FILTERS DESIGN
BY POLE-ZERO PLACEMENT

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Poles/Zeros

- When a zero is placed at a given point on the z-plane, the frequency response will be zero at the corresponding point. A pole on the other hand produces a peak at the corresponding frequency point.

- Poles that are close to the unit circle give rise large peaks, where as zeros close to or on the unit circle produces troughs or minima. Thus, by strategically placing poles and zeros on the z-plane, we can obtain sample low pass or other frequency selective filters

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A Narrowband Bandpass Filter

- Also known as a resonator
- Pass a single frequency $0 < F_0 < \frac{F_S}{2}$
- Place a pole at the point inside the unit circle that corresponds to the resonant freq F_0
- The corresponding angle is $\omega_0 = \frac{2\pi}{F_S} F_0$
- Place zeros at the two end frequencies, i.e., $z = 1$ ($F = 0$) and $z = -1$ ($F = F_S/2$)

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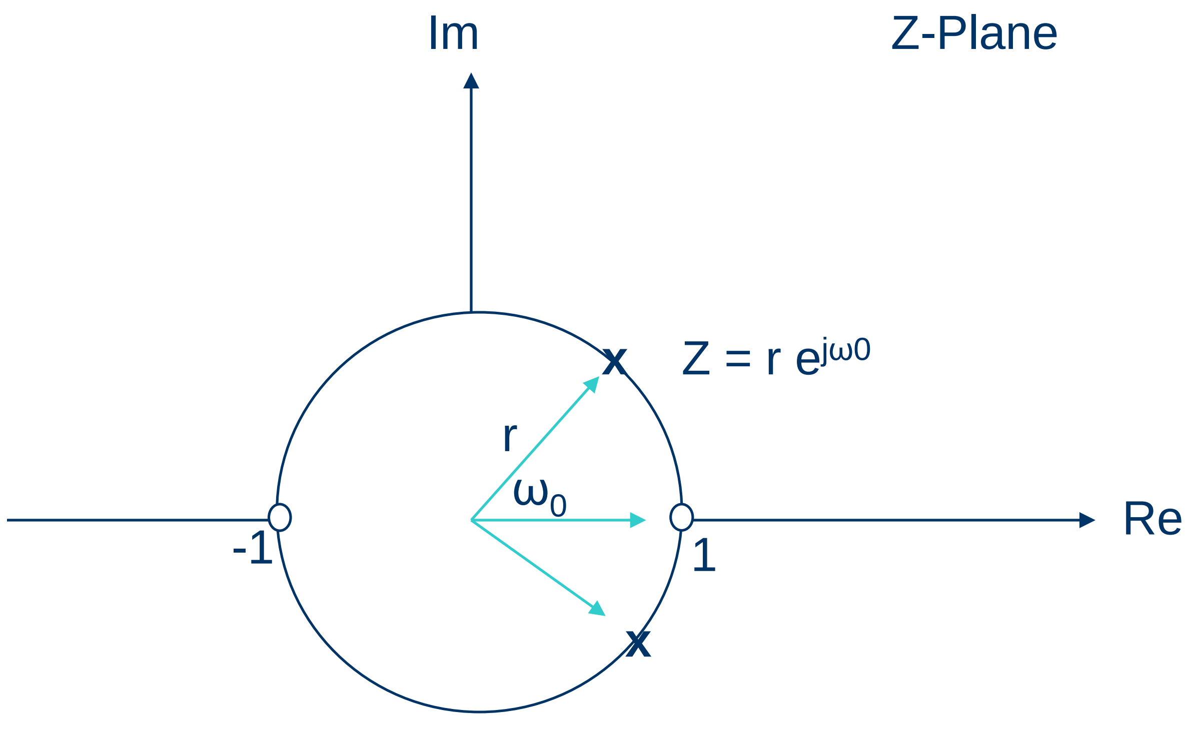
A Narrowband Bandpass Filter



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$$H_r(z) = \frac{K(z-1)(z+1)}{(z-re^{j\omega_0})(z-re^{-j\omega_0})} = \frac{K(z^2-1)}{z^2-2r\cos\omega_0 z+r^2}$$

r is chosen such that the filter is highly selective, i.e., small 3-dB BW of the passband, ΔF.

$$r \approx 1-\frac{(\Delta F)\pi}{F_S}, \quad 0.9<r<1$$

The gain factor K is inserted to ensure that the passband gain is 1, i.e., no amplification.

$$K = \frac{\left|e^{j2\omega_0}-2r\cos\omega_0 e^{j\omega_0}+r^2\right|}{\left|e^{j2\omega_0}-1\right|} = \frac{(1-r)\sqrt{1-2r\cos 2\omega_0+r^2}}{2\,|\sin\omega_0|}$$

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Example 8.17 Design a second-order bandpass filter using the pole-zero placement method and satisfying the following specifications:

Sampling rate = 8000 Hz
3-dB bandwidth = 200 Hz
Passband center frequency = 1000 Hz
Zero gain at zero and 4000 Hz.

