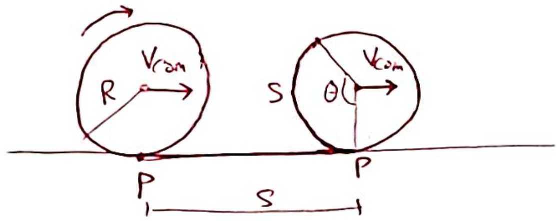
Ch 11: Rolling, Torque, & Angular Momentum.

11.1 Rolling as translation & rotation combined:-

→ smoothly rolling motion :→ No slipping
→ No bouncing

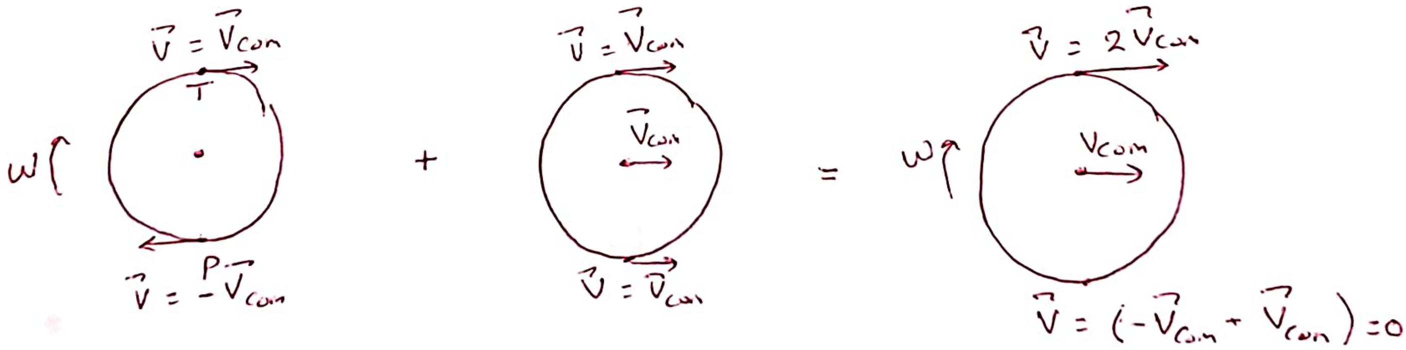


V_{com} : speed of the Center of mass

$$S = R\theta$$

$$V_{com} = \frac{ds}{dt} = R\frac{d\theta}{dt} \Rightarrow V_{com} = R\omega$$

→ Rolling motion is a combination of purely translational & purely rotational motions.



pure Rotation + pure translation = Rolling motion

→ in pure Rotation : $\vec{V} = R\omega$

⇒ at the top $\vec{V} = R\omega = \vec{V}_{com}$ (to the right)

⇒ at the bottom $\vec{V} = R\omega = -\vec{V}_{com}$ (to the left)

11-2 Forces & kinetic energy of rolling

$$K = K_{rotation} + K_{translation}$$

$$\boxed{K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M V_{com}^2}$$

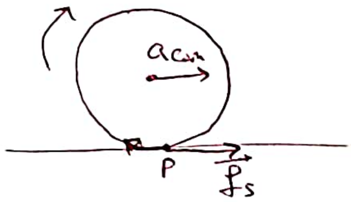
→ The Forces of Rolling:

* if $V_{com} = \text{Const} \Rightarrow a_{com} = 0 \Rightarrow F_{net} = 0$

  $\Rightarrow$ No frictional force.

* if $F_{net} \neq 0 \Rightarrow$ speed it up

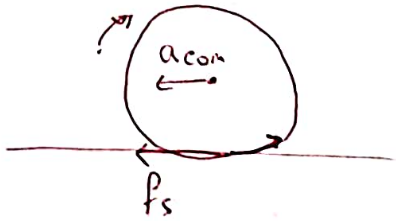
  a_{com}



static frictional force $\vec{f}_s$ acts on the wheel at P, opposing its tendency to slide.

$\Rightarrow$ slow it down

$-a_{com}$



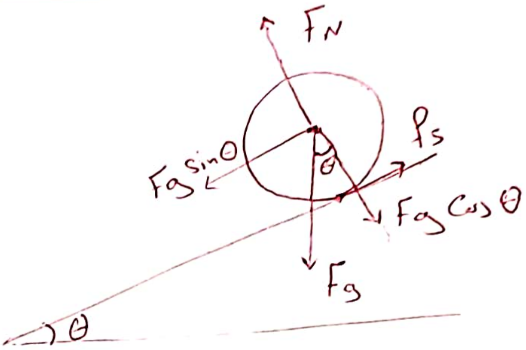
* if the wheel doesnt slide, the force is static friction f_s

$$a_{com} = R\alpha \quad (\text{smooth rolling})$$

* a_{com} & α tends to cause slipping, static friction must present to oppose the tendency of slipping.

