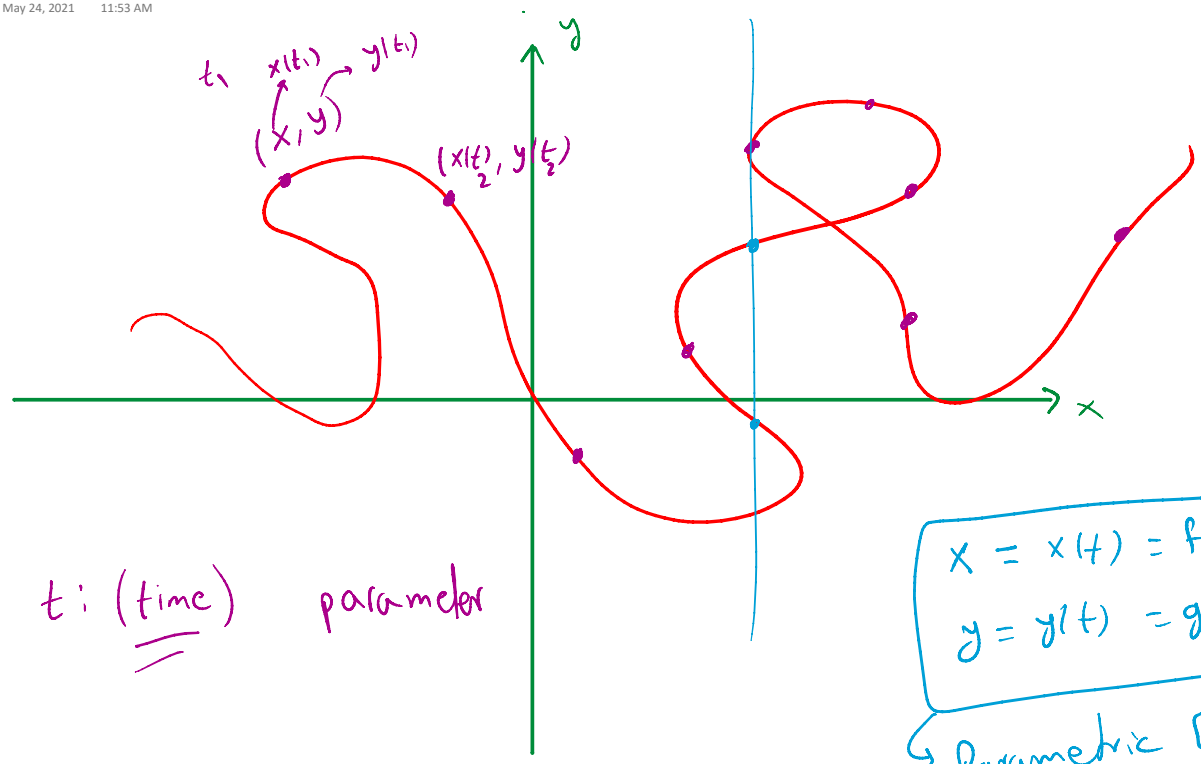
11.1

Monday, May 24, 2021 11:53 AM




not function

curve

Parametric curve

t: (time) parameter

Parameter interval

$t \in I$

$x = x(t) = f(t)$

$y = y(t) = g(t)$

Parametric Eq.

I

$I = [a,b]$

$a \le t \le b$

$t = a \Rightarrow (x(a), y(a))$

starting point

(IP): Initial point

$t = b \Rightarrow (x(b), y(b))$

(TP) Terminal Point

$I = (a,b)$

$a < t < b$

~~TP~~

~~IP~~

$I = [a,b)$

$a \le t < b$

IP ✓

TP X

$I = (a,b]$

$a < t \le b$

IP X

TP ✓

Exp Assume particle moves along the curve whose Parametric Equations are

Parametric Equations are

① $x = t^2$ , $y = t+1$ , $-\infty < t < \infty$

Find (A) Cartesian equation (relation between x and y)

