Shell Method

$$V = r^2 \pi h$$

$h \quad A4$

$2r_2\pi$

[A] The volume of solid results by revolving the region about $\boxed{x-axis}$ is

$$V = \int_c^d 2\pi \left(\begin{array}{c} \text{shell} \\ \text{radius} \end{array} \right) \left(\begin{array}{c} \text{shell} \\ \text{length} \end{array} \right) dy$$

موازية لمحور الدوران

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[B] $\boxed{y-axis}$ is

$$V = \int_a^b 2\pi \left(\begin{array}{c} \text{shell} \\ \text{radius} \end{array} \right) \left(\begin{array}{c} \text{shell} \\ \text{height} \end{array} \right) dx$$
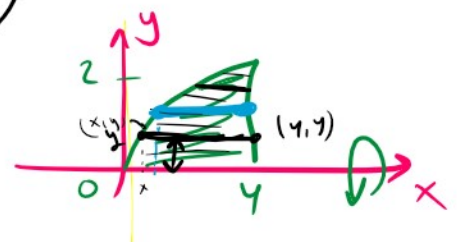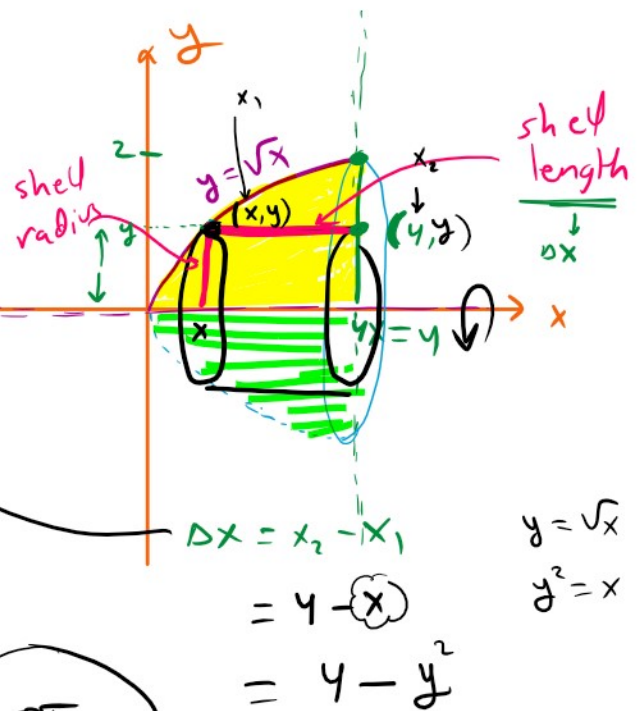
$$\int 2\pi \ (\text{radius})(\boxed{\text{height}})$$

Use Shell Method to

Exp / Find the volume of the solid generated by revolving the region bounded by

① $y = \sqrt{x}$, x-axis / x = 4 about **x-axis**

$$V = \int_c^d \left(2\pi \left(\begin{array}{c}\text{shell}\\\text{radius}\end{array}\right)\left(\begin{array}{c}\text{shell}\\\text{length}\end{array}\right)\right) dy$$

$$= \int_0^2 2\pi \ (y)(4-y^2) \, dy$$

$$= 2\pi \int_0^2 (4y - y^3) \, dy$$

$$= 2\pi \left(2y^2 - \frac{y^4}{4}\right)\Big|_0^2 = \cdots = \boxed{8\pi}$$

$\Delta x = x_2 - x_1$

$= 4 - \{x\}$

$= 4 - y^2$

$y = \sqrt{x}$
$y^2 = x$

② $y = \sqrt{x}$, x-axis , x = 4 about **y-axis**

$$V = \int_a^b \left(\begin{array}{c}\text{shell}\\\end{array}\right)\left(\begin{array}{c}\text{shell}\\\end{array}\right) dx$$

$$V = \int_a^b 2\pi \left(\begin{matrix}\text{shell}\\\text{radius}\end{matrix}\right)\left(\begin{matrix}\text{shell}\\\text{height}\end{matrix}\right) dx$$

$$= \int_0^4 2\pi (x)(\sqrt{x}) \, dx$$

$$= 2\pi \int_0^4 x^{\frac{3}{2}} \, dx$$

$$= 2\pi \left. \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right|_0^4 = \cdots = \frac{128\,\pi}{5}$$

