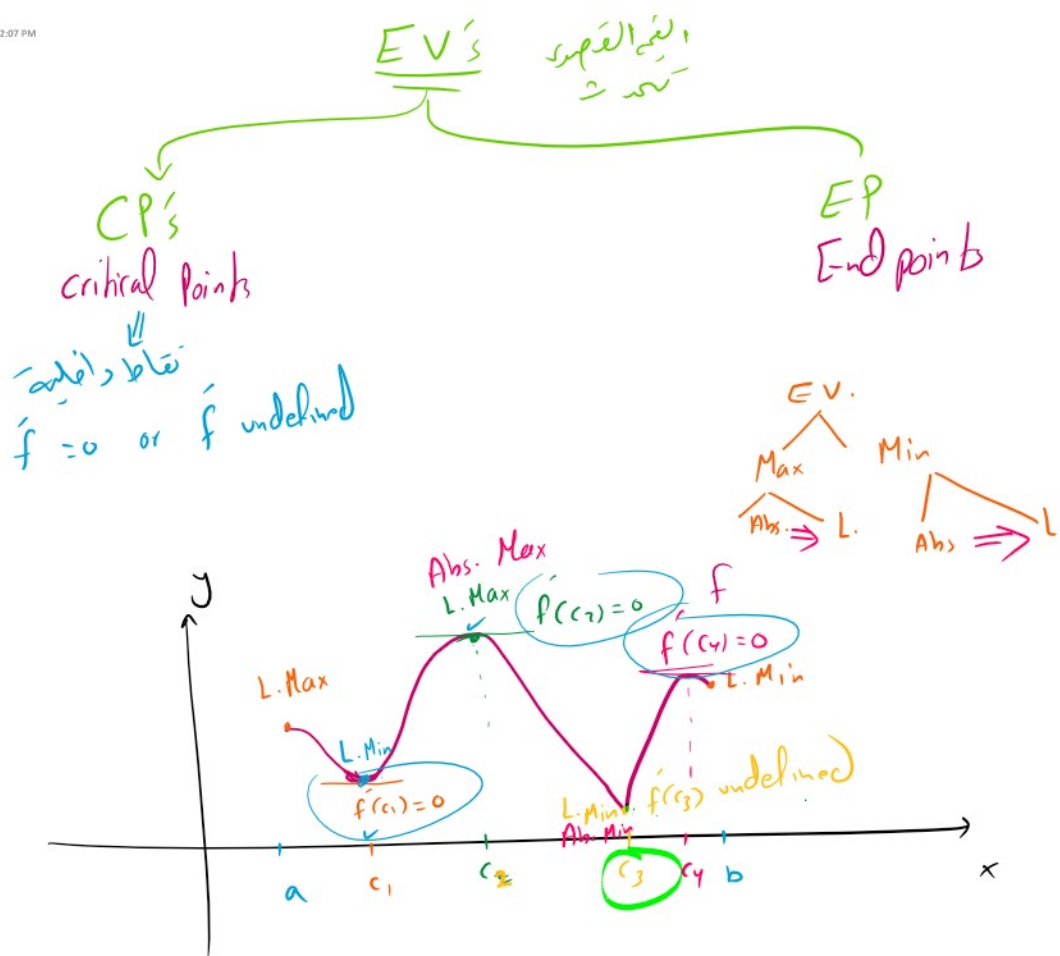




- f has L. Max of $f(a)$ at $x=a$
- f has L. Min of $f(c_1)$ at $x=c_1$
- f has L. Max (Abs) of $f(c_2)$ at $x=c_2$
- f has L. Min (Abs) of $f(c_3)$ at $x=c_3$
- f has L. Min of $f(b)$ at $x=b$
- f has L. Max of $f(c_4)$ at $x=c_4$

Th If f diff at c and f has EV at c then $f'(c) = 0$

Exp $f(x) = x^2$ on $[-2, 3]$
Find EV's

Find EV's

check CP's and check Endpoint

$$\begin{aligned} f' = 2x = 0 &\Rightarrow \boxed{x=0} \in [-2, 3] \\ &\Rightarrow x=0 \text{ is the only CP} \end{aligned}$$

$$\Rightarrow f(0) = 0^2 = 0 \quad \text{Abs. Min} \Rightarrow \text{L. Min}$$

(0, 0)

