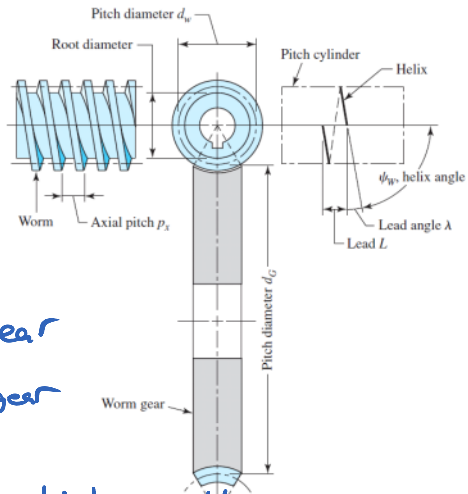
Worm Gears (1)

Wednesday, June 23, 2021 12:22 AM

⇒ Geometry:



* The pitch diameter of worm gear is:

$$\boxed{d_g = \frac{N_g\, P_c}{\pi}}$$, where:

P_c = circular pitch of gear

P = diametral pitch of gear

$$\boxed{P_g = \frac{N_g}{d_g}}$$

* The pitch diameter of the worm should be in the range:

$$\boxed{\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{1.7}} \text{ [in]} \quad \text{or} \quad \boxed{\frac{C^{0.875}}{2.002} \le d_w \le \frac{C^{0.875}}{1.68}} \text{ [mm]}$$

* The velocity ratio is:

$$\boxed{VR = \frac{n_w}{n_g} = \frac{N_g}{N_w}}$$ where: $N_g > 24$, $N_w + N_g > 40$

* The center distance is:

$$\boxed{C = \frac{d_w + d_g}{2}}$$

* The tangential velocity of worm gear is:

$$\boxed{V_w = \frac{\pi\, d_w\, n_w}{12}} , \quad \boxed{V_g = \frac{\pi\, d_g\, n_g}{12}}$$

* The sliding velocity is: $$\boxed{V_s = \frac{V_w}{\cos\lambda} = \frac{V_g}{\sin\lambda}}$$

* The lead angle of the worm is: $$\boxed{\tan\lambda = \frac{L}{\pi d_W}}$$

* The lead (L) is: $$\boxed{L = P_a\, N_w}$$

* Single threaded worm: $N_w = 1$
* Double threaded worm: $N_w = 2$
* Tripple threaded worm: $N_w = 3$

* For a 90° shaft angle ⇒ axial pitch $(P_a)_w$ = circular pitch $(P_c)_g$

$$\boxed{P_g = \frac{\pi}{P_a}} \qquad \boxed{P_a = P_c \Rightarrow \frac{L}{N_w} = \frac{\pi\, d_g}{N_g}}$$

* The axial pitch (P_a) has standard values which are:

$\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, 2 inch

STUDENTS-HUB.com Uploaded By: anonymous

Wednesday, June 23, 2021 8:24 PM

* The addendum & dedendum are:

where: $p_x = p_a$ = axial pitch

$$\boxed{a = \frac{p_x}{\pi} = 0.3183 p_x} \qquad (15\text{–}39)$$

$$\boxed{b = \frac{1.157 p_x}{\pi} = 0.3683 p_x} \qquad (15\text{–}40)$$

* The full depth h_t is: $P_x = P_a$

$$h_t = \begin{cases} \dfrac{2.157 p_x}{\pi} = 0.6866 p_x & p_x \ge 0.16 \text{ in} \\ \dfrac{2.200 p_x}{\pi} + 0.002 = 0.7003 p_x + 0.002 & p_x < 0.16 \text{ in} \end{cases} \qquad (15\text{–}41)$$

* The worm outside diameter d_o is:

$$d_o = d_w + 2a \qquad (15\text{–}42)$$

* The worm root diameter d_r is:

$$d_r = d_w - 2b \qquad (15\text{–}43)$$

* The worm-gear throat diameter D_t is:

$$D_t = \underset{\to d_g}{D} + 2a \qquad (15\text{–}44)$$

* The worm-gear root diameter D_r is:

$$D_r = \underset{\to d_g}{D} - 2b \qquad (15\text{–}45)$$

* The clearance c is:

$$c = b - a \qquad (15\text{–}46)$$

* The worm face width (maximum) $(F_W)_{max}$ is:

$$(F_W)_{max} = 2\sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D}{2} - a\right)^2} = 2\sqrt{2Da} \qquad (15\text{–}47)$$

which was simplified using Eq. (15–44). * The worm-gear face width F_G is:

