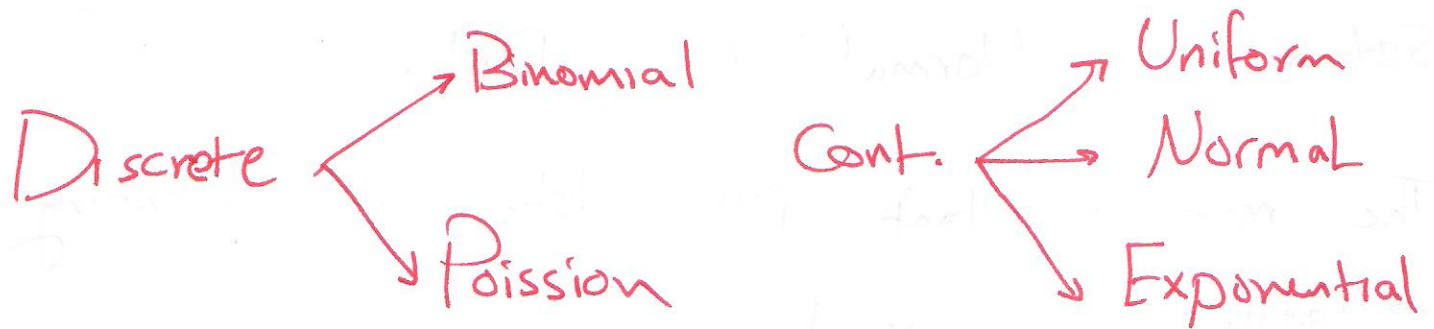


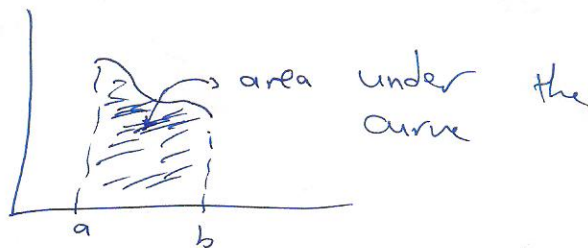
Ch.6 Continuous Probability Dist.



Continuous prob Dist: A prob. Dist in which the Random Variable X can take on any value

- IF X is a cont. random variable, then:-

$$P(a \leq X \leq b) = \int_a^b f(x) dx \quad \text{where } f(x) \text{ is the P.D.F.}$$



The total Area = 1
under the curve

* The total area under the curve $f(x)$, where it's the prob. Density

Section 6.1 :- Skip.

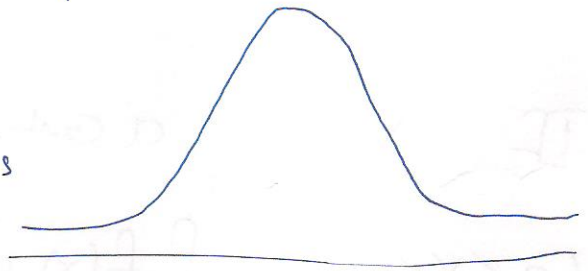
Section 6.2 :- Normal prob. Dist.

The most important prob. dist. for describing
a Continuous random variable

Normal curve :-

* It has a bell-shaped Dist.