(B) sketch the curve

(C) Find direction

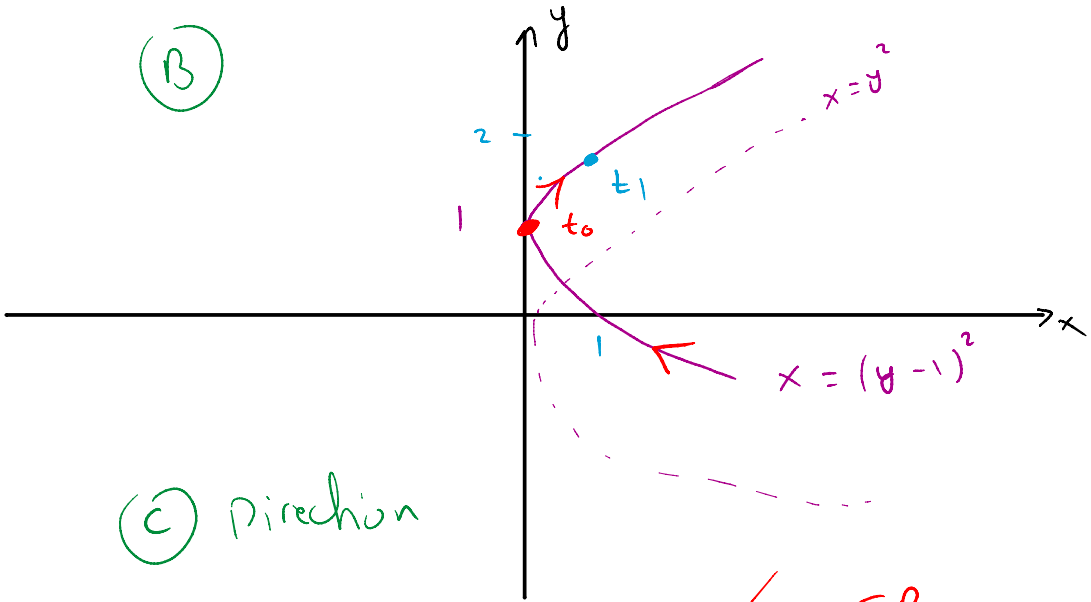
$y = t+1$

$t = y-1$

(A) $x = t^2 = (y-1)^2$

$x = (y-1)^2$ → Cartesian Eq..

(B)




$t_1 > t_0$

(C) Direction

$-\infty < t < \infty$ ⇒ ~~I~~ IP, ~~I~~ TP

$x = t^2$

$y = t+1$

$t_0 = 0$ ⇒ $(x,y) = (0^2, 0+1) = (0,1)$

$t_1 = 1$ ⇒ $(x,y) = (1^2, 1+1) = (1,2)$

$t_1 = 1 \Rightarrow (x,y) = (1, 1+1) = (1,1)$

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(13) (2) $x = t$ , $y = \sqrt{1-t^2}$ , $-1 \le t \le 0$

(A) Cartesian Eq.

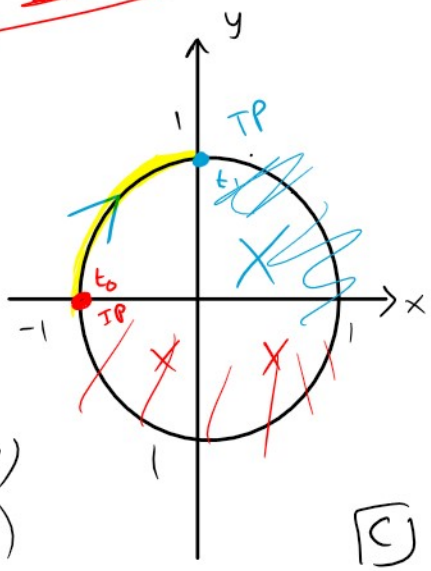
$y = \sqrt{1-t^2}$

$x = t$

$x^2 = t^2$

$y = \sqrt{1-x^2}$ ✓

$y \ge 0$




(B) $y = \sqrt{1-x^2}$

$y^2 + x^2 = 1$

$x = t$

$y = \sqrt{1-t^2}$

$-1 \le t \le 0$

IP , TP

IP : $t_0 = -1 \Rightarrow (x,y) = (-1, \sqrt{1-(-1)^2})$

$= (-1, \sqrt{1-1})$

$= (-1, 0)$

(C) Direction

TP : $t_1 = 0 \Rightarrow (x,y) = (0, \sqrt{1-0^2}) = (0, \sqrt{1}) = (0,1)$

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(16) $x = -\sec t$ , $y = \tan t$ , $-\frac{\pi}{2} < t < \frac{\pi}{2}$

$\sec^2 t = 1 + \tan^2 t$

$\sec^2 t - \tan^2 t = 1$

(A) $x^2 - y^2 = (-\sec t)^2 - (\tan t)^2$

$= \sec^2 t - \tan^2 t$

$= 1$

$x^2 - y^2 = 1$ → Cartesian Eq.