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A Narrowband Bandstop Filter (or known as Notch filter)

- Also known as a notch filter
- Remove a single frequency $0 < F_0 < \frac{F_s}{2}$
- Place zeros at the points on the unit circle that correspond to the notch freq F_0
- The corresponding angle is $\omega_0 = \frac{2\pi}{F_s} F_0$
- Place poles at the points inside the unit circle that correspond to the notch freq F_0

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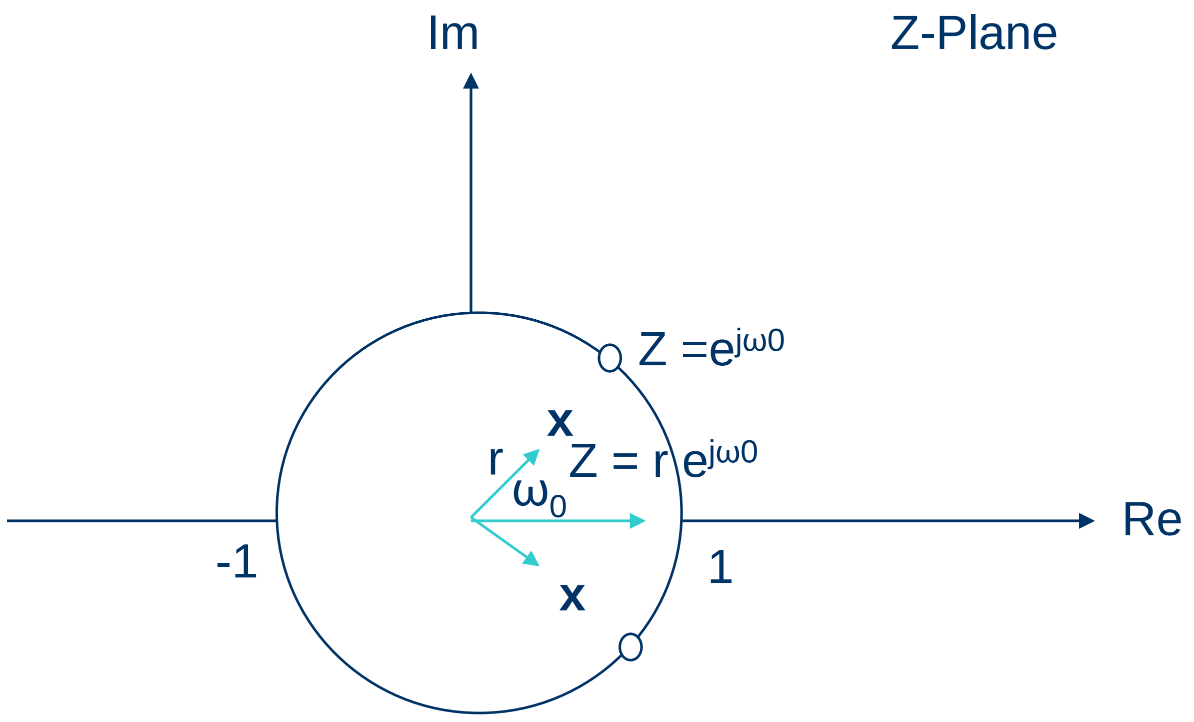
2^{nd} order Notch Filter



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$$H_n(z) = \frac{K(z - e^{j\omega_0})(z - e^{-j\omega_0})}{(z - re^{j\omega_0})(z - re^{-j\omega_0})} = \frac{K(z^2 - 2\cos\omega_0 z + 1)}{z^2 - 2r\cos\omega_0 z + r^2}$$

r is chosen such that the filter is highly selective, i.e., small 3-dB BW of the passband, ΔF.

$$r \approx 1 - \frac{(\Delta F)\pi}{F_S}, \quad 0.9 < r < 1$$

The gain factor K is inserted to ensure that the passband gain is 1, i.e., no amplification.