* X : Cont. Normal R.V has
a prob. function P.D.F.,



$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ = Expected value

σ = Standard deviation

$$\pi = 3.1415$$

$$e = 2.718$$

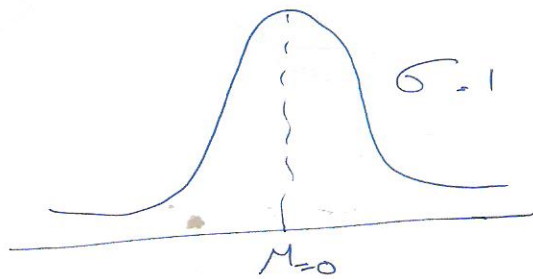
* The normal curve has 2 parameter μ, σ

$$N(\mu, \sigma)$$

Standard normal Dist

* Normal Dist $\boxed{\mu=0}$ and $\boxed{\sigma=1}$

$$N(\mu, \sigma) \longrightarrow N(0, 1)$$

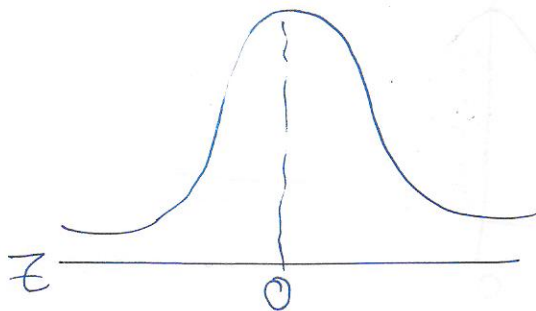


$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Standard normal Dist. func.

* To transform from normal prob. Dist. to Standard normal Dist [Use Z-score]

* Z table:- a table that is used to find the prob. for the Standard normal Dist.



- [1] The total Area under the curve = 1
- [2] The Area to the left of 0 = 0.5
- [3] The Area to the Right of 0 = 0.5
- [4] prob = Area under the curve.

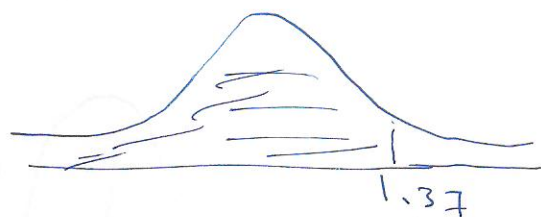
Entries in the table give the Area under