$x^2 - 1 = y^2$

$y = \pm\sqrt{x^2-1}$

$x^2 - y^2 = 1$ ✓

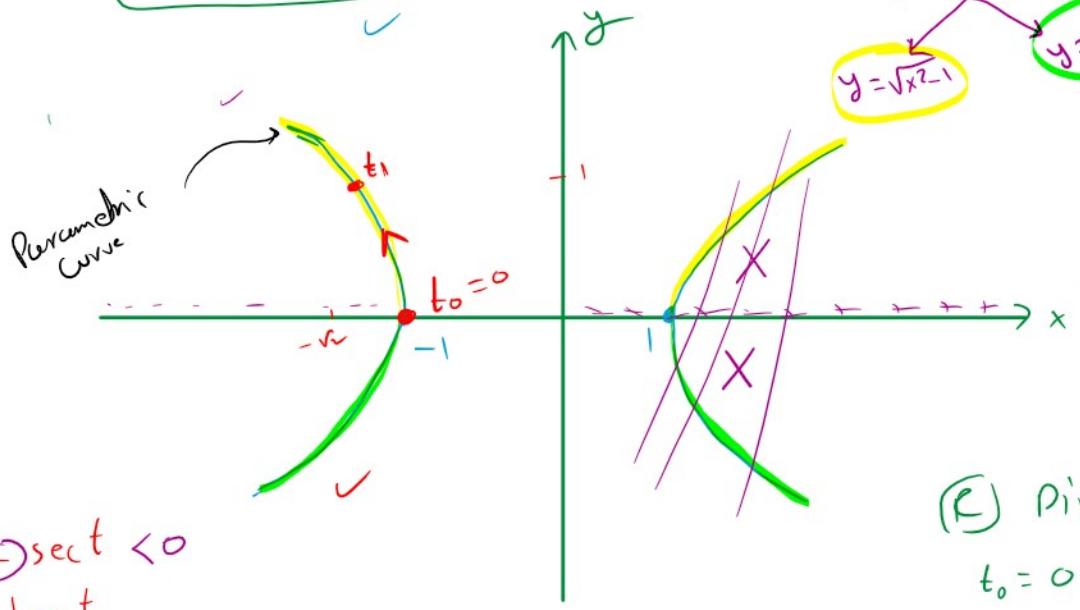
$x^2 - 1 = 0$

$y = \pm\sqrt{x^2-1}$

$y = \sqrt{x^2-1}$

$y = -\sqrt{x^2-1}$

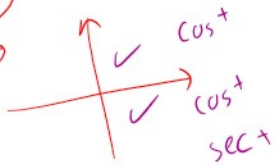
(B)




$x = -\sec t < 0$

$y = \tan t$

$-\frac{\pi}{2} < t < \frac{\pi}{2}$




~~I P~~

~~I P~~

$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

(C) Direction

$t_0 = 0 \Rightarrow (x, y) =$

$(-\sec 0, \tan 0) =$

$(-1, 0)$

$t_1 = \frac{\pi}{4} \Rightarrow (x, y) = (-\sec\frac{\pi}{4}, \tan\frac{\pi}{4})$

$= (-\sqrt{2}, 1)$

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(4) $x = t + \frac{1}{t}$

$y = t - \frac{1}{t}$

$t > 0$

(A) Cartesian Eq.

