Ch 0

0.1 Sets

Def: A set is well-defined collection of objects

→ we use letters A, B, C, D, ..., X, Y, N, ... to give names for sets

→ Members of any set are called elements

→ The elements of any set are ordered between braces { }

Exp Consider the set

① $A = \{1, 2, 3, 4, 5\}$

A has 5 elements
so A is finite set

$\Rightarrow$ 1 is member of A $\Rightarrow 1 \in A$
2 " " " A $\Rightarrow 2 \in A$
3 " " " A $\Rightarrow 3 \in A$
4 " " " A $\Rightarrow 4 \in A$
5 " " " A $\Rightarrow 5 \in A$

$\Rightarrow$ 6 is not member of A $\Rightarrow 6 \notin A$

② $B = \{x, y, z\}$

B has 3 elements $\Rightarrow$ so B is finite set

③ $C = \{x : x \text{ is even between 1 and 17}\}$

$= \{2, 4, 6, 8, 10, 12, 14, 16\}$

C has 8 elements $\Rightarrow$ so C is finite set

(4) $N = \{t : t \text{ is natural number}\}$

$= \{1, 2, 3, 4, \dots\}$

we can not list all elements ⇒ so N is infinite set

(5) $D = \{t : t \text{ is odd natural number}\}$

$= \{1, 3, 5, 7, 9, \dots\}$

D is infinite set

(6) E is the set of all natural numbers less than 7

$E = \{1, 2, 3, 4, 5, 6\} = \{x : x \text{ is natural less than } 7\}$

E is finite set

(7) $F = \{x : x \text{ is even between 7 and 9}\}$

$= \{8\}$ ⇒ F is finite

$= \{x : x = 8\}$ ⇒ F is singleton set

(8) $X = \{y : y \text{ is natural odd less than } 1\}$

$= \emptyset$

$= \{\}$

X is empty set or Null set

(9) Write the set $\{x : x \text{ is natural number less than } 8\}$ using second way

$\{1, 2, 3, 4, 5, 6, 7\}$

* Two sets X and Y are equal if they contain the same elements

Exp $X = \{a, b, c, d\}$ and $Y = \{c, b, a, d\}$ are equal $\Rightarrow X = Y$

$A = \{Red, Blue, Yellow\}$ and $Y = \{Yellow, Red, Blue\}$ are equal

$\Rightarrow A = B$

* The set A is subset of B, $A \subseteq B$, if every element in A is in B

Exp $A = \{2, 4, red, blue\}$, $B = \{2, 4, 6, red, Yellow, blue\}$

$\Rightarrow A \subseteq B$, $A \subseteq A$, $B \subseteq B$, $\emptyset \subseteq A$, $\emptyset \subseteq B$

* The empty set is subset of every set
* Each set is subset of itself

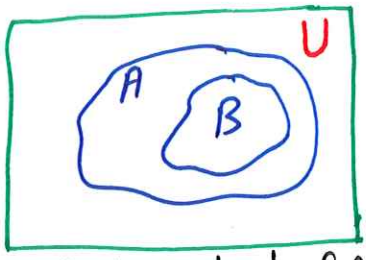
* If the sets C and D have no elements in common, then they are called disjoint.

Exp $C = \{1, 3, 5, 7\}$, $D = \{2, 4, 6, 8\}$

$\Rightarrow$ C and D are disjoint

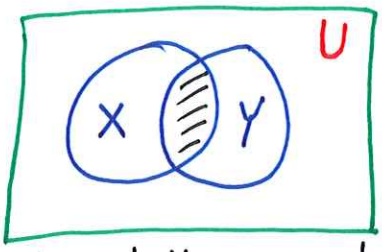
* The largest set which contains all subsets is called **universal set** denoted by U

* To illustrate the relationships among sets we use **Venn diagrams**



B is subset of A
$B \subseteq A$

R and T are disjoint



X and Y are not disjoint

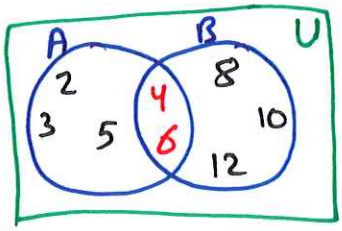
Set Intersection

* The set intersection of A and B is defined by

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Exp Find $A \cap B$ if ① $A = \{2,3,4,5,6\}$ and $B = \{4,6,8,10,12\}$

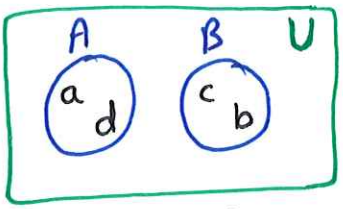
$$A \cap B = \{4,6\}$$



Note that $A \cap B \subseteq A$
$A \cap B \subseteq B$

② $A = \{a, d\}$ and $B = \{b, c\}$

$$A \cap B = \emptyset$$



Note that $A \cap B \subseteq A$
$A \cap B \subseteq B$

③ $A = \{3,6\}$ and $B = \{1,2,3,4,5,6\}$

$$A \cap B = A = \{3,6\}$$

set Union

* The union of A and B is defined by

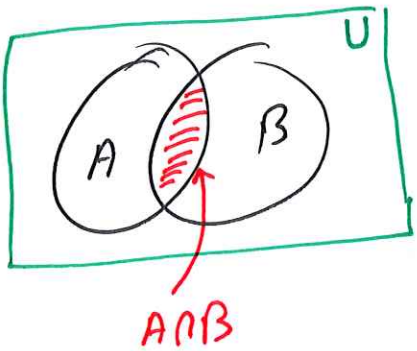
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

or means one or the other or both

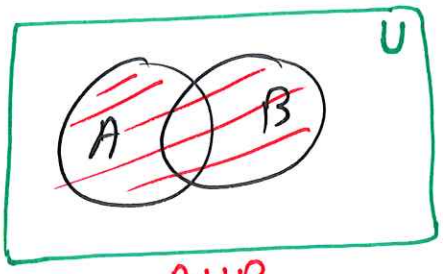
Exp Find $A \cup B$ if $A = \{a, b, c, f\}$ and $B = \{e, f, a, b\}$

$$A \cup B = \{a, b, c, e, f\}$$

Exp Use Venn diagrams to illustrate the intersection and union of any two sets A and B



