STAT2361

ملاحظات المحاضرات

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BZU-HUB

Chapter 5

ملاحظة: المحاضرات من شرح الدكتور محمد مضية

5.1: Discrete random variabds
5.-3

- Medical Test $\to$ positive $\xrightarrow[\text{بنربطهم}]{\text{نحولها}}$ # رقم
  $\to$ negative

- Roll 2 dice : outcomes $\xrightarrow{\text{بنحولها}}$ numbers

* Random Variable: numerical description of the outcomes of a random expirement

*Example: Toss 2 Coins

$$S = \{HH, HT, TH, TT\}$$

# of Tails $= X$ $\qquad$ X = Count the number of heads

$$0 \leftarrow HH \rightarrow 2$$
$$1 \leftarrow HT \rightarrow 1$$
$$2 \leftarrow TT \rightarrow 0$$

* 2 types of random variables

$\boxed{1}$ منفصلة Discreate Random Variables (D.R.V)

$$X = x_1, x_2, x_3 \dots x_n$$

*Ex: ① # of children in a family

0, 1, 2, --- 15

② # of pass students

0, 1, --- 31

$\boxed{2}$ Continuous Random Varibles (Intervals)

ch6 *Ex: time, Income, distance, weight ---

* **Example:** Toss 2 Coins نفس المثال السابق

$S = \{HH, HT, TH, TT\}$

Define the random variable X

$\Rightarrow X$ can assume the values

$X = 0, 1, 2 \Rightarrow X$ is discrete

* **Ex:** Roll 2 dice

$S = \{(1,1), \ldots (6,6)\}$ let $X =$ Sum of the 2 faces

$(1,1) \rightarrow 2 \quad \ldots \quad (6,6) \rightarrow 12$

$X = 2, 3, \ldots 12 \Rightarrow$ Discrete

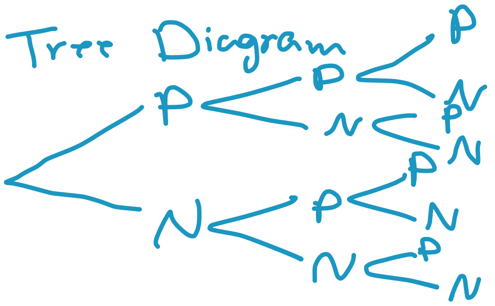
$Y =$ Difference (face 2 - face 1)

$Y = 0, \pm 1, \ldots, \pm 5$

* **Ex:** 3 Test Results (Positive (P), Negative (N))