⇒ Rolling down a ramp:

Mass (M), Radius R



To find a_{com}:

Using Newton's 2^{nd} law:

$$f_s - Mg\sin\theta = M(-a_{com}) \quad - (1)$$

$\boxed{f_s \text{ is not } f_{s\,max}}$

but $\tau_{net} = I\alpha$

$$\tau_f = \vec{R} \times \vec{f}_s = R f_s = I_{com}\alpha \quad - (2) \quad (\tau\,(+),\ \alpha\,(+))$$

$$a_{com} = R\alpha \Rightarrow \boxed{\alpha = \frac{a_{com}}{R}} \quad \text{sub. in (2)}$$

$$R f_s = I_{com}\left(\frac{a_{com}}{R}\right) \Rightarrow \boxed{f_s = I_{com}\left(\frac{a_{com}}{R^2}\right)} \quad \text{sub in (1)}$$

$$I_{com}\frac{a_{com}}{R^2} - Mg\sin\theta = -M a_{com}$$

$$a_{com} = \frac{Mg\sin\theta}{M + \frac{I_{com}}{R^2}} = \frac{g\sin\theta}{1 + \frac{I_{com}}{MR^2}}$$

dir of a_{com} is downward (−)

P4

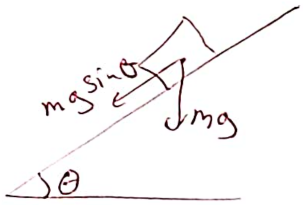
a) if $a_{com} = 0.15g$, Find θ?

$$a_{com} = \frac{g\sin\theta}{1 + \frac{I_{com}}{MR^2}} \quad , \quad I_{com\,(sphere)} = \frac{2}{5}MR^2$$

$$0.15\cancel{g} = \frac{g\sin\theta}{1 + \frac{\frac{2}{5}\cancel{MR^2}}{\cancel{MR^2}}} = \frac{\cancel{g}\sin\theta}{1 + 2/5}$$

$$\sin\theta = 0.15\left(\frac{7}{5}\right) \Rightarrow \boxed{\theta = 12.12^\circ}$$

b)



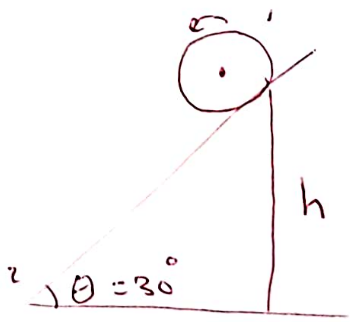
$$\Sigma F = ma$$

$$\cancel{m}g\sin\theta = \cancel{m}a$$

$$\boxed{a = 0.21\,g}$$

the accelration will be larger

S.P 11.1



$M = 6\,kg$, $h = 1.2\,m$

R

$I_{com} = \frac{2}{5}MR^2$

a) Find v_2?

using Conservation of E_{mech}

$$E_1 = E_2$$

$$(K + U)_1 = (K + U)_2$$

$$0 + Mgh = \frac{1}{2} I_{com} w^2 + \frac{1}{2} M V_{com}^2 + 0$$

but $V_{com} = Rw \Rightarrow w = \frac{V_{com}}{R}$

$I_{com} = \frac{2}{5} MR^2$

$$\Rightarrow \cancel{M} gh = \frac{1}{2}\left(\frac{2}{5} \cancel{M} \cancel{R^2}\right) \frac{V_{com}^2}{\cancel{R^2}} + \frac{1}{2} \cancel{M} V_{com}^2$$

$$gh = \frac{7}{10} V_{com}^2 \Rightarrow V_{com} = 4.1 \text{ m/s}$$

b) Find $\vec{f_s}$?

$$a_{com} = \frac{g \sin\theta}{1 + \frac{I_{com}}{MR^2}} = \frac{g \sin 30}{1 + \frac{2}{5} \frac{M\cancel{R^2}}{M\cancel{R^2}}} = 3.5 \text{ m/s}^2$$

