

12.1: Three dimensional Coordinate systems

①

• If $P(x_1, y_1, z_1) = Q(x_2, y_2, z_2)$

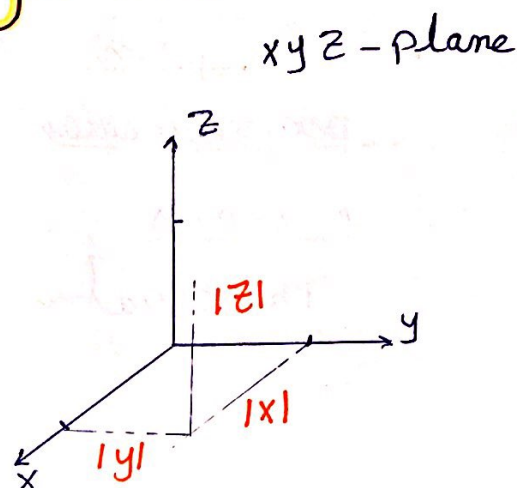
Then $x_1 = x_2, y_1 = y_2, z_1 = z_2$

• xyz plane :-

If $y=0 \rightarrow$ plane (xz)

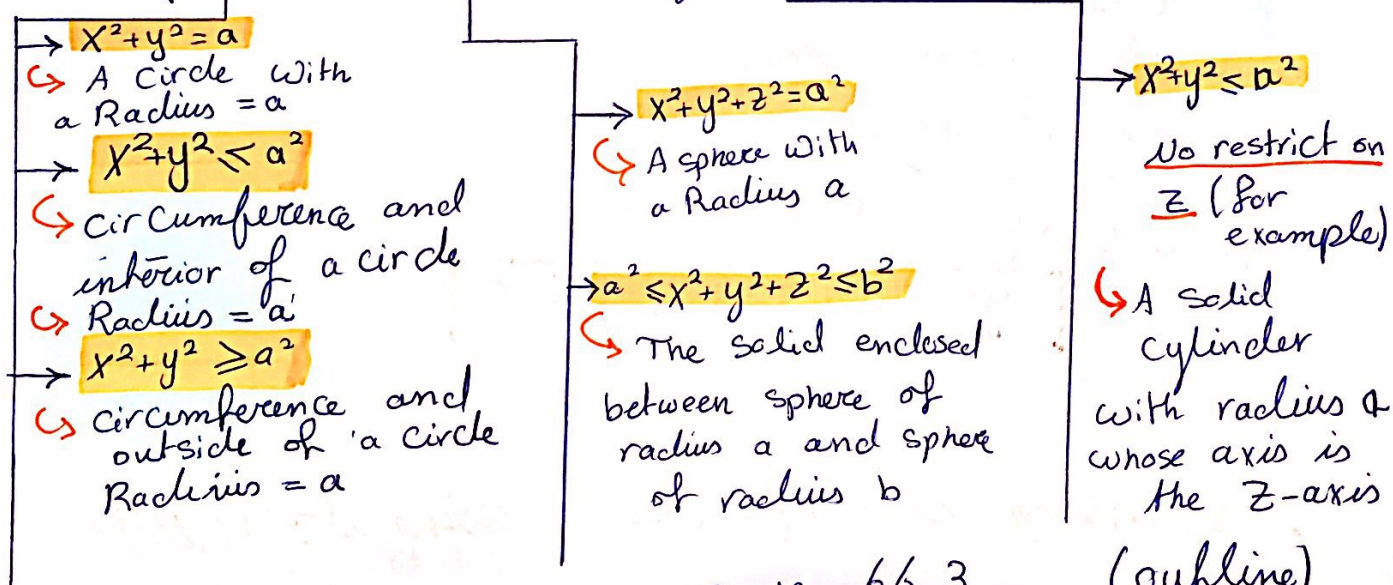
If $x=0 \rightarrow$ plane (yz)

If $z=0 \rightarrow$ plane (xy)



• How to describe Geometry ?