$x + y = 2t$

$x - y = \frac{2}{t}$

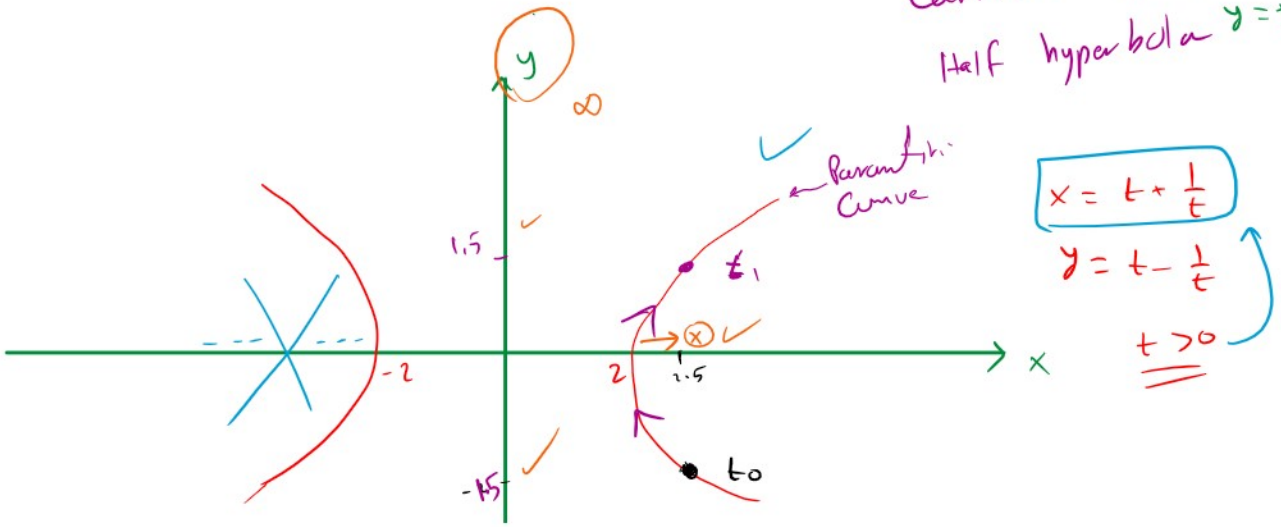
$\Rightarrow (x+y)(x-y) = (2t)\left(\frac{2}{t}\right)$

$x^2 - y^2 = 4$

Cartesian Eq. $y^2 = x^2 - 4$

Half hyperbola $y = \pm\sqrt{x^2-4}$

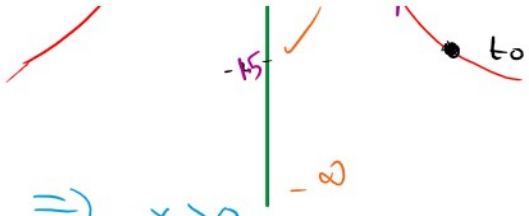
(B)




$x = t + \frac{1}{t}$

$y = t - \frac{1}{t}$

$t > 0$




when $t > 0 \Rightarrow x > 0$

(c) Direction $t_0 = \frac{1}{2} \Rightarrow x = \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{1}{2} + 2 = 2.5$

$y = \frac{1}{2} - \frac{1}{\frac{1}{2}} = \frac{1}{2} - 2 = -1.5$

$(x, y) = (2.5, -1.5)$

$t_1 = 2 \Rightarrow x = 2 + \frac{1}{2} = 2.5$

$y = 2 - \frac{1}{2} = 1.5$ $\Rightarrow (x, y) = (2.5, 1.5)$

**Remark** We can write more than one Parametrization for the same curve

Parametric Eq: $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$

Parameter interval: $t \in I$

$x^2 - y^2 = 4 \Rightarrow x^2 = y^2 + 4$

$x = \pm\sqrt{y^2 + 4}$

Exp

Parametrization 1 $= \left\{ x = t + \frac{1}{t},\ y = t - \frac{1}{t},\ t > 0 \right\}$

Parametrization 2 $= \left\{ x = \sqrt{4 + t^2},\ y = t,\ -\infty < t < \infty \right\}$

Parametrization 3 $= \left\{ x = 2\sec t,\ y = 2\tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2} \right\}$

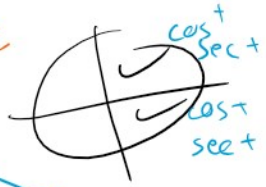
$x^2 - y^2 = 4$

$(2\sec t)^2 - (2\tan t)^2 =$

$4\sec^2 t - 4\tan^2 t =$

$4(1 + \tan^2 t - \tan^2 t) =$

$x > 0$



$4\sec^2 t - \cdots$

$4(\sec^2 t - \tan^2 t) =$ (4)