$$K = \frac{\left|1 - 2r\cos\omega_0 + r^2\right|}{2\left|1 - \cos\omega_0\right|}$$

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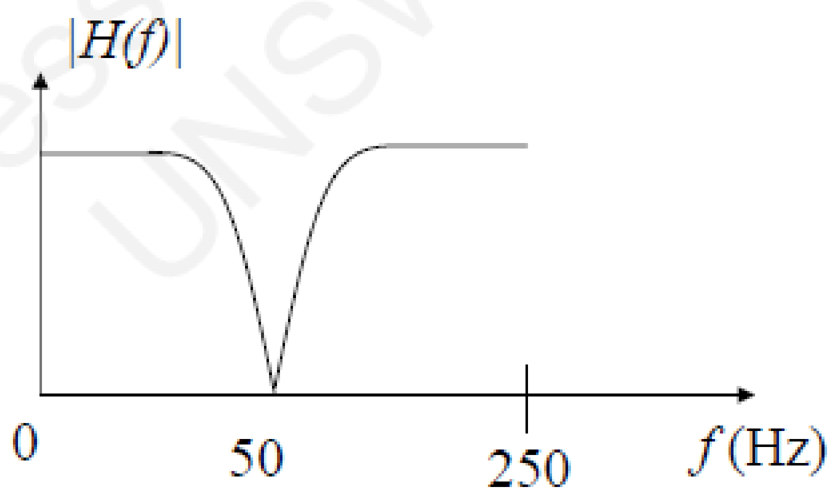
Example:

- Obtain, by the pole-zero placement method, the transfer function of a sample digital notch filter (see figure below) that meets the following specifications: [4]

  - Notch Frequency: 50Hz
  - 3db width of the Notch: ±5Hz
  - Sampling frequency: 500 Hz

The radius , r of the poles is determined by : $r = 1 - \left(\frac{\Delta f}{f_s}\right)\pi$



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Example 8.18 Design a second-order notch filter using the pole-zero placement method and satisfying the following specifications: Sampling rate = 8000 Hz

3-dB bandwidth = 100 Hz

Stopband center frequency = 1500 Hz

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A First-Order Lowpass Filter

- A wideband filter
- Pass frequency components from 0 to F_c, the cutoff frequency
- Place a zero on the unit circle at $z = -1$
- Place *a* pole on the real axis and inside the unit circle

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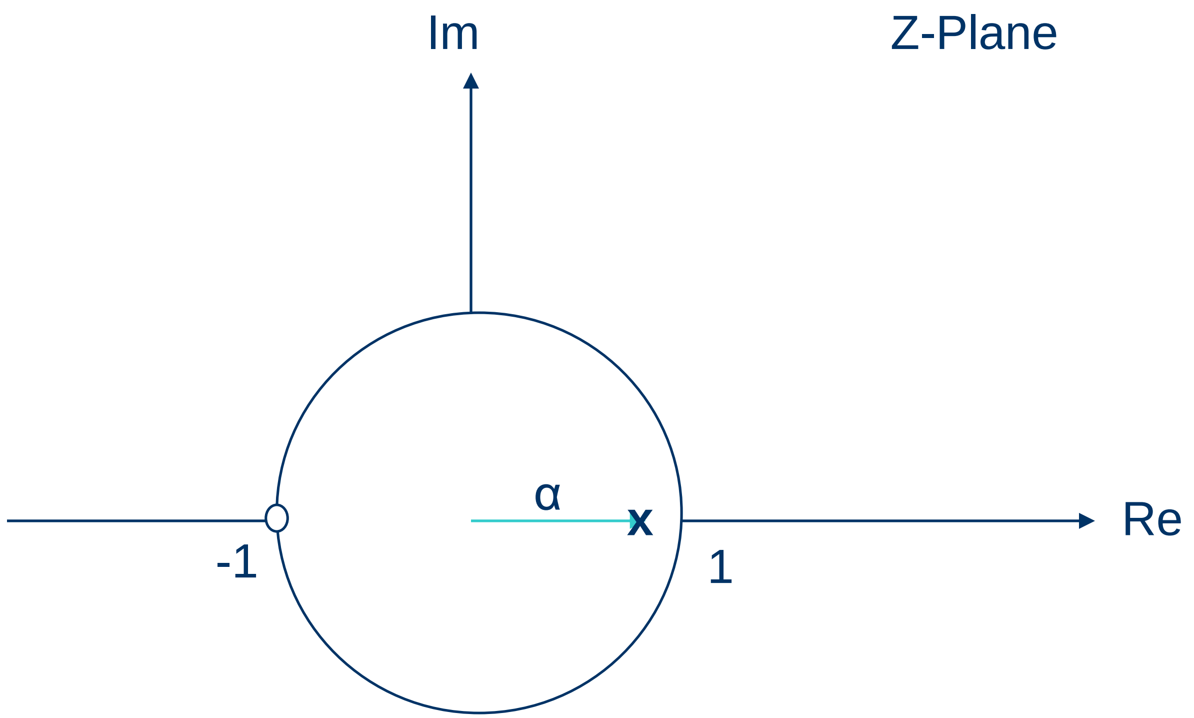
1^{st} order LowPassFilter