(diagram top right)

$$\Delta y = y_2 - y_1$$
$$= y - 0$$
$$= y$$
$$= \sqrt{x}$$

$$y = \sqrt{x}$$

Q9
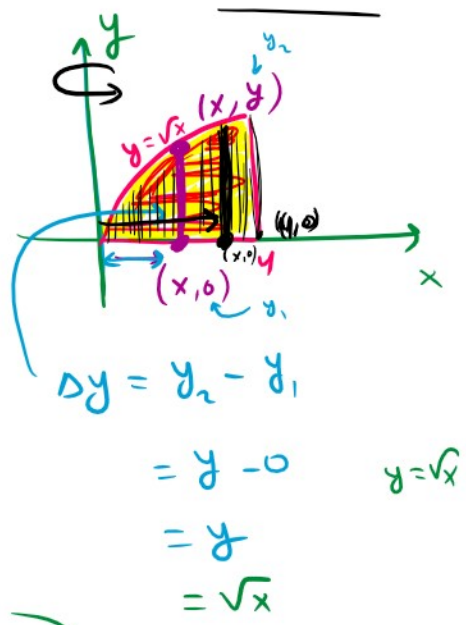
③ $y = x^2$, $y = 2-x$, $x = 0$, $x \geq 0$

about y-axis
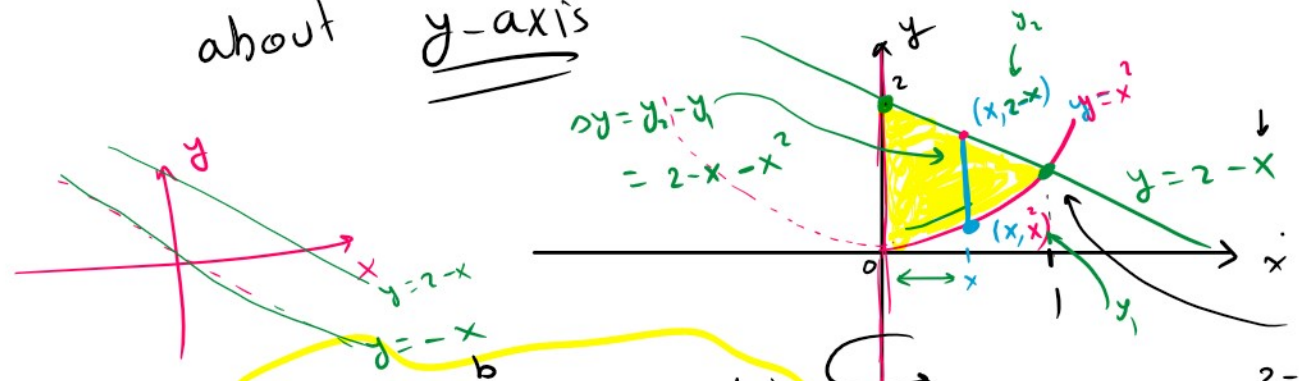
(diagrams)

$$\Delta y = y_1 - y_2$$
$$= 2 - x - x^2$$

$$y = y$$
$$2 - x = x^2$$
$$x^2 + x - 2 = 0$$
$$(x+2)(x-1) = 0$$
$$x = -2 \;,\; \boxed{x = 1}$$

$$\Rightarrow V = 2\pi \int_a^b \left(\begin{matrix}\text{shell}\\\text{radius}\end{matrix}\right)\left(\begin{matrix}\text{shell}\\\text{height}\end{matrix}\right) dx \qquad x = 0$$

$$= 2\pi \int_0^1 x(2 - x - x^2) \, dx$$

$$= 2\pi \int (2x - x^2 - x^3) \, dx = 2\pi \left. \left(x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right) \right|_0^1$$

$$= 2\pi \int_0^{} (2x - x^2 - x^3)\,dx = 2\pi \left(x - \frac{x^2}{3} - \frac{x^4}{4} \right) \Big|_0$$

