

6.3 Normal Approximation of Binomial probabilities "continuous"

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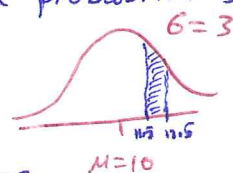
- Recall the discrete binomial distribution $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ where the discrete random variable x is the number of successes in n trials within a binomial experiment.
- When the number of trials become large, evaluating the binomial probability function by hand or with calculator is difficult.
 - $\Rightarrow$ In cases where $np \geq 5$ and $n(1-p) \geq 5$, the normal distribution provides an easy to use approximation of binomial probabilities, by using $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.

Example: A company has a history of making errors in 10% of its invoices. A sample of 100 invoices has been taken.

[a] Find the prob. that 12 invoices contain error.

- We need the binomial prob. of 12 successes $= f(12) = \binom{100}{12} (0.1)^{12} (0.9)^{88}$ which is hard to calculate by hand or calculator.
- So we use the normal approximation: $\mu = np = 100(0.1) = 10$
 $\sigma = \sqrt{10(0.9)} = \sqrt{9} = 3$

Since here we are using continuous prob. distribution, the probabilities are computed as areas under the prob. density function.



$\Rightarrow$ So, the prob. of any single value is zero

$\Rightarrow$ Thus, to approximate the binomial prob. of 12 successes, we compute the area under the normal curve between 11.5 and 12.5.

$\Rightarrow$ The 0.5 we add and subtract from 12 is called a continuity correction factor.

$$\begin{aligned}
 f(12) &\approx P(11.5 \leq x \leq 12.5) \\
 &= P(0.5 \leq z \leq 0.83) \\
 &= P(z \leq 0.83) - P(z \leq 0.5) \\
 &= 0.7967 - 0.6915 = 0.1052
 \end{aligned}
 \quad \left| \quad \begin{aligned}
 z &= \frac{x - \mu}{\sigma} = \frac{11.5 - 10}{3} = 0.5 \\
 z &= \frac{x - \mu}{\sigma} = \frac{12.5 - 10}{3} = 0.83
 \end{aligned}
 \right.$$

b Find the prob. of 13 or fewer errors.
 invoices contain

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$$P(X \leq 13.5)$$

$$= P(Z \leq 1.17) = 0.8790$$

$$Z = \frac{X - \mu}{\sigma} = \frac{13.5 - 10}{3} = 1.17$$



Example (Q26 page 244) A binomial prob. distribution has $p = 0.2$ and $n = 100$

a) What are the mean and the standard deviation?

$$\mu = np = 100(0.2) = 20$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{20(0.8)} = \sqrt{16} = 4$$

b) Is this situation one in which binomial probabilities can be approximated by the normal distribution? Explain.

Yes because $\mu = np = 20 \geq 5$ and $n(1-p) = 100(0.8) = 80 \geq 5$

c) What is the prob. of exactly 24 successes?

$$P(23.5 \leq X \leq 24.5)$$

$$Z = \frac{X - \mu}{\sigma} = \frac{23.5 - 20}{4} = 0.88$$

$$= P(0.88 \leq Z \leq 1.13)$$

$$Z = \frac{X - \mu}{\sigma} = \frac{24.5 - 20}{4} = 1.13$$

$$= P(Z \leq 1.13) - P(Z \leq 0.88)$$

$$= 0.8708 - 0.8106 = 0.0602$$

d) What is the prob. of 18 to 22 successes?

$$P(17.5 \leq X \leq 22.5)$$

$$Z = \frac{X - \mu}{\sigma} = \frac{17.5 - 20}{4} = -0.63$$

$$Z = \frac{X - \mu}{\sigma} = \frac{22.5 - 20}{4} = 0.63$$

$$= P(-0.63 \leq Z \leq 0.63)$$

$$= P(Z \leq 0.63) - P(Z \leq -0.63) = 0.7357 - 0.2643 = 0.4714$$

e) What is the prob. of 15 or fewer successes? "at most 15"

$$P(X \leq 15.5)$$

$$Z = \frac{X - \mu}{\sigma} = \frac{15.5 - 20}{4} = -1.13$$

$$= P(Z \leq -1.13) = 0.1292$$

f) What is the prob. of 15 or greater successes? "at least 15"

$$Z = \frac{X - \mu}{\sigma} = \frac{14.5 - 20}{4} = -1.38$$

$$P(X \geq 14.5) = P(Z \geq -1.38) = 1 - P(Z \leq -1.38) = 1 - 0.0838 = 0.9162$$