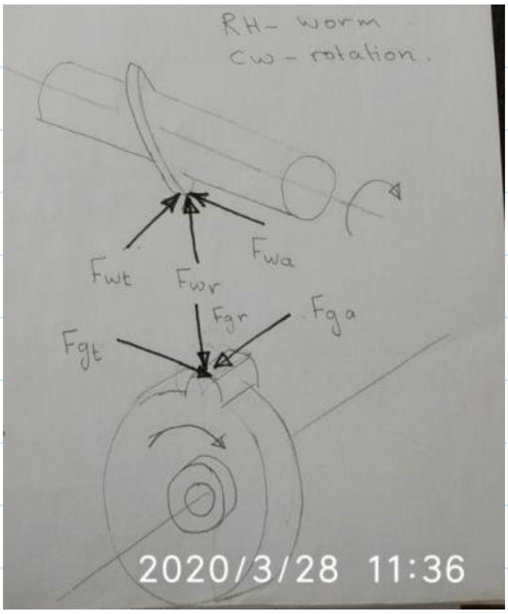
$$b_g = F_G = \begin{cases} 2d_m/3 & p_x > 0.16 \text{ in} \\ 1.125\sqrt{(d_o + 2c)^2 - (d_o - 4a)^2} & p_x \le 0.16 \text{ in} \end{cases} \qquad (15\text{–}48)$$

where: $d_m = d_w$

* The normal diametrical pitch (P_n) is: $\boxed{P_n = \dfrac{P_g}{\cos\lambda}}$

* The normal circular pitch (p_n) is: $\boxed{p_n = \dfrac{\pi}{P_n}}$

STUDENTS-HUB.com Uploaded By: anonymous



→ Force Analysis:

+ For a shaft angle of 90°:

$$\begin{cases} F_{wt} = F_{ga} \\ F_{gt} = F_{wa} \\ F_{wr} = F_{gr} \end{cases}$$

+ The tangential force is:

$$\boxed{F_{gt} = F_{wt}\,\frac{\cos\phi_n \cos\lambda - \mu \sin\lambda}{\cos\phi_n \sin\lambda + \mu \cos\lambda}}$$

or

→ It can be found from: $\boxed{H = \frac{F_t V}{33000}}$

+ The radial forces are: $\boxed{F_{gr} = F_{wr} = F_{gt}\,\frac{\sin\phi_n}{\cos\phi_n \cos\lambda - \mu \sin\lambda} = F_{wt}\,\frac{\sin\phi_n}{\cos\phi_n \sin\lambda + \mu \cos\lambda}}$

* The frictional force is: $\boxed{F_f = \frac{\mu F_{gt}}{\mu \sin\lambda - \cos\phi_n \cos\lambda}}$

+ The total force (F_n) is: $\boxed{F_n = \frac{F_{wt}}{\cos\phi_n \sin\lambda + \mu \cos\lambda}}$

+ To find ($F_{gr} = F_{wr}$): $\boxed{F_{wr} = F_{gr} = F_n \sin\phi_n}$

+ To find ($F_{wa} = F_{gt}$): $\boxed{F_{wa} = F_{gt} = F_n(\cos\phi_n \cos\lambda - \mu \sin\lambda)}$

+ The efficiency is:

$$e_w = \boxed{\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} = \frac{H_{out}}{H_{in}}} \qquad (13\text{–}46)$$

+ Increasing ($p_a = p_c$) → increase (C) then calculate $(d_w)_{min}$ and check if $d_w > (d_w)_{min}$

STUDENTS-HUB.com Uploaded By: anonymous

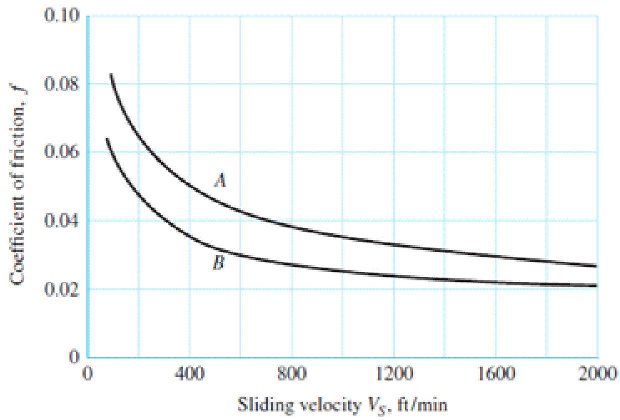
(4)

