BIRZEIT UNIVERSITY

Faculty of Engineering

Electrical and Computer Engineering Department

Digital Signal Processing, ENCS4310

Suggested Problems on Chapter 3

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Problem#1:

Determine the z-transform, including the region of convergence, for each of the following sequences:

(a) $\left(\frac{1}{2}\right)^n u[n]$

(b) $-\left(\frac{1}{2}\right)^n u[-n-1]$

(c) $\left(\frac{1}{2}\right)^n u[-n]$

(d) $\delta[n]$

(e) $\delta[n-1]$

(f) $\delta[n+1]$

(g) $\left(\frac{1}{2}\right)^n (u[n] - u[n-10])$

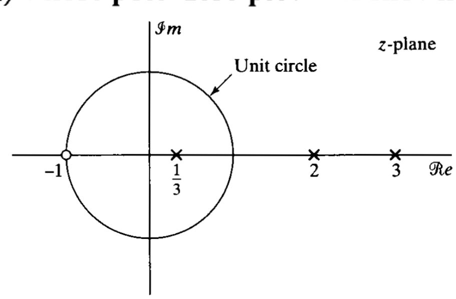
Problem #2:

Determine the z-transform of the sequence

$$x[n] = \begin{cases} n, & 0 \le n \le N-1, \\ N, & N \le n. \end{cases}$$

Problem #3:

Consider the z-transform $X(z)$ whose pole–zero plot is as shown in Figure



(a) Determine the region of convergence of $X(z)$ if it is known that the Fourier transform exists. For this case, determine whether the corresponding sequence $x[n]$ is right sided, left sided, or two sided.

(b) How many possible two-sided sequences have the pole–zero plot

(c) Is it is possible for the pole-zero plot in Figure to be associated with a sequence that is both stable and causal? If so, give the appropriate region of convergence.

Problem #4:

Determine the sequence $x[n]$ with z-transform

$$X(z) = (1 + 2z)(1 + 3z^{-1})(1 - z^{-1}).$$

Problem #5:

For the following z −transfrom functions. Determine the inverse z-transform. In addition, in each case whether the Fourier Transform exists

(a) $X(z) = \dfrac{1}{1 + \frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$

(b) $X(z) = \dfrac{1}{1 + \frac{1}{2}z^{-1}}, \quad |z| < \frac{1}{2}$

(c) $X(z) = \dfrac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}, \quad |z| > \frac{1}{2}$

(d) $X(z) = \dfrac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-2}}, \quad |z| > \frac{1}{2}$

(e) $X(z) = \dfrac{1 - az^{-1}}{z^{-1} - a}, \quad |z| > |1/a|$

Problem#6:

The input to a causal linear time-invariant system is

$$x[n] = u[-n-1] + \left(\tfrac{1}{2}\right)^n u[n].$$

The z-transform of the output of this system is

$$Y(z) = \frac{-\frac{1}{2}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + z^{-1}\right)}.$$

(a) Determine $H(z)$, the z-transform of the system impulse response. Be sure to specify the region of convergence.
(b) What is the region of convergence for $Y(z)$?
(c) Determine $y[n]$.

Problem #7:

The system function of a causal linear time-invariant system is

$$H(z) = \frac{1 - z^{-1}}{1 + \frac{3}{4}z^{-1}}.$$

The input to this system is

$$x[n] = \left(\tfrac{1}{3}\right)^n u[n] + u[-n-1].$$

(a) Find the impulse response of the system, $h[n]$.
(b) Find the output $y[n]$.
(c) Is the system stable? That is, is $h[n]$ absolutely summable?

Problem #8:

When the input to an LTI system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + (2)^n u[-n-1],$$

the corresponding output is

$$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n].$$

(a) Find the system function $H(z)$ of the system. Plot the pole(s) and zero(s) of $H(z)$ and indicate the region of convergence.
(b) Find the impulse response $h[n]$ of the system.
(c) Write a difference equation that is satisfied by the given input and output.
(d) Is the system stable? Is it causal?

Problem #9:

Determine the inverse z-transform of

$$X(z) = \log 2\left(\tfrac{1}{2} - z\right), \qquad |z| < \tfrac{1}{2},$$

Problem #10:

For each of the following pairs of input z-transform $X(z)$ and system function $H(z)$, determine the region of convergence for the output z-transform $Y(z)$:

(a)

$$X(z) = \frac{1}{1 + \frac{1}{2}z^{-1}}, \qquad |z| > \frac{1}{2}$$

$$H(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}, \qquad |z| > \frac{1}{4}$$

(b)

$$X(z) = \frac{1}{1 - 2z^{-1}}, \qquad |z| < 2$$

$$H(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}, \qquad |z| > \frac{1}{3}$$

(c)

$$X(z) = \frac{1}{\left(1 - \frac{1}{5}z^{-1}\right)\left(1 + 3z^{-1}\right)}, \qquad \frac{1}{5} < |z| < 3$$

$$H(z) = \frac{1 + 3z^{-1}}{1 + \frac{1}{3}z^{-1}}, \qquad |z| > \frac{1}{3}$$

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