the curve to the left of the positive Z value

Exp:- Find

① $P(Z < 1.37) = f(Z < 1.37)$

	0.07
1.3	0.9147



② $P(Z \leq 1.37) =$

③ $P(Z < 2.45) = \boxed{0.9929}$

④ $P(Z < 1.5) =$

	0.00
1.5	

⑤ $P(Z < 0.4) =$

⑥ $P(Z < 0) = \boxed{0.5}$

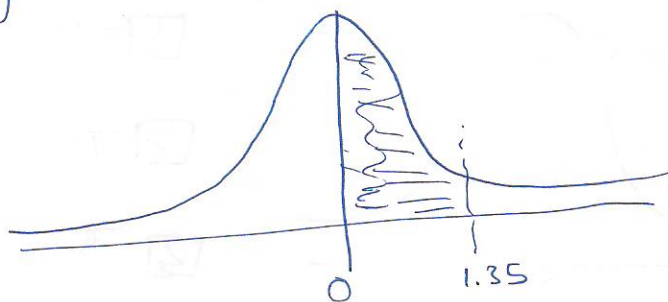
	0.00
0.4	

⑦ $P(0 < Z < 1.35)$

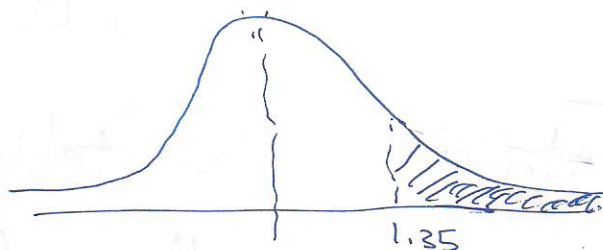
$P(Z < 1.35) - P(Z < 0)$

$0.9115 - 0.5$

0.4115



⑧ $P(Z > 1.35)$

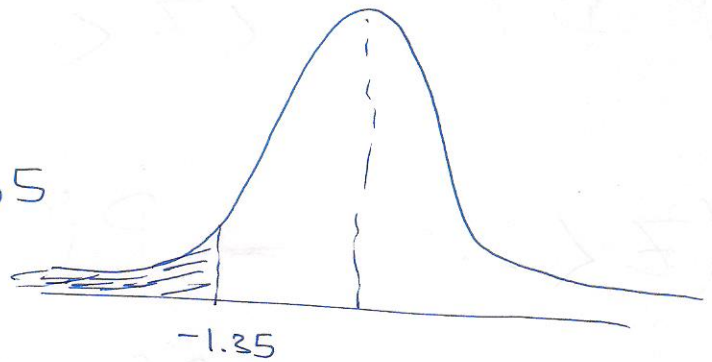


$P(Z < 1.35) = 0.9885$

$$\textcircled{9} P(Z \leq -1.35)$$

$$= P(Z > 1.35) = 0.0885$$

from ~~the~~ part 8



We know Normal Dist symmetric

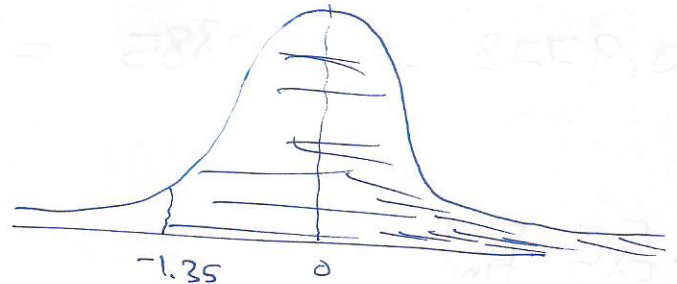
$$\text{So } P(Z \leq -1.35) = P(Z \geq 1.35) \\ = P(Z > 1.35)$$

$\textcircled{10}$

$$P(Z > -1.35)$$

$$= P(Z < 1.35)$$

$$= 0.9115$$



We can find $P(Z < 1.35)$ from the Z-table and from the symmetry $P(Z > 1.35) = P(Z < -1.35)$

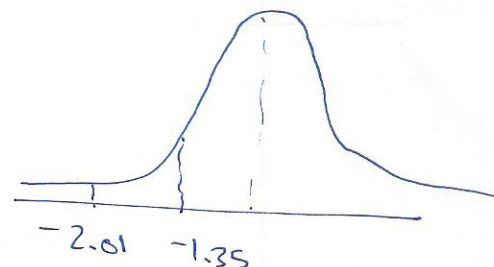
$$\textcircled{11} P(1.35 < Z < 2.01)$$

$$P(Z < 2.01) - P(Z < 1.35)$$

$$0.9778 - 0.9115 = 0.0663$$

$$\textcircled{12} P(-2.01 < Z < -1.35)$$

$$= P(1.35 < Z < 2.01) = 0.0663$$



$$⑬ P(-1.35 < Z < 2.01)$$

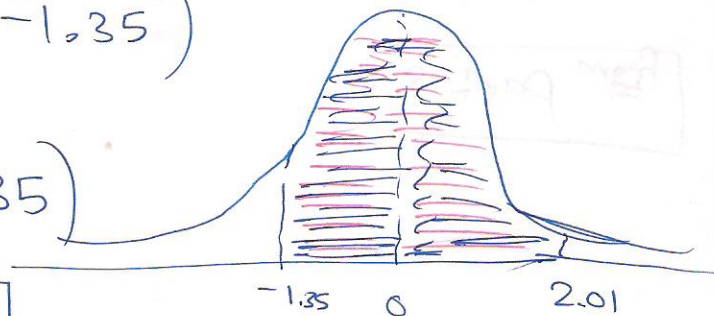
$$P(Z < 2.01) - P(Z < -1.35)$$

$$0.9778 - P(Z > 1.35)$$

$$0.9778 - [1 - P(Z < 1.35)]$$

$$0.9778 - [1 - 0.9115]$$

$$0.9778 - 0.0885 = \boxed{0.8893}$$



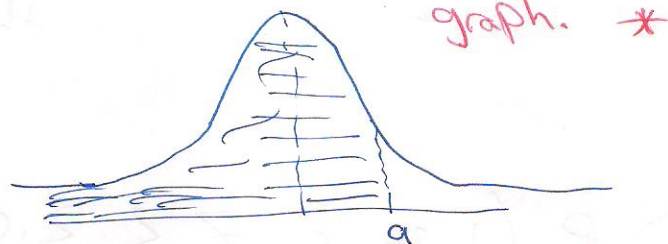
* Ex:- Find a

$$① P(Z < a) = 0.8997$$

$$\boxed{a = 1.28}$$

إذا كانت المساحة = الاحتمال معروف

(جد قيمه Z)