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$$H_{LP}(z) = \frac{K(z+1)}{z-\alpha}$$

$$\alpha \approx \begin{cases} 1 - 2\pi\left(\dfrac{F_c}{F_s}\right) & F_c < \left(\dfrac{F_s}{4}\right) \\ \pi - 1 - 2\pi\left(\dfrac{F_c}{F_s}\right) & F_c > \left(\dfrac{F_s}{4}\right) \end{cases}$$

The gain factor K is inserted to ensure that the passband gain is 1, i.e., no amplification

$$K = \frac{1-\alpha}{2}$$

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Example 8.19 Design a first-order lowpass filter using the pole-zero placement method and satisfying the following specifications: Sampling rate = 8000 Hz

3-dB cutoff frequency = 100 Hz

zero gain at 4000 Hz

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A First-Order Highpass Filter

- A wideband filter
- Suppress frequency components from 0 to F_c, the cutoff frequency
- Place a zero on the unit circle at $z = 1$
- Place *a* pole on the real axis and inside the unit circle

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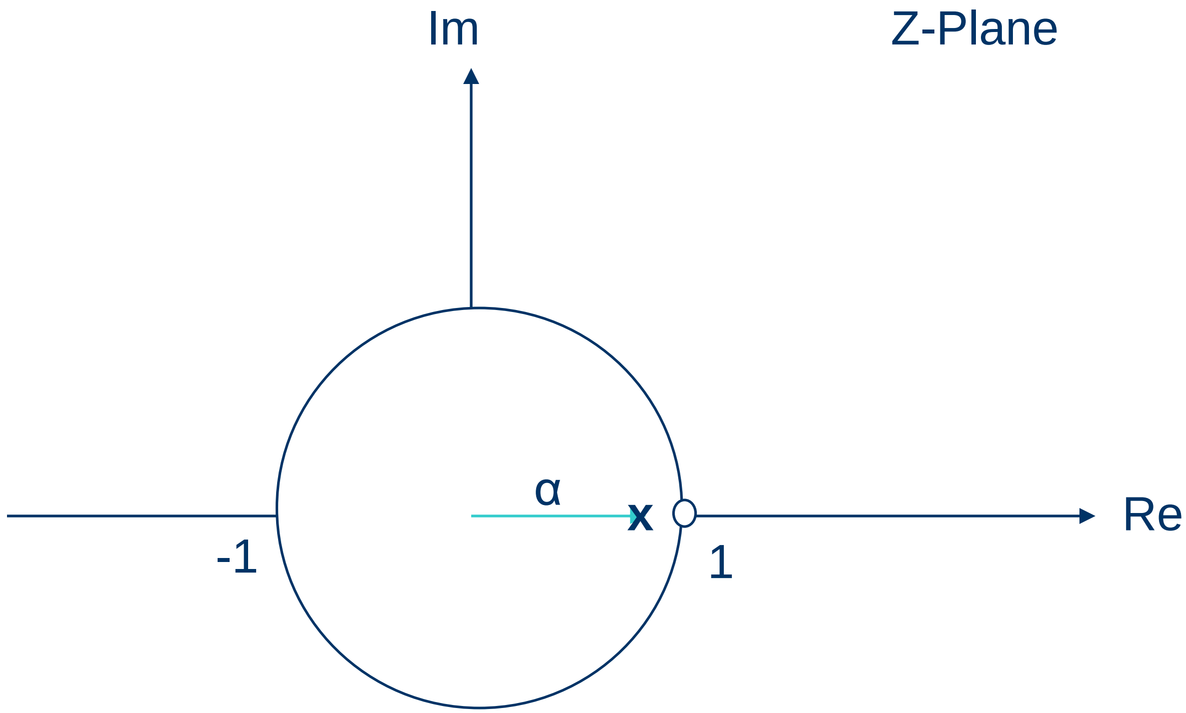
1st order HighPassFilter