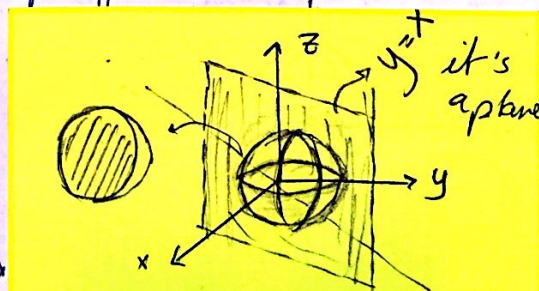
(circle, sphere, cylinder)



• Example: Question 14

$x^2 + y^2 + z^2 = 4$ $y = x$

The description:-
it's a circle formed by the intersection of the sphere and yz



page 663 (outline)

• Describing a set given with an Equation

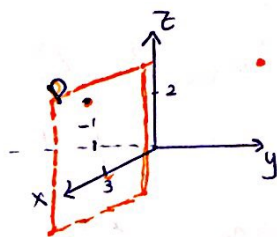
Example: Question 26 page 664 (outline)

- The plane through the point $(3, -1, 2)$ perpendicular to the

x -axis

The equation is $x=3$

• The plane should be perpendicular to the x -axis so $x = \boxed{3}$



• It has to go through the point so $x=3$

- Example: The circle of Radius 2 centered at $(0, 0, 0)$ and lying in

b) yz -plane

(Question 28 Page 664)

$a=2$

center = $(0, 0, 0)$ origin

lying at yz -plane ($x=0$)

$$(y-0)^2 + (z-0)^2 = a^2$$



$$\boxed{y^2 + z^2 = 4, \quad x=0}$$

• Describing sets of points using inequalities

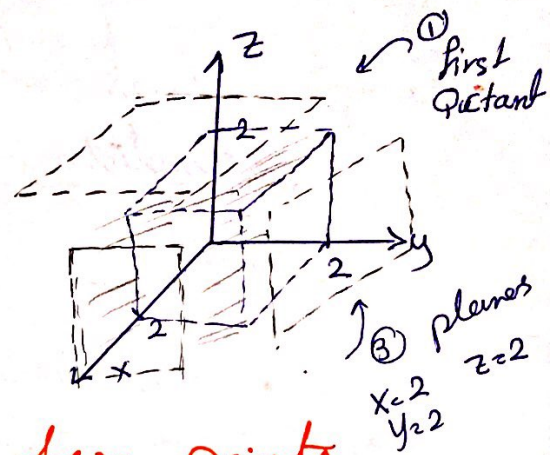
Example: Question 36 page 664 (outline)

- Solid Cube in the first octant bounded by the coordinate planes and the planes $x=2$, $y=2$, $z=2$

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So the inequalities are:-

$$0 \leq x \leq 2, \quad 0 \leq y \leq 2, \quad 0 \leq z \leq 2$$



• finding Distance between two points

Example:- Question 43 page 664 (outline)

$$P_1 (1, 4, 5)$$

$$P_2 (4, -2, 7)$$

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$d = \sqrt{(4-1)^2 + (-2-4)^2 + (7-5)^2}$$

$$= \sqrt{9 + 36 + 4} = \sqrt{49} = 7$$

• finding Radius of a sphere

Example:- Question 55 page 664 (outline)

$$x^2 + y^2 + z^2 + 4x - 4z = 0$$

• يجب ان نكمل كل من x, y, z بدلالة $(x-a)^2, (y-b)^2, (z-c)^2$
 • اكمل المثلث المربع

$$x^2 + 4x + y^2 + z^2 - 4z = 0$$

$$(x^2 + 4x + 4) - 4 + y^2 + (z^2 - 4z + 4) - 4 = 0$$

$$(x+2)^2 - 4 + y^2 + (z-2)^2 - 4 = 0$$

$$(x+2)^2 + y^2 + (z-2)^2 = 8 \rightarrow \text{radius} = \sqrt{8} = 2\sqrt{2}$$

$$\text{center} = (-2, 0, 2)$$

- Question 64 (outline)
- Example: find an equation for all the points equidistant from the point $(0,0,2)$ and the xy plane

assuming the point is (x,y,z)
 from $(0,0,2)$ $d_1 = d_2$ ← From xy -plane

$$\sqrt{(x-0)^2 + (y-0)^2 + (z-2)^2} = \sqrt{(x-x)^2 + (y-y)^2 + (z-0)^2}$$

$$x^2 + y^2 + (z-2)^2 = z^2$$

$$x^2 + y^2 + z^2 - 4z + 4 = z^2$$

$$x^2 + y^2 - 4z + 4 = 0$$

$$z = \frac{x^2 + y^2}{4} + 1$$

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