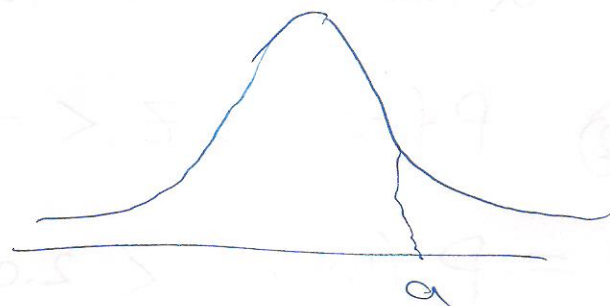


كما ان الاحتمال أكبر من 0.5

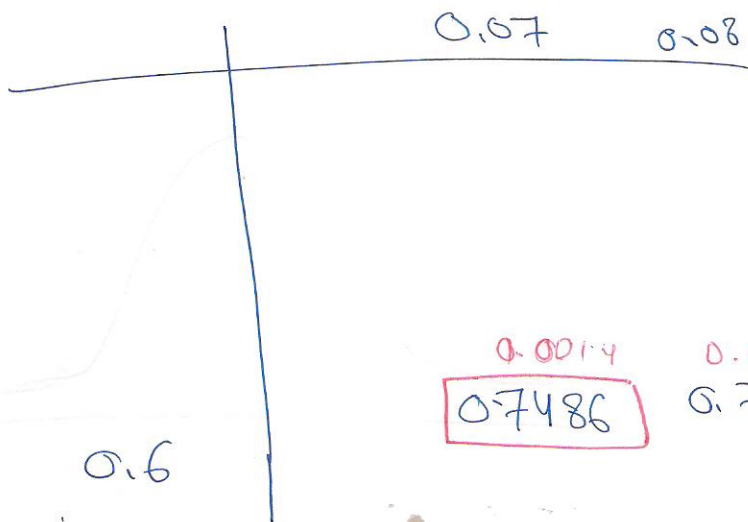
$$② P(Z \leq a) = 0.9 \quad \boxed{a = 1.28}$$