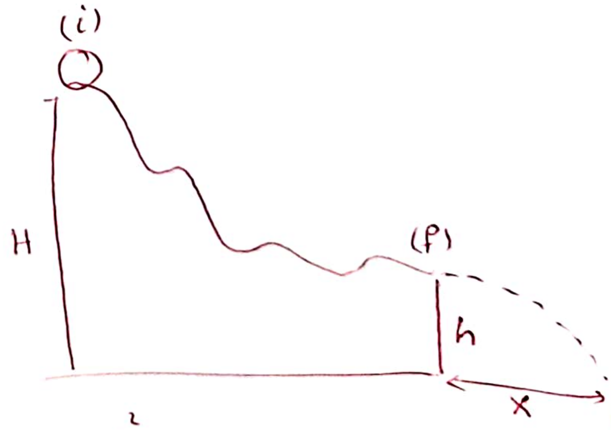
$$f_s - Mg \sin\theta = M(-a_{com})$$

$$f_s = mg \sin\theta - M a_{com}$$

$$= 6(9.8) \sin 30 - 6(3.5)$$

$$= 8.4 \text{ N}$$

P9 $H = 6m$, $h = 2m$



$$(E_{mech})_i = (E_{mech})_f$$

$$U_i + k_i = U_f + k_f$$

$$mgH + 0 = mgh + \frac{1}{2} I_{com} w^2 + \frac{1}{2} m V_{com}^2$$

$$\boxed{w = \frac{V_{com}}{R}, \quad I_{com} = \frac{2}{5} mR^2}$$

$$mgH = \frac{1}{2}\left(\frac{2}{5}m\cancel{R^2}\right)\left(\frac{V_{com}}{\cancel{R}}\right)^2 + \frac{1}{2}mV_{com}^2 + mgh$$

$$gH = \frac{1}{2}V_{com}^2\left(\frac{7}{2}\right) + gh$$

$$V_{com} = \sqrt{\frac{10g}{7}(H-h)}$$

$$= 7.48 \text{ m/s}$$

To find x :

$$\Delta y = V_{oy}t - \frac{1}{2}gt^2$$

$$-2 = 0 - \frac{1}{2}(9.8)t^2 \Rightarrow t = 0.639 \text{ sec}$$

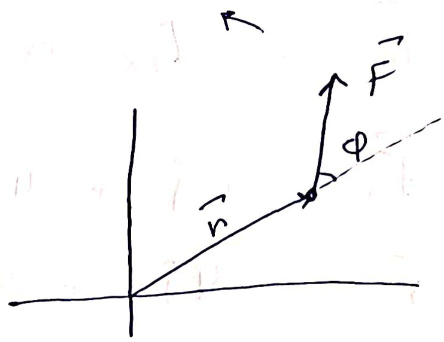
$$x = V_x t$$

$$= V_{com} t = (7.48)(0.639)$$

$$\boxed{x = 4.8 \text{ m}}$$

chill Lec 2

11.4 Torque:



$$\vec{\tau} = \vec{r} \times \vec{F}$$

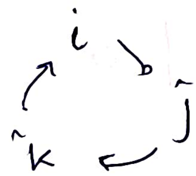
$$= r F \sin \phi$$

- $\vec{\tau} \perp \vec{r}$, $\vec{\tau} \perp \vec{F}$

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{j} = -\hat{i}$

⋮ ⋮



P21 Find the torque about the origin on a particle located at (0, -4, 3) m, if that torque due to

a) $\vec{F}_1 \Rightarrow F_{1x} = 2\,N$, $F_{1y} = F_{1z} = 0$

$\vec{F}_1 = 2\hat{i}$

$\vec{r} = -4\hat{j} + 3\hat{k}$

$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1$

$= (-4\hat{j} + 3\hat{k}) \times (2\hat{i})$

$= -8(\hat{j} \times \hat{i}) + 6(\hat{k} \times \hat{i})$

$= -8(-\hat{k}) + 6\hat{j}$

$= 8\hat{k} + 6\hat{j}$ N.m

b) $\vec{F}_2$; $F_{2x} = 0$, $F_{2y} = 2N$, $F_{2z} = 4N$

$\vec{F}_2 = 2\hat{j} + 4\hat{k}$

$\vec{r} = -4\hat{j} + 3\hat{k}$

$$\vec{\tau}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 3 \\ 0 & 2 & 4 \end{vmatrix} = \hat{i}(-16-6) - \hat{j}(0) + \hat{k}(0)$$

$= -22\hat{i}$ N.m

P 23| $\vec{F} = 2\hat{i} - 3\hat{k}$

a) if $\vec{r} = 0.5\hat{j} - 2\hat{k}$, Find $\vec{\tau}$?

