بِسۡمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيمِ

MATH1321

Calculus 2

Chapter 10.4

Ahmad Ouri

10.4 Comparison Test

Direct Comparison Test:

$d_n \leq a_n \leq c_n$

result two	result one
$\sum d_n$ div.	$\sum c_n$ conv.
$\sum d_n$ div. $\xleftarrow{\text{then}}$	$\sum a_n$ conv.

c_n and d_n are given whether conv. or div.

a_n, d_n and c_n are positive $\forall$ large n.

Ex. Check for conv. or div. ?

1) $\sum_{n=1}^{\infty} \frac{3+\sin n}{n^2}$

$-1 \leq \sin n \leq 1$

$\underset{\text{div.}}{\frac{3}{n^2}} \leq \frac{3+\sin n}{n^2} \leq \underset{\text{conv.}}{\frac{4}{n^2}}$

$\sum_{n=1}^{\infty} \frac{4}{n^2}$ conv. using P-series test.

$\longrightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.

2) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$

$\left(\frac{n}{3n+1}\right)^n \leq \left(\frac{n}{3n}\right)^n$ conv.

$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ convergent geometric series.

converges to $\frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$.

$\longrightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.

3) $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$

$n > \sqrt{n} \longrightarrow 2n > \sqrt{n} + n$

$\underset{\text{div.}}{\frac{3}{2n}} < \frac{3}{n+\sqrt{n}}$.

$\sum_{n=1}^{\infty} \frac{3}{2n}$, div by P-series.

Harmonic Series.

$\longrightarrow \sum_{n=1}^{\infty} a_n$ div. by DCT.

Limit Comparison Test:

$\sum_{n=1}^{\infty} a_n$? Find b_n s.t $\sum b_n$ is known.

a_n and b_n are positive.

- $\lim_{n\to\infty} \frac{a_n}{b_n} = \underset{\text{finite}}{c} > 0$ then both $\sum a_n$ and $\sum b_n$ conv. or both div.
- $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ conv. then $\sum a_n$ conv.
- $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ div. then $\sum a_n$ div.

Ex. Check for conv. or div. ?

1) $\sum_{n=1}^{\infty} \tan\frac{1}{n}$ $b_n = \frac{1}{n}$ div. (Harmonic. s)

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\tan\frac{1}{n}}{\frac{1}{n}} = 1.$

$\longrightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

2) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$ $b_n = \frac{1}{n^2}$ conv. P-series

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+2^n}{n^2 2^n}}{?} = \lim_{n\to\infty} \frac{\frac{n+2^n}{n^2 2^n}}{\frac{1}{n^2}}$

$= \lim_{n\to\infty} \frac{n}{2^n} + 1 = \lim_{n\to\infty} \frac{1}{2^n \ln 2} + 1 = 0 + 1 = 1.$

$\longrightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

5) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$ $b_n = \frac{1}{\sqrt{n}}$ div. P-series.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \sqrt{\frac{n^2+n}{n^2+2}} = \sqrt{1} = 1.$

$\longrightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

7) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$ $b_n = \frac{1}{n}$ div. Harmonic Series.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{1+\ln n} = \lim_{n\to\infty} \frac{1}{0+\frac{1}{n}} = \infty$

$\longrightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

8) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{9/2}}$ $b_n = \frac{1}{n^{3/2}}$ conv. P-series.

$\lim_{n\to\infty} \frac{a_n}{b_n} = 0.$

$\longrightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

3) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{n 2^n} + \frac{1}{n^2}\right) \leq \overset{\text{conv.}}{\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{n^2}\right)}$

$\sum \frac{1}{n^2}$ is conv. by P-series test.

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ conv. geometric series. $(r = \frac{1}{2})$

$\longrightarrow$ conv. + conv. = conv. so $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{n^2}\right)$ conv.

Hence, $\sum_{n=1}^{\infty} a_n$ conv. by DCT.

4) $\sum_{n=1}^{\infty} \frac{n+2}{n^3+n^2+5}$ $b_n = \frac{1}{n^2}$ conv. by P-series Test.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2(n+2)}{n^3+n^2+5} = 1.$

$\longrightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

6) $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$ $b_n = \frac{1}{\sqrt{n^3}}$ conv. P-series

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n+1}{\sqrt{n^5}} \cdot \sqrt{n^3} = \lim_{n\to\infty} \frac{n+1}{n} = 1.$

$\longrightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

or $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}} \leq \sum_{n=1}^{\infty} \frac{n+n}{n^2\sqrt{n}} = \frac{2n}{n^2\sqrt{n}} = \frac{2}{n^{\frac{3}{2}}}$

$\sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$ conv. P-series.

$\longrightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.