

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

MATH1321

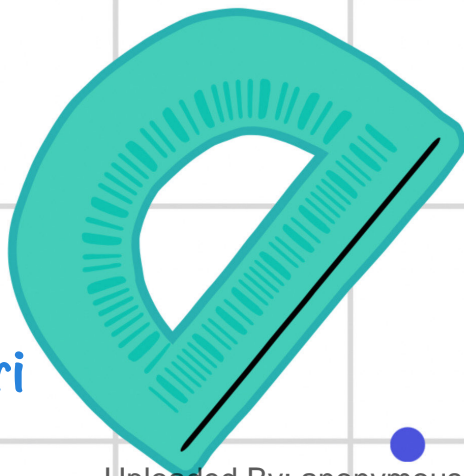


Calculus 2

Chapter 10.4

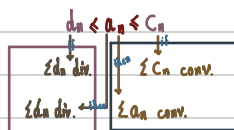


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10.4 Comparison Test

Direct Comparison Test:



c_n and d_n are given whether
conv. or div.

a_n, d_n and c_n are positive \forall large n .

Ex. Check for conv. or div.?

$$1) \sum_{n=1}^{\infty} \frac{3 + \sin n}{n^2} \quad -1 \leq \sin n \leq 1 \quad \text{conv.}$$

$$\sum_{n=1}^{\infty} \frac{4}{n^2} \text{ conv. using } \frac{3}{n^2} \leq \frac{3 + \sin n}{n^2} \leq \frac{4}{n^2} \quad \text{P-series test.}$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.

$$2) \sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^n \quad \left(\frac{n}{3n+1} \right)^n \leq \left(\frac{n}{2n} \right)^n \quad \text{conv.}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^n \text{ convergent geometric series.}$$

converges to $\frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1}{2}$.

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.

$$3) \sum_{n=1}^{\infty} \frac{3}{n + \sqrt{n}} \quad n > \sqrt{n} \rightarrow 2n > \sqrt{n} + n$$

$$\sum_{n=1}^{\infty} \frac{3}{2n} \text{ div. by P-series. } \frac{3}{2n} < \frac{3}{n + \sqrt{n}} \quad \text{div.}$$

Harmonic Series.

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by DCT.

Limit Comparison Test:

$\sum_{n=1}^{\infty} a_n$? Find b_n s.t. $\sum b_n$ is known.

a_n and b_n are positive.

$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \text{finite } c > 0$ then both $\sum a_n$ and $\sum b_n$ conv. or both div.

$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ conv. then $\sum a_n$ conv.

$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ div. then $\sum a_n$ div.

Ex. Check for conv. or div. ?

1) $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ $b_n = \frac{1}{n}$ div. (Harmonic s.)

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

2) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$ $b_n = \frac{1}{n^2}$ conv. P-series

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+2^n}{n^2 2^n}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n+2^n}{n^2} = \lim_{n \rightarrow \infty} \frac{1+2^n/n^2}{2^n/n^2} = \lim_{n \rightarrow \infty} \frac{1}{2^n/n^2} + 1 = 0 + 1 = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

3) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^3+2}}$ $b_n = \frac{1}{\sqrt{n^3}}$ div. P-series.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n+1}{n^3+2}}}{\frac{1}{\sqrt{n^3}}} = \lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{n^3+2}} \cdot \sqrt{n^3} = \lim_{n \rightarrow \infty} \sqrt{\frac{n^3+n^2+1}{n^3+2}} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

7) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$ $b_n = \frac{1}{n}$ div. Harmonic Series.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{1}{1+\ln n} \cdot n = \lim_{n \rightarrow \infty} \frac{n}{1+\ln n} = \lim_{n \rightarrow \infty} \frac{1}{0+\frac{1}{n}} = \infty.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ div. by LCT.

8) $\sum_{n=1}^{\infty} \frac{(\ln n)^4}{n^{9/2}}$ $b_n = \frac{1}{n^{9/2}}$ conv. P-series.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

3) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{n 2^n} + \frac{1}{n^2} \right) \leq \sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{n^2} \right)$ conv.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \text{ is conv. by P-series test.}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \text{ conv. geometric series. } (r = \frac{1}{2})$$

$$\rightarrow \text{Conv.} + \text{Conv.} = \text{Conv. so } \sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{n^2} \right) \text{ conv.}$$

Hence, $\sum_{n=1}^{\infty} a_n$ conv. by DCT.

4) $\sum_{n=1}^{\infty} \frac{n+2}{n^3 + n^{3/2} + 5}$ $b_n = \frac{1}{n^2}$ conv. by P-series Test.

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{n^2(n+2)}{n^3 + n^{3/2} + 5} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

6) $\sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}}$ $b_n = \frac{1}{\sqrt{n^3}}$ conv. P-series

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{n+1}{n^2} \cdot \sqrt{n^3} = \lim_{n \rightarrow \infty} \frac{n+1}{n} = 1.$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by LCT.

$$\text{or } \sum_{n=1}^{\infty} \frac{n+1}{n^2 \sqrt{n}} \leq \sum_{n=1}^{\infty} \frac{n+n}{n^2 \sqrt{n}} = \sum_{n=1}^{\infty} \frac{2n}{n^{5/2}} = \sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$$

$$\sum_{n=1}^{\infty} \frac{2}{n^{3/2}} \text{ conv. P-series.}$$

$\rightarrow \sum_{n=1}^{\infty} a_n$ conv. by DCT.