4.4 Conditional probability :-

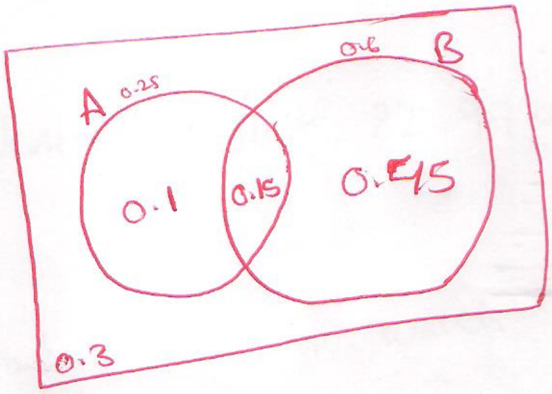
Exp:- let A, B two events such that $P(A) = 0.25$

$P(B) = 0.6$

$P(A \cap B) = 0.15$

1. Find $P(A \cup B)$
2. Find $P(A \mid B)$
3. Find $P(B \mid A)$
4. Are the event A, B independent, Why?
5. Are the event A, B mutually exclusive, why?
6. $P(A \mid B')$
7. $P(B' \mid A)$

Solution :-



$\boxed{1}$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.6 - 0.15$

$P(A \cup B) = 0.7$

$\boxed{2}$ $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.15}{0.6} = \boxed{0.25}$

$\boxed{3}$ $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.15}{0.25} = \boxed{0.6}$

$\boxed{6}$ $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{P(A - B)}{0.4} = \dfrac{P(A) - P(A \cap B)}{0.4} = \dfrac{0.1}{0.4} = \boxed{0.25}$

$\boxed{7}$ $P(B' \mid A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{P(A - B)}{P(A)} = \dfrac{0.1}{0.25} = \boxed{0.4}$

$\boxed{4}$ Yes, A, B : Independent event since $P(A \mid B) = P(A)$

$\boxed{5}$ No, A, B : not mutually exclusive since $P(A \cap B) \neq 0$

Extra question :-

A survey about a sample of B.ZU Students is summarized below :-

	Watch Horror Movies		
weight	Yes	No	Total
50 – 65	6	8	14
66 – 81	8	10	18
82 – 97	9	5	14
More than 97	12	2	14
Total	35	25	60

① What is the prob. that student weight 82 – 97 kg

② What is the prob. that a student does not watch horror movies ??

③ What is the prob. that a student weight more that 65 kg?

④ What is the prob. that a student watchs horror movies and weights less than 82 kg ?

⑤ If a student watchs horror movies, what is the prob. that she/he weights more than 97

⑥ Are the Variables Independent, Explain your answer ?

Chapter 5 : Random variable

Random Variable :- is a numerical discription of the outcomes of an experiment, usually denoted by X

Random variable → Discrete
Random variable → Cont.

Discrete random variable : assume finit number of values or infinite sequence of values (usually no decimal)

Experiment	Random variable X	Value of X
Tossing three coins	number of H	0, 1, 2, 3
20 questions exam	number of questions answered correctly	0, 1, 2, ... 20
Inspect 20 radios	number of defective radios	0, 1, 2, ... 20

Continous random variable : assumes numerical values in an interval

Example : Age, time, weight, distance, temperature
$20 \le x \le 30$ $t \le 3$ $50 \le x \le 60$ $x \ge 0$
Area, volume.

Exp:– In the experiment of tossing two coins

Random variable X – number of Heads

Total outcome = 2.2 = 4 "Multiplication Rule"

S = {HH, HT, ~~H~~TH, TT}

Define X = number of Heads observed

HH ⟶ 2

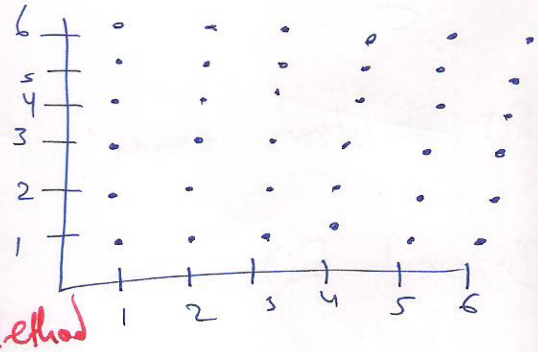
TH ⟶ 1

HT ⟶ 1

TT ⟶ 0

$\underline{X}$ = {0, 1, 2} Discrete Random variable

Exp: Rolling 2 dice :–



Total outcomes : 36

Sample space = {(1,1), . . — (6,6)}

$P(1,6) = \frac{1}{36}$ $P(6,3) = \frac{1}{36}$ Classical Method (equally likely)

Define : Random variable X : Sum of the two faces

(1,1) ⟶ 2

(2,3) ⟶ 5

(6,6) ⟶ 12

X = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

5.2 , ~~5~~5.3

Discrete Probability Distribution

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Each random variable X has a probability distribution

* Probability distribution :- describes how prob. are distributed over the value of X.

* Probability function $f(x) \equiv P(X=x)$ defines the probability distribution of X

* $f(x)$ can be formula, table, graph.

Two condition for a probability function $f(x)$

* $0 \leq f(x) \leq 1 \quad , \forall x$

* $\sum_{i=1}^{n} f(x_i) = 1 \qquad X = \{x_1, x_2, x_3 \text{ --- } x_n\}$

Exp :- Tossing two coins :- $\# S = 4$

$S = \{HH, HT, TH, TT\}$

let X = number of tails $\longrightarrow$ $X = \{0, 1, 2\}$ Discrete

$X=0$: when outcomes is $(H,H) \longrightarrow P(0) = \frac{1}{4} = f(0) = P(x=0)$

$X=1$: when outcomes is $(H,T)\ (TH) \longrightarrow f(1) = P(X=1) = P(1) = \frac{2}{4}$

$X=2$: when outcomes is $(TT) \longrightarrow f(2) = P(X=2) = P(2) = \frac{1}{4}$