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8.7

$$\int_1^{\infty} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1} & \text{if } p>1 \quad \text{conv} \\ \infty & \text{if } p\leq 1 \quad \text{div} \end{cases}$$

$$\int_0^{1} \frac{dx}{x^p} = \begin{cases} \frac{1}{1-p} & \text{if } p<1 \\ \infty & \text{if } p\geq 1 \end{cases}$$

$\lim_{n\to\infty} \frac{\ln n}{n}=0$,, $\lim_{n\to\infty} \sqrt[n]{n}=1$,,, $\lim_{n\to\infty} x^{\frac{1}{n}}=1$, $x>0$,,, $\lim_{n\to\infty} x^n=0$, $|x|<1$ (div او ∞ غير ذلك) ,,, $\lim_{n\to\infty} (1+\frac{x}{n})^n=e^x$,,, $\lim_{n\to\infty} \frac{x^n}{n!}=0$ "Taylor Series"

* A seqence $\{a_n\}$ is bounded from above if $\exists$ a number M st $a_n \leq M$ for all $n \Rightarrow$ least upper bound ← أول رقم بكون يساوي او أكبر حده
* A seqence $\{a_n\}$ is bounded from below if $\exists$ a number m. st $a_n \geq m$ for all $n \Rightarrow$ greatest lower bound ← أول رقم بكون يساوي او حده
* A sequnce $\{a_n\}$ is bounded if it is bounded from above and is bounde from below.
* A sequnce $\{a_n\}$ is not bounded if it is not bounded from above and is not bounded from below.
* A sequnce $\{a_n\}$ is nondecreasing if $a_n \leq a_{n+1}$ $\forall n$ $a_1 \leq a_2 \leq a_3 \leq \dots$
* A sequnce $\{a_n\}$ is nonincreasing if $a_n \geq a_{n+1}$ $\forall n$ $a_1 \geq a_2 \geq a_3 \geq \dots$
* A sequnce $\{a_n\}$ is montonic if it's either nondecreasing or nonincreasing
* If a sequnce $\{a_n\}$ is both bounded and montonic then $\{a_n\}$ converge

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Test 1 (n^{th} partial sum test)

$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$

Find $S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$

- if $\lim_{n\to\infty} S_n = L$ then $\sum_{n=1}^{\infty} a_n$ converge to L
- if $\lim_{n\to\infty} S_n = \text{div}$ then $\sum_{n=1}^{\infty} a_n$ diverge

* Telscoping series $\Rightarrow$ A series which terms cancel together to leave only two terms.

Test 2 n^{th} term test (for div)

$\sum_{n=1}^{\infty} a_n$

- If $\lim_{n\to\infty} a_n \neq 0$ (or $\infty, -\infty$) then $\sum_{n=1}^{\infty} a_n$ div
- If $\lim_{n\to\infty} a_n = 0 \Rightarrow$ The test fail

* Harmonic Series div $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$

*

Sequence a_n	series $\sum_{n=1}^{\infty} a_n$
$a_n = \frac{1}{n}$	$\sum \frac{1}{n}$ diverge
$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$	
a_n converge to zero	

Test 3 Geometric Series Test

- If $|r|<1 \Rightarrow$ converg to sum $\Rightarrow$ sum $= \frac{a}{1-r}$ (أول حد)
- If $|r| \geq 1 \Rightarrow$ diverge

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Integral Test (IT)

consider $\sum_{n=k}^{\infty} a_n$, wher:

إذا عدد (0, 1, 2, . .) بمعنى conv

إذا ∞ بمعنى div

- a_n positiv terms
- $a_n = f(n)$ is cont, positive, decreasing on $[k, \infty)$

Then $\sum_{n=k}^{\infty} a_n$ and $\int_k^{\infty} f(x)\,dx$ both conv or both div

P-series Test:

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ conv if $p > 1$ by Integral test

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ div if $p \leqslant 1$ by Integral Test

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Direct Comparison Test (DC)

let $\sum a_n, \sum c_n, \sum d_n$ be series with nonnegative terms.

suppos that $d_n \leqslant a_n \leqslant c_n$ for all n large

- if c_n converges, then $\sum a_n$ also converge.
- if $\sum d_n$ diverge, then $\sum a_n$ also diverges.