$$= 2\pi \left(1 - \frac{1}{3} - \frac{1}{4} \right) = \boxed{\frac{5\pi}{6}}$$

④ $\quad x = 2y - y^2 \quad , \quad y\text{-axis} \qquad \text{about} \qquad x\text{-axis}$

$$\overset{\theta}{\longrightarrow} \quad \Rightarrow \quad V = 2\pi \int_c^d \binom{shell}{radius}\binom{shell}{length} dy$$

$$x = 2y - y^2 = -[y^2 - 2y] = -[y^2 - 2y + 1 \; -1]$$

$$= -[(y-1)^2 - 1] = 1 - (y-1)^2$$

$x = -(y-1)^2 \qquad x = (y-1)^2$

$x = 1-(y-1)^2 \qquad x = y^2$

$y = x^2$

$y = (x-1)^2$

$y = -(x-1)^2 + 1$

$y = -(x-1)^2$
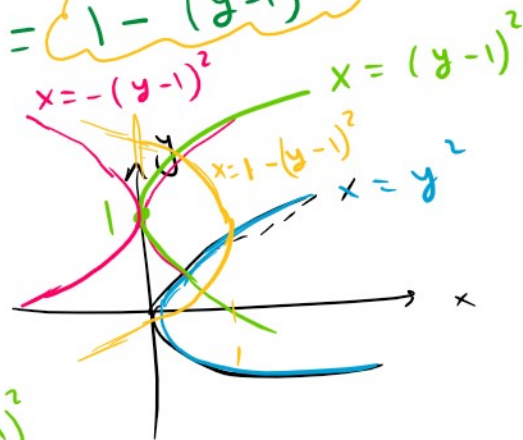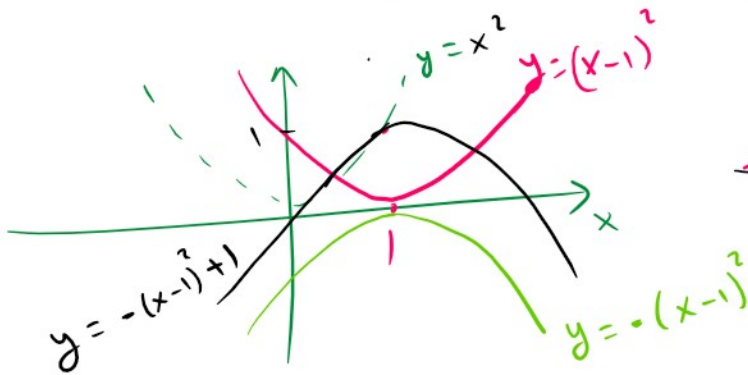
$$x = 1 - (y-1)^2$$

$$V = 2\pi \int_0^2 (\,y\,)(2y - y^2)\,dy$$

shell length

$$= 2\pi \int^2 (2y^2 - y^3)\,dy$$

$x_1 \quad x_2$

$(x,y) \; x = 1-(y-1)^2$

$= 2y - y^2$

$(0,y)$

$Dx = x_2 - x_1$

when $x = 0 \Rightarrow 0 = 2y - y^2$

$$= 2\pi \int_0^{} \left(\right)$$

$$= 2\pi \quad 2\frac{y^3}{3} - \frac{y^4}{4} \Big|_0^2$$
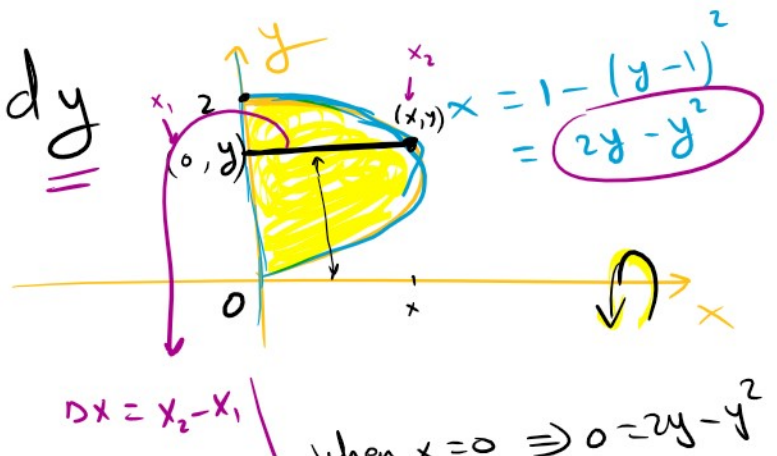
$$= \cdots = \boxed{\frac{8\pi}{3}}$$