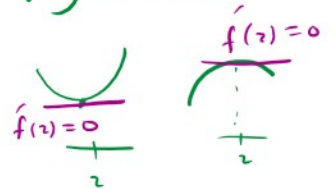
Endpoints

$$\begin{aligned} (-2, f(-2)) &= (-2, (-2)^2) = (-2, 4) \\ (3, f(3)) &= (3, 3^2) = (3, 9) \end{aligned}$$

(3, 9) $\Rightarrow$ Abs. Max $\Rightarrow$ L. Max

Exp Suppose $f(x) = \alpha x^3 - 12x$ has EV at $x=2$
Find α

f cont. $\Rightarrow f$ diff since f is polynomial
 $\Rightarrow f'(2) = 0$



$$f'(x) = 3\alpha x^2 - 12$$

$$f'(2) = 3\alpha(2)^2 - 12 = 0$$

$$3\alpha(4) - 12 = 0$$

$$12\alpha - 12 = 0$$

$$12\alpha = 12$$

$$\boxed{\alpha = 1}$$

Th says if f diff and has EV at $c \Rightarrow f'(c) = 0$

Th enforces at this Th is not true

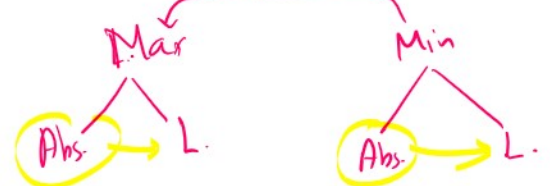
The converse of this Th is not true
 if $f'(c) = 0$, then f may not have EV at c

Exp $f(x) = x^3$ | $f'(0) = 0$ ~~$\Rightarrow$~~ f has EV
 $f'(x) = 3x^2 = 0$
 $3x^2 = 0$
 $x = 0$
 $f(0)$ is not EV



Q. How to classify the EV's

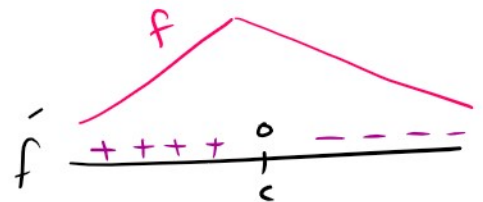
A. We can use FDT
S D T



Th FDT: (First Derivative Test)

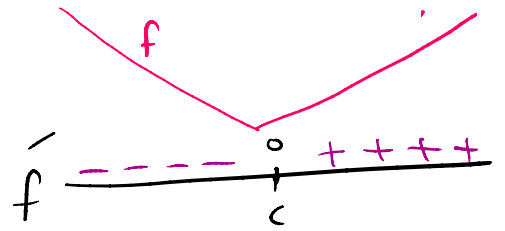
Assume f has CP at c
 f exists about c

(1) If f' changes sign
 from $+$ to $-$ at $x=c$
 then $f(c)$ is L. Max

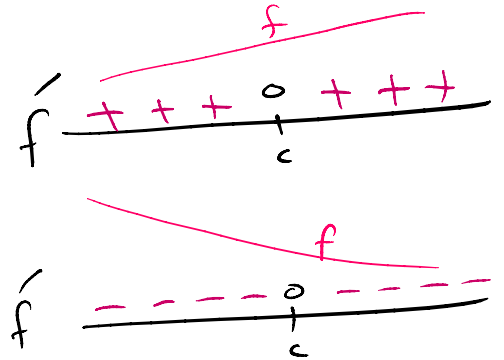


then $f(c)$ is L. Min

- (2) If f' changes sign from $-$ to $+$ at $x=c$ then $f(c)$ is L. Min



- (3) If f does not change sign at $x=c$ then f has no EV's



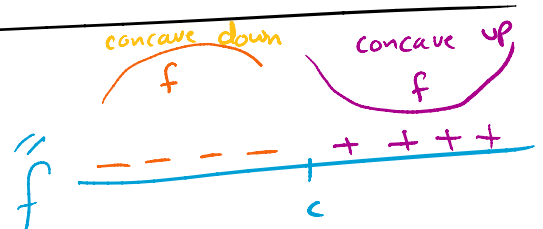
Th (SDT: Second Derivative Test)