∴ عدد موجب a
نبحث عنه قيمه 0.8997

0.08 0.09

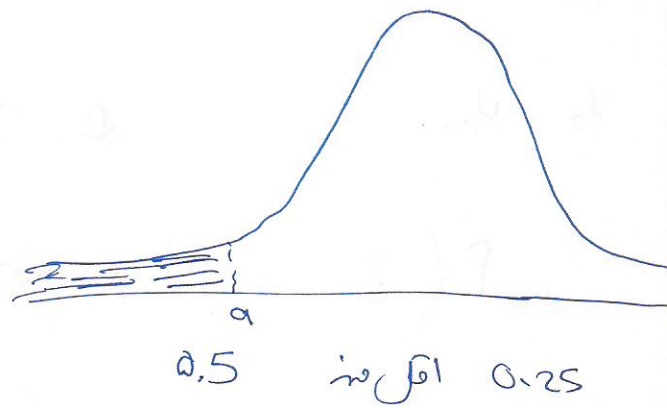


③ $P(Z < a) = 0.25$



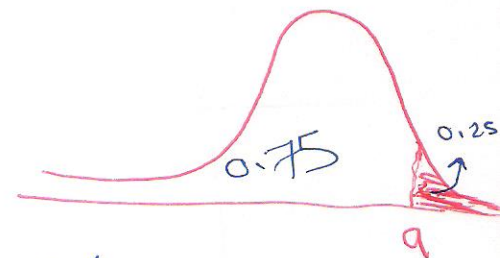
0.0014 0.0015
0.7486 0.7515

$a = -0.67$



عند a 0.25

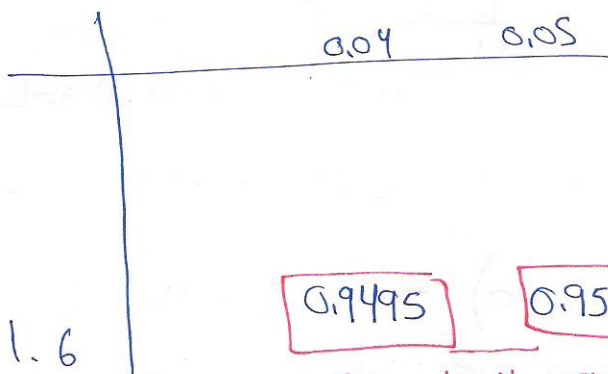
المساحة الكلية هي 0.5



$P(Z > a) = P(Z < -a)$

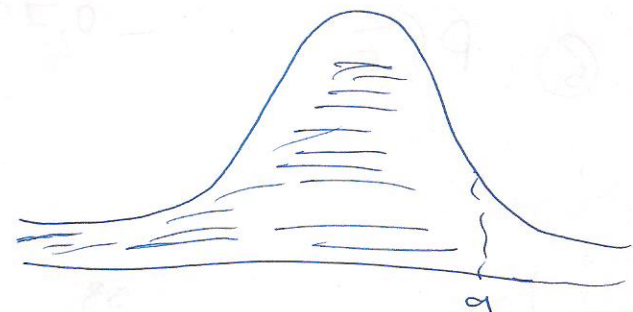
$1 - 0.25$
 $= 0.75$

④ $P(Z < a) = 0.95$



0.9495 **0.9505**

0.95 نفس الجدول العكسي



$0.5 < 0.95$

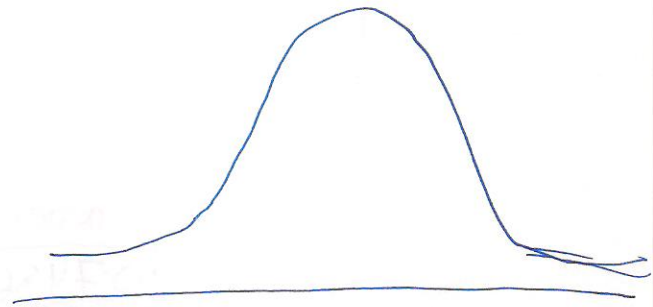
a : موجهه شكل مباشر نبحث في الجدول

$a = \frac{1.64 + 1.65}{2} = 1.645$

⑤ Find z from z table such that the Area to the right of z is 0.05

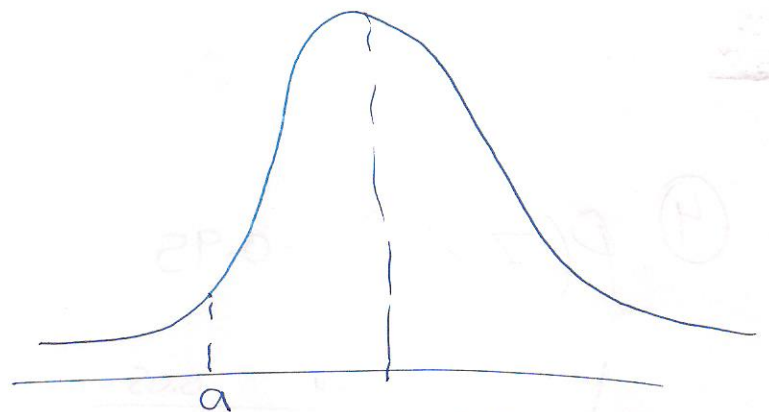
$$P(Z) = 0.05$$

$$\rightarrow P(Z) = 0.95$$



	0.04	0.05
1.6	0.9495	0.9505

⑥ $P(Z > a) = 0.75$



الاحاحه التي فوقها 0.75

∴ a ← z value

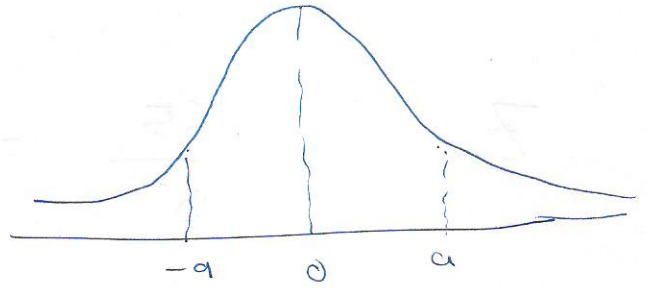
$$P(Z > -a) = P(Z < a)$$

نبحث في الجبل عن 0.75

	0.07	0.08
0.6	0.7486	0.7517

$$a = 0.67$$

8 $P(-a < Z < a)$ or $P(|Z| < a) = 0.97$



لكن تقريبي من ايجاد a من جدول

$$0.485 = \frac{0.97}{2}$$

* رقم على 2

$$0.485 + 0.5 = 0.985$$

* نصف اليمين 0.5

نبحث في هذا الجدول

الكل

$$a = 2.17$$

$$-a = -2.17$$

Ex:- The final exam in Stat 2311 were normally Dist
With a mean of 65 and standard deviation of 10

a Find the prob. that a randomly selected student
scored is less than 78 on the exam.