$Dx = x_2 - x_1$

$= x - 0$

$= \boxed{x}$

$= 2y - y^2$

when $x = 0 \Rightarrow 0 = 2y - y^2$
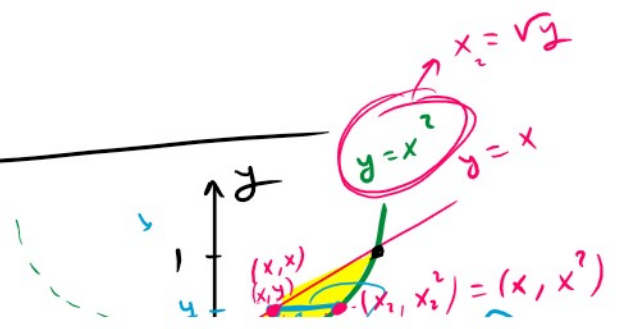
$0 = y(2-y)$

$y = 0, \quad y = 2$

DM
SM
WM

SM   W.M
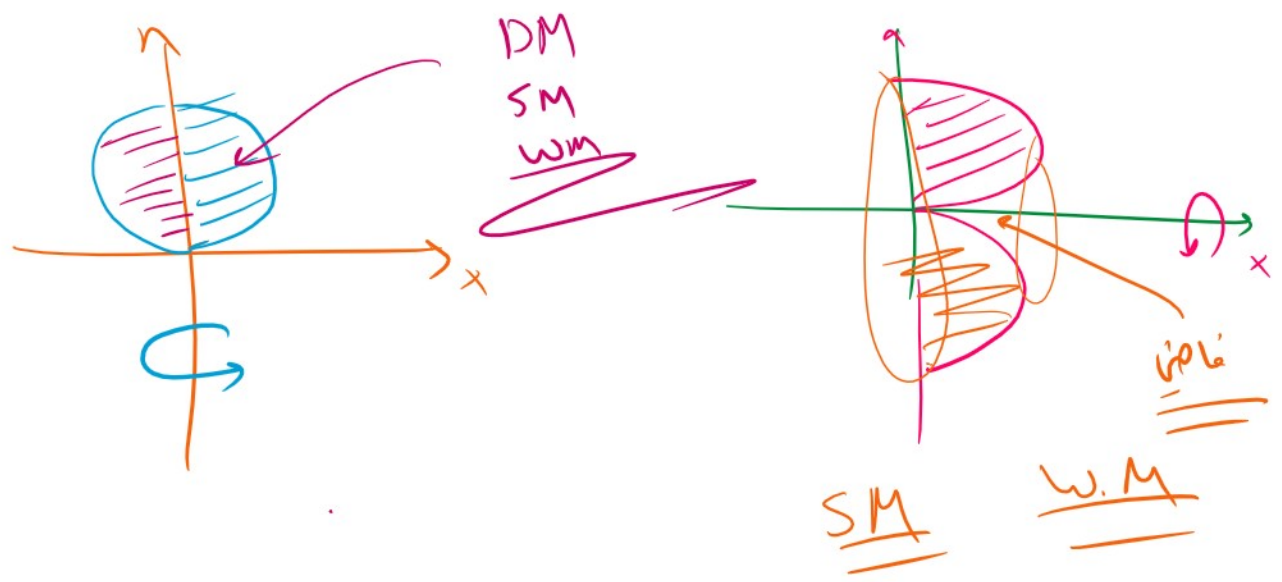
Q29   Find V of solid generated by rotating the region bounded by