Wednesday, June 23, 2021 12:22 AM

To find (μ):

Figure 13–42

Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. * Use curve B for high-quality materials, such as a case-hardened steel worm mating with a phosphor-bronze gear. * Use curve A when more friction is expected, as with a cast-iron worm mating with a cast-iron worm gear.



$$f = \begin{cases} 0.15 & V_s = 0 \\ 0.124\exp(-0.074V_s^{0.645}) & 0 < V_s \le 10 \text{ ft/min} \\ 0.103\exp(-0.110V_s^{0.450}) + 0.012 & V_s > 10 \text{ ft/min} \end{cases} \qquad (15\text{–}38)$$

⇒ To find the allowable tangential force $(F_{gt})_a$:

$$\boxed{(F_{gt})_a = C_s\, C_m\, C_v\, d_g^{0.8}\, b}$$

To find (C_s):

The parameters in Eq. (15–28) are, quantitatively,

$$C_s = 720 + 10.37C^3 \qquad C \le 3 \text{ in} \qquad (15\text{–}32)$$

* For sand-cast gears,: $D_m = d_g$

$$C_s = \begin{cases} 1000 & C > 3 \quad D_m \le 2.5 \text{ in} \\ 1190 - 477\log D_m & C > 3 \quad D_m > 2.5 \text{ in} \end{cases} \qquad (15\text{–}33)$$

* For chilled-cast gears,: $D_m = d_g$

$$C_s = \begin{cases} 1000 & C > 3 \quad D_m \le 8 \text{ in} \\ 1412 - 456\log D_m & C > 3 \quad D_m > 8 \text{ in} \end{cases} \qquad (15\text{–}34)$$

* For centrifugally cast gears,: $D_m = d_g$

$$C_s = \begin{cases} 1000 & C > 3 \quad D_m \le 25 \text{ in} \\ 1251 - 180\log D_m & C > 3 \quad D_m > 25 \text{ in} \end{cases} \qquad (15\text{–}35)$$

* To find (C_m):

The ratio correction factor C_m for gear ratio m_G is given by $m_G = VR = \frac{N_w}{N_g}$

$$C_m = \begin{cases} 0.02\sqrt{-m_G^2 + 40m_G - 76} + 0.46 & 3 < m_G \le 20 \\ 0.0107\sqrt{-m_G^2 + 56m_G + 5145} & 20 < m_G \le 76 \\ 1.1483 - 0.006\,58m_G & m_G > 76 \end{cases} \qquad (15\text{–}36)$$

STUDENTS-HUB.com Uploaded By: anonymous

Wednesday, June 23, 2021 12:22 AM

To find (C_v):

The velocity factor C_v is given by

$$C_v = \begin{cases} 0.659 \exp(-0.0011 V_s) & V_s < 700 \text{ ft/min} \\ 13.31\, V_s^{-0.571} & 700 \le V_s < 3000 \text{ ft/min} \\ 65.52\, V_s^{-0.774} & V_s > 3000 \text{ ft/min} \end{cases} \quad (15\text{–}37)$$

+ The torque of the worm-gear is: $\boxed{T_g = \frac{F_{gt}\, d_g}{2}}$

⇒ Worm gear efficiency & Power losses:

* The efficiency of worm is:

$$\boxed{e_W = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}} \quad (15\text{–}54)$$

+ The efficiency of the worm gear is:

$$\boxed{e_G = \frac{\cos\phi_n - f\cot\lambda}{\cos\phi_n + f\tan\lambda}} \quad (15\text{–}55)$$

* To ensure that the worm gear will drive the worm:

$$\underset{(\mu)}{f_{\text{stat}}} < \cos\phi_n \tan\lambda \quad (15\text{–}56)$$

* The heat loss rate H_{loss} from the worm-gear case in ft · lbf/min is:

$$H_{\text{loss}} = 33\,000(1 - e_{\omega})H_{\text{in}} \quad (15\text{–}49)$$

* The temperature of the oil sump t_s is given by:

