BIRZEIT UNIVERSITY
Faculty of Engineering
Electrical And Computer Engineering Department
Communication Systems– ENEE 3309
Suggested Problems

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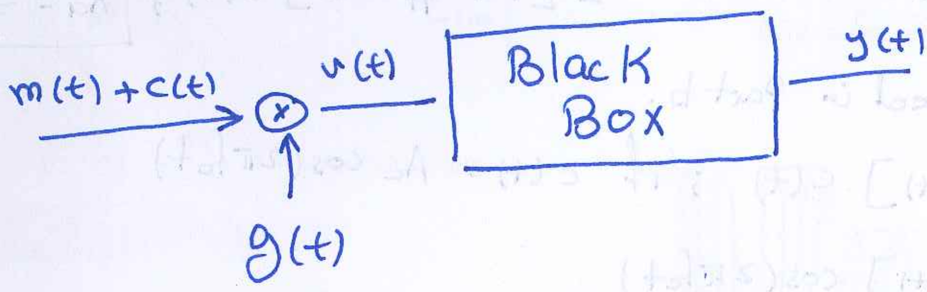
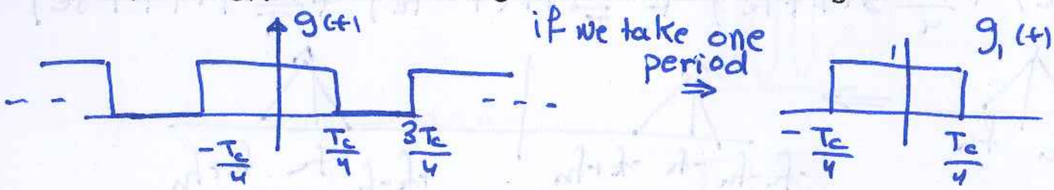
Problem #1 (10 points): Consider the following modulator system shown below. Assume $c(t) = A_c \cos(2\pi f_c t)$, and

$$g(t) = f(x) = \begin{cases} 1, & -\frac{T_c}{4} \le t < \frac{T_c}{4} \\ 0, & \frac{T_c}{4} \le t < \frac{3T_c}{4} \end{cases}$$

$g(t + T_c) = g(t)$.



a. Represent g(t) in terms of the trigonometric Fourier series signal.



$g_1(t) = \Pi\left(\frac{t}{\tau}\right), \tau = \frac{T_c}{2}$

For periodic signal

$g(t) = \sum_{n=\infty}^{\infty} g_n e^{j2\pi n f_c t}$ where g_n can be evaluated by using Transform

method;

$g_n = f_c G_1(nf_c)$; since $g_1(t) = \Pi\left(\frac{t}{\tau}\right) \Rightarrow G_1(f) = \tau \operatorname{sinc}(\tau f)$

$\Rightarrow g_n = f_c G_1(nf_c) = f_c \tau \operatorname{sinc}(\tau n f_c) = f_c \frac{T_c}{2} \operatorname{sinc}\left(n \frac{T_c}{2} f_c\right) = \frac{1}{2} \cdot \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi/2}$

$$\Rightarrow g_n = \begin{cases} \frac{1}{2} & n = 0 \\ \frac{1}{n\pi} & n = 1, 5, 9, \ldots \\ -\frac{1}{n\pi} & n = 3, 7, \ldots \\ 0 & \text{o.w} \end{cases}$$

if we take only $n > 0$

$\Rightarrow a_n = 2\,\mathrm{Re}\{X_n\} = \begin{cases} \frac{2}{n\pi} & n = 1, 5, \ldots \\ -\frac{2}{n\pi} & n = 3, 7, \ldots \\ 0 & n \text{ even} \end{cases}$; $a_0 = X_0 = \frac{1}{2}$ and $b_n = 0$ (even function)

b. Implement a suitable system inside Black Box to get a signal $y(t) = A_c\,[1 + k_a m(t)]\cos(2\pi f_c t)$. Specify the value of constant k_a.

By using Fourier series $g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_c t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_c t)$

$\Rightarrow g(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi f_c (3) t) + \frac{2}{5\pi}\cos(2\pi f_c (5) t) + \cdots$

$\downarrow$ if we use B.P.F centered at f_c on the following system

$v(t) = [m(t) + c(t)]\, g(t) = [m(t)+c(t)]\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \cdots\right]$

$\downarrow$ after B.P.F centered at f_c

$y(t) = \frac{1}{2}c(t) + \frac{2}{\pi} m(t)\, c(t) = \frac{1}{2}\left[1 + \frac{4}{\pi} m(t)\right] c(t)$; $\boxed{k_a = \frac{4}{\pi}}$

c. Plot the spectrum of the signal $y(t)$

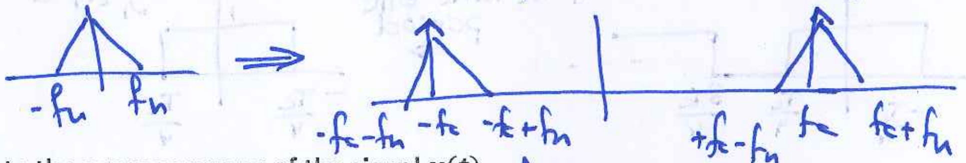
Based on result obtained in part b.

$y(t) = \frac{1}{2}\left[1 + \frac{4}{\pi} m(t)\right] c(t)$; if $c(t) = A_c \cos(2\pi f_c t)$

$= \frac{A_c}{2}\left[1 + \frac{4}{\pi} m(t)\right]\cos(2\pi f_c t)$

$Y(f) = \frac{A_c}{4}\delta(f - f_c) + \frac{A_c}{4}\delta(f + f_c) + \frac{2}{\pi} M(f - f_c) + \frac{2}{\pi} M(f + f_c)$

if $M(f)$



d. Evaluate the average power of the signal $y(t)$. ~~Assume~~

average Power $= \left(\frac{A_c}{4}\right)^2 \cdot 2 + \left(\frac{2}{\pi}\right)^2 |M(f - f_c)|^2 + \left(\frac{2}{\pi}\right)^2 |M(f + f_c)|^2$

If we assume $m(t) = A_m \cos(2\pi f_m t) \Rightarrow M(f) = \frac{A_m}{2}\delta(f - f_m) + \frac{A_m}{2}\delta(f + f_m)$

$\Rightarrow [M(f - f_c) + M(f + f_c)] = \frac{A_m}{2}\delta(f - f_c + f_m) + \frac{A_m}{2}\delta(f - f_c - f_m) + \frac{A_m}{2}\delta(f + f_c - f_m) + \frac{A_m}{2}\delta(f + f_c + f_m)$

$\Rightarrow P_{avg} = \left(\frac{A_c}{4}\right)^2 \cdot 2 + \left(\frac{2}{\pi}\right)^2 \cdot \left(\frac{A_m}{2}\right)^2 \cdot 4$

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