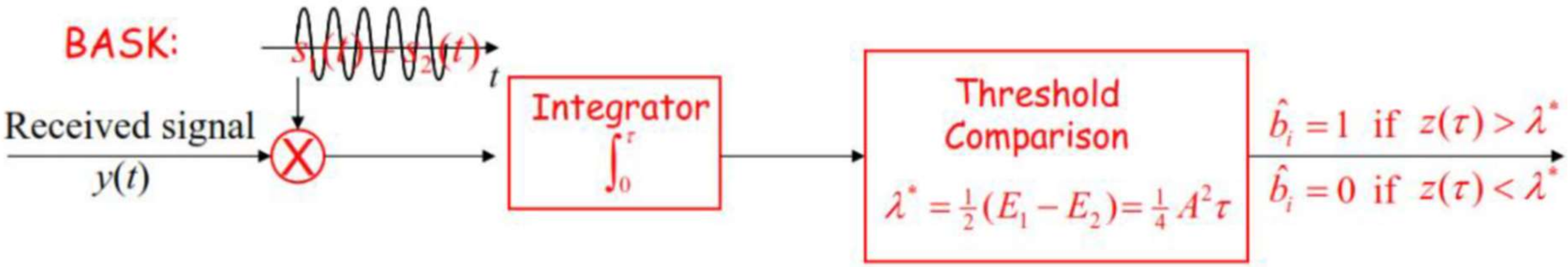


(twice the data rate); Same as that of BPSK

The 90% power bandwidth $= \frac{2}{\tau} = 2R_b$

Non-coherent Demodulation of the Binary ASK Signal

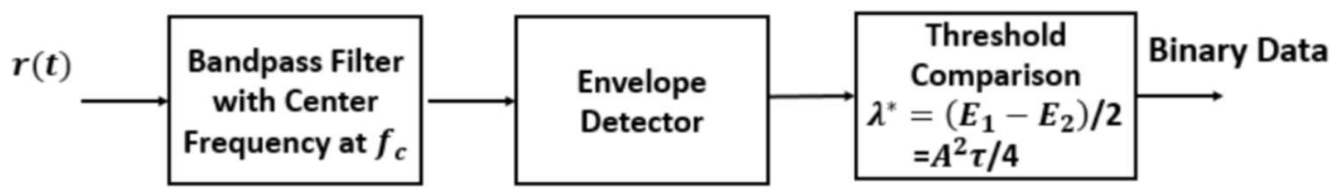
The demodulator which uses the signal difference $s_1(t) - s_2(t) = Acos(2\pi f_c t)$ is called coherent demodulator



In non-coherent demodulation, there is no need for the carrier frequency at the receiver. The basic elements of the receiver are a bandpass filter with center frequency at the carrier, an envelope detector, and a threshold comparator. The receiver is simple, however it is not optimal in terms of the probability of error. The details are shown in the following block diagram

$\mathbf{r(t)} = Acos(2\pi f_c t) + n(t)$

$\mathbf{r(t)} = n(t)$



Non-Coherent Binary ASK Demodulation

Binary Frequency Shift Keying (BFSK): Signal Representation

In binary FSK, the frequency of the carrier signal is varied to represent the binary digits and 0 by two distinct frequencies. The amplitude and frequency remain constant during each bit interval.

Signal Representation (coherent FSK)

Send: $s_1(t) = Acos(2\pi(f_c + \Delta f))t)$ if the information bit is "1";

Send: $s_2(t) = Acos(2\pi(f_c - \Delta f))t)$ if the information bit is "0";

Δf is an offset frequency (from the unmodulated carrier f_c) chosen so that $s_1(t)$ and $s_2(t)$ are orthogonal, i.e.,

$$\int_0^\tau s_1(t)s_2(t)dt = 0$$

$$\frac{sin(2\pi(2f_c)\tau}{2f_c} + \frac{sin(2\pi(2\Delta f)\tau}{2\Delta f} = 0$$