The limit Comparison Test (LCT)

Suppos that $a_n \geqslant 0$ and $b_n \geqslant 0$ for all $n \geqslant N$ (large n)

* من الطرق لـ إيجاد bn هي اخذ القوى بين القوى في an مثل

1) if $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then $\sum a_n$ and $\sum b_n$ both converges or both diverges

$\sum_{n=1}^{\infty} \frac{n^1+2}{n^3+n^2+5}$ ∴ $b_n = \frac{1}{n^2}$

2) if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges

3) if $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges

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Ratio test (RT)

عادة" بستخدم عند وجود المضروب (!n)

Consider the infinite series $\sum a_n$ with positive terms. Assume $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$ Then:

- if $\rho < 1$, Then the series converges.
- if $\rho > 1$, Then the series diverges.
- if $\rho = 1$, Then the test is inconclusive.

The Root test

Consider the infinite series $\sum a_n$ with $a_n \geqslant 0$ for $n \geqslant N$. Assume $\lim_{n\to\infty} \sqrt[n]{a_n} = \rho$ then:

- if $\rho < 1$, Then the series converges.
- if $\rho > 1$, Then the series diverges.
- if $\rho = 1$, Then the test is inconclusive.

Alternating series Test (AST)

$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} U_n = U_1 - U_2 + U_3 - U_4 + \dots$

AST: $\sum (-1)^{n+1} U_n$ conv if

* إذا تحققت جميع الشروط ← conv

1) $U_n > 0 \ \forall n$, $U_n = |a_n|$

* إذا لم يتحقق الثالث ← $\sum a_n$ div by n^{th} term test

2) $U_n \downarrow$ for large n ($U_{n+1} \leq U_n$)

* اذا لم يتحقق الثاني فقط ← we can't say $\sum a_n$ conv or div (Test fail)

3) $\lim_{n\to\infty} U_n = 0$ "if not ⇒ $\sum (-1)^{n+1} U_n$ div by n^{th} term test"

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Conv Abs

if $\lim_{n\to\infty} \sum |a_n|$ conv, Then $\sum_{n=1} a_n$ Conv Abs

Converge Conditionally

The infinite series $\sum a_n$ converge conditionally if it converge by AST but not Abs

Alternating P-series Test

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p} = \begin{cases} \text{conv. Abs} & \text{if } P>1 \\ \text{conv. condi} & \text{if } 0<P\leq 1 \\ \text{div by } n^{th} \text{ term Test} & \text{if } P\leq 0 \end{cases}$$

Alternating Estimation Th

- Assume $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} U_n = U_1 - U_2 + U_3 - U_4 + \cdots + (-1)^{n+1} U_n = L$
- If we approximate L by $S_n = U_1 - U_2 + U_3 + \cdots + (-1)^{n+1} U_n$ Then:

1) The remainder $L - S_n$ has same sign as a_{n+1}

2) The error $|L - S_n| < U_{n+1} = |a_{n+1}|$

3) $\min\{S_n, S_{n+1}\} < L < \max\{S_n, S_{n+1}\}$ $\forall n$

* الإيرور إشارته = إشارة الحد اللي بعده يعني لو أخذت S_4 الإيرور إله إشارة S_5

* الـ L يقدر أوجدها من خلال انه آخذ عدد حدود معينة S_n واجمعهم من خلال قاعدة الـ Geometric و
اللي هي $sum = \frac{1}{a-r}$ وهيك بطلع الـ L وبعدها بجمع الحدود بشكل طبيعي وبطلع S_n

* واذا اعطاني الـ Series وطلب أوجد الـ sum اللي بعطيني error أقل من قيمة معينة بروح بعمل هاي الخطوات
1) القيمة اللي اعطاني اياها > U_n، وبوجد من خلالها قيمة n مثلاً "لنفترض" $n \geq 5$
2) بوجد الـ S_n للـ n اللي طلعت معي ⇐ S_5
3) $error = |L - S_n| < U_{n+1}$

Power Series

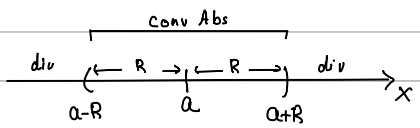
$$\sum_{n=0}^{\infty} a_n (x-a)^n = \underbrace{a_0}_{a_i} + \underbrace{a_1}_{a_i}(x-a) + \underbrace{a_2}_{a_i}(x-a)^2 + \underbrace{a_3}_{a_i}(x-a)^3 + \ldots$$

a_i : cofficients

a : center

R : Radias of convergence

IC : Interval of convergence



$IC = |x - a| < R \Rightarrow -R < x - a < R \rightarrow a - R < x < a + R$

* To Find R and IC ⇒ we apply Ratio Test (أغلب الأوقات)