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$$H_{HP}(z) = \frac{K(z-1)}{z-\alpha}$$

$$\alpha \approx \begin{cases} 1 - 2\pi\left(\dfrac{F_c}{F_s}\right) & F_c < \left(\dfrac{F_s}{4}\right) \\ \pi - 1 - 2\pi\left(\dfrac{F_c}{F_s}\right) & F_c > \left(\dfrac{F_s}{4}\right) \end{cases}$$

The gain factor K is inserted to ensure that the passband gain is 1, i.e., no amplification

$$K = \frac{1+\alpha}{2}$$

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Example 8.20 Design a first-order highpass filter using the pole-zero placement method and satisfying the following specifications:

Sampling rate = 8000 Hz

3-dB cutoff frequency = 3800 Hz

zero gain at zero Hz

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A Comb Filter

- Has several equally spaced passbands
- Passes DC, F_0, and its harmonics for a total of $\left\lfloor \frac{n}{2} \right\rfloor + 1$ resonant frequencies, where n is the order of the filter and $F_0 = F_S/n$
- Place n zeros at the origin
- Place n poles at the points equally spaced and inside the unit circle

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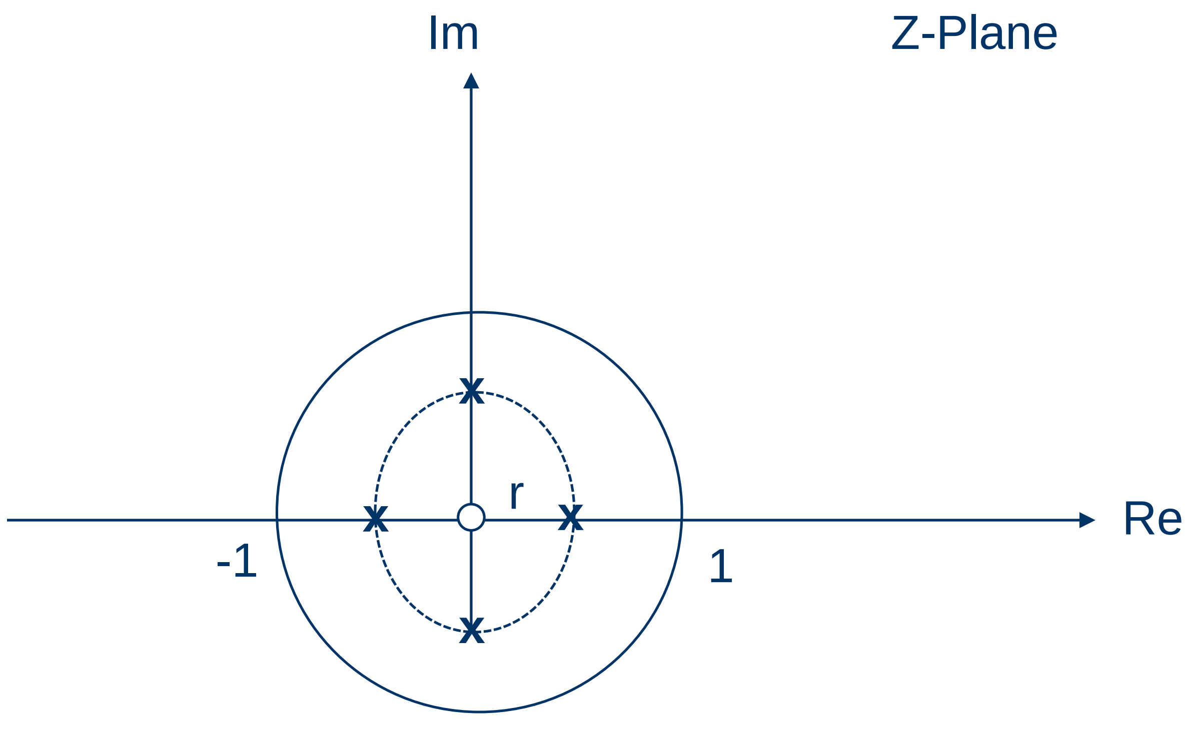
4^{th} order Comb Filter