$4(1)$

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Parametrization

line through the points $(\overset{x_0}{a}, \overset{y_0}{b})$, $(c,d)$

$y - y_0 = m(x - x_0)$

$y - b = m(x - a)$, $m = \frac{d-b}{c-a}$

$t$

Let $t = x - a \Rightarrow$ $x = t + a$

$y = b + mt$

$-\infty < t < \infty$

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Exp Find Parametrization for

① line through $(\overset{a}{-1}, \overset{b}{3})$, $(3, -2)$

$m = \frac{-2-3}{3-(-1)} = \frac{-5}{4}$

$x = t + a = t - 1$

$y = b + mt = 3 + \frac{-5}{4}t$

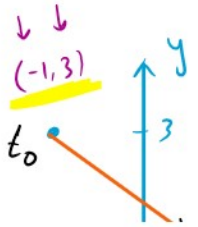
$-\infty < t < \infty$

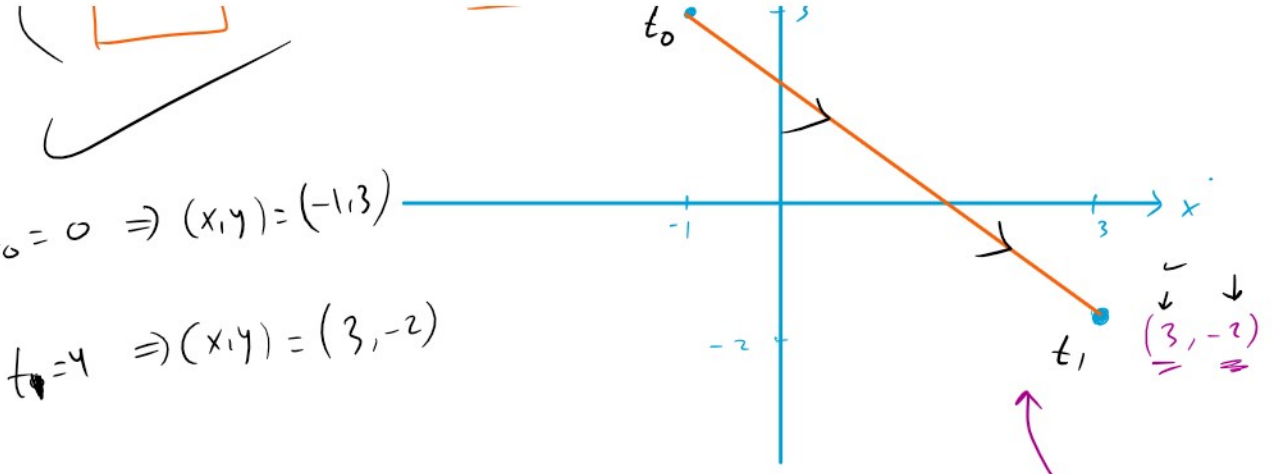
② segment with endpoint $(-1,3)$, $(3,-2)$

$x = t - 1$

$y = 3 - \frac{5}{4}t$

$0 \leq t \leq 4$






IP $\quad t_0 = 0 \Rightarrow (x,y) = (-1,3)$

TP $\quad t_1 = 4 \Rightarrow (x,y) = (3,-2)$

P1 $\leftarrow \begin{cases} x = -1 + t \\ y = 3 - \frac{5}{4}t \\ 0 \leq t \leq 4 \end{cases}$

$t \rightarrow 4t$

$x = -1 + 4t \Rightarrow$

$y = 3 - \frac{5}{4}(4t) \Rightarrow$

$0 \leq 4t \leq 4 \Rightarrow$

P2 $\rightarrow \begin{cases} x = -1 + 4t \\ y = 3 - 5t \\ 0 \leq t \leq 1 \end{cases}$

$t \rightarrow t+1$

P3 $\rightarrow \begin{cases} x = -1 + (t+1) \\ y = 3 - \frac{5}{4}(t+1) \\ 0 \leq t+1 \leq 4 \end{cases}$

$\Rightarrow \begin{cases} x = t \\ y = 3 - \frac{5}{4}t - \frac{5}{4} \\ -1 \leq t \leq 3 \end{cases}$