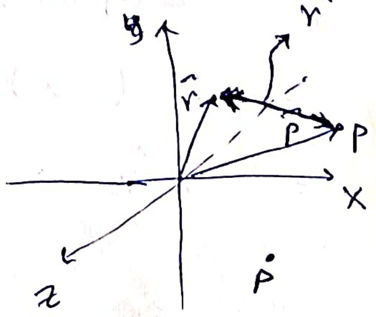
$\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.5 & -2 \\ 2 & 0 & -3 \end{vmatrix}$$

$= \hat{i}\big((0.5)(-3) - 0\big) - \hat{j}\big(0 - (-2)(2)\big) + \hat{k}\big(0 - (0.5)(2)\big)$

$= -1.5\hat{i} - 4\hat{j} - 1\hat{k}$ N.m

b) The resulting torque on the pebble about the point $(2, 0, -3)$ $\Rightarrow$ $P : (2, 0, -3)$



$\vec{r}' = \vec{r} - \vec{P}$

$\vec{P} = 2\hat{i} - 3\hat{k}$

$\vec{r}' = (0.5\hat{j} - 2\hat{k}) - (2\hat{i} - 3\hat{k})$

$= -2\hat{i} + 0.5\hat{j} + \hat{k}$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ -2 & 0.5 & 1 \\ 2 & 0 & -3 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}\left((0.5)(-3) - 0\right) - \hat{j}\left((-2)(-3) - (2)(1)\right) + \hat{k}\left(0 - (0.5)(2)\right)$$

$$= -1.5\,\hat{i} - 4\hat{j} - \hat{k} \quad N.m$$

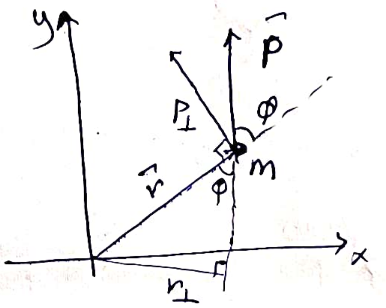
11.5| Angular Momentum:

$$\vec{l} = \vec{r} \times \vec{p}$$

$$= r p \sin\phi = r m v \sin\phi$$

$$= r P_\perp \quad ; \quad P_\perp = p \sin\phi$$

$$\text{or} = r_\perp p \quad ; \quad r_\perp = r \sin\phi$$



$$\vec{l} \perp \vec{r} \;\&\; \vec{p} \qquad [\vec{l}] = kg\,m^2/s$$

P26|

$m = 2\,kg$

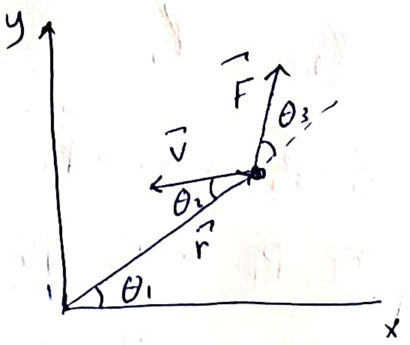
$\vec{r} = 5m$

$\theta_1 = 45°$

$\theta_2 = \theta_3 = 30°$

$\vec{V} = 4\,m/s$

$\vec{F} = 2N$

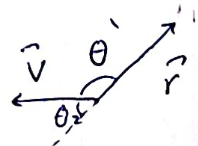


a) Find $\vec{l}$ about the origin?

$$\vec{l} = \vec{r} \times \vec{p} = r p \sin\theta'$$

$$= (5)(2)(4)\sin 150$$

$$= 20\;kg\,m^2/s$$



$\theta' = 180 - \theta_2$

dir ($\vec{l}$) outward , (+ve rotation ccw) (positive)

b) Find $\vec{\tau}$?

$\vec{\tau} = \vec{r} \times \vec{F}$

$= (5)(2) \sin \theta_3$

$\vec{\tau} = 3$ N.m

dir ($\vec{\tau}$) outward , (+ve rotation ccw)

$\vec{\tau} = 3\hat{k}$ N.m

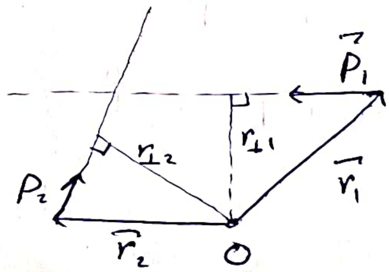
S.P 11.3

$P_1 = 5$ kg.m/s

$r_{\perp 1} = 2$ m

$P_2 = 2$ kg m/s

$r_{\perp 2} = 4$ m



Find $\vec{L}_{tot}$?

$\vec{L}_{tot} = l_1 + l_2$

$= \vec{r}_1 \times \vec{P}_1 + \vec{r}_2 \times \vec{P}_2$

$= r_{\perp 1} P_1 + r_{\perp 2} P_2$

$= (2)(5) + {}^{-}(4)(2)$

$= +2$ kg.m²/s (outward)

$\vec{l}_1 = +10$ kg.m²/s (outward

$\vec{l}_2 = -8$ kg.m²/s (inward)