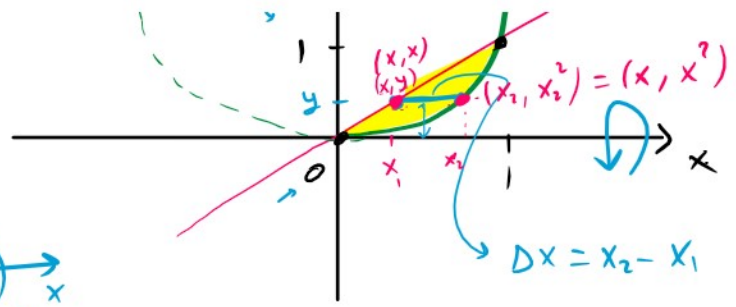$y = x$, $y = x^2$ about x-axis and y-axis

using SM and WM

$y = y$

$x_2 = \sqrt{y}$

$y = x^2$ $y = x$

(x, x)
(x, y) $(x_1, x_2^2) = (x, x^2)$

$$y = y$$
$$x^2 = x$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, \quad x = 1$$

$$\boxed{SM} \, \circlearrowright \to x$$

$$V = 2\pi \int_c^d \binom{shell}{radius}\binom{shell}{length} dy \quad = \sqrt{y} - y$$

$$= 2\pi \int_0^1 y(\sqrt{y} - y)\, dy$$

$$= 2\pi \int_0^1 (y^{\frac{3}{2}} - y^2)\, dy$$

$$= 2\pi \left. \frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{y^3}{3} \right|_0^1 = \cdots = \boxed{\frac{2\pi}{15}}$$

$$\boxed{DX = x^2 - x}$$
$$= \sqrt{y} - y$$
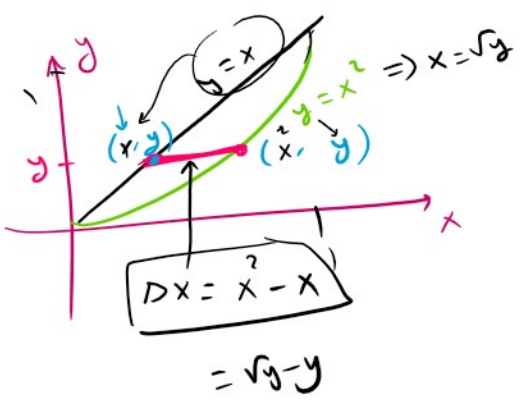
$$R(x) = \Delta y$$
$$= y_2 - y_1$$
$$= y - 0$$
$$= y$$
$$= x$$

$$r(x) = \Delta y = y_2 - y_1$$
$$= x^2 - 0$$
$$= x^2$$

$$\boxed{WM} \, \circlearrowright \to x$$

$$V = \pi \int_a^b [R^2(x) - r^2(x)]\, dx$$

$$= \pi \int_0^1 [(x)^2 - (x^2)^2]\, dx$$

$$= \pi \int_0^1 (x^2 - x^4)\, dx = \pi \left. \left(\frac{x^3}{3} - \frac{x^5}{5}\right) \right|_0^1$$