Binary Frequency Shift Keying (BFSK): Signal Representation

Orthogonality condition

$$\frac{sin(2\pi(2f_c)\tau}{2f_c} + \frac{sin(2\pi(2\Delta f)\tau}{2\Delta f} = 0$$

$$2f_c = \frac{n}{2\tau} = \frac{nR_b}{2}, n = 1, 2, \dots \quad \boldsymbol{f_c = \frac{nR_b}{4} = kR_b}$$

$$2\Delta f = \frac{mR_b}{2}, m = 1, 2, \dots \quad \boldsymbol{\Delta f = \frac{mR_b}{4}}$$

Note that $sin(x) = 0$ when $x = n\pi$

The minimum frequency separation $\boldsymbol{2\Delta f = R_b/2}$.

"1" "0"

$s_1(t) = A\cos(2\pi(f_c + \Delta f)t)$ $s_2(t) = A\cos(2\pi(f_c - \Delta f)t)$

$0 \le t \le \tau$ $0 \le t \le \tau$

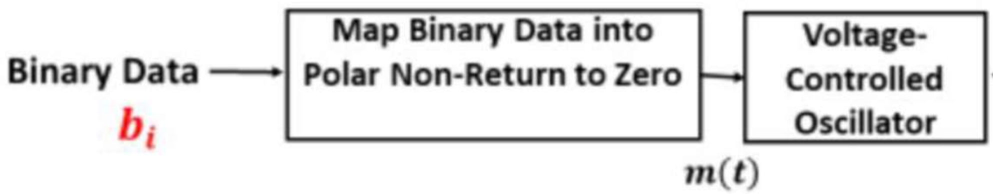
(τ is an integer number of $1/(f_c \pm \Delta f)$)

τ: is the time allocated to transmit the binary digit.