b What is the percentage of the scores that are
between 56 and 70 ??

$$a) P(X < 78)$$

$$\text{Score} = X$$

$$Z = \frac{78 - 65}{10} = 1.3$$

$$Z_{\text{score}} = Z$$

$$P(Z < 1.3) = 0.9032.$$

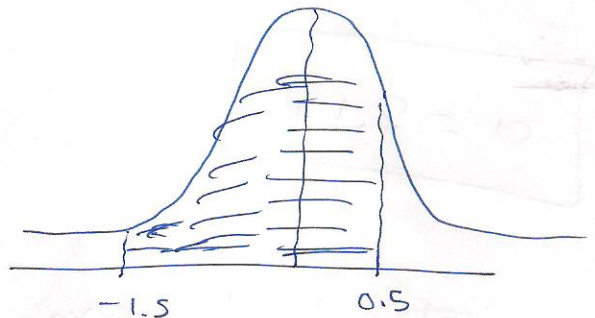
Z table.

$$b) P(50 < X < 70)$$

$$X = 50 \longrightarrow Z = -1.5$$

$$X = 60 \longrightarrow Z = 0.5$$

$$P(-1.5 < Z < 0.5)$$



$$\frac{P(Z < 0.5)}{\text{Table}} - P(Z < -1.5)$$

$$0.6915 - [P(Z > 1.5)]$$

$$0.6915 - [1 - \frac{P(Z < 1.5)}{\text{Table}}]$$

$$0.6915 - [1 - (0.9332)] = 0.6247$$

Percentage = 62.47%

© Min score ?? $X = ??$

$$P(Z > a) = 0.15$$

النسبة في الطرف اليمين

$$P(Z < a) = \boxed{0.85}$$

النسبة في الطرف الأيسر

$$\boxed{a = 1.04}$$

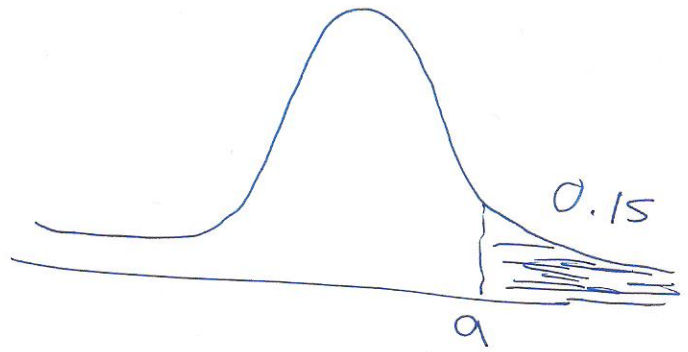
Z Score



X Score

$$Z = \frac{X - \mu}{\sigma}$$

$$1.04 = \frac{X - 65}{10} \implies \boxed{X = 75.4}$$



	0.03	0.04
1.0	0.8485	0.8508

Ex:- The weight of newborn babies are Normally Dist with

$$\boxed{\mu = 3 \text{ kg}} \quad , \quad \boxed{\sigma = 0.8 \text{ kg}}$$

① What is the percentage of babies who weight between 3.5 kg and 4.5 kg

② If there were 100 babies this week, How many of them do you expect to weight at least 4?

© A baby is consider unhealthy if its weights is in the bottom 5%, What is the weight of an unhealthy baby?

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986