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{CR} A} = \frac{33\,000(1-e)(H)_{\text{in}}}{\hbar_{CR} A} + t_a \quad (15\text{–}51)$$

Where:

$$\hbar_{CR} = \begin{cases} \frac{n_W}{6494} + 0.13 & \text{no fan on worm shaft} \\ \frac{n_W}{3939} + 0.13 & \text{fan on worm shaft} \end{cases} \quad (15\text{–}50)$$

Lateral case area ← $\boxed{A_{\text{min}} = 43.20\,C^{1.7}}$ [in²]   (15–52)

Center distance

t_a = ambient temp. in [F°]

STUDENTS-HUB.com Uploaded By: anonymous

Wednesday, June 23, 2021 9:22 PM

+ The power of the worm is:

$$\boxed{H_w = \frac{F_{wt} V_w}{33000}}$$

+ The power of the worm-gear is:

$$\boxed{H_g = \frac{F_{gt} V_g}{33000}}$$

+ We can use the output power (H_o) to find (F_{gt}) by:

$$\boxed{F_{gt} = \frac{33000\, n_d\, H_o\, K_a}{V_g\, e_w}}$$

where: K_a = application factor

+ Relating input & output powers:

$$\boxed{e_w = \frac{H_o}{H_i}}$$

+ Power loss due to friction:

$$\boxed{H_f = \frac{|F_f|\, V_s}{33000}}$$

$$\boxed{H_f = (1 - e_w) H_w}$$

+ The bending stress:

$$\boxed{\sigma_b = \frac{F_{gt}}{p_n\, b\, y}} \longrightarrow (\sigma_b)_{allow} = \frac{(F_{gt})_a}{p_n\, b\, y}$$

where ($p_n = p_a \cos\lambda$) and y is the Lewis form factor related to circular pitch. For $\phi_n = 14.5°$, $y = 0.100$; $\phi_n = 20°$, $y = 0.125$; $\phi_n = 25°$, $y = 0.150$; $\phi_n = 30°$, $y = 0.175$

$\phi_n =$

STUDENTS-HUB.com
Uploaded By: anonymous

Thursday, June 24, 2021 12:04 AM

* The wear load is :

$$\boxed{(F_{gt})_{allow} = K_w \, d_g \, b}$$

To find K_w :

Table 15–11

Wear Factor K_w for Worm Gearing

Source: Earle Buckingham, *Design of Worm and Spiral Gears,* Industrial Press, New York, 1981.

Material		Thread Angle ϕ_n			
Worm	Gear	$14\frac{1}{2}°$	20°	25°	30°
Hardened steel*	Chilled bronze	90	125	150	180
Hardened steel*	Bronze	60	80	100	120
Steel, 250 BHN (min.)	Bronze	36	50	60	72
High-test cast iron	Bronze	80	115	140	165
Gray iron†	Aluminum	10	12	15	18
High-test cast iron	Gray iron	90	125	150	180
High-test cast iron	Cast steel	22	31	37	45
High-test cast iron	High-test cast iron	135	185	225	270
Steel 250 BHN (min.)	Laminated phenolic	47	64	80	95
Gray iron	Laminated phenolic	70	96	120	140

*Over 500 BHN surface.

†For steel worms, multiply given values by 0.6.

* For the design to be safe : $F_{gt} < (F_{gt})_{allow}$

* Useful tables for design :

Table 15–9

Largest Lead Angle Associated with a Normal Pressure Angle ϕ_n for Worm Gearing

ϕ_n	Maximum Lead Angle λ_{max}
14.5°	16°
20°	25°
25°	35°
30°	45°

Table 15–10

Minimum Number of Gear Teeth for Normal Pressure Angle ϕ_n

ϕ_n	$(N_G)_{min}$
14.5	40
17.5	27
20	21
22.5	17
25	14
27.5	12
30	10

* Design Steps :

Assume :

1. n_d
2. Material type
3. N_w
4. ϕ_n and check for $(N_G)_{min}$.
5. p_a
6. mean worm diameter (d_w) then find (C), then find the min. & max. values of (d_w) and check if the assumption is correct
7. Compute everything necessary to find (b) from $(F_{gt})_a$

STUDENTS-HUB.com Uploaded By: anonymous