$A \cap B$



$A \cup B$

Exp Find $A \cup B$ if

① $A = \emptyset$, $B = \{1, 2, 3, 4\}$ $\Rightarrow$ $A \cup B = \{1, 2, 3, 4\} = B$

② $A = \{x : x$ is natural number greater than $5\}$

$B = \{x : x$ is natural number less than $5\}$

$A = \{6, 7, 8, 9, \dots\}$, $B = \{1, 2, 3, 4\}$

$A \cup B = \{1, 2, 3, 4, 6, 7, 8, 9, \dots\} = \mathbb{N} \setminus \{5\}$

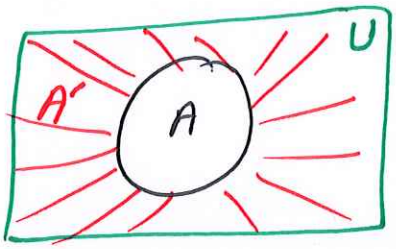
Note that $A \cap B = \emptyset$

set complement

* The complement of A is defined by

$$A' = \{x : x \in U \text{ and } x \notin A\}$$

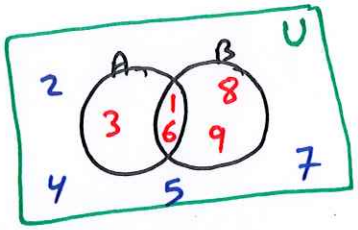
Exp Use Venn diagram to illustrate the complement of any set.



Exp Let $U = \{x \in N : x < 10\}$, $A = \{1,3,6\}$, $B = \{1,6,8,9\}$

Find ① A' ② B' ③ $(A \cap B)'$ ④ $A' \cup B'$ ⑤ $(A \cup B)'$ ⑥ $A' \cap B'$

$U = \{1,2,3,4,5,6,7,8,9\}$



① $A' = \{2,4,5,7,8,9\}$

② $B' = \{2,3,4,5,7\}$

③ $A \cap B = \{1,6\} \Rightarrow (A \cap B)' = \{2,3,4,5,7,8,9\}$

④ $A' \cup B' = \{2,3,4,5,7,8,9\}$

⑤ $A \cup B = \{1,3,6\} \cup \{1,6,8,9\} = \{1,3,6,8,9\}$

$(A \cup B)' = \{2,4,5,7\}$ ✓

⑥ $A' \cap B' = \{2,4,5,7,8,9\} \cap \{2,4,3,5,7\}$

$= \{2,4,5,7\}$ ✓

⑦ $A - B = A \cap B' = \{x : x \in A \text{ and } x \notin B\} = \{3\}$

Exp (Application)

Records office at small college shows the following data about the enrollments of 50 first year students in Math and Economics:

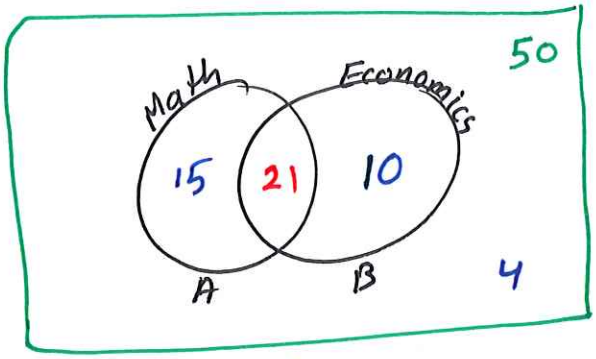
36 students take math
21 students take math and economics
4 students do not take neither math nor economics

1. How many students take only math
2. How many students take economics

1. $A \cap B$ contains 21 students

A contains 36 students

$36 - 21 = 15$ students take only math



A: Math

B: Economics

2. $50 - 15 - 21 - 4 = 10$

10 students take only economics

21 students take economics and math

31 students take economics

Exp Let A be set of multiple of 2

P be set of all prime numbers

$B = \{x: x = 4n+1 \text{ where } n \text{ is natural } 1 \le n \le 5\}$

Find ① $A \cap B$ ② $P \cap B$ ③ which sets are disjoint

④ Find elements in B but not in A and not in P

$A = \{2, 4, 6, 8, 10, 12, \dots\}$

$P = \{2, 3, 5, 7, 11, 13, 17, \dots\}$

$B = \{5, 9, 13, 17, 21\}$ since $n = 1, 2, 3, 4, 5$

① $A \cap B = \{\} = \emptyset$

② $P \cap B = \{5, 13, 17\}$

③ A and B are disjoint since $A \cap B = \emptyset$

$A \cap P = \{2\}$ so A and P are not disjoint

④ $9 \in B$ but $9 \notin A$ and $9 \notin P$

$21 \in B$ but $21 \notin A$ and $21 \notin P$ $\Rightarrow \{9, 21\}$