Assume $f'(c) = 0$ and f'' is cont. about c

- (1) If $f''(c) < 0$ then $f(c)$ is L. Max
 (2) If $f''(c) > 0$ then $f(c)$ is L. Min
 (3) If $f''(c) = 0$ then Test fail

Remark (1) If $f''(x) \geq 0$
 $\forall x \in I$

then f concave up on I



- (2) If $f''(x) \leq 0$

② If $f'(x) \leq 0$
 $\forall x \in I$
 then f concave down

Def A point where f has tangent
 and f changes concavity
 is called inflection point

To find inflection point $\Rightarrow$ we start where
 $f''(x) = 0$
 and check

Exp $f(x) = x^4 - 4x^3 + 10$ Find

① $D(f) = \mathbb{R} = (-\infty, \infty)$

② CP's $\Rightarrow f'(x) = 4x^3 - 12x^2$

$f'(x) = 0 \Rightarrow 4x^2[x - 3] = 0$

$x = 0$ or $x = 3$

$\in D(f)$

$f(0) = 10$

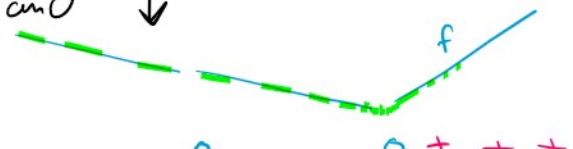
$\in D(f)$

$f(3) = 3^4 - 4(3)^3 + 10$
 $= 81 - 4(27) + 10$
 $= -17$

$(0, 10)$
 $(3, -17)$ $\rightarrow$ Critical Point

③ intervals where $f \uparrow$ and $\downarrow$

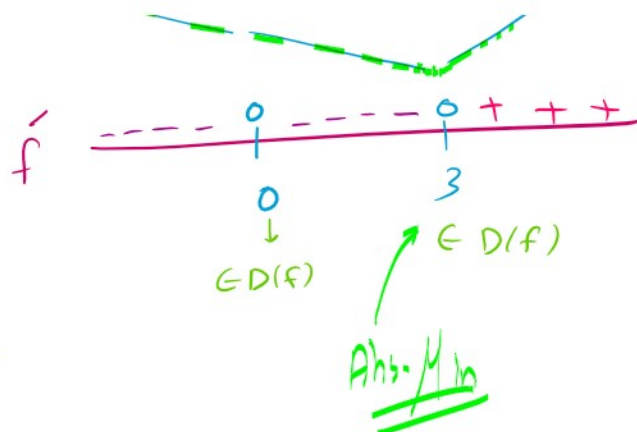
$f'(x) = 4x^3 - 12x^2$



$$f'(x) = 4x^3 - 12x^2$$

$$= 4x^2(x-3)$$

بسط $(x-3)$



$f \uparrow$ on $[3, \infty)$ ✓

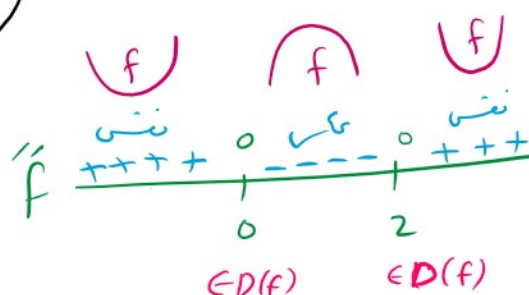
$f \downarrow$ on $(-\infty, 3]$ ✓

④ intervals of concavity
 $f'(x) = 4x^3 - 12x^2$

$$f''(x) = 12x^2 - 24x$$

$$f''(x) = 0 \Rightarrow 12x(x-2) = 0$$

$$x=0, x=2$$



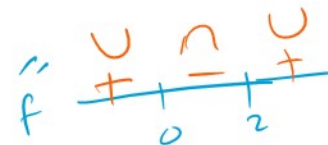
f is concave up on $(-\infty, 0] \cup [2, \infty)$