$$= \cdots = \boxed{\frac{2\pi}{15}}$$

$$\oint SM$$
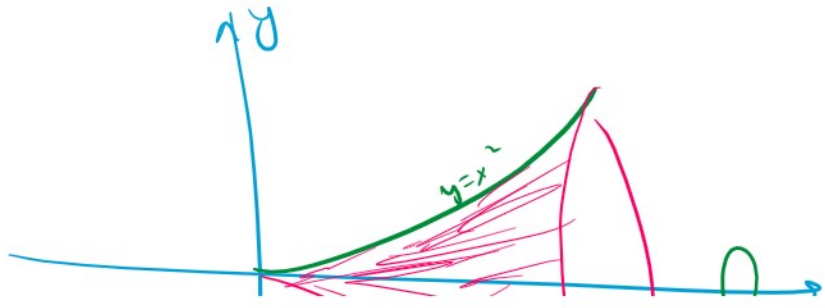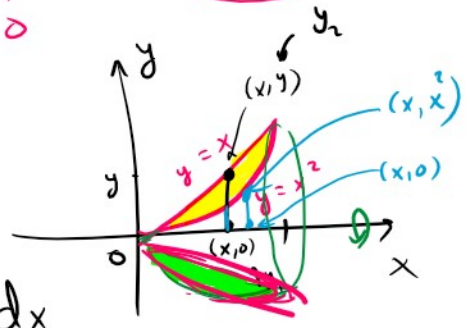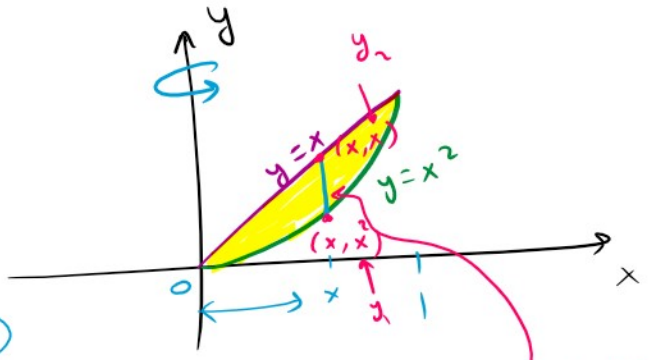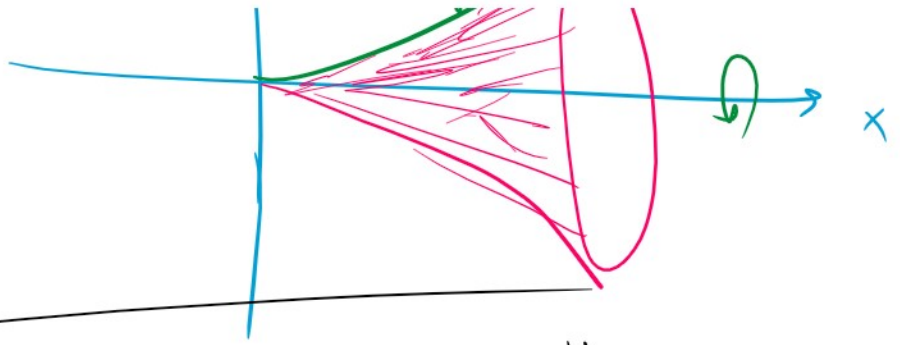
$$V = 2\pi \int_a^b \left(\begin{array}{c}\text{shell}\\\text{radius}\end{array}\right)\left(\begin{array}{c}\text{shell}\\\text{height}\end{array}\right) dx$$
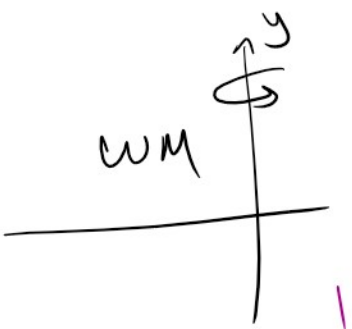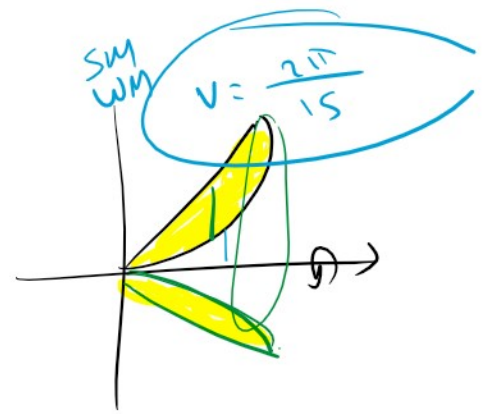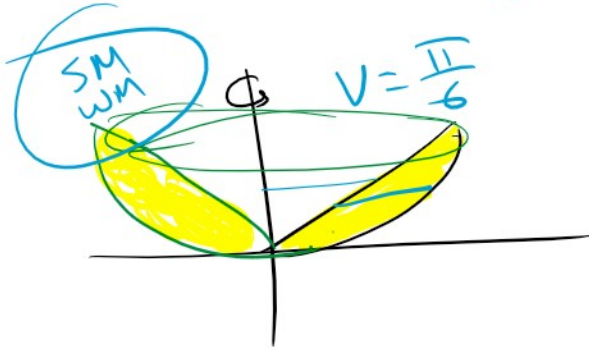
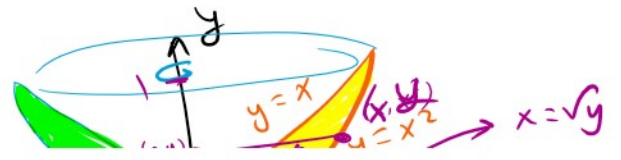$$= 2\pi \int_0^1 (x)(x - x^2)\, dx$$