$\# = 2^3 = 8$

Tree Diagram



$S = \{(PPP), (PPN), \ldots (NNN)\}$

Define $X = \#$ of positive results

$X = 0, 1, 2, 3 \Rightarrow$ Discrete

Table

x	$P(X=x)$
0	1/8
1	3/8
2	3/8
3	1/8

$\leftarrow P(X=0)$

مع $\leftarrow$ Probability distribution

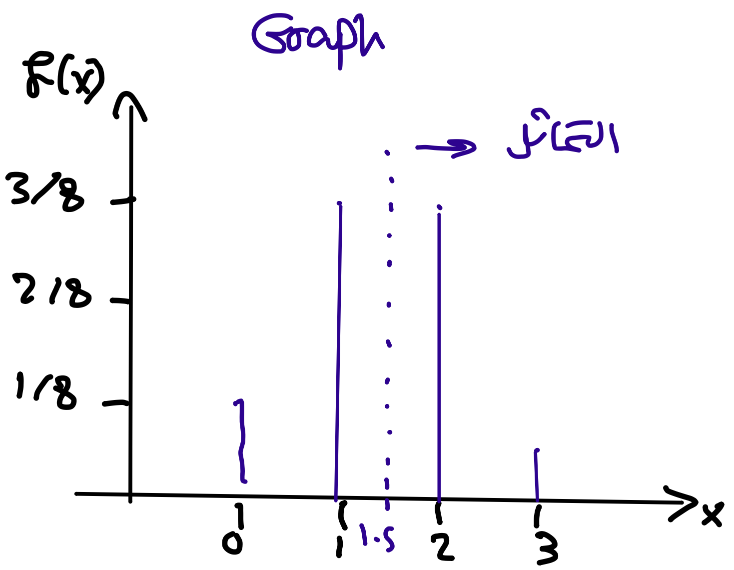
$P(X=x) = f(x) =$ probability function

$f(1) = 3/8$

* 2 Conditions

$\boxed{1}$ $0 \le f(x) \le 1$

$\boxed{2}$ $\sum f(x) = 1$



احتمال متساوي

*** Ex:** Uniforms discrete random Variable $f(x_i) = \frac{1}{n}$

Roll a die $P(s_i) = \frac{1}{6}$ Uniform

*** Ex:** Consider a discrete random Variable X with probability Function $f(x) = \frac{x+1}{10}$, $x = 1, 2, 4$

هل تحقق الشرطين ؟

$f(1) = \frac{2}{10}$, $f(2) = \frac{3}{10}$, $f(4) = \frac{5}{10}$ ✓ أقل من 1

$\sum f(x) = 1$ ✓

Let $X = D.R.V = \{x_1, x_2, \dots x_n\}$ with a probability function $f(x)$, Then:

1. The expected Value of x, $\underset{\mu}{E(x)} = \sum x \cdot f(x)$

2. The Variance of X

$$Var(x) = \sum (x - E(x))^2 \cdot f(x)$$

$$\sigma(x) = \sqrt{\sum (x - E(x))^2 \cdot f(x)}$$

***Ex:** نفس المثال السابق

x	$P(X=x)$	$x \cdot P(x)$	$(x-E(x))$	$(x-E(x))^2$	$(x-E(x))^2 \cdot P(x)$
0	$1/8$	0	$-3/2$	$\frac{9}{4}$	$\frac{9}{32}$
1	$3/8$	$3/8$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{32}$
2	$3/8$	$6/8$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{32}$
3	$1/8$	$3/8$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{9}{32}$
		$\frac{12}{8}$	0		$\frac{24}{32} = \frac{3}{4}$

$\boxed{1}$ Find $E(x)$

$E(x) = \sum x \cdot P(x) = \frac{12}{8} = 1.5 = \frac{3}{2}$

$\boxed{2}$ Find $Var(x)$

$= \frac{3}{4}$

$\sigma(x) = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

17. A volunteer ambulance service handles 0 to 5 service calls on any given day. The probability distribution for the number of service calls is as follows.

Number of Service Calls	Probability	Number of Service Calls	Probability
0	.10	3	.20
1	.15	4	.15
2	.30	5	.10

Discrete

a. What is the expected number of service calls?
b. What is the variance in the number of service calls? What is the standard deviation?

$\boxed{1}$ What is the probability of at least 4 calls?

$P(X \geq 4) = P(4) + P(5)$

$= 0.15 + 0.1$

$= 0.25$

$\boxed{2}$ At least one call

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - 0.1$

$= 0.9$

$\boxed{3}$ Find $E(x)$

By calculator $= 2.45$

$\boxed{4}$ Find $Var(x)$

By calculator $= 1.43$

CALCULATOR: SD (weighted mean)

- Clean (Shift mode 3 = =)
- mode 2
- Data Entry:

$$\left.\begin{array}{l} 0 \text{ Shift}, \ 0.1 \ M+ \\ \vdots \\ 5 \text{ Shift}, \ 0.1 \ M+ \end{array}\right\} n=1$$

- Shift 2

$E(x) =$ Shift 2 1 $= 2.45$

$\sigma(x) =$ Shift 2 2 $= 1.43$

- Shift 2 3 → math error

error ← 0 ← n=1, (n−1) لأنه ↵

5.4: The Binomial Distribution
توزيع ذو الحدين

* Two possible outcomes : → success, → failure
  - Coin H, T
  - Birth B, G
  - Exam Pass, fail

* Binomial Experiment
  1. n-trials
  2. in each trial 2 outcomes : Success or failure
  3. P(success) = p , P(failure) = 1 − p → are the same on each trial
  4. The trials are independent