f is concave down on $[0, 2]$

⑤ inflection points

$$f'' = 0 \Rightarrow \begin{matrix} x=0 \in D(f) \\ x=2 \in D(f) \end{matrix}$$

change concavity ✓



$$f(x) = 4x^3 - 12x^2$$

$$f'(0) \checkmark$$

$$f'(2) \checkmark$$

$$f(0) = 0, f(2) = 16$$

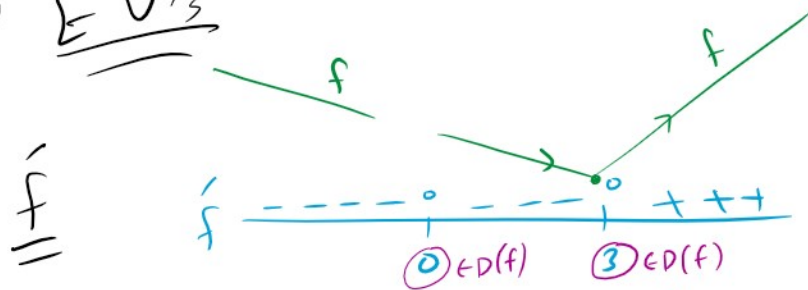
inflection point $(0, f(0)) = (0, 10)$
 $(2, f(2)) = (2, -6)$

