Chapter 7.1, Problem 51E

Problem

Exercise refers to the Euler phi function, denoted ϕ, which is defined as follows: For each integer $n \geq 1$, $\phi(n)$ is the number of positive integers less than or equal to n that have no common factors with n except ±1. For example, $\phi(10) = 4$ because there are four positive integers less than or equal to 10 that have no common factors with 10 except ±1; namely, 1, 3, 7, and 9.

Exercise

Find each of the following:

a. $\phi(15)$

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b. $\phi(2)$

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c. $\phi(5)$

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d. $\phi(12)$

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e. $\phi(11)$

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f. $\phi(1)$

Step-by-step solution

Step 1 of 7

If n is a positive integer, then the number of positive integers less than n, and relatively prime to n is denoted by $\phi(n)$, which is called the Euler totient function .

Step 2 of 7

(a) $\phi(15)$

$$\because (1,15)=1,(2,15)=1,(3,15)=3,(4,15)=1$$
$$(5,15)=1,(6,15)=3,(7,15)=1,(8,15)=1,(9,15)=3$$
$$(10,15)=5,(11,15)=1,(12,15)=3,(13,15)=1,(14,15)=1$$

$$\therefore \phi(15)=8$$

Step 3 of 7

(b) $\phi(2)=1$

$\because (1,2)=1,(2,2)$ is not allowed

Step 4 of 7

(c) $\phi(5)=4$

$\because (1,15)=1,(2,15)=1,(3,15)=1,(4,15)=1$

$\therefore$ Every positive integer less than *n* is co-prime to *n* when *n* is prime

In general $\phi(n)=(n-1)$ when *n* is a prime number

Step 5 of 7

(d) $\phi(12)=4$

Step 6 of 7

(e) $\phi(11)=10$, while 11 is prime

Step 7 of 7

(f) $\phi(1)=1$