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$$H_C(z) = \frac{Kz^n}{z^n - r^n}$$

r is chosen such that the filter is highly selective, i.e., small 3-dB BW of the passband, ΔF.

$$r \approx 1 - \frac{(\Delta F)\pi}{F_S}, \quad 0.9 < r < 1$$

The gain factor K is inserted to ensure that the passband gain is 1, i.e., no amplification

$$K = 1 - r^n$$

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Example Design a comb filter using the pole-zero placement method and satisfying the following specifications:

Sampling rate = 200 Hz

3-dB bandwidth = 1 Hz

Order of the filter is 10

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An Inverse Comb Filter

- A generalization of the notch filter
- Eliminates DC, F_0, and its harmonics for a total of $\left\lfloor \frac{n}{2} \right\rfloor + 1$ resonant frequencies, where n is the order of the filter and $F_0 = F_S/n$
- Place n zeros at the points equally spaced on the unit circle
- Place n poles at the points equally spaced and inside the unit circle

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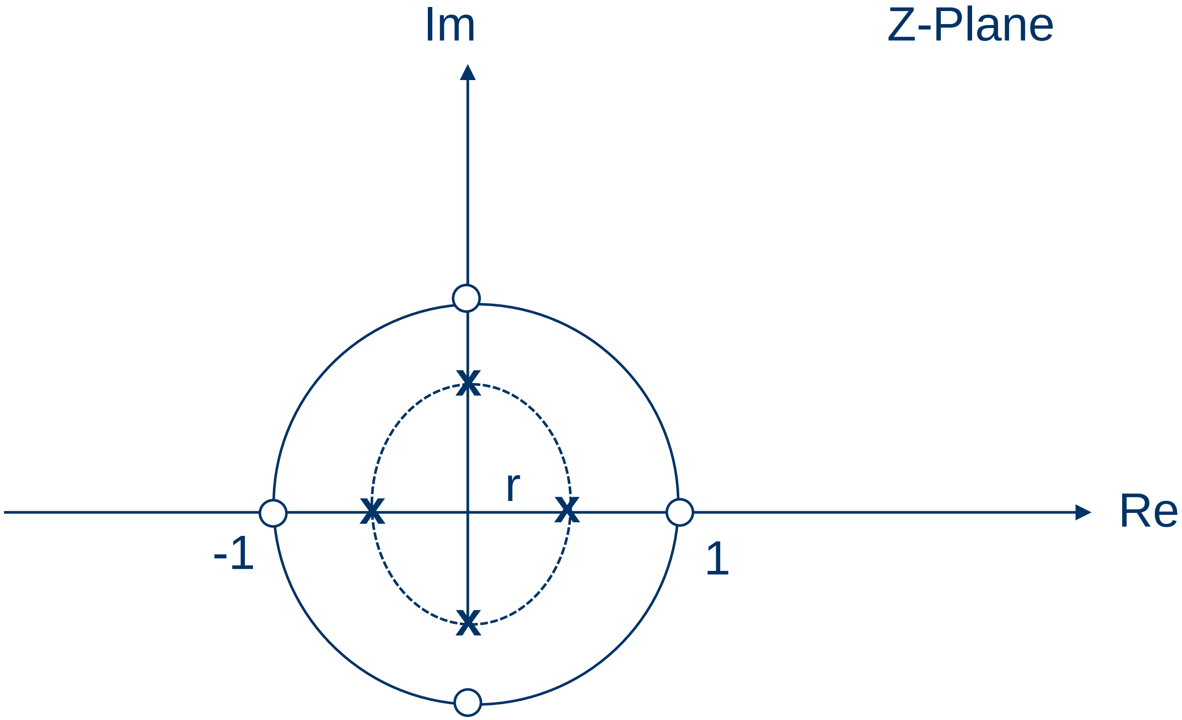
4^{th} order Comb Filter



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$$H_{IC}(z) = \frac{K(z^n - 1)}{z^n - r^n}$$

r is chosen such that the filter is highly selective, i.e., small 3-dB BW of the passband, ΔF.

$$r \approx 1 - \frac{(\Delta F)\pi}{F_S}, \quad 0.9 < r < 1$$

The gain factor K is inserted to ensure that the passband gain is 1, i.e., no amplification.

$$K = \frac{1 + r^n}{2}$$

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Example Design an inverse comb filter using the pole-zero placement method and satisfying the following specifications:

Sampling rate = 2200 Hz

3-dB bandwidth = 10 Hz

Order of the filter is 11

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