$$f(2) = 2^4 - 4(2^3) + 10$$

$$= 16 - 4(8) + 10$$

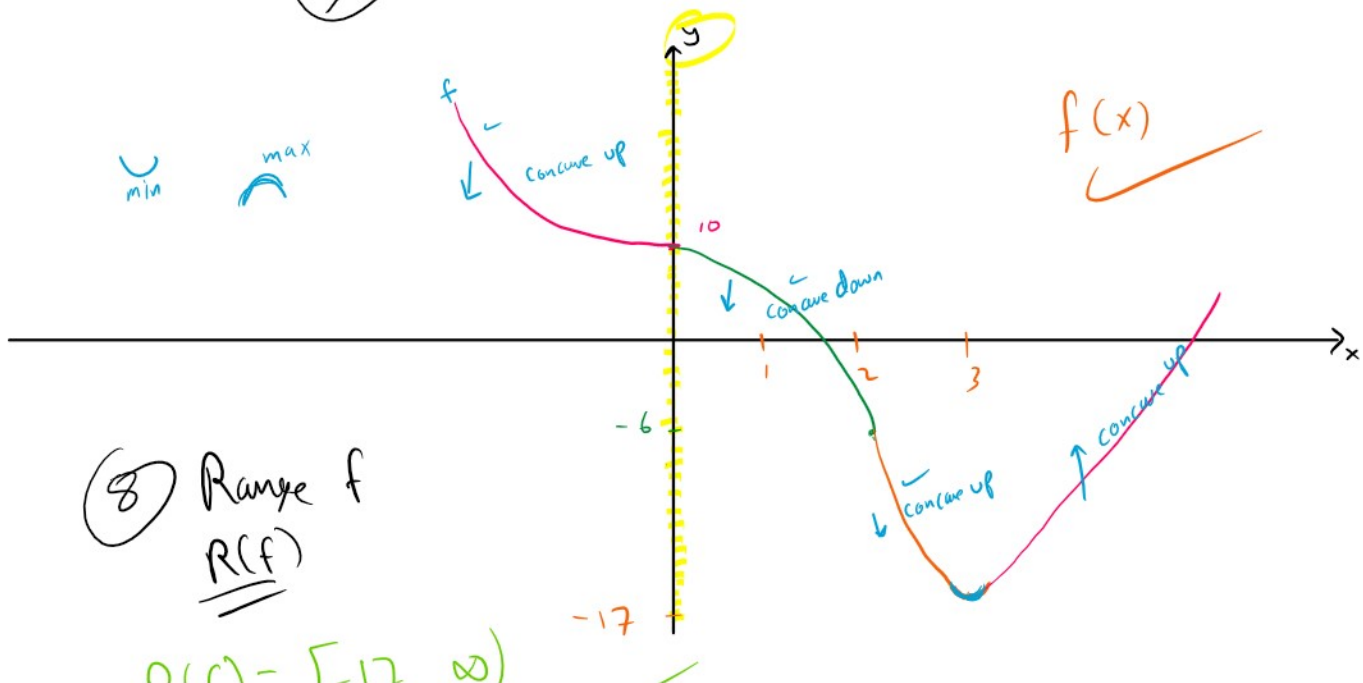
$$= -6$$

⑥ EV's



$(3, f(3)) = (3, -17)$ L Min
 (Abs. Min)
 $f(3) = -17$

⑦ sketch f



⑧ Range f
 $R(f)$

$$(-\infty, -17 \cup \infty)$$

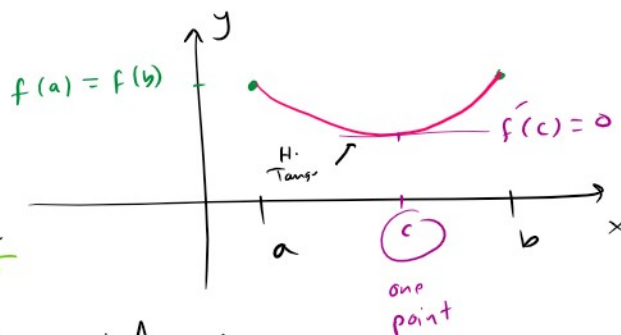
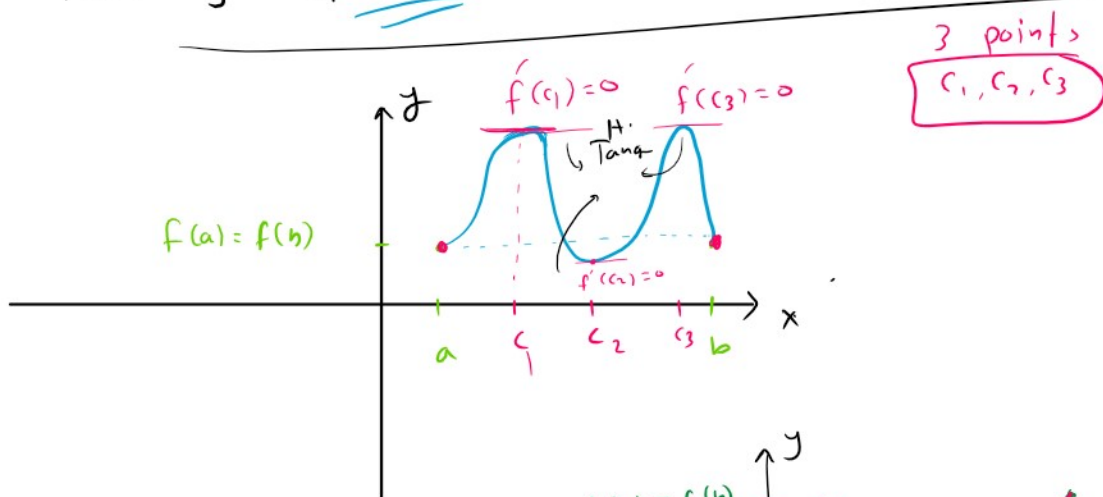
$$R(f) = [-17, \infty)$$

Th (Rolle's Th)

If f cont. on $[a, b]$

f diff on (a, b) s.t. $f(a) = f(b)$

then $\exists$ at least one point $c \in (a, b)$ s.t. $f'(c) = 0$



Exp $f(x) = \underline{\sin x} + \underline{2}$

① Does f have horizontal tangent on $[0, 2\pi]$

- ✓ f cont. on $[0, 2\pi]$
 - ✓ f diff on $(0, 2\pi)$
 - ✓ $f(0) = f(2\pi)$
- $$\begin{aligned} \sin 0 + 2 &= \sin 2\pi + 2 \\ 2 &= 2 \end{aligned}$$
- Rolle's Th

$\exists$ at least one point $c \in (0, 2\pi)$
 such that $\boxed{f'(c) = 0}$

Yes $\exists$ Horizontal tangent

(2) Find point where f has H. tangent
 $f'(x) = 0$

$$\cos x + 0 = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \dots \in \underline{\underline{(0, 2\pi)}}$$

(3) Sketch

$$f(x) = \sin x + 2$$

$[0, 2\pi)$