T_c=1/f_c is the carrier period

$R_b = \frac{1}{\tau}$: $Data\ rate$ bits/sec

Binary FSK : Generation using the Single Oscillator Method

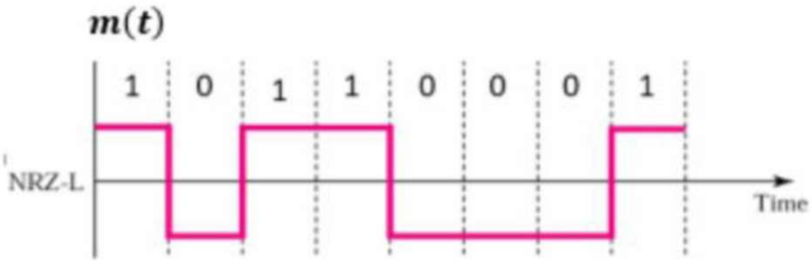


$f_1(t) = f_c + \Delta f;\ for\ digit\ 1$

$f_2(t) = f_c - \Delta f;\ for\ digit\ 0$

m(t): Polar non-return to zero

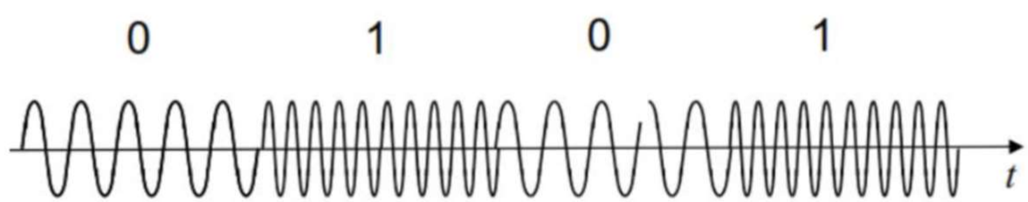
$$f_i(t) = f_c + k_f m(t);\ for\ VCO$$



Polar Non-return to Zero

$$s_1(t) = A\cos(2\pi(f_c + \Delta f))t)$$

$$s_2(t) = A\cos(2\pi(f_c - \Delta f))t)$$

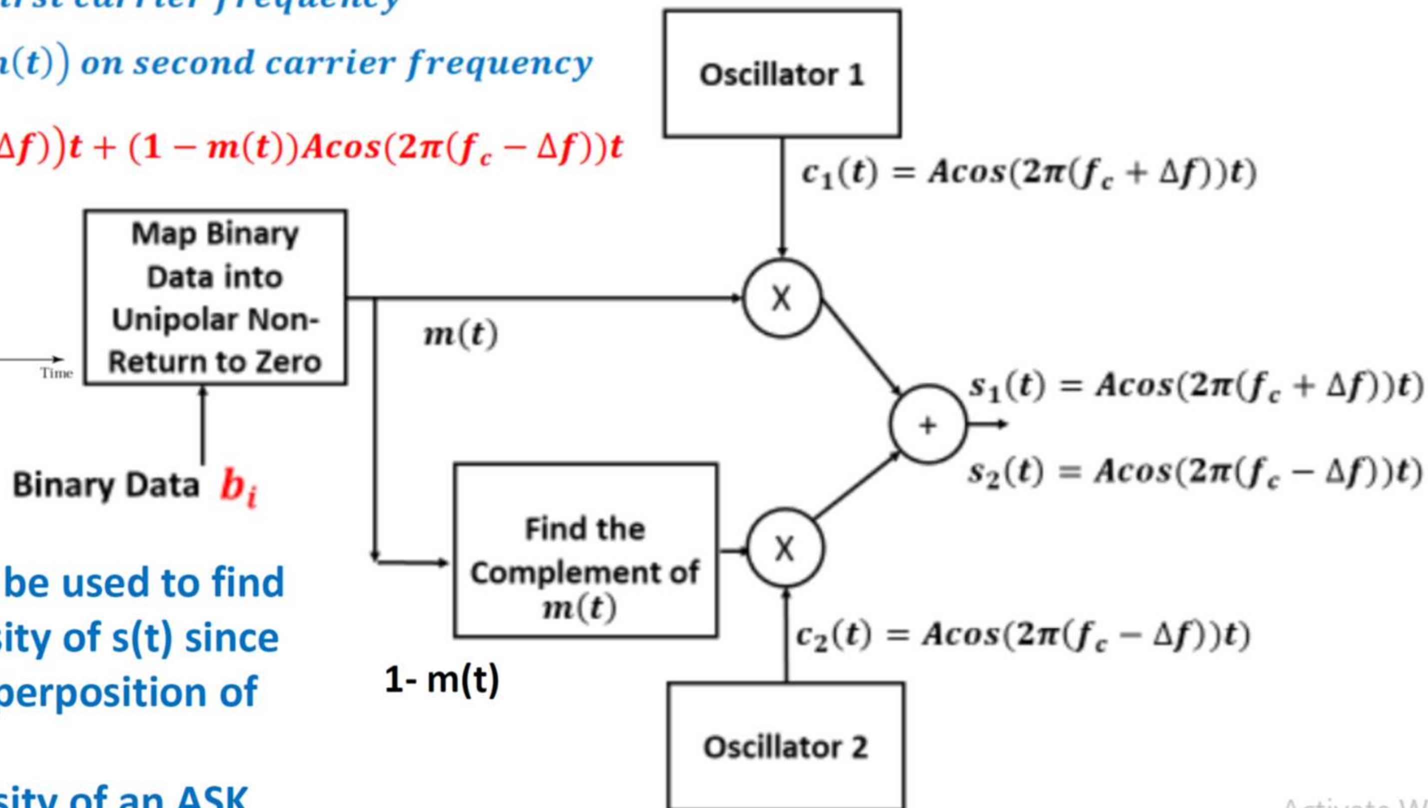


Binary FSK : Generation using the Two-oscillator Method

$$s_{BFSK}(t) = ASK\ of\ m(t)\ on\ first\ carrier\ frequency$$
$$+ ASK\ of\ \left(1 - m(t)\right)\ on\ second\ carrier\ frequency$$

$$s_{BFSK}(t) = m(t)Acos\left(2\pi(f_c + \Delta f)\right)t + (1 - m(t))Acos(2\pi(f_c - \Delta f))t$$