$$= 2\pi \int_0^1 (x^2 - x^3)\, dx$$

$$= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = \cdots = \boxed{\frac{\pi}{6}}$$
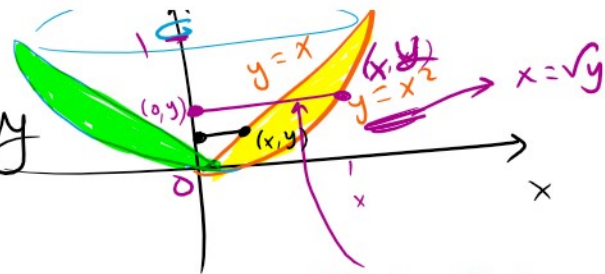
$$dy = y_2 - y_1 = x - x^2$$

SM
WM $V = \frac{\pi}{6}$

SM
WM $V = \frac{2\pi}{15}$

$$WM$$

$$V = \pi \int_c^d \left[R^2_{(y)} - r^2_{(y)} \right] dy$$

$y = x$ (x, y) $y = x^2$ $x = \sqrt{y}$

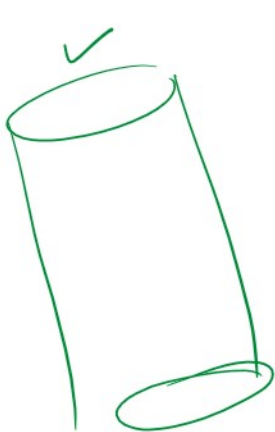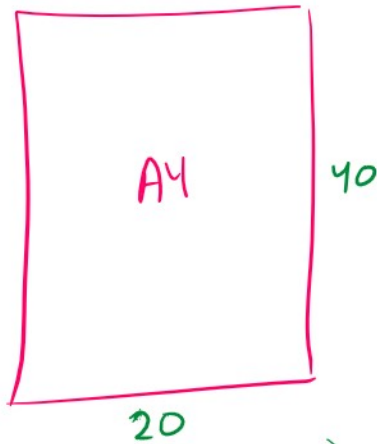$$V = \pi \int_0^1 \left[(\sqrt{y})^2 - (y)^2 \right] dy$$

$$= \pi \int_0^1 (y - y^2) \, dy$$

$$= \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1 = \boxed{\frac{\pi}{6}}$$
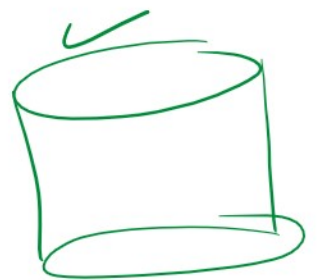
$r(y) = \Delta x$
$= x_2 - x_1$
$= y - 0$

$R(y) = \Delta x$
$= x_2 - x_1$
$= \sqrt{y} - 0$
$= \sqrt{y}$

حالة واحدة

Aربعون 40

20

الطول

العرض

$20 = المحيط = 2r\pi$ $\Rightarrow r\pi = 10$

$$\boxed{r = \frac{10}{\pi}}$$

40

$$V = (r^2 \pi) \, 40$$

$$(100 \; \tfrac{1}{\pi}) \, 40$$

A4   40

AY   40

20

→

40

$V = (\ )$

$= \left(\dfrac{100}{\pi^2}\, \pi\right) 40$

$= \dfrac{4000}{\pi}$

AY   40

20

$40 = $ المحيط $= 2r\pi$ $r\pi = 20$

$r = \dfrac{20}{\pi}$

20   نصف المحيط

$V_2 = (r^2\pi)\, 20$

$= \left(\dfrac{400}{\pi^2}\right) \pi \, 20$

$= \dfrac{8000}{\pi}$

$r^2\pi\, h$ r ← نصف قطر المحيط