*Ex: Toss a coin 100 times
- n = 100
- Head, tail
- $P(H) = P(T) = \frac{1}{2}$
- independent

* Binomial Random Variable
Consider a binomial experiment. # of trials = n , P(success) = p

Define the random variable X to be the number of success in n-trials
- X = 0, 1, 2 ..., 100 ⇒ discrete variables
- X is called a binomial random variable
- $X = B(n, p)$
  ↳ $X = B(100, \frac{1}{2})$

Let $X = B(n,p)$, then :

1. The probability function for X is given by.

مهم ← $f(x) = (n\,C\,x)\,(p^x)\,((1-p)^{n-x})$

- n = # of trials
- x = # of success
- f(x) ≡ the probability of x-success in trials
- p = p(success)
- p(failure) = 1 − p

2. The expected Value of x:

$E(x) = n \cdot p$

3. The Variance of X

$Var(x) = n \cdot p\,(1-P)$

$\sigma(x) = \sqrt{var(x)}$

Ex: 5% of the selected from a production line are defective

1. In a sample of 1,000 items, what is the expected number of defective items? what is the variance?

- $E(x) = n \cdot p = 1{,}000 * 0.05 = 50$
- $Var(x) = n \cdot p\,(1-p) = 50\,(0.95) = 47.5$

2. In a sample of 2,000 items, what is the expected number of nondefective items? what is the variance?

$Y = \#$ of nondefective

$n = 2{,}000 \qquad p = 0.95$

$E(Y) = 1900$

3. In a sample of 10 items, what is the probability of exactly 6 defective?

$f(x) = (n\,C\,x)\,(p^x)\,((1-p)^{n-x})$

$f(6) = (10\,C\,6)\,(0.05)^6\,(0.95)^4$

$\cong 0$

$2.67 * 10^{-6}$

$= 0.\underbrace{00000}_{5\text{ zeros}}267$

4. In a sample of 8 Items, what is the probability of at least 2 defective?

$X = B(8, 0.05)$

$p(x \geqslant 2) = f(2) + f(3) + \dots + f(8)$

$= 1 - p(x < 2)$

$= 1 - [f(0) + f(1)]$

$= 1 - [(8\,C\,0)(0.05)^0(0.95)^8] + [(8\,C\,1)\,(0.05)^1 \cdot (0.95)^7]$

$= 0.0572$

[B] What is the probability of at most (no more than) 2 non-defective items in a sample of 6 items

$X = B(6, 0.95)$

$P(X \le 2) = f(2) + f(1) + f(0)$

5.5:

The poisson Distribution

of occurences of an event in a certain period

*** Poisson Experiment :**

[1] The probability of an occurrence is the same at any 2 equal intervals.

[2] The probability of occurence or non-occurence is independent of the probability of occurence or non-occurence on any other interval

*** The probability function for a poisson Distribution is given by:**

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \xrightarrow{} \text{Shift } \ln -\mu$$

- x = # of occurence of an event in a certain period. $x = 0, 1, 2 \ldots$
- μ = The expected number of occurences in a given period.

*** Ex:** X = Poisson random Variable, $\mu = 10$ occurance per period

[1] $f(5) = \frac{10^5 \cdot e^{-10}}{5!}$

$= 0.0378$

[2] At least 2 occurances

$P(X \ge 2) = f(2) + f(3) + \ldots f(\infty)$

or $= 1 - P(X < 2)$

$= 1 - [f(0) + f(1)]$

$= 0.9995$

* Q: 44, P: 213 :-

15 accidents each year $= \mu$

$\boxed{a}$ Average per month

$\frac{15}{12} = 1.25$ per month

$\boxed{b}$ No accident $\rightarrow x = 0$, $\mu = 1.25$, $f(0) = ?$ تقريب لأربع منازل

$f(0) = \frac{1 \cdot e^{-1.25}}{1} = 0.2865$

(c) $x = 1$, $f(1) = 0.3581$

$\boxed{d}$ pr more than 1

$P(x > 1) = 1 - [f(0) + f(1)]$

$= 0.3554$