Unipolar Non-Return to Zero Signal

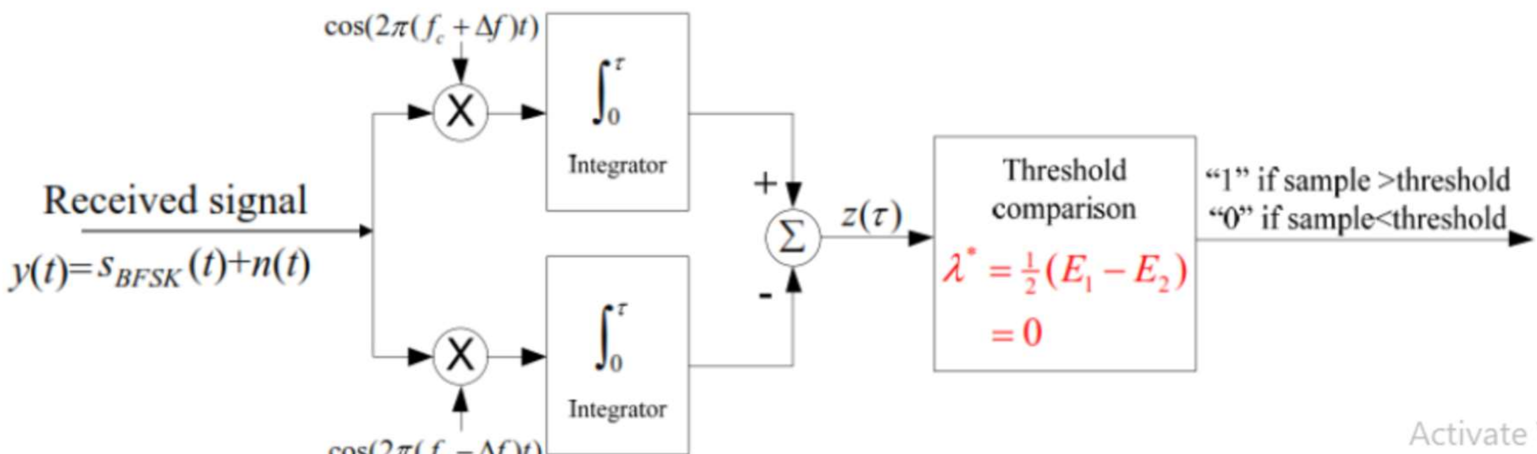


- This representation will be used to find the power spectral density of s(t) since it is envisaged as the superposition of two ASK signals.
- The power spectral density of an ASK signal was derived in a previous video titled: Binary ASK

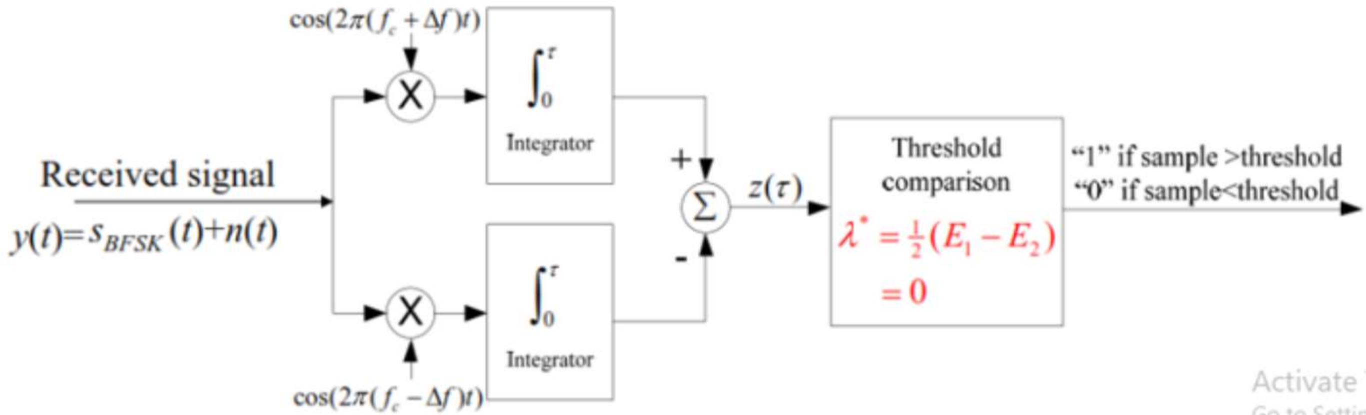
FSK: modeled as a sum of two ASK signals

Binary FSK : Coherent Demodulation

The optimum coherent receiver consists of two correlators. The operation of the receiver makes use of the orthogonality condition imposed on the signals $s_1(t)$ and $s_2(t)$. In the absence of noise, if $s_1(t)$ is received, then the ouput of the upper correlator will have a value greater than zero, while the output of the lower correlator is zero. The converse is true when $s_2(t)$ is received. In the presence of noise, the system decides 1 when $z(\tau) > 0$. That is, when the output of the upper correlator is greater than the output of the lower one. Otherwise, it decides 0.



Binary FSK : Probability of Error



Probability of Error

Energy of $s_i(t)$: $E_1 = E_2 = \frac{1}{2}A^2\tau$

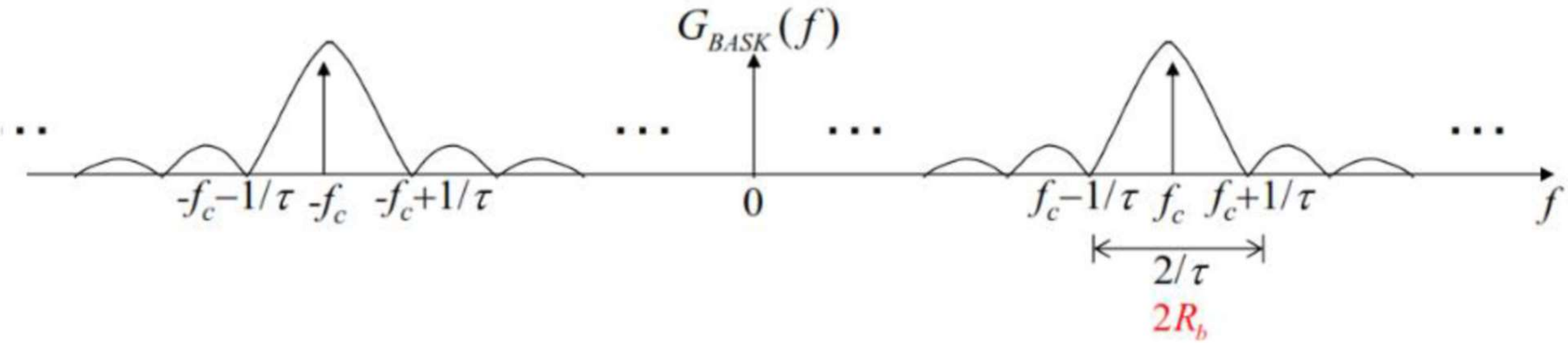
Average Energy per bit: $E_b = \frac{1}{2}(E_1 + E_2) = \frac{1}{2}A^2\tau$

When the signals are orthogonal, i.e., when $\int_0^\tau s_1(t)s_2(t)dt = 0$, the probability of error is given by

$$P_b^* = Q\left(\sqrt{\frac{A^2\tau}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Binary FSK : Power Spectral Density

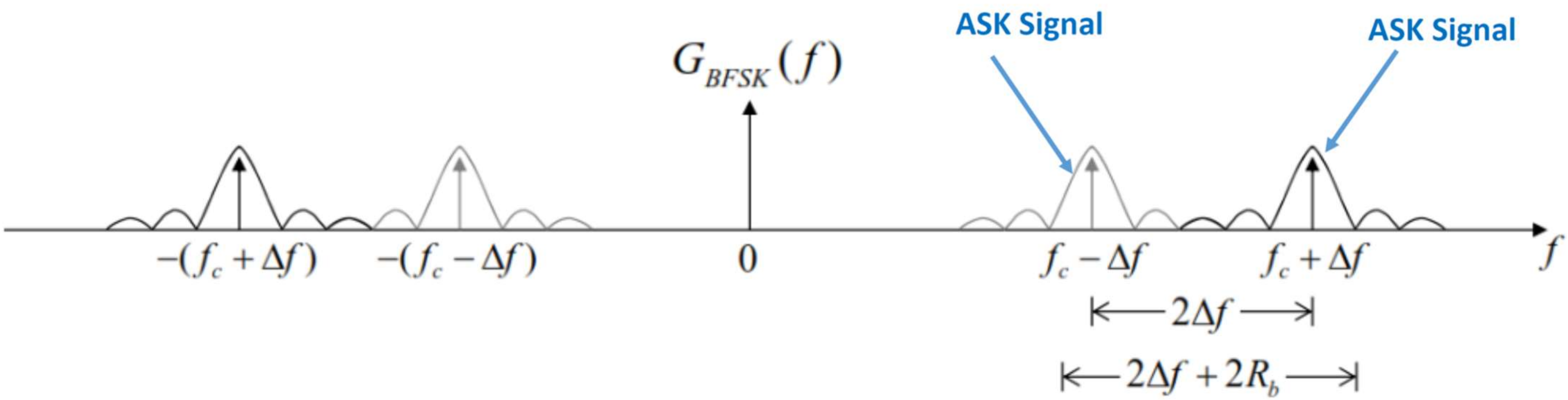
Since the FSK signal is the superposition of two ASK signals on two orthogonal frequencies, the spectrum is also the superposition of that of the ASK signals. We recall that the spectrum of the ASK signal is as shown below



$$s_{BFSK}(t) = ASK\ of\ m(t)\ on\ first\ carrier\ frequency$$
$$+\ ASK\ of\ \big(1 - m(t)\big)\ on\ second\ carrier\ frequency$$

$$s_{BFSK}(t) = m(t)A\cos\big(2\pi(f_c + \Delta f)\big)t + (1 - m(t))A\cos(2\pi(f_c - \Delta f))t$$

Binary FSK : Power Spectral Density and Bandwidth

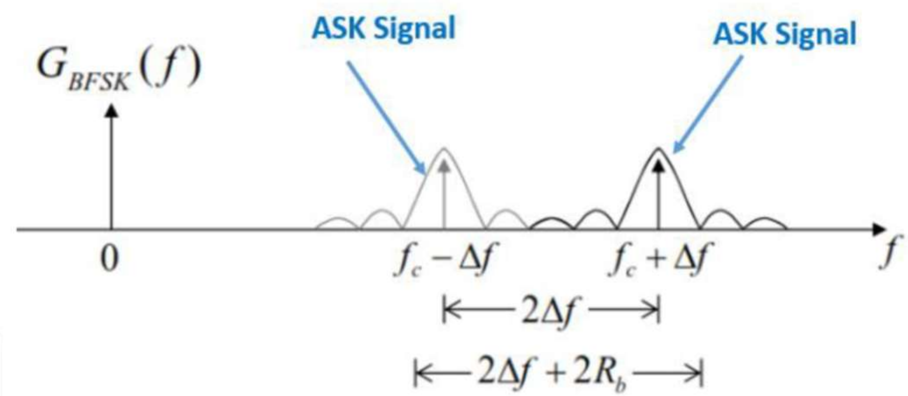


The required channel bandwidth for 90% in-band power

$$B_{h\_90\%} = 2\Delta f + 2R_b$$

$$\boldsymbol{B.W = (f_1 - f_2) + 2R_b = \frac{R_b}{2} + 2R_b}$$

Binary FSK : Non-coherent Demodulation



$$\mathbf{r(t)} = A\cos(2\pi(f_c + \Delta f)t) + n(t)$$

$$\mathbf{r(t)} = A\cos(2\pi(f_c - \Delta f)t) + n(t)$$