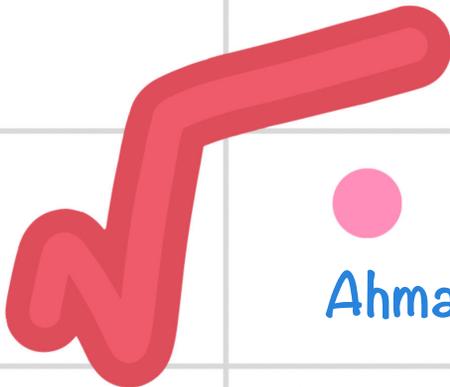


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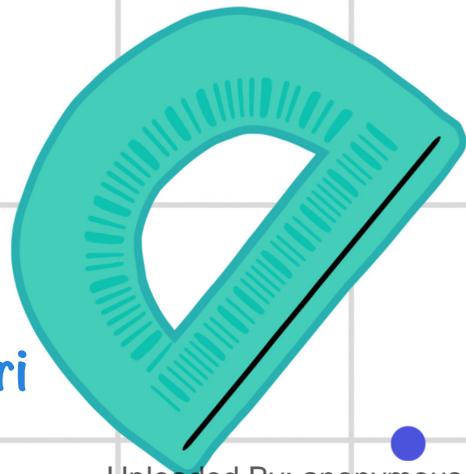


Calculus 2

Chapter 10.4

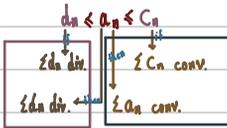


Ahmad Ouri



10.4 Comparison Test

Direct Comparison Test:



result two. result one.

C_n and d_n are given whether conv. or div.

a_n, d_n and C_n are positive \forall large n .

Ex. Check for conv. or div.?

$$1) \sum_{n=1}^{\infty} \frac{3 + \sin n}{n^2} \quad -1 \leq \sin n \leq 1$$

$\sum_{n=1}^{\infty} \frac{4}{n^2}$ conv. using $\frac{3}{n^2} \leq \frac{3 + \sin n}{n^2} \leq \frac{4}{n^2}$ P-series test.

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.

$$2) \sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^n \quad \left(\frac{n}{3n+1} \right)^n \leq \left(\frac{n}{2n} \right)^n$$

$\sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^n$ convergent geometric series.

converges to $\frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$.

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.

$$3) \sum_{n=1}^{\infty} \frac{3}{n + \sqrt{n}} \quad n > \sqrt{n} \rightarrow 2n > n + \sqrt{n}$$

$\sum_{n=1}^{\infty} \frac{3}{2n}$ div. by P-series. $\frac{3}{2n} < \frac{3}{n + \sqrt{n}}$ Harmonic Series.

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by DCT.

Limit Comparison Test:

$\sum_{n=1}^{\infty} a_n$? Find b_n s.t. $\sum b_n$ is known.

a_n and b_n are positive.

$-\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$ then both $\sum a_n$ and $\sum b_n$ conv. or both div.

$-\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ conv. then $\sum a_n$ conv.

$-\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ div. then $\sum a_n$ div.

Ex. Check for conv. or div. ?

1) $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ $b_n = \frac{1}{n}$ div. (Harmonic. s)

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

2) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$ $b_n = \frac{1}{n^2}$ conv. P-series

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+2^n}{n^2 2^n}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n+2^n}{n^2 2^n} \cdot \frac{n^2}{1} = \lim_{n \rightarrow \infty} \frac{n}{2^n} + 1 = 0 + 1 = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

3) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^3+2}}$ $b_n = \frac{1}{\sqrt{n}}$ div. P-series.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n+1}{n^3+2}}}{\frac{1}{\sqrt{n}}} = \sqrt{1} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

3) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$ $b_n = \frac{1}{n}$ div. Harmonic Series.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{1}{1+\ln n} \cdot n = \lim_{n \rightarrow \infty} \frac{n}{1+\ln n} = \infty$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

3) $\sum_{n=1}^{\infty} \frac{1 \ln n!}{n^{4n}}$ $b_n = \frac{1}{n^{4n}}$ conv. P-series.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

3) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{n^2 2^n} + \frac{1}{n^2} \right) < \sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{n^2} \right)$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ is conv. by P-series test.

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ conv. geometric series. ($r = \frac{1}{2}$)

\rightarrow conv. + conv. = conv. so $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{n^2} \right)$ conv.

Hence, $\sum_{n=1}^{\infty} a_n$ conv. by DCT.

4) $\sum_{n=1}^{\infty} \frac{n+2}{n^2 \cdot n^{3/5}}$ $b_n = \frac{1}{n^2}$ conv. by P-series Test.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{n^2(n+2)}{n^2 \cdot n^{3/5}} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

6) $\sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}}$ $b_n = \frac{1}{\sqrt{n^3}}$ conv. P-series

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{n+1}{n^2} \cdot \sqrt{n^3} = \lim_{n \rightarrow \infty} \frac{n+1}{n} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

OR $\sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}} < \sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}} = \sum_{n=1}^{\infty} \frac{2}{n^{3/2}} = \frac{2}